Reactor Theory

~~SECRET~~

MUC-FLF-5
This document consists of 4 pages. No. 13 of 25 copies. Series A.

Young

METALLURGICAL LABORATORY
P.O. Box 5207
Chicago 80, Illinois

December 21, 1944

DECLASSIFIED
CLASSIFICATION CHANGED TO:
BY AUTHORITY OF: AEC-Sub #5 10-28-64
BY: 6-29-65

TO: E. P. Wigner

FROM: F. L. Friedman and A. T. Monk

ORNL Technical Information Division
Y-12 Technical Library
Document Reference Section
Bldg. 9711-1

RE: Preliminary Report on Critical Sizes and Wall Losses for Breeders and Converters: P-9 Systems

Section I: Critical Sizes

We consider the problem of finding the saving in pile size effected by reflectors for spherical piles. The reflector considered is assumed to be infinite. The sphere is characterized by its Laplacian and by the thermal diffusion length, L_1; the reflectors by the thermal diffusion length, L_2. The slowing down distance is assumed to be the same for both pile and reflector and is maintained constant throughout all the calculations with the Fermi age equal to that of P-9; i.e. 114 cm^2.

The limitation to a single description of slowing down restricts us to a consideration of piles and reflectors in which any other materials than P-9 occupy only a small fraction of the volume. This limitation, however, is of rather little significance as far as the pile is concerned. Furthermore, we are still able to treat in this way the useful range of thorium slurry reflectors; and since in any practical case the reflector must be held down to a finite size, it is not impossible that all practical reflectors will have essentially this characteristic slowing down distance.

The preliminary considerations which we present here are restricted to P-9 because its exceptionally low absorption minimizes the losses due to the presence of the moderator. Since P-9 occupies the major portion of the volume, we have assumed that the diffusion constants are the same in the pile and in the reflectors. This assumption results in a problem which appears to be sufficiently simple so that the Hopf-Wiener treatment as developed by Volkoff in reports MT-21, 30 and 38 might be applied without change. We have used the formal results of these reports in our computations of critical size. Despite the appearance, however, this procedure is not strictly permissible.

[redacted]

Volkoff's results are based on a model which confines itself to the case of thermal fissions only. In using his formulae, therefore, we tacitly assume that the difference between the distribution of resonance fissions and that of thermal fissions does not affect the critical dimensions. For values of the resonance escape probability which are near one, our assumption is substantially true. Unfortunately, as resonance fissions increase our results become uncertain. One other assumption is involved in our use of this formalism. The resonance capture in the reflector is assumed to be the same as that in the pile. This assumption cannot be valid for all the cases which we consider but without a detailed knowledge of the constituents of the pile and the reflector it can only be remarked that for cases in which the resonance capture is small this error again leaves the critical dimension unaffected.

The methods of calculating diffusion lengths are too well-known to concern us here. The calculation of the Laplacian for systems in which only thermal diffusion takes place are similarly familiar. For systems in which there is also resonance fission, the appropriate formulae are to be found in CP-1729. In this report we assume that the diffusion lengths and Laplacians are known and we present graphs from which, with this knowledge, the critical sizes may be found. They have been computed both on the Fermi and on the three-group theory. As a function of Laplacian and diffusion lengths, there is practically no divergence of the results computed on the basis of these two theories. The main difference between these theories arises in the computation of the Laplacian. If the Laplacian is known, for example from experiment, the predictions of critical size including the reflector effect are essentially unique. A crude summary of the results which are presented in detail on the graphs is embodied in the statement that the saving in the critical radius due to the presence of the reflector is equal to $L_2 + 3.5$. The range over which this formula has been checked by our computations extends from $L_1^2 = 50$ to $L_1^2 = 125$ and from $\sqrt{(114 + L_1^2)} \times$ Laplacian = .6 to 1.2. This saving it will be noticed, is crudely independent of L_1 and the Laplacian in this range.

Section II: Wall Effect

If the pile and the reflectors considered in Section I are separated by an absorbing wall, a loss of neutrons to the wall will occur. The purpose of this section is to give an estimate of the magnitude of this loss. To a first approximation, the number of neutrons absorbed by the wall is

$$\sigma_{at} l_t n v_t$$

where σ_{at} is the absorption cross section of the wall per cm^3, ℓ_t is its thickness and nv_t is the thermal neutron density times velocity at the pile reflector interface. Of all the neutrons not absorbed in the pile, the fraction which is absorbed in the wall is approximately

$$\frac{\sigma_{at}\ell_t nv_t}{\text{total abs. in reflector}} .$$

To within a factor of the order of the resonance escape probability in the reflector, the total absorption in the reflector may be replaced by the thermal absorption in the reflector. Since $(k - 1)/k$ is the ratio of neutrons absorbed outside the pile to those absorbed inside the pile, the fraction of all neutrons absorbed which are lost to the wall is

$$F_t = \frac{k - 1}{k} \times \frac{\sigma_{at}\ell_t nv_t}{\text{thermal abs. in reflector}} .$$

