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INVENTORY 1953

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For The Atomic Energy Commission
H. F. Carroll
Chief, Declassification Branch

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SECRET

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PILE THEORY LECTURE NOTES

* * *

A. M. Weinberg

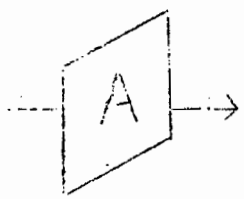
April 13 and 15, 1943

* * *

I. Elementary Diffusion Theory

A. Definition of cross section:

Consider a beam of particles of velocity v which strikes a thin slab of material of area A.



Number of particles entering A and passing per sec = n v A. n = beam density = number/cc.

N_a = no. of atoms in A
N = atoms/c.c.

If each atom presents a target area σ, then

No. of events occurring in A per second = $n\ v\ A \times N_A \sigma / A$

No. of events occurring in A per second = $n\ v\ N_A \sigma$

σ = atomic cross section

$N\sigma$ = macroscopic cross section

No. of events occurring in 1 cc per second = $\dfrac{\text{nx velocity}}{\text{distance between events}}$ = $\dfrac{nv}{\lambda}$

$$\therefore \boxed{\lambda = \frac{1}{N\sigma}}$$

λ ≡ "mean free path"

[No. travelling x without process = $e^{-x/\lambda}$] Loeb, kinetic theory, p. 41.
Can speak of $\frac{\sigma}{\lambda}$ for each process under discussion. $\sigma_{total} = \Sigma\sigma_i$

[redacted]

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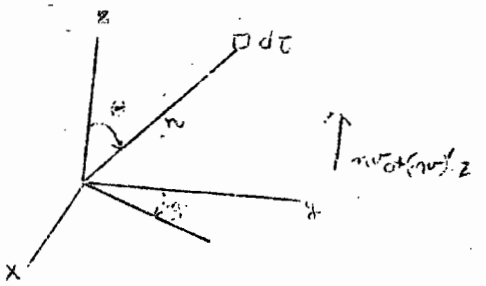
B. Fick's Law (Loeb, kinetic theory of gases, p. 224)



Assume a linear distribution of nv, nv = (vn)'z. Such distribution is sufficiently general because if diffusion theory is to be applicable, gradient must not vary in distance of order of λ, otherwise would be looking at individual particles.

To calculate flow through ds in xy plane,

number of particles starting flight per sec/cc = $\left(\frac{nv}{\lambda}\right) d\tau$

number of particles starting flight per sec/cc in right directions =

$$\frac{nv}{\lambda} \frac{ds \cos\theta}{4\pi r^2} d\tau$$

number of particles starting flight per sec/cc surviving =

$$\frac{nv}{\lambda} \frac{ds \cos\theta}{4\pi r^2} e^{-r/\lambda} d\tau$$

$$d\tau = r^2 \sin\theta \, d\varphi \, d\theta dr \qquad z = r\cos\theta$$

∴ number of particles striking from above =

$$\frac{ds}{4\pi\lambda} \int_0^{\pi/2} \int_0^{2\pi} \int_0^{\infty} e^{-r/\lambda} \sin\theta \cos\theta \, d\theta \, d\varphi \, dr (vn)_0$$

$$+ \frac{(vn)'}{4\pi\lambda} ds \int_0^{\pi/2} \int_0^{2\pi} \int_0^{\infty} r e^{-r/\lambda} \sin\theta \cos^2\theta \, d\theta \, d\varphi \, dr$$

$$= \left[\frac{vn_0}{4\pi\lambda} \cdot \lambda \cdot 2\pi \cdot \frac{1}{2} + \frac{(vn)'}{4\pi\lambda} \cdot \lambda^2 \cdot 2\pi \cdot \left(+\frac{1}{3}\right)\right] ds = \left[\frac{vn_0}{4} + \frac{(vn)'}{6}\lambda\right] ds$$

From below, $\left[\frac{vn_0}{4} - \frac{(vn)'}{6}\lambda\right] ds$

∴ Flow from above to below is

$$\boxed{F = \text{number of particles/cm}^2\text{/sec} = -\frac{\lambda}{3}(vn)'} \quad \text{"Fick's Law"}$$

For constant v, define $D = \frac{\lambda v}{3}$,

$$F = -Dn'$$

Note: This can be generalized for transfer of property g to amount of property g/cm²/sec = $-\frac{\lambda}{3}(g)'$ [illegible]

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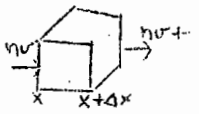
C. The diffusion equation



Consider small element dx dy dz
Number entering face at x =

$$-\frac{\lambda}{3}\frac{d(nv)}{dx}dy\,dz$$

Number leaving face at x + dx = $-\frac{\lambda}{3}\frac{d(nv)}{dx}(x+dx)dydz$ =

$$-\frac{\lambda}{3}\left[\frac{d(nv)}{dx}x\,dy\,dz + \frac{d^2(nv)}{dx^2}dx\,dy\,dz\right]$$

∴ Net flow into volume element is

$$+\frac{\lambda}{3}\left[\frac{d^2(nv)}{dx^2}+\frac{d^2(nv)}{dy^2}+\frac{d^2(nv)}{dz^2}\right]dx\,dy\,dz = +\frac{\lambda}{3}\Delta nv\,dx\,dy\,dz$$

Net production in particles per cc per sec is q; production in dx dy dz is (per sec)

$$q\,dx\,dy\,dz$$

Hence

$$\boxed{\frac{dn}{dt} = \frac{\lambda}{3}\Delta nv + q}$$

Laplacian in various coordinate systems:

cylindrical — cyl. symmetric $\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}+\frac{d^2}{dz^2}$
spherical
spheric. symmetric $\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}$

1. Diffusion in absorbing medium

Suppose σ_a = atomic absorption cross section
$nN\sigma_a v$ = number absorbed per second/cm^3 $= N\sigma_a v = \frac{1}{\tau}$
τ = "lifetime", $\Lambda = \frac{\lambda}{N\sigma_a}$ = "m.f.p. for capture"

Hence in steady state,

$$\frac{\lambda}{3}\Delta nv - \frac{nv}{\tau} + q = 0$$

$$\boxed{\Delta nv = \kappa^2 nv + \frac{3}{\lambda}q} = 0$$

where $\kappa = \sqrt{\frac{3}{\lambda\tau v}} = N\sqrt{3\sigma\sigma_a}$ is the reciprocal of the "diffusion length", L.

Note that $D = \frac{1}{3}\lambda v = \frac{N\sigma_a v}{\kappa^2}$

C.1 In spherical coordinates, radial symmetry,

$$\frac{d^2n}{dr^2} + \frac{2}{r}\frac{dn}{dr} + \frac{3}{\lambda v} q = \varkappa^2 n = o$$

$q = o$ except at $r = o$; $\int 4\pi r^2 q(r)\, dr = Q$

Solution of equation C.1 is

$n = A \frac{e^{-\varkappa r}}{r}$ satisfies homog. eqn. except at o.

To determine A, we must have total absorbed = total produced

$$4\pi N\sigma_a vA \int_0^\infty r\, e^{-\varkappa r}\, dr = \frac{4\pi N\sigma_a vA}{\varkappa^2} = Q$$

$$A = \frac{\varkappa^2}{N\sigma_a v}\frac{Q}{4\pi} \quad \therefore \quad n = \frac{\varkappa^2}{N\sigma_a v}\frac{Q}{4\pi}\frac{e^{-\varkappa r}}{r} = \frac{Q}{4\pi D}\frac{e^{-\varkappa r}}{r}$$