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Metallurgical Project

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ACTIVITY IN PIPES EXTENDING FROM HOMOGENEOUS PILE

F. Murray and G. Young

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ABSTRACT

A calculation is made for the heat generated in a slurry pocket at the edge of a homogeneous pile.

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[redacted] ACTIVITY IN PIPES EXTENDING FROM HOMOGENEOUS PILE

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Pipes full of slurry extending from the pile tank are to some extent in the pile field and there will thus be fissions and heat generation within them. If a pump and heat exchanger is cut out of service there will be stagnant slurry in the pipes between the tank and the closed valves, and there arises the question as to the cooling that may be needed for such places.

Let us suppose that the effect of the pipe is to extend the pile boundary (where the neutron density vanishes) by a cylinder of radius, a, and length L, open at the pile end and closed at the outer end. The radius a is, a, cm. or two larger than the actual inside radius of the pipe. Let $-A$ be the outward derivative of the neutron density at the pile surface before the pipe was added, i.e., for $L = 0$. Then for L small compared to, a, this value of the derivative will persist throughout the cylinder, and for the average value of the neutron density throughout the cylinder we thus have

$$\bar{\varphi} = \frac{L}{2} A \tag{1}$$

As L increases from zero the neutron density at the open mouth of the cylinder increases and the flow of neutrons into the cylinder from the pile decreases. We therefore overestimate the flux into the cylinder in taking it to be the same as it was for $L = 0$, i.e., before the cylinder was added.

The solution of $\nabla^2\varphi + b^2\varphi = 0$ which vanishes at $x = L$ and at $r = a$ is

$$\varphi = \sum_{1}^{\infty} B_n \sinh \mu_n (L-x)\, J_0(\lambda_n r) \tag{2}$$

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where the $\lambda_n = \frac{\alpha_n}{a}$ are the roots of $J_o(\lambda a) = 0$, and

$$\mu_n^2 = \lambda_n^2 - b^2 \tag{3}$$

The average of this throughout the cylinder is

$$\bar{\varphi} = 2 \sum B_n \frac{\cosh \mu_n L - 1}{\mu_n L} \frac{J_1(\alpha_n)}{\alpha_n} \tag{4}$$

The neutron flux into the base of the cylinder is proportional to

$$F(n) = - \frac{\partial \varphi}{\partial x}\bigg|_{x = 0} = \sum B_n \mu_n \cosh \mu_n L \cdot J_o(\lambda_n r), \tag{5}$$

and if this be pre-assigned as the series

$$F(r) = \sum_1^\infty A_n J_o(\lambda_n r), \tag{6}$$

then

$$B_n = \frac{A_n}{\mu_n \cosh \mu_n L} \tag{7}$$

and (4) becomes

$$\bar{\varphi} = 2 \sum \frac{A_n}{\mu_n^2 L} \frac{\cosh \mu_n L - 1}{\cosh \mu_n L} \frac{J_1(\alpha_n)}{\alpha_n} \tag{8}$$

If now we take $F(r) =$ constant $= A$, we obtain

$$A_n = \frac{2A}{\alpha_n J_1(\alpha_n)} . \tag{9}$$

$$\bar{\varphi} = \frac{4A}{L} \sum \frac{1}{\mu_n^2 \alpha_n^2} \frac{\cosh \mu_n L - 1}{\cosh \mu_n L} \tag{10}$$

In application to practical cases it is found that $\lambda_n >> b$ so that with (3) we have $\mu_n = \lambda_n = \frac{\alpha_n}{a}$. Then $\mu_n L = \alpha_n \frac{L}{a}$, and for L of the order of a or larger the above cosh terms are $>> 1$ and we obtain

$$\bar{\varphi} = \frac{4a^2}{L} A \sum \frac{1}{\alpha_n^4} \sim \frac{a^2}{8L} A \tag{11}$$

On the other hand, for $y << 1$ we have $\frac{\cosh y - 1}{\cosh y} = \frac{y^2}{2}$ and applying this in (10), together with $\sum \frac{1}{\alpha_n^2} = \frac{1}{4}$, leads again to (1) for L small compared to a.

If φ_o be the neutron density at the center of the pile, then at the edge of a spherical pile without reflector and of radius R

$$A = \frac{\varphi_o}{R} \tag{12}$$

and (11) becomes our principal result

$$\frac{\bar{\varphi}}{\varphi_o} = \frac{a^2}{8RL} \tag{13}$$

This gives an over-estimate of the mean density in the pipe since, as mentioned earlier, the actual flux into the cylinder will be less (for a pile without reflector) than here assumed.

A typical set of dimensions is R = 200 cm, a = 15 cm., L = 30 cm., and this gives a half percent as the value of the ratio in (13). If the pile is operating at 12,000 kw. per ton of tank water, the rate of heat generation at its center is about 10 cal per sec. per cm^3, and thus that in the pipe is .05 cal per sec. per cm^3 of volume or .4 cal per sec. per cm^2 of surface. We estimate that this heat can be gotten rid of by natural convection if the pipe wall is 16° colder than the slurry, the heat transfer coefficient being about .025 in this case. If the pipe be surrounded by water some 40° colder than the slurry, natural convection operating on both sides of the pipe wall would appear to suffice.

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