In order to evaluate this fraction, we have calculated the following quantity on the basis of the three-group theory for the various piles and reflectors considered in Section I:

$$G_t = \frac{nv_t\sigma_{a2}L_2}{\text{thermal abs. in reflector}} .$$

Here σ_{a2} is the absorption cross section per cm^3 for thermal neutrons in the reflector. Graphs of G_t are appended and from them F_t may be calculated by the following formula

$$F_t = \frac{k - 1}{k} \frac{\sigma_{at}\ell_t}{\sigma_{a2}L_2} G_t .$$

The approximate formula for F_t which covers our computations from $L_2 = 4$ to $L_2 = 7$, from $L_1^2 = 50$ to 125, and from $\varkappa = \sqrt{(114 + L_1^2)} \times \text{Laplacian} = .6$ to 1.2 is

$$F_t = \frac{k - 1}{k} \frac{\sigma_{at}\ell_t}{\sigma_{a2}L_2} \frac{1}{1 + \varkappa} \frac{3}{2} \frac{L_1^2}{60 + L_1^2} .$$

Simplicity rather than maximum accuracy has been aimed at in writing this formula and it should be remembered that even the graphs of G_t must be used with caution since we have again in this section neglected the perturbation due to resonance fission and to resonance absorption in the reflector.

The questions raised by the existance of resonance fission and because of the difference in resonance capture in the pile and in the reflector are under investigation at present. We hope to deal with them in a future report.

FLF:jjp

RECEIVED
MAY 27 1948
OAK RIDGE NATIONAL
LABORATORIES
CENTRAL FILES

~~S E C R E T~~

SECRET

MUC-FLF-5—APPENDIX
This document consists of
1 page and 15 figures.
No. 13 of 25 copies. Series A.
G-7 1-26-45

APPENDIX

Attached are fifteen graphs which should be appended to MUC-FLF-5.

Complete

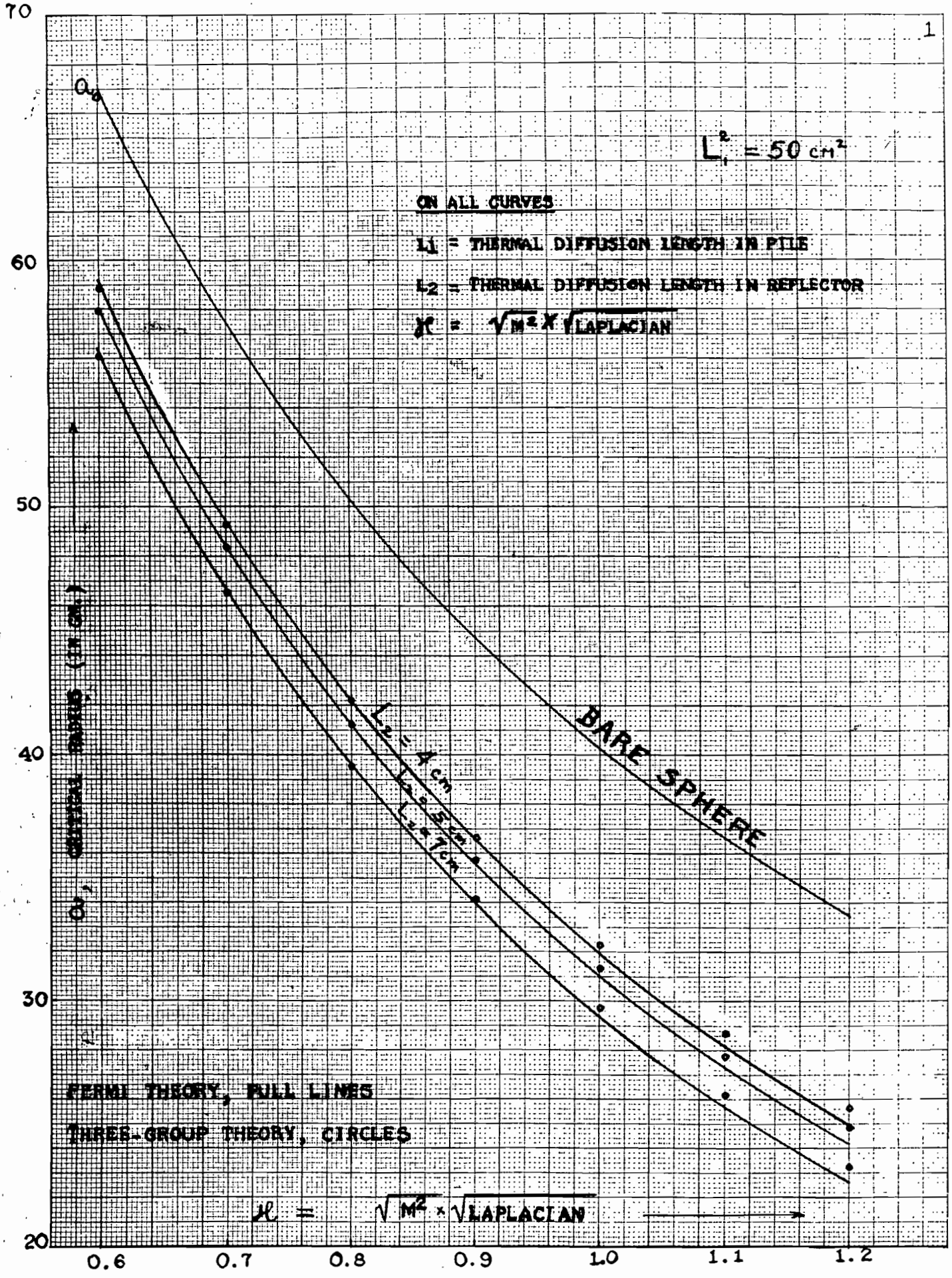
CORREGENDUM:

