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CL-574(14)
2-B

[redacted] PILE THEORY LECTURE NOTES

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June 22, 1943

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Calculation of the Multiplication Factor

The discussion thus far in this series has been largely phenomenological (and somewhat formal) insofar as the multiplication factor k has been assumed as a given datum. In the remaining lectures we shall review the definition of the multiplication factor, and we shall show how it can be calculated, at least approximately, in some simple cases.

The multiplication factor for an infinite medium, k, is defined as the number of slow neutrons produced per slow neutron absorbed anywhere in the medium. It has already been expressed as the product of four factors,

$$k = \eta \epsilon p f$$

where

$f \equiv$ thermal utilization = fraction of thermal neutrons absorbed in metal

η = number of fast neutrons produced by thermal fission per thermal neutron absorbed in the metal.

ϵ = fast fission multiplication = total number of fast neutrons produced per fast neutrons created by thermal fission.

p = fraction of neutrons which escape resonance absorption and thus become thermal.

The calculation of f, ϵ, and p in a homogeneous mixture will illustrate most of the important principles involved in estimating k, and we shall turn to a discussion of such systems first.

Homogeneous Mixtures

1) The thermal utilization, f

As has already been pointed out [CL-574 (3)] the thermal utilization f is

$$f = \frac{\text{thermal neutrons absorbed in metal}}{\text{thermal neutrons absorbed anywhere in system}} = \frac{N_0 \sigma_{a_0}}{N_0 \sigma_{a_0} + N_1 \sigma_{a_1}}$$

where o refers to the metal and 1 refers to the moderator atoms associated with the metal. If the moderator is a compound, σ_{a_1} is the absorption cross-section per molecule of moderator; e.g. for CH_2, $\sigma_{a_1} = 2\,\sigma_{a_H} + \sigma_{ac}$. The thermal utilization can be written in terms of the molecular ratio of metal to moderator, $N_u = N_o/N_1$, as

$$f = 1 - \sigma_{a_1}/(N_u\,\sigma_{a_o} + \sigma_{a_1}) \quad .$$

2) The contribution of fast fission, ε

The phenomenon of fast fission occurs with about the same cross-section in both 238 and 235, the 238 threshold being about 900 KeV. However because the 238 is 140 times as plentiful as 235, fast fission occurs much more frequently in 238 than in 235. In the discussion which follows we shall assume, for want of better evidence that ν, the number of neutrons produced per fission of 238, is the same as the number produced per thermal fission of 235.

The multiplication due to fast fission is a multiple process; that is, fast neutrons produced in the original fission create second generation fast neutrons which create third generation neutrons, and so on. It is therefore necessary to sum the contributions to the process from all generations in order to compute the total number of fast neutrons arising from an original thermal fission.

Suppose the probability that a fast neutron escape from the medium before it makes a collision is p; for an infinite medium, or in fact any medium which is large in all dimensions compared with the mean free path, p is unity. We can calculate in terms of p the total number of neutrons arising from one original fast neutron (created by thermal fission) as a result of the cascading of fast fissions. After one collision there will be:

$(1 - p)$ neutrons escaped from medium

$\frac{\sigma_i}{\sigma}\,p$ neutrons slowed below the fast fission threshold

$\frac{\nu\sigma_f + \sigma_e}{\sigma}\,p$ fast neutrons capable of producing further fast fission

In these expressions σ_i is the cross-section for inelastic scattering (from above to below the fast fission threshold), σ_e ($\approx 1.5 \times 10^{-24}$) is the elastic scattering cross-section, σ_f ($\approx 0.4 \times 10^{-24}$) is the fast fission cross-section for neutrons whose energy distribution is that of fission neutrons, σ is the total cross-section ($\sigma = \sigma_e + \sigma_i + \sigma_f$), and ν is the number of neutrons produced per fast fission (≈ 2.2). The inelastic scattering is partly ($\sim 2.4 \times 10^{-24}$) made up of true inelastic scattering in the 238 nucleus, a process in which a 238 nucleus is raised to an excited state by collision with a high energy neutron; and partly of elastic scattering by any light atom associated with the U in the mixture, the elastic scattering causing sufficient energy transfer to bring the scattered neutron below the fast fission threshold.

The $\frac{\nu\sigma_f + \sigma_e}{\sigma}$ p fast neutrons capable of producing fast fission after the first collision will give rise, after two collisions to

$$\frac{\nu\sigma_f + \sigma_e}{\sigma} p \left(1 - p + \frac{\sigma_i}{\sigma} p\right)$$ neutrons escaped or slowed inelastically

and

$$\left(\frac{\nu\sigma_f + \sigma_e}{\sigma}\right)^2 p^2$$ fast neutrons capable of further fast fission

After n collisions there will be

$$\left[\frac{\nu\sigma_f + \sigma_e}{\sigma} p\right]^{n-1} \left(1 - p + \frac{\sigma_i}{\sigma} p\right)$$ neutrons escaped or slowed

and

$$\left(\frac{\nu\sigma_f + \sigma_e}{\sigma}\right)^n p^n$$ fast neutrons capable of further fission

The total ~~added~~ number of neutrons due to the fast fission process (plus the original neutron) is the sum of all the slowed or escaped neutrons; hence

$$\epsilon = \left[1 - p + \frac{\sigma_i}{\sigma} p\right]\left[1 + \frac{\nu\sigma_f + \sigma_e}{\sigma} p + \left(\frac{\nu\sigma_f + \sigma_e}{\sigma}\right)^2 p^2 + \cdots\right]$$

$$= \frac{1 - p + \frac{\sigma_i}{\sigma} p}{1 - \frac{\nu\sigma_f + \sigma_e}{\sigma} p}$$

For an infinite homogeneous medium, p = 1, and we have

$$\epsilon = \frac{\sigma_i}{\sigma - (\nu\sigma_f + \sigma_e)} = 1 + \frac{(\nu - 1)\sigma_f}{\sigma_i - (\nu - 1)\sigma_f}$$

If the ratio of uranium to moderator atoms per c.c. is N_u, we may rewrite the expression for ϵ as

$$\epsilon = 1 + \frac{(\nu - 1)\,\sigma_f}{\sigma_{i_o} + \frac{\sigma_{i_M}}{N_u} - (\nu - 1)\,\sigma_f}$$

where we have denoted the true inelastic cross-section per atom of 238 by σ_{i_o} and the "slowing down" inelastic cross-section per atom of the moderator by σ_{i_M}.

To determine the appropriate σ_{i_M} to use for a moderator whose scattering cross-section is σ_M, we use the relation

$\frac{\sigma_M}{\sigma_{i_M}}$ = number of collisions with moderator atom required to reduce energy of neutron below the fission threshold

If the average fission neutron has energy 2 MeV, and if the fast fission threshold is 900 KeV, the number of collisions required to reduce the energy of the neutron below 900 KeV is

$$\frac{\ln 2.23}{\alpha} = \frac{.80}{\alpha}$$

where α = average logarithmic energy loss per collision. Hence

$$\sigma_{i_M} = \frac{\alpha}{.80}\,\sigma_M ,$$

provided, of course, that σ_{i_M} so determined is less than σ_{e_M}. If as in the case of H, $\alpha = 1$ and $\alpha/.80 = 1.25$, we take $\sigma_{i_M} = \sigma_{e_M}$.