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PILE THEORY LECTURE NOTES

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SLOWING DOWN AND DIFFUSION OF NEUTRONS

Neutrons drop from 3 - 5 Mev (vel. 2×10^4 km/sec) to thermal energies (vel. 2 km/sec) primarily by losing their KE in elastic collisions with stationary nuclei.

Two exceptions are the cases of 1.) very fast neutrons, 2.) very low energy neutrons. The very fast may effect an inelastic collision, with consequent rearrangement of the nucleus, emitting a γ-ray. This case allows for much more rapid energy loss, but it is important only in heavy elements. Again, at very low energies, the neutron does not have enough KE to free an atom from the chemical bonds holding it to its neighbors, so that the neutron effectively rebounds off the whole crystal; this effect occurs from 1.2 volts on down in graphite.

In the equilibrium state, neutrons have approximately equal probability of gaining energy from or losing energy to lattice. Since low energy neutrons are absorbed very rapidly, the Maxwell distribution of energies expected in the equilibrium state is distorted, so that neutron temperature is greater than the lattice temperature.

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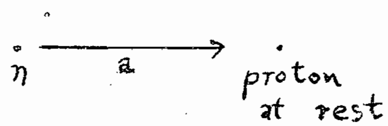
SECRET

LOSS OF ENERGY IN ELASTIC COLLISIONS WITH ATOMS

1. Collision with hydrogen - mass equal to neutron mass

Considering the proton to be stationary and the neutron to have a velocity of $\underline{a}$, assume:

proton at rest 1.) Conservation of momentum: $\underline{v} + \underline{u} = \underline{a}$

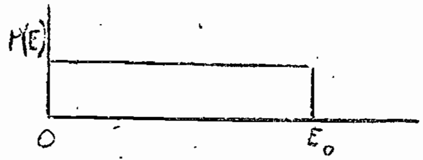


2.) Conservation of KE: $\frac{mv^2}{2} + \frac{mu^2}{2} = \frac{ma^2}{2}$

$\therefore$ θ must be a right angle and vectors will slide on a circle, but n will never bounce back.

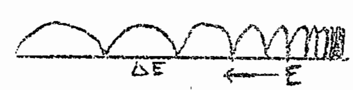
Here the minimum final KE of the neutron occurs when the neutron is stopped, whereas it has a maximum energy equal to the initial energy.

All energies are equally likely since in the center of mass system all directions of recoil are equally likely. (In the c. of m. system mv = 0; therefore, only rotation is possible. Here the n may go backward after a collision.) Then the chance to find any recoil energy may be shown, as in adjacent figure, where the chance of finding a neutron with an energy at E is given by P(E). From the diagram, the (arithmetical) average energy loss is Eo/2.



Angular distribution in proton collision.

If the energy values that the neutron has after successive collisions are plotted, the result is that the values of the KE at thermal energies will be crowded closely together



since the neutron loses 1/2 of its energy each collision, on the average. It is much more convenient, therefore, to use a logarithmic scale.

The average logarithmic energy loss per collision, ξ , is defined by:

$$\xi \equiv \overline{\ln \frac{E_0}{E}} = \frac{\int_0^{E_0} \ln \frac{E_0}{E} dE}{\int_0^{E_0} dE} = 1 \quad \text{for hydrogen}$$

2. Collision with a heavy atom, mass A:

<u>Lab system</u>

Mass 1 mass A, at rest

$\xrightarrow{\underline{a}}$ •

Position of c. of m. $= \rho \equiv \dfrac{1\,\underline{r} + A\underline{R}}{1+A}$

Vel. of c. of m. $= \underline{V} = \dot{\rho} = \dfrac{a}{1+A}$

<u>C. of M. system</u>

before collision

after collision

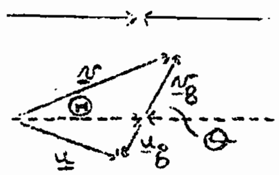


Fig. 1.

where $\underline{v}$ = vel. of ~~nucleus~~ neutron and subscripts refer to c. of m. system.

From conservation of momentum, $\underline{v}_g = A\underline{u}_g$

From conservation of energy, $\underline{v}_g^{\,2} + \underline{u}_g^{\,2} A + (A+1) V^2 = a^2$

from which $v_g = a \dfrac{A}{A+1}$

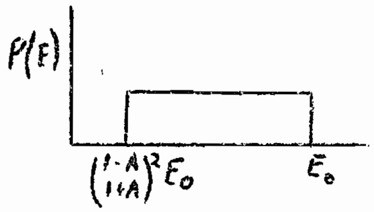
Hence the range of velocities of the neutron after collision include a

minimum, $v = V - \dfrac{A}{A+1} a = \dfrac{a}{A+1} - \dfrac{Aa}{A+1} = \dfrac{1-A}{1+A} a$

and a maximum, $v = V + \dfrac{A}{A+1} a = a$

Thus again, if all directions in the c. of m. system are equally probable, (this is not true for very fast neutrons)

$$\xi = \overline{\ln \frac{E_0}{E}}$$



$$\xi = \overline{\ln \frac{E_0}{E}} = \frac{\int_{\left(\frac{1-A}{1+A}\right)^2 E_0}^{E_0} \ln \frac{E_0}{E}\, dE}{\int_{\left(\frac{1-A}{1+A}\right)^2 E_0}^{E_0} dE}$$

$$= \left\{E_0 - \left(\frac{1-A}{1+A}\right)^2 E_0 \left[\ln \frac{E_0}{\left(\frac{1-A}{1+A}\right)^2 E_0} + 1\right]\right\} \Big/ E_0 - \left(\frac{1-A}{1+A}\right)^2 E_0$$

$$\xi = 1 - \frac{(1-A)^2}{2A} \ln \frac{A+1}{A-1}$$

or using the approximation

$$\ln \frac{A+1}{A-1} \simeq \frac{2}{A},$$

$$\xi \cong \frac{2}{A}$$

To obtain the angular distribution in the laboratory system from Figure 1.