In Section II, page 3, the absorption in the reflector is always normalized to a square centimeter of wall area. The absorption in the wall, it will be noted, is similarly normalized.

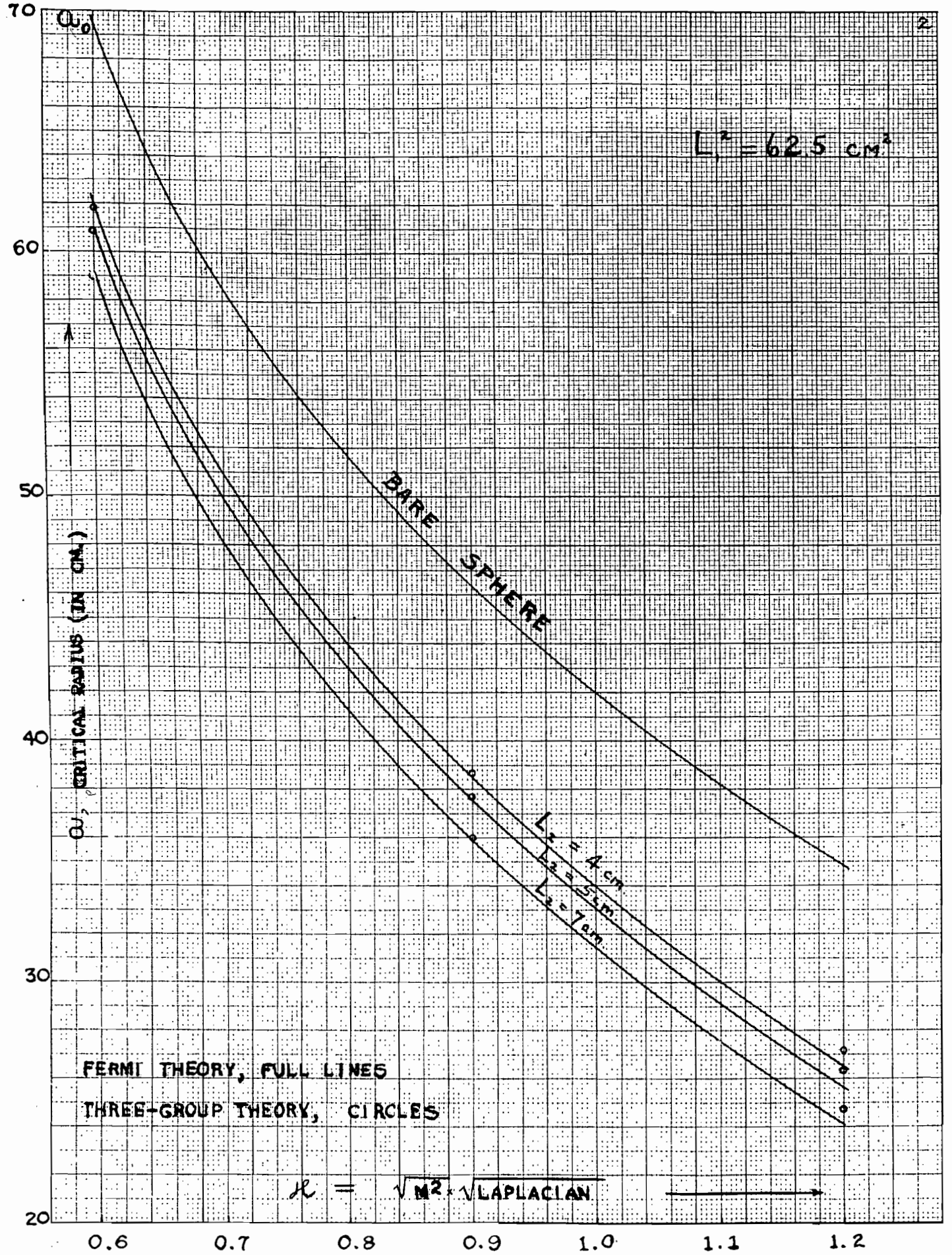
FLF:jjp F. L. Friedman

[redacted]

