DECLASSIFIED Per Letter Instructions Of TID-1040
M. S. for N. T. Bray
10-5-54 SUPERVISOR LABORATORY

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PILE THEORY LECTURE NOTES

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P. Morrison

April 29, 1943

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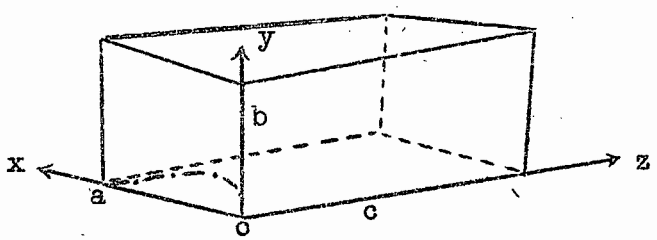
3) The Slowing Down Density in a Finite Block



A finite block of moderator such as graphite

(For .1 dimension)

$e^{iwx-w^2\tau}$ is not as good a form for the solution as $\sin wx e^{-w^2\tau}$. These two solutions are related by the well-known trigonometric expression

$$\sin wx = \frac{e^{iwx} - e^{-iwx}}{2i}$$

If we assume that the augmentation distance is = 0 (i.e., q = 0 on all faces), this solution satisfies the boundary conditions

$$q = \sin wx e^{-w^2\tau}$$

$$w = \frac{\ell\pi}{a}$$

where ℓ = any integer and a = a constant. Then

$$\boxed{q_\ell = \sin\left(\frac{\ell\pi}{a}\right) x e^{-\left(\frac{\ell^2\pi^2}{a^2}\right)\tau}}$$

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We can write for 3 dimensions

$$q_{\ell mn} = \sin\left(\frac{\ell \pi x}{a}\right) \sin\left(\frac{m \pi y}{b}\right) \sin\left(\frac{n \pi z}{c}\right) e^{-\tau \pi^2 B}$$

The classical solution of the problem gives

$$q(\vec{r}, \tau) = \sum_{\substack{\ell=1 \\ m=1 \\ n=1}}^{\infty} q_{\ell mn} A_{\ell mn}$$

where $A_{\ell mn} = \frac{8}{abc} \int_0^a dx \int_0^b dy \int_0^c dz\, q(\vec{r}, \tau = 0) \cos \frac{\ell \pi x}{a} \cos \frac{m \pi y}{b} \cos \frac{n \pi z}{c}$

Consider a point source at the middle:

$$(x,y,z) = (a/2,\ b/2,\ c/2)$$

$$q(\vec{r}, \tau = 0) = Q$$

$$A_{\ell mn} = \frac{8}{abc} Q \sin\left(\frac{\ell \pi}{2}\right) \sin\left(\frac{m \pi}{2}\right) \sin\left(\frac{n \pi}{2}\right)$$

which gives only odd harmonics (111, 333, 555, ...). This series converges quite rapidly and is suitable for this expression. At indium resonance, $\tau_{In} = 300$ cm.^{2} ~ 2 Mev. ℓ, m, n are odd integers (0, 1, 3, 5, ...). Only a couple of terms (or harmonics) are used.

Infinite column

Consider an infinite column of graphite where $\tau = 300$ cm.^{2} which is approximately the mean square distance.

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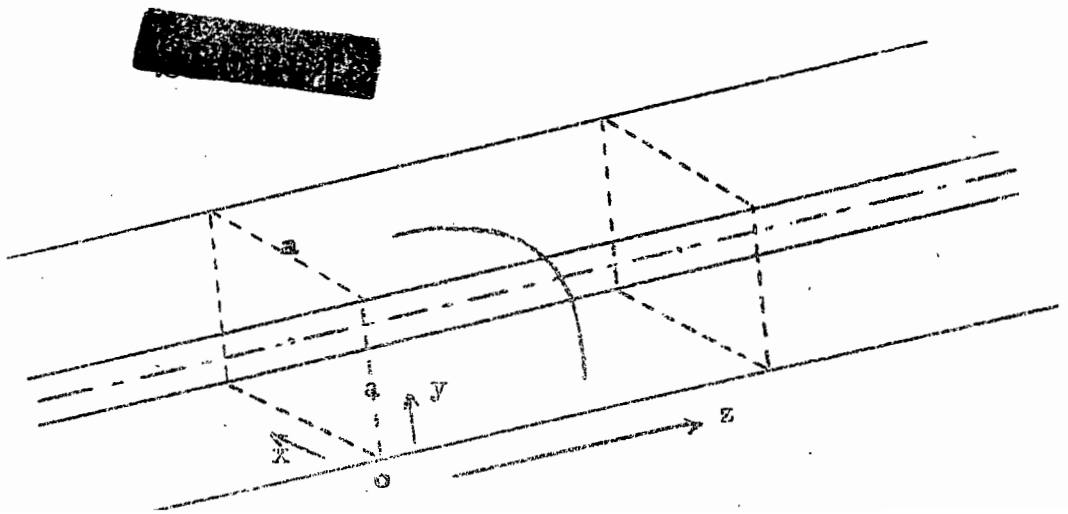

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$$q = q(z)\ f(x, y)$$

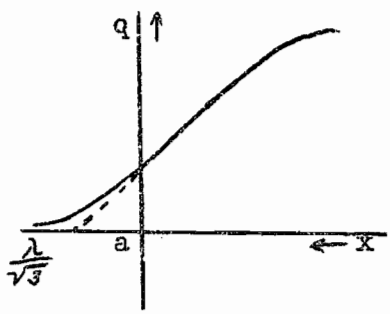
The Fourier Series terms in the x, y directions vanish at the boundaries. The neutrons are thus "pushed out" in the z direction.

$$q = \frac{Q}{2\sqrt{\pi z}}\ e^{-z^2/4\tau} \sum \sin\frac{\ell\pi x}{a} \sin\frac{m\pi y}{b}\ e^{-\pi^2/a^2\tau(\ell^2 + m^2)}\ A_{\ell m}$$

Let $4\tau = r_o^2$ the range of the Gaussian function.

$$a = a_{\text{geometric}} + \frac{\partial\lambda}{\sqrt{3}}$$

because of the augmentation distance. (For a piece of graphite 5 feet on one edge $\frac{\lambda}{\sqrt{3}} = 3$ to 4 cm.)



$$\frac{I_{\text{surface}}}{I_{\text{effective}}} = \cos\left(\frac{\pi}{2}\right)\frac{a}{a_{\text{effective}}}$$

(I = neutron intensity) is the experimental measurement of the neutron intensity at the surface of the geometrical body compared to that at a distance $\lambda/\sqrt{3}$ away from that surface.

<u>Faults with the preceding solution</u>

(1) For very large distances away from the source

$$q \sim ke^{-z/\lambda}$$

Neutrons very far away from the source made few, if any, collisions and their intensity $\sim e^{-z/\lambda}$. Hence they dominate over the Gaussian $e^{-z^2/4\tau}$.

(2) Neutrons come from inhomogeneous (non-monochromatic) sources. Even photo-neutron sources are inhomogeneous and therefore no single neutron age can give a detailed picture of the situation.

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