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PILE THEORY LECTURE NOTES

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PILE KINETICS

I. Chronology of a neutron

Time

0 sec Born as a fast neutron $\sim$ 2 Mev energy

5.10^{-5} Slows down to thermal energy by elastic collision:

$$\text{no. of coll.} = \ln \frac{E_o}{E_{th}} \cdot \frac{1}{\alpha_c}$$

$$= \frac{1}{116} \ln \frac{2.10^6}{2.5 \cdot 10^{-2}} = 110$$

Free path $\sim$ 3.5 cm; ave vel. at least 50 times thermal vel.

Lives as thermal, and is finally absorbed.

$$\text{no. of coll.} = 1080 \text{ in graphite}$$

$$= 1080\ (1-f) \sim 120 \text{ in a pile}$$

Free path $\sim$ 2.7 cm; vel. $\sim$ 800 m/sec.

$1.5 \cdot 10^{-3}$ sec. new neutron is born

Thus one generation lasts $1.5 \cdot 10^{-3}$ sec. $= \ell$

II. Simple theory of reacting pile:

Therefore, we can write for the time variation of the thermal neutron density n in a going pile with an effective reproduction factor k_e

$$\frac{dn}{dt} = (k_e-1)\frac{n}{\ell} \qquad \therefore\ n = n_o\, e^{(k_e-1)t/\ell}$$

The "relaxation" time T is given by:

$$1/T = (k_e-1)/\ell$$

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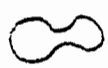
For Argonne pile, $(k_e-1)_{max} \sim .004$, $T \sim \frac{1.5 \cdot 10^{-3}}{4 \cdot 10^{-3}} \sim .4$ sec.

(Note: with a ten percent excess k, $T \sim 1.5 \cdot 10^{-2}$ sec -- rather fast!)

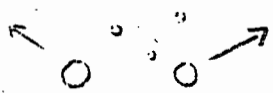
Actually, something is wrong: the minimum Argonne period is ~ 7 sec.

III. Delayed neutrons.

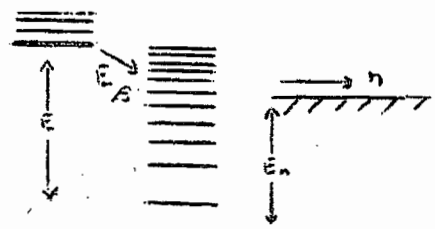
It was early recognized that all neutrons produced by fission did not follow the simple chronology of I. Some neutrons appear a few seconds or even a minute after fission has occured. Neutron emission from the excited fragments is known to require times $\sim 10^{-16}$. The delayed neutrons come out long after the fission products have been formed, and result from the chance for high excitation following the β-decay of the fission products. The process is described in the accompanying sketches:



Nucleus in act of division.



Fission fragments with neutron "splash" released at moment of fission. 10^{-15} sec.



Level diagram of one of fission products - charge Z, and its daughter element after β-decay. En = neutron binding energy; E total excitation of nucleus Z.

After some time during which the fission products lose energy by β-decay, we may have a nucleus whose excitation energy exceeds the neutron binding energy. The β-decay probability increases with available energy like E^5; but the number of states in the product nucleus increases very rapidly as the excitation energy grows, so that some of the transitions will occur as shown, leaving the product nucleus excited with more than neutron binding energy, and a neutron will go off in a matter of 10^{-14} seconds or less. But this all awaited β-decay which takes some seconds for the few million volts here involved.

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Direct measurement (see C-81) shows that about 1% of all fission neutrons are delayed, with a good representation of the delayed neutrons with periods .01 sec < period < 20 minutes given by:

$$\frac{\text{no. of delayed neutrons}}{\text{total fission neutrons}} = .01(.037e^{-.0122t} + .283e^{-.029t} + .34e^{-.099t} + .34e^{-.28t})$$

The relative amounts and lifetimes here are rather accurate, but the total number of delayed neutrons is given better by indirect methods which we will discuss later.

IV. Simple theory with delayed neutrons of a single life-time.

Write n = density of thermal neutrons

c = density of active atoms which will eventually emit a neutron

δ = - fraction of neutrons which are delayed

T = lifetime of delayed neutrons.

