CL-574(10)
Copy 41A

~~SECRET~~

PILE THEORY LECTURE NOTES

* * *

P. Morrison

June 10, 1943

V. Improved theory of period of going pile

We write the fundamental pile equations for delayed and normal neutrons:

a) $$\Delta q = \partial q/\partial \tau$$

$$q_{th} = e^{\Delta \tau_0} q(0) \cdot p$$

p is the prob. that a neutron escapes resonance capture in slowing, and τ_0 is the age of thermal neutrons; Δ is the value of the Laplacian.

b) $$\Delta n - \frac{n}{L^2} + \frac{3q_{th}}{\lambda v} = \frac{3}{\lambda v}\frac{dn}{dt}$$ (If the density is not steady, $\frac{dn}{dt} \neq 0$.)

c) $$q(0) = (k - \Sigma \alpha_i)\frac{nv}{p\Lambda} + \frac{\Sigma \lambda_i c_i}{p}$$

Where α_i is the number of delayed neutrons of mean lifetime $1/\lambda_i$ per fission; c_i is the density of potential emitters of such neutrons; k is the reproduction factor.

d) $$\frac{dc_i}{dt} = -\lambda_i c_i + \frac{\alpha_i n v}{\Lambda}$$ for all i.

Assume $n = n_0(\vec{r})e^{\omega t}$; $c = \bar{c}_i n_0(\vec{r}) e^{\omega t}$

$$\therefore \Delta n - \frac{n}{L^2} + e^{\Delta \tau}\left((k - \Sigma\alpha_i)\frac{n}{L^2} + \Sigma \lambda_i c_i\right) = \frac{dn}{dt}\cdot\frac{3}{\lambda v}$$

Substituting from a):

$$\Delta - \frac{1}{L^2} + e^{\Delta\tau}\frac{(k - \Sigma\alpha_i)}{L^2} + \frac{e^{\Delta\tau}}{L^2}\sum_i \frac{\lambda_i \alpha_i}{\lambda_i + \omega} = \omega \cdot \frac{3}{\lambda v}$$

If we measure Δ from the value at the critical condition, Δ_0, we get

$$k e^{\Delta_0 \tau_0} = (1 - \Delta L^2)$$

and for $\Delta - \Delta_0$ not too large (ω not too large):

$$(\Delta - \Delta_0)(L^2 + \tau_0) = \omega \cdot \frac{\Lambda}{v} + \Sigma \frac{\omega \alpha_i}{\omega + \lambda_i}$$

~~SECRET~~

Or, writing the relaxation time T instead of $\omega = 1/T$, and writing $t_i = 1/\lambda_i$

$$(\Delta-\Delta_0)(L^2+\tau_0) = \frac{1}{T}\frac{\Delta}{v} + \sum_i \frac{\alpha_i t_i}{t_i+T}$$

This result has the same properties as that of IV, and gives a curve like that shown before. It is more adequate for experimental comparison since in a pile like CP-2 accurate settings can be made in the whole region of k near 1. Using the measured value for $(\Delta-\Delta_0)$ and $(\tau_0+L^2) = 700\ cm^2$, using the ratios of α's and the ti's as given in II, the value for the percentage of delayed neutrons comes out to be .61% instead of the directly measured 1 ± .2%. The calibration of the controls at Argonne has been made in terms of the period for a given setting, using a unit, the reciprocal hour or "inhour" which corresponds to a shift of the pile from the critical setting to one with a period of one hour. An inhour near the critical position means a change of $3 \cdot 10^{-5}$ in k. This is rather close to the arbitrary unit - the conventional inch or "cinch" used in CP-1; 1 inhour = 1.23 cinch. The inhour is an absolute unit of reactivity, and will correspond to the same excess k for all piles in which the number and lifetime of the delayed neutrons and the time per generation are like those of CP-2.

VI. Transient Phenomena

So far we have discussed the undisturbed exponential growth or fall of neutron density. It is interesting to consider the result of rapid changes in the going pile. We shall use the simplified theory of IV for this; the results are entirely analogous to those obtained with V and the formulas are much simpler.

The relaxation time $1/\omega$ in the practical case of a pile operated with $k_e-1 \ll$ delayed neutron percentage is given by the two roots:

$$\omega_+ = \varepsilon / (\delta - \varepsilon) T$$

$$\omega_- = -\frac{(\delta - \varepsilon)}{l}$$

With these values, one gets two independent solutions:

$$c_+ = \frac{T}{l}(\delta - \varepsilon)\, n_-$$

and $c_- = -\frac{\delta}{(\delta-\varepsilon)} n_-$

The solution with ω_- is transient, dies away in time, and cannot exist alone, since it implies a <u>negative</u> density of potential neutrons. Thus we discarded it when we looked for the undisturbed rise. Now we want to consider its effect on transient operation. Note the relation: $n_+ n_- + \frac{l}{\delta T} c_+ c_- = 0$. (This is an orthogonality relation for the two solutions, and is a special case of the more detailed treatment using 4 lifetimes, where five roots, and five orthogonal solutions exist. See C-65, CP-351 for details.)

a) Neutron burst.