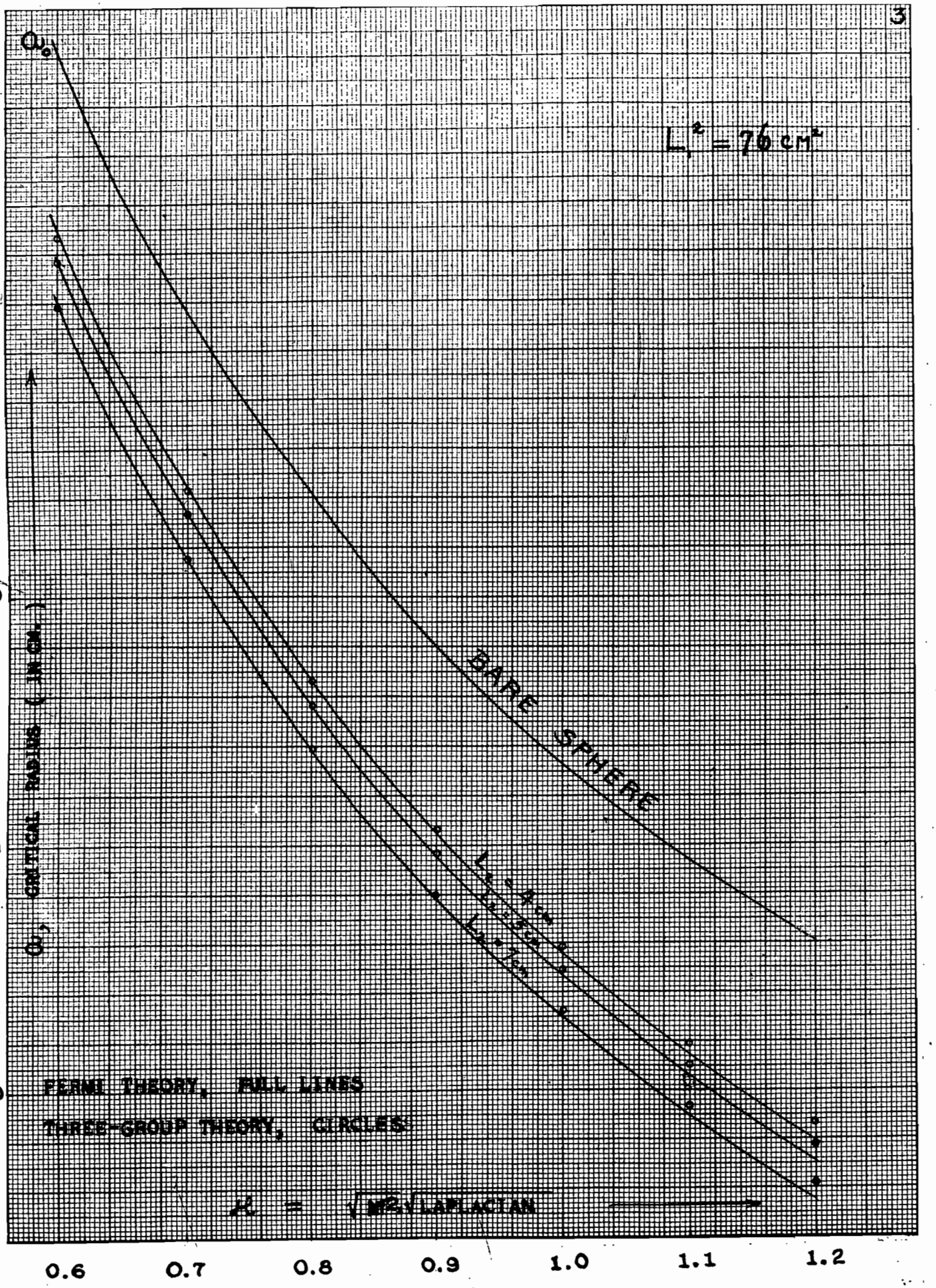
dgr
6-16-48
CA-10A



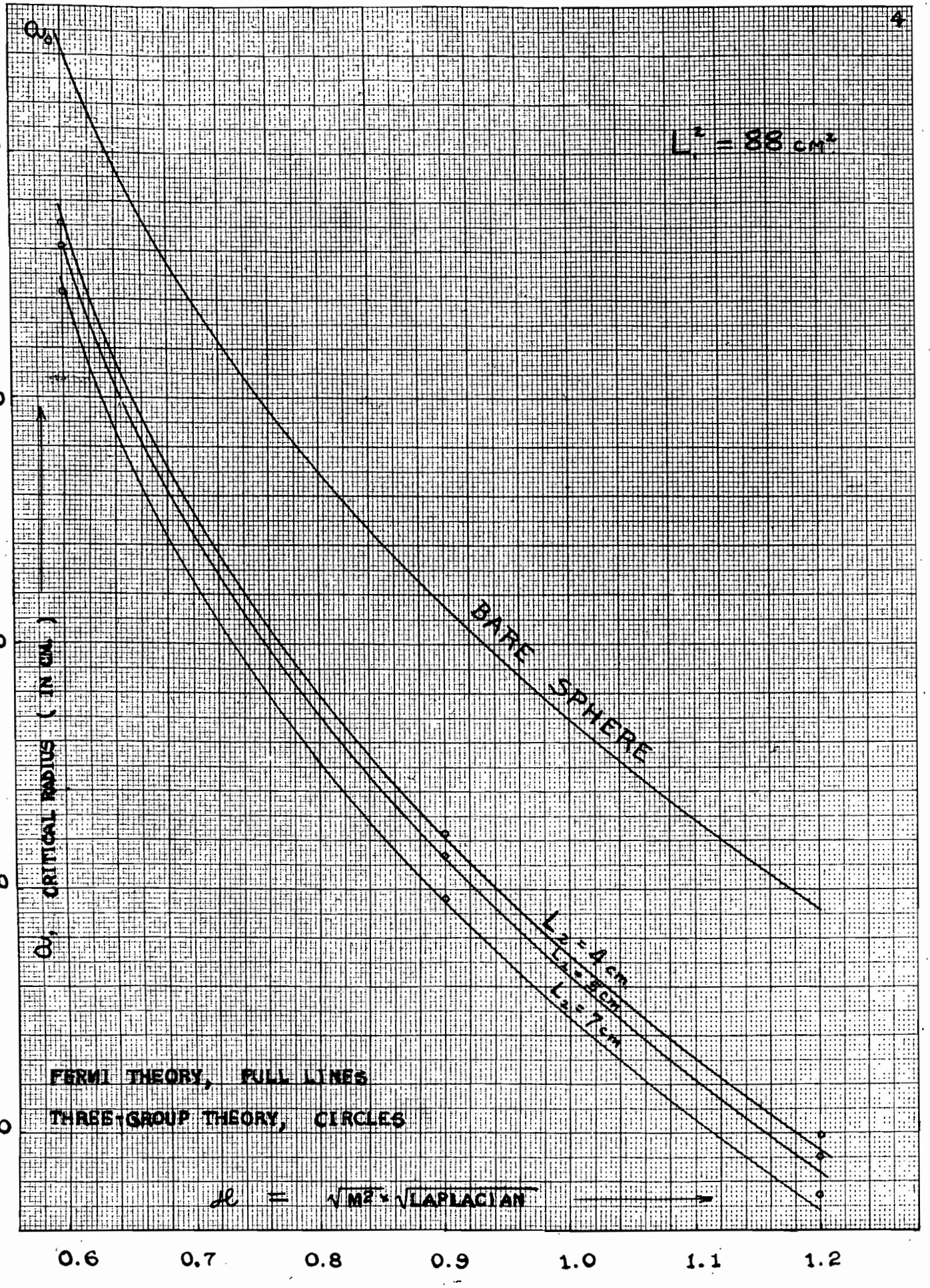
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U. S. A.



