PILE THEORY LECTURE NOTES

P. Morrison

June 15, 1943

Control of the pile:

Control has turned out to be relatively easier than at first feared. The general method is to vary k (in experimental piles, variation $\sim 1\%$ or less) by moving a rod bearing Cd or B into or out of the pile. In practice several rods are provided; one will have a fine adjustment and is the measuring instrument, the others are arranged as safety devices, more or less automatic in operation.

Intensity ($\sim n$, the thermal density) is measured with a BF_3 ionization chamber, corrected for saturation by comparing the readings against the known exponential rise of n with time for given setting of the control rod.

a) Sensitivity

Two methods of measuring k have been used; measuring the relaxation time directly, and measuring the position of the rod for which the pile is steady. The first case gives a sensitivity depending on the accuracy of the density measurements; the second is independent of instruments, being in fact a null method, especially if a galv. with a constant biasing current is used.

In case two there is an interesting relation. We have for the period (when delays control):

$$P = \frac{\delta T}{\varepsilon}$$

If we set for $\varepsilon \approx 0$, with an error Δ_ε, and observe that the intensity does not change within our sensitivity, for ΔP seconds, we have $P > \Delta P$; $P = \dfrac{\delta T}{\Delta \varepsilon}$ or $\Delta P \cdot \Delta \varepsilon < \delta T$; ($\delta T \sim .1$ sec.)

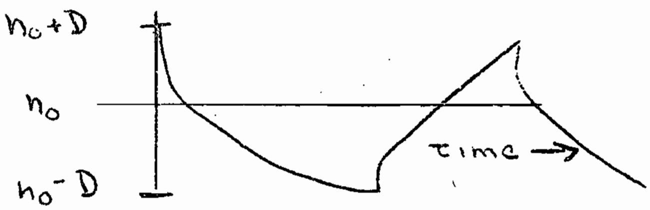
[redacted]

If we suppose we can detect a charge of 1% in density, then $\Delta P > .01 P$; and we have $\Delta P \Delta \epsilon < .001$ sec. Thus a ten-minute observation gives $\Delta \epsilon \sim 2 \cdot 10^{-6}$. This is not very different from the practical case.

b) Control

Two general types of control have been suggested:

1) Limit control: This control operates to actuate a control rod where the neutron density crosses certain pre-set limits above or below normal operating intensity. Using the simple theory of paragraph VI b, we find the time t between rod movements for a limit at $n = (1 \pm D) n_0$; $t = \frac{\delta T}{2\epsilon} \cdot D$



plot of density versus time

Thus for a 5% D, $t = .05 \cdot \frac{.1}{2 \cdot .001}$ sec ~ 3 sec. in a typical case. This represents rather serious hunting.

2) Derivative control:

For $\epsilon = 0$, n may have any value. But $dn/dt \neq 0$ only when $\epsilon \neq 0$. Thus detecting the derivative of the density should be a better approach than simply the variations in n. Suppose we have a control rod which works so that its velocity is determined partly by neutron density and partly by the derivative:

$$\frac{dx}{dt} = -\frac{C_1'}{l}\left(\frac{n - n_0}{n_0}\right) - C_2' \frac{d(n - n_0)}{dt}$$

Now $\epsilon \sim x$, $\therefore d\epsilon/dt = -C_1\left(\frac{n}{n_0} - 1\right) - \frac{C_2}{n_0}\frac{dn}{dt}$ where C_1, C_2 are constants of the control mechanism.

If C_1, C_2 are both positive, the control opposes any change in n.

We have:
$$\frac{dn}{dt} = \frac{\epsilon n}{l} + \frac{c}{T}$$

$$\frac{dc}{dt} = -\frac{c}{T} + \frac{\delta n}{l}$$ as before.

Introduce the fractional change in neutron density: $N = \frac{n - n_0}{n_0}$, $C = \bar{C} N$

and restricting to N [redacted]

$$\therefore \frac{dN}{dt} = \frac{\varepsilon}{l} + \frac{\bar{C}N}{T}$$

$$\bar{C}\frac{dN}{dt} = -\frac{\bar{C}N}{T} + \frac{\delta N}{l}$$

Differentiating: $$\frac{d^2N}{dt^2} = \frac{1}{l}\frac{d\varepsilon}{dt} + \bar{C}\frac{dN}{dt}$$

$$= \frac{\delta N}{lT} + \frac{1}{l}\frac{d\varepsilon}{dt} - \frac{\bar{C}N}{T^2}$$

$$= \frac{\delta N}{lT} - \frac{\bar{C}N}{T^2} + \frac{1}{l}\left(-\frac{C_1}{l}N - C_2\frac{dN}{dt}\right)$$