Suppose we have a pile with k near 1 (ε may be + or -). With an external source we induce a large neutron density, then suddenly remove the source. We may write for all times:

$$n(t) = a n_+(t) + b n_-(t) \quad \text{Take } n_+ = 1 \cdot e^{\omega_+ t},\ n_- = 1 \cdot e^{\omega_- t}$$

$$c(t) = a \frac{T}{l}(\delta - \varepsilon)\, n_+(t) - b\left(\frac{\delta}{\delta - \varepsilon}\right) n_-(t)$$

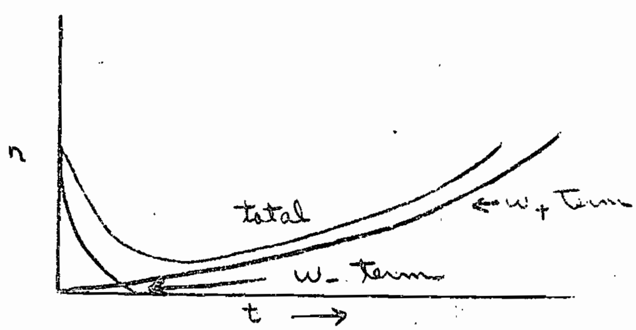
The initial condition is:

$$n(0) = a + b$$

$$c(0) = 0 = a\frac{T}{l}(\delta - \varepsilon) - b\frac{\delta}{\delta - \varepsilon}$$

$$\therefore\ n(t) = \frac{n(0)}{\left(1 + \frac{(\delta-\varepsilon)^2}{l\delta} T\right)} \left(e^{\omega_+ t} + \frac{(\delta - \varepsilon)^2}{l\delta} T e^{\omega_- t}\right).$$

If $\varepsilon > 0$, a plot of two terms and the total would look like:



The thermal density <u>decreases</u>, even though k is greater than 1, as the transient disappears. Physically, the delayed neutrons leave no chance to contribute for time of the order of T. After that, the density settles into the steady exponential rise corresponding to ω_+.

b) Control rod shift.

Suppose we operate with $\varepsilon = 0$, establishing a steady value of n and c:

$$n = n_0; \quad c = \frac{T\delta}{\ell} n_0 .$$

Than - say by moving the control rod - we change suddenly to a new value of ε.

Then:

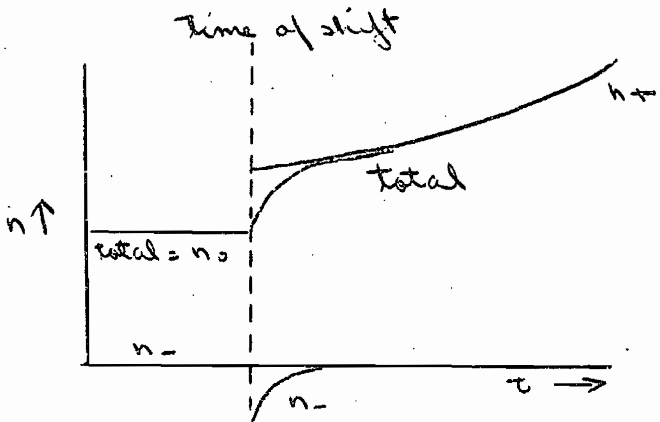
$$n_0 = a + b$$

$$\frac{T\delta}{\ell} n_0 = aT \frac{(\delta - \varepsilon)}{\ell} - \frac{b\delta}{(\delta - \varepsilon)}$$

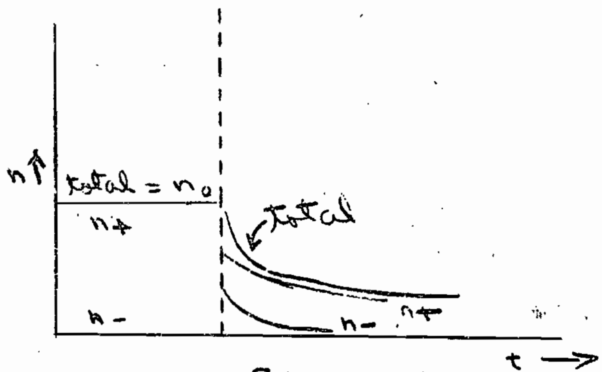
Solving, we get:

$$n(t) = n_0 \frac{\delta \ell}{\delta\ell + T(\delta - \varepsilon)^2} \left[\left[\frac{\delta}{\delta - \varepsilon} + \frac{T\delta}{\ell} \right] e^{\omega_+ t} - \frac{T}{\ell} \varepsilon e^{\omega_- t} \right]$$

and a plot like that in a) looks like:



Here $\delta K > 0$



Same, $\delta K < 0$