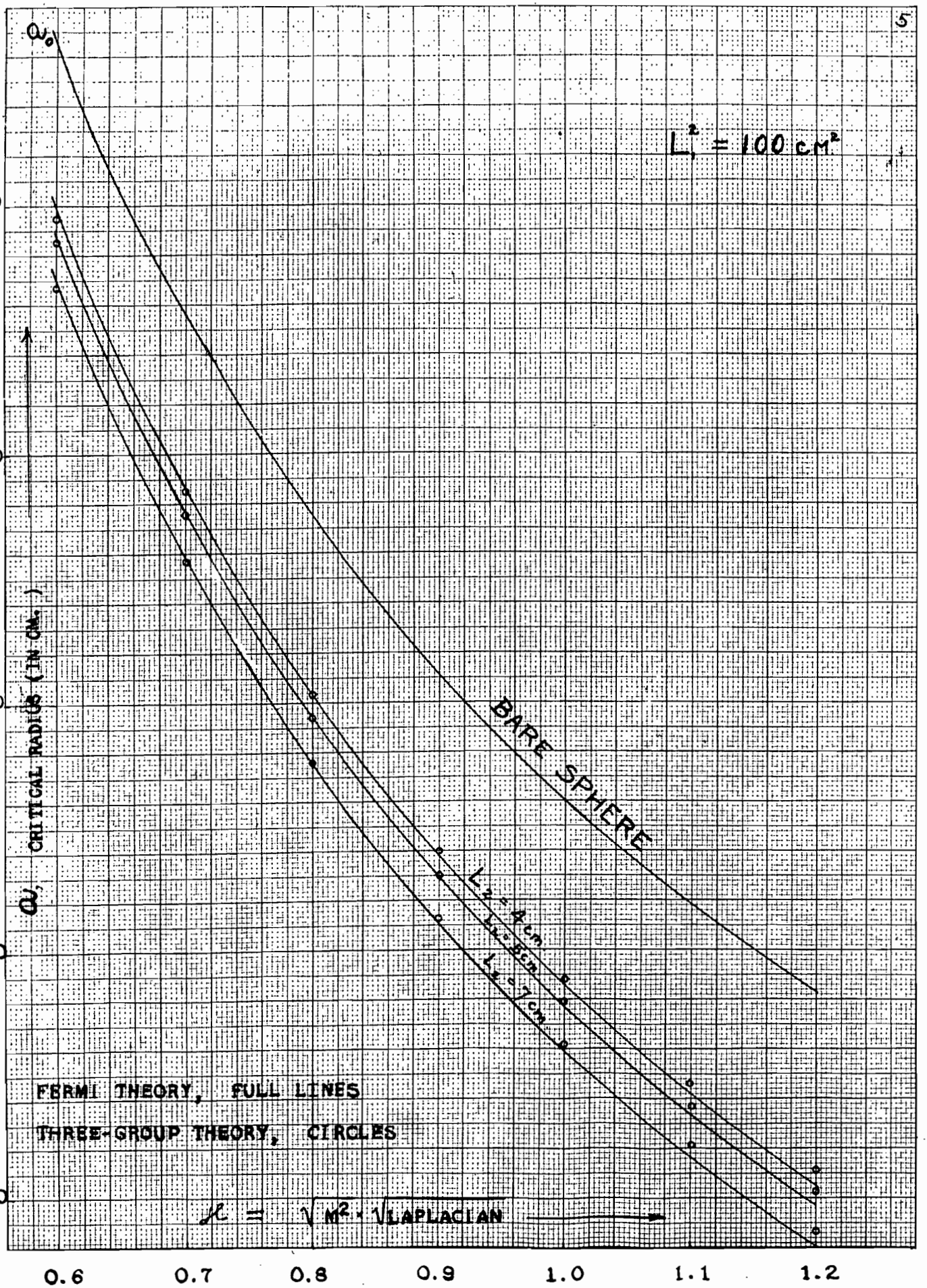
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy



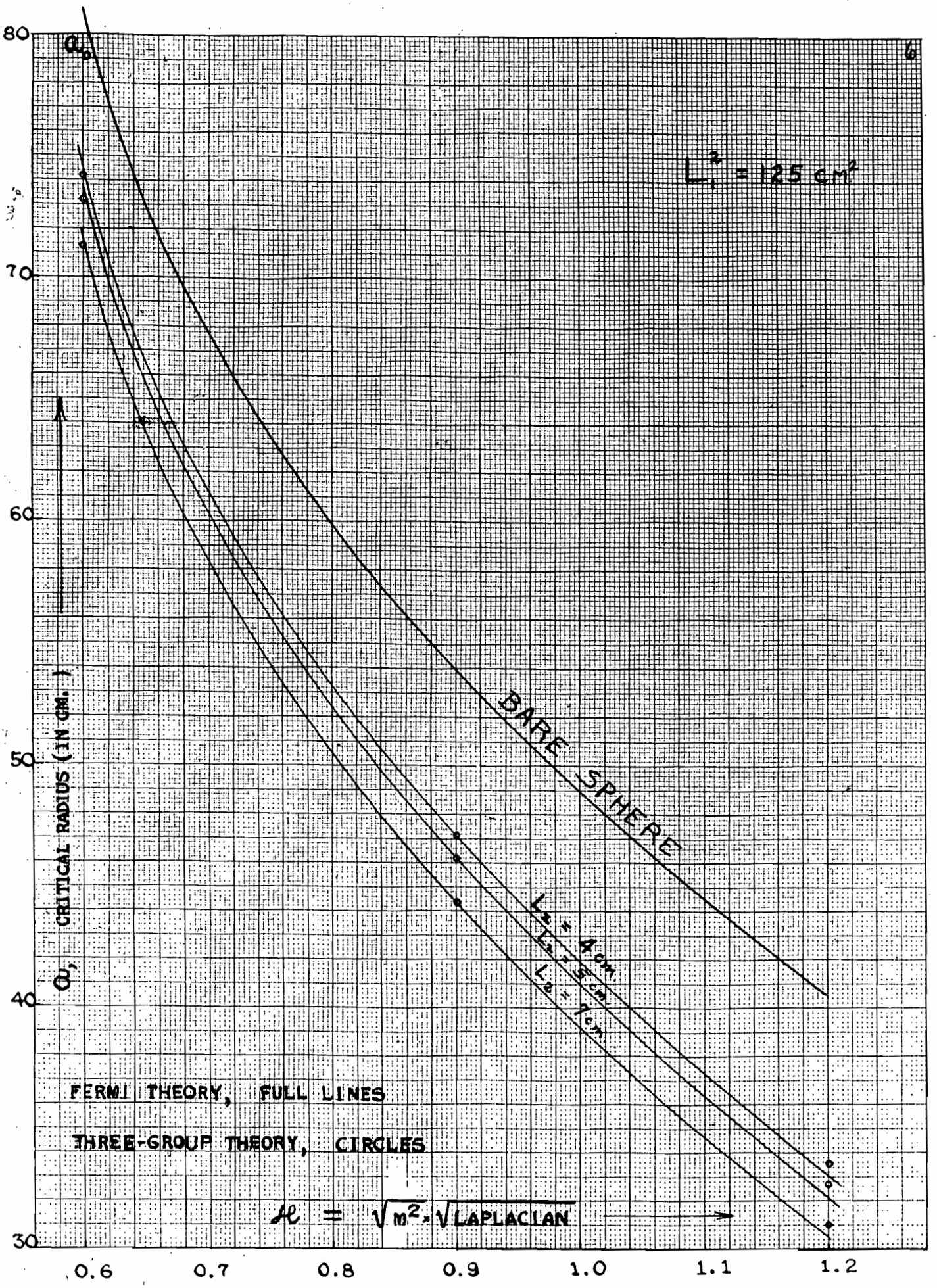
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U. S. A.