ℓ = time for a generation

Then
$$\begin{cases} \dfrac{dn}{dt} = -n/\ell + k(1-\delta)\dfrac{n}{\ell} + c/T \\[2ex] \dfrac{dc}{dt} = -c/T + k\delta n/\ell \end{cases}$$

We solve these equations with the assumption: $n = n_0 e^{\omega t}$

$c = c_0 e^{\omega t}$

Then ω must be a solution of the equation:

$$\left(\omega + \frac{\delta-(k-1)}{\ell}\right)\left(\omega + \frac{1}{T}\right) - \frac{\delta k}{\ell T} = 0$$

which gives:

$$\omega = \frac{\ell + (\delta-(\varepsilon))T}{2\ell T}\left[-1 \pm \sqrt{1 + 4(\varepsilon)\ell T/(\ell+(\delta-(\varepsilon))T)^2}\right]$$

under the squareroot $4\varepsilon\ell T$ should be multiplied by $(1+\delta)$ which makes very little difference.

writing ε for k-1.

Let us consider several cases of this result:

a) Delayed neutrons control: $T(\delta - \varepsilon) >> \ell$ (practical case, for any pile run with $\varepsilon < 1\%$)

$$\omega = 1/\text{relax. time} \cong \varepsilon/(\delta - \varepsilon)T$$

This is easy to interpret and gives a qualitative explanation of the contradiction mentioned in II.

Problem: How do you decide which root of the quadratic is the physically meaningful one?

b) Excess k large compared to δ: delayed neutrons unimportant.

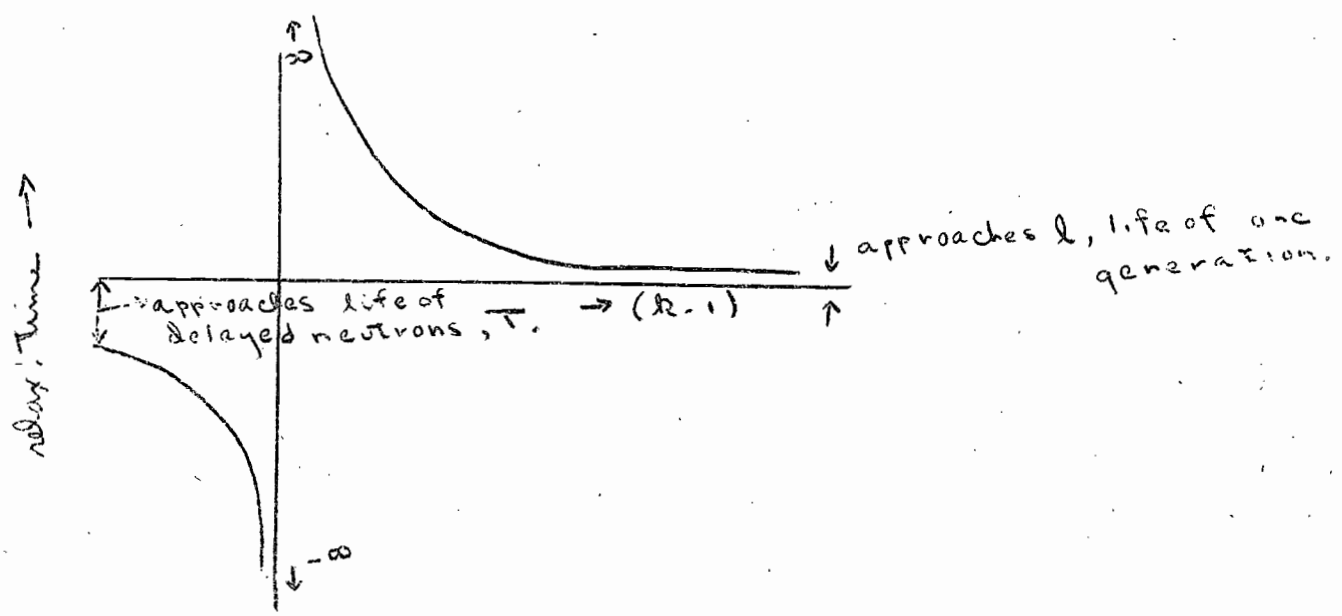
$$\omega \simeq -\frac{\varepsilon}{2\ell}\left(-1 \pm \left(1 + \frac{2\ell}{\varepsilon T}\right)\right) \simeq \varepsilon/\ell \text{ as in paragraph II.}$$

c) $\varepsilon = k-1 < 0$; pile below critical size. In this case, ω is negative and n falls exponentially with time.

For control by delayed neutrons (a) holds unchanged.

If pile very far from critical, the other root is required in b), and the min. relaxation time is given by $|\omega| \sim 1/T$.

In the accompanying figure, we plot the relaxation time against the excess k.



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