$$v \cos \Theta = V + v_g \cos \theta$$

$$v \sin \Theta = v_g \sin \theta$$

giving $\tan \Theta = \dfrac{A \sin \theta}{1 + A \cos \theta}$

and $\cos \Theta = \dfrac{1 + A \cos \theta}{(1 + A^2 + 2A \cos \theta)^{\frac{1}{2}}}$

Hence the average value of the cos of the angle of scattering is

$$\overline{\cos \Theta} = \frac{\int_{-1}^{+1} \cos \Theta \, d(\cos \theta)}{\int_{-1}^{+1} d(\cos \theta)}$$

$$= \frac{2}{3A} - \cancel{\frac{1}{3A^2}}$$

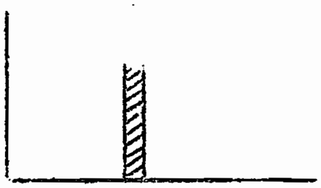
$$= 2/3A$$

SLOWING DOWN (OR AGING) EQUATION (GOOD FROM SOURCE TO THERMAL ENERGIES)

Consider neutrons in a region where the density in space is uniform. Assume that every neutron with energy E has experienced the same number of collisions, and that to reach this E a certain time must have elapsed since leaving the source. (Actually, a spread exists, and if this process is to be treated as a continuous one there must be many collisions.) Then, if there is a constant supply of neutrons, and if there is no absorption, the number of neutrons entering an energy range $\Delta(\ln E)$ is equal to the number that leave per sec.

Now the number in the range $\Delta(\ln E)$ per cm.^{3} is

$$n(E)\,\Delta(\ln E)/\text{cm}^3 = \text{number entering/cm}^3\text{ sec.} \times \text{time to cross range}$$


$\Delta(\ln E)$

$$= q(E) \times \text{number of collisions to cross} \times \text{time/collision}$$

$$= q(E) \times \frac{\Delta \ln E}{\xi} \times \frac{\lambda}{v}$$

where $q(E)$ = slowing down density

= number of neutrons/cm^3 sec. entering a given log energy range

From the last equation we have

$q(E) = \xi n(E)\,\frac{v}{\lambda}$ and Fick's law will hold for it.

$\therefore\; D\Delta q = \partial q/\partial t,$

which is difficult to solve directly because D is a function of time $[D = D_{(t)}]$

Hence the substitution

$D(E)dt = d\tau$ is made, where τ is (Fermi's) "age" and has the dimensions of length^2. However, using the relation on a previous page

$$d\tau = \frac{\lambda v}{3}\cdot\frac{1}{1-\overline{\cos\theta}}\cdot dt = \frac{\lambda v}{3}\cdot\frac{1}{1-\overline{\cos\theta}}\underbrace{\frac{\lambda}{v}\cdot\frac{1}{\xi}\cdot d\ln E}_{\text{Time required to cross range } d(\ln E)}$$

or,

(1) $$d\tau = \frac{\lambda^2}{3(1-\overline{\cos\theta})}\;\frac{d(\ln E)}{\xi}$$

Integrating this equation with the boundary condition that initially

$\tau = 0$,

$$\tau(E) = \int_{E_0}^{E} \frac{\lambda^2(E)}{3(1-\overline{\cos\theta})} \; \frac{d(\ln E)}{\xi}$$

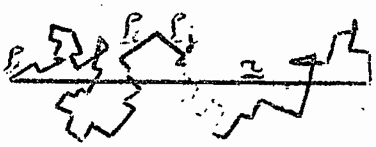
$$\cong \frac{\bar{\lambda}^2}{3\xi} \int_{E_0}^{E} d(\ln E) \quad \text{for heavy element}$$

$$\cong \frac{\bar{\lambda}^2}{3\xi} \ln \frac{E}{E_0} = \frac{\bar{\lambda}^2}{3} \quad \text{for 1 collision}$$

$$\cong \frac{\bar{\lambda}^2 N}{3} \quad \text{for } N \text{ collisions}$$

Since, the distance the individual neutron travels $\sim \sqrt{N}\lambda$ one may relate τ to the displacement, $\underline{r}$, of the neutron from its initial position by using the

THEORY OF THE RANDOM WALK



ρ = displacement of the neutron between any two successive collisions

$$\underline{r} = \sum_i \underline{\rho}_i$$

$$\overline{r^2} = \overline{\left(\sum_i \rho_i^2 + 2 \sum_{ij} \underline{\rho}_i \cdot \underline{\rho}_j\right)}$$

But $\left(\overline{\sum_{ij} \underline{\rho}_i \cdot \underline{\rho}_j}\right)$ becomes zero for heavy elements, since here the distribution after collision is isotropic. Since the probability of making a collision at the end of a displacement ρ is the product:

probability of surviving a distance ρ · probability of making a collision during distance $d\rho = e^{-\rho/\lambda} \cdot d\rho/\lambda$

from which we can obtain the average value of ρ^2.

$$\frac{\int_0^\infty \rho^2 e^{-\rho/\lambda}\, d\rho/\lambda}{\int_0^\infty e^{-\rho/\lambda}\, d\rho/\lambda} = 2\lambda^2$$

so that

$$\overline{r^2} = \sum_i \overline{\rho^2} = 2\lambda^2 N$$

$$= 2\lambda^2 \int_{E_0}^{E} \frac{d(\ln E)}{\xi} = 6\tau$$