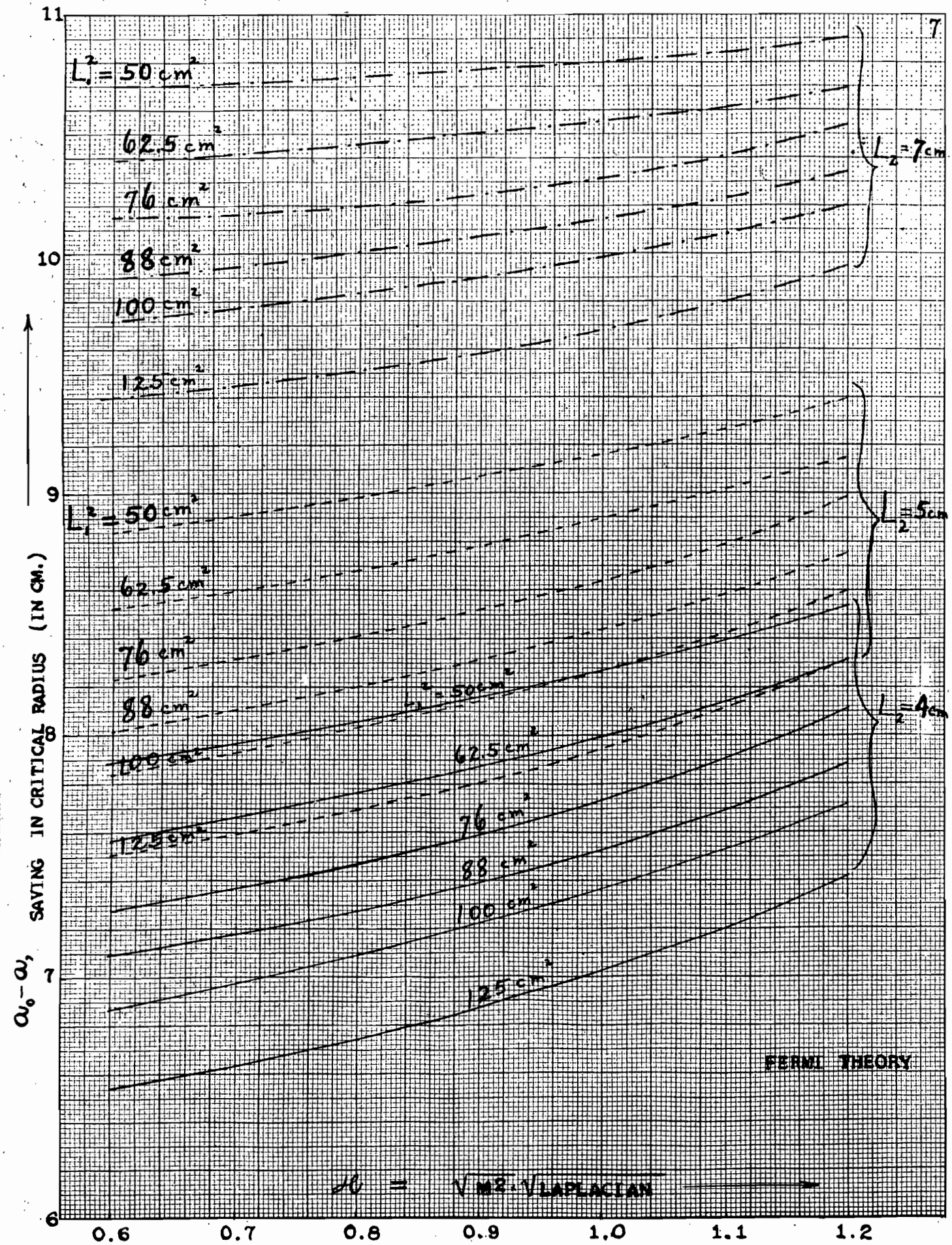
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U. S. A.



