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THE LEAKAGE OF NEUTRONS FROM A HETEROGENEOUS PILE

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ABSTRACT

The leakage of neutrons from a heterogenous pile with finite sides is calculated developing the method used in CP-992. The change in the fraction of neutrons absorbed by the moderator is obtained using the results of the calculation of the leakage.

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THE LEAKAGE OF NEUTRONS FROM A HETEROGENEOUS PILE

The number of neutrons diffusing from a heterogeneous pile with finite sides can be calculated more accurately than in the usual simple theory by extending the calculations given in CP-992, "The Diffusion Length and the Utilization of Thermal Neutrons in a Heterogeneous Pile" (referred to hereafter as I). The same notation will be used in this report as in I without further explanation. The method used here takes account of changes in the behavior of a neutron because of the finite overall gradient of n across the pile and because of the variation of the density near the individual multiplying lumps.

Let e be the leakage per cc per second from the pile divided by the production per cc per second (let P be this production). Then it is evident that e can be calculated by obtaining the total leakage from a unit cell and dividing by the volume of the cell. Therefore we have that

$$e = -\frac{3}{4\pi (r_0+r_1)^3 P}\int_0^{2\pi}\int_0^{\pi}\frac{N_1 \sigma_{a1} v}{\varkappa_1^2}\left[\frac{\partial n}{\partial r}\right]_{r=r_0+r_1}(r_0+r_1)^2 \sin\theta \, d\theta \, d\phi . \tag{1}$$

From the equation immediately after Eq. (40) in I, we have, expanding the $e^{i\vec{s}\cdot\vec{r}}$ in powers of $\vec{s}$, that

$$n = \left\{f_s(r) + i\vec{s}\cdot\hat{m} f_p(r) + \left[3(\vec{s}\cdot\hat{m})^2 - \vec{s}^{\,2}\right] f_d(r)\right\}\left\{1 + i r\vec{s}\cdot\hat{m} - \frac{r^2}{2}(\vec{s}\cdot\hat{m})^2\right\} e^{i\vec{s}\cdot\vec{k}} . \tag{2}$$

Taking the derivative of this with respect to r and noting that all

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except two of the terms vanish when substituted in the integral of (1), we have that

$$e = \frac{3}{4\pi}\,\frac{N_1 \sigma_{a1} v}{\varkappa_1^2 P}\left[f_{s1}(r_0+r_1) + f_{p1}'(r_0+r_1)\right] e^{i\vec{s}\cdot\vec{k}} \int_0^{2\pi}\int_0^{\pi} (\vec{s}\cdot\hat{m})^2 \sin\theta \, d\theta \, d\phi . \tag{3}$$

Evaluating the integral gives us that

$$e = \frac{N_1 \sigma_{a1} v}{\varkappa_1^2 P}\, s^2 \left[f_{s1}(r_0+r_1) + f_{p1}'(r_0+r_1)\right] e^{i\vec{s}\cdot\vec{k}} . \tag{4}$$

Since we are only interested in calculating "e" correct to terms in s^2, P is given to the desired degree of approximation by

$$P = \frac{(r_0+r_1)^3 - r_0^3}{(r_0+r_1)^3}\, e^{i\vec{s}\cdot\vec{k}} . \tag{5}$$

Therefore

$$e = \frac{N_1 \sigma_{a1} v s^2}{\varkappa_1^2}\,\frac{1}{1 - \dfrac{r_0^3}{(r_0+r_1)^3}}\left[f_{s1}(r_0+r_1) + f_{p1}'(r_0+r_1)\right] . \tag{6}$$

f_{s1} and f_{p1}' can be obtained from Eqs. (26) and (27) of I. After some algebraic cancelation and rearrangement of terms, it is found that the combination of f_{s1} and f_{p1}' occurring in Eq. (6) can be written simply in terms of four of the constants, α_0, β_0, γ, ϵ, used in I.

The result is that

$$e = \left[\frac{N_1 \sigma_{a1} v s^2}{\varkappa_1^2}\right]\left[\frac{1}{1-\frac{r_0^3}{(r_0+r_1)^3}}\right]\left[\frac{1}{r_0+r_1}\left\{\gamma \operatorname{ch}\varkappa_1 r_1 + \epsilon \operatorname{sh}\varkappa_1 r_1 - \right.\right. \tag{7}$$

$$\left.\left. - 2\alpha_0 \operatorname{ch}\varkappa_1 r_1 - 2\beta_0 \operatorname{sh}\varkappa_1 r_1\right\} + \frac{1}{v N_1 \sigma_{a1}}\right].$$