We can find solutions of this equation, which is just the damped simple harmonic motion equation, in the form $e^{\omega t}$. ω has a real part and an imaginary part, and represents a damped oscillation.

$$\omega^2 = \frac{C_2}{l}\omega + \left(\frac{\bar{C}}{T^2} + \frac{C_1}{l^2} - \delta/lT\right) = 0$$

For small real ω, for example, $\omega = -C_1/C_2$, which represents a slow drift back to zero. The derivative term would compensate for a change in k by letting N increase to about $N = \frac{\Delta k}{C_2}$, and then N would drift back to 0 with relaxation time C_1/C_2. Such a control is very good in principal, but limited available sensitivity seems to prevent its operation except for large changes in k. It is still useful to include some C_2 to prevent disaster. In practice, a combination of both methods would be used. The oscillations of a) are not bad because of their relative slowness, and b) aids the control when accidental deviations are considerable. In the experimental piles, manual control seem entirely adequate.

Approach to critical conditions:

As a pile grows, it comes closer and closer to a self maintaining condition. For general reasons, one would like to know how he is progressing, so measurements of several kinds are made at this time.

The presence of spontaneous fission (amounting to 15 neutrons/sec/kilo of U as oxide) provides a background intensity. This will rise to dangerous amounts as the pile goes to completion. A number of Cd pieces inserted during construction keep this activity down. They are removed periodically to make the tests.

I. Extrapolated activity

Perhaps the best method, which serves quantitatively to predict the critical size, runs as follows:

A measurement of activity ($\sim$n) is made at a point as near as possible to the effective center. If we call the no. of fast natural neutrons produced per second per cm^3 Q, we have:

$$1)\ \Delta q = \frac{\partial q}{\partial t}$$

$$2)\ \Delta n - \frac{nv}{L^2} + \frac{3 q_{th}}{\lambda v} = 0$$

$$\text{and } 3)\ q(0) = pQ + \frac{Knv}{\Lambda}$$

Expand Q in Fourier series, vanishing at the boundaries (take a square pile, edge a, height h).

$$Q = \sum^{\infty} Q_{lmn} \sin\frac{l\pi x}{a} \sin\frac{m\pi y}{a} \sin\frac{n\pi z}{h}$$

$$n = \sum^{\infty} n_{lmn} \sin \sin \sin$$ (Q_{lmn}, n_{lmn} coefficient of a Fourier expansion.)

Substituting, we get:

$$\Delta_{lmn} - \frac{1}{L^2} + \frac{3\rho Q_{lmn}}{n\lambda v)_{lmn}} e^{\Delta\tau} + \frac{k e^{\Delta\tau}}{L^2} = 0$$

$$\left.\frac{\rho Q}{nv\lambda}\right)_{lmn} = \frac{1}{3L^2}\left(\left(1 - L^2\Delta_{lmn}\right)e^{-\Delta\tau_{lmn}} - K\right)\right) ; \text{ Take } \Delta\tau_{lmn} << 1.$$

But $K = (1 - M^2\Delta_0)$, with $M^2 = L^2 + T_0$, Q
Δ_0 the critical Laplacian for the given lattice.

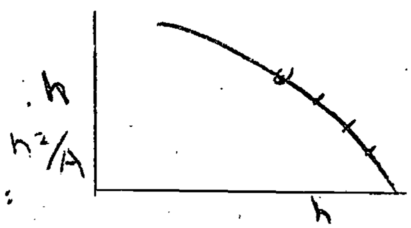
$$\therefore \left.\frac{\rho Q}{nv\lambda}\right)_{lmn} = \frac{1}{3L^2}\left(M^2(\Delta_0 - \Delta_{lmn})\right)$$

Taking main harmonic

$$\left.\frac{\rho Q}{nv\lambda}\right)_{111} = \frac{M^2}{3L^2}\left(-\frac{2\pi^2}{a^2} - \frac{\pi^2}{h_c^2} + \cancel{\frac{2\pi^2}{a^2}} + \frac{\pi^2}{h^2}\right)$$

$$\frac{h^2\rho Q}{nv\lambda} = \frac{\pi^2 M^2}{3L^2}\left(1 - h^2/h_c^2\right)$$

Plotting $h^2/(A_{al\,In\,cd} - 1.21\,A_{cd\,In\,cd})$ vs. h



[The natural neutrons in an exponential pile are proportional to:

$$\left.nv\lambda\right)_{lmn} = \frac{3L^2\rho Q}{M^2(\Delta_0 - \Delta_{lmn})}$$

given a rough check on Δ_0 with first reading.]

Other methods

1) Introduction of sources, measurement of the time. (See part VI a.)

2) Introduction of standard source measure activity of standard foils. (Calibration method - CP-430)