Let us now define p_{2c} as the number of neutrons absorbed in the moderator per cc per sec in a pile with finite sides divided by the number produced per cc per sec. Let p_{20c} be the corresponding quantity for a pile with infinite sides. Let p_{2u} and p_{20u} be the thermal utilizations usually denoted by these symbols without the subscript "u". Then evidently we have that

$$p_{20c} + p_{20u} = 1 \quad , \tag{8}$$

$$e + p_{2c} + p_{2u} = 1 \quad . \tag{9}$$

By subtraction we immediately obtain that

$$p_{20c} - p_{2c} = e - (p_{20u} - p_{2u}) \tag{10}$$

The quantity $p_{20u} - p_{2u}$ can be obtained from the exact numerical calculation of L^2 for the ten particular radii chosen in I by merely using Eq. (11) of I. The quantity e can be obtained from Eq. (7) using the values of the constants α_0, β_0, γ, ϵ obtained from the calculations of I. The numerical calculation of e is thus straightforward. One must be careful, however, to compute each individual term to a sufficient degree of accuracy as there is a large amount of cancelation when the individual terms are added and subtracted in Eq. (7). Table I gives the values computed in this way for the three sets of quantities of interest in Eq. (10).

TABLE I

r_0	r_1	$\frac{e}{s^2}$	$\frac{P_{20u} - P_{2u}}{s^2}$	$\frac{P_{20c} - P_{2c}}{s^2}$
2	4	111.7	106.1	5.6
2	5	177.4	163.3	14.1
2	6	260.2	229.9	30.3
2	7	358.6	301.3	57.3
2	8	469.7	371.6	98.1
3	6	138.7	129.8	8.9
3	7.5	221.8	199.1	22.7
3	9	325.7	277.0	48.7
3	10.5	446.6	355.6	91.0
3	12	580.1	427.3	152.8

This calculation gives accurately the difference between $\frac{P_{20u} - P_{2u}}{s^2 P_{20u}}$ and $\frac{P_{20c} - P_{2c}}{s^2 P_{20c}}$. These quantities are given in Table II. The last column gives the percentage by which $\frac{P_{20c} - P_{2c}}{s^2 P_{20c}}$ is greater than $\frac{P_{20u} - P_{2u}}{s^2 P_{20u}}$.

TABLE II

r_0	r_1	$\frac{P_{20u} - P_{2u}}{s^2 P_{20u}}$	$\frac{P_{20c} - P_{2c}}{s^2 P_{20c}}$	% difference
2	4	111.6	113.6*	1.8%*
2	5	177.2	179.8*	1.5%*
2	6	259.7	263.7*	1.5%*
2	7	357.9	362.2*	1.2%*
2	8	468.5	474.1	1.2%
3	6	138.6	140.4*	1.3%*
3	7.5	221.5	224.5*	1.4%*
3	9	325.0	329.5*	1.4%*
3	10.5	445.4	451.2	1.3%
3	12	578.2	585.4	1.2%

*The final figure of the values marked with an asterik is not significant.

It is seen that the relative change in the fraction of neutrons absorbed by the carbon is a little more than 1% greater than the same fraction for the uranium.

APPENDIX I

The "usual" values for the constants were used in both CP-992 and in this report. They are

$$N_0 \sigma_{a0} = 0.277$$

$$\mathcal{H}_0 = 0.732$$

$$N_1 \sigma_{a1} = 0.000418$$

$$\mathcal{H}_1 = 0.0208$$

APPENDIX II

Table III gives the values for the seven constants, a_0, α_0, β_0, b, γ, ϵ, a_1, divided by ν that were obtained from the numerical calculations described in CP-992. These results should properly have been included in that report.

TABLE III

		Values of the constants divided by ν as determined by the boundary conditions						
r_0	r_1	a_0	α_0	β_0	b	γ	ϵ	a_1
2	4	99.02205	-4,582.093	-108,506.1	444.3566	-13,621.43	-327,626.1	-11,697.3
2	5	154.6295	-4,468.325	-104,850.1	703.6322	-13,192.65	-317,312.5	-28,612.6
2	6	223.4698	-4,327.480	-100,323.98	1,027.694	-12,656.73	-304,421.7	-60,112.7
2	7	304.1444	-4,162.423	-95,019.80	1,410.324	-12,023.96	-289,201.2	-112,131.8
2	8	394.3314	-3,977.906	-89,090.27	1,840.654	-11,312.28	-272,083.2	-189,651.3
3	6	76.44377	-6,837.709	-105,938.6	418.3604	-20,088.82	-322,354.1	-11,480.4
3	9	168.8366	-6,427.589	-94,966.70	962.2016	-18,214.88	-292,283.9	-57,785.4
3	10.5	226.4551	-6,171.828	-88,124.32	1,308.148	-17,022.84	-273,155.8	-105,460
3	12	288.7732	-5,895.206	-80,723.85	1,686.295	-15,719.838	-252,247.2	-173,688