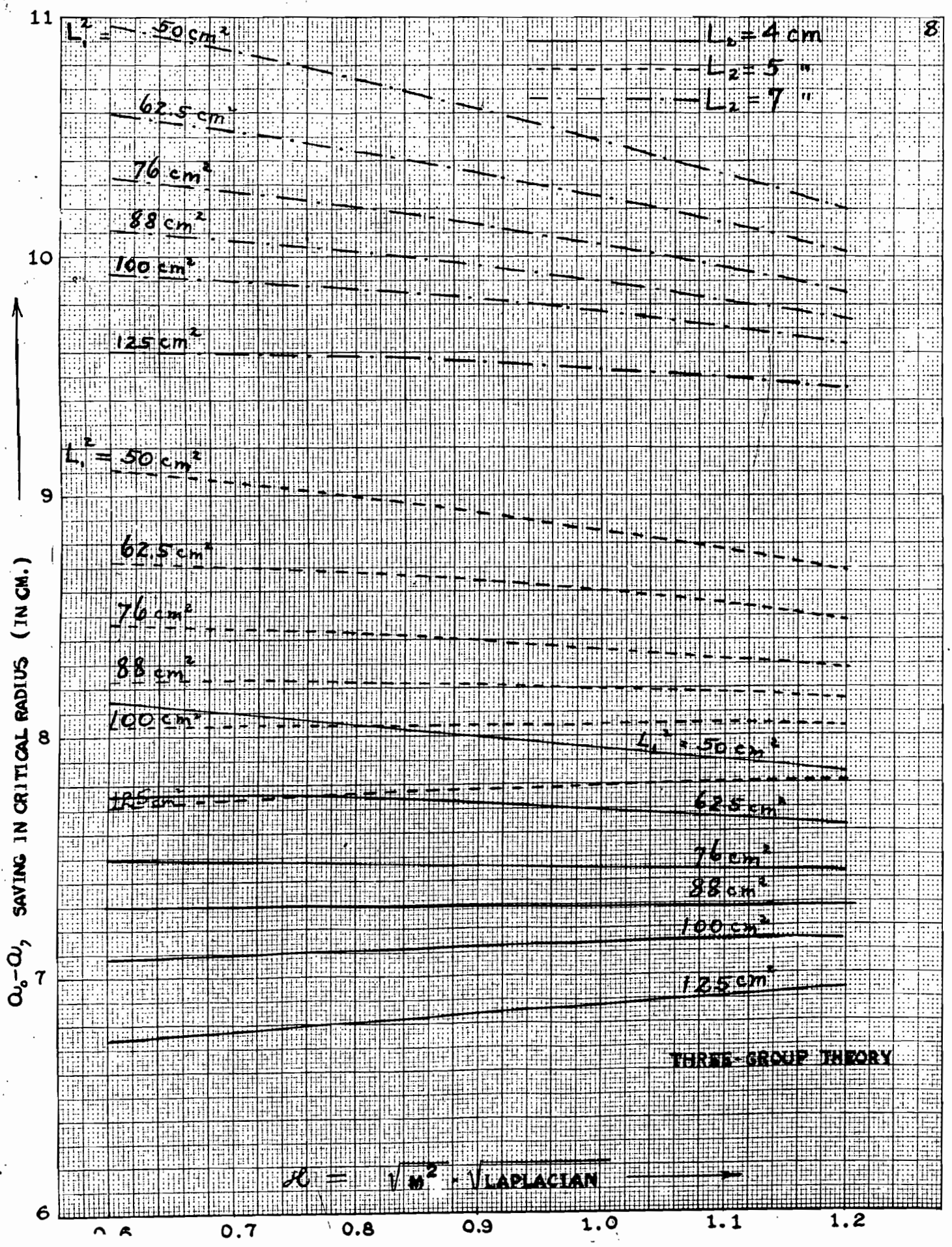
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm, lines accented, cm, lined heavy
MADE IN U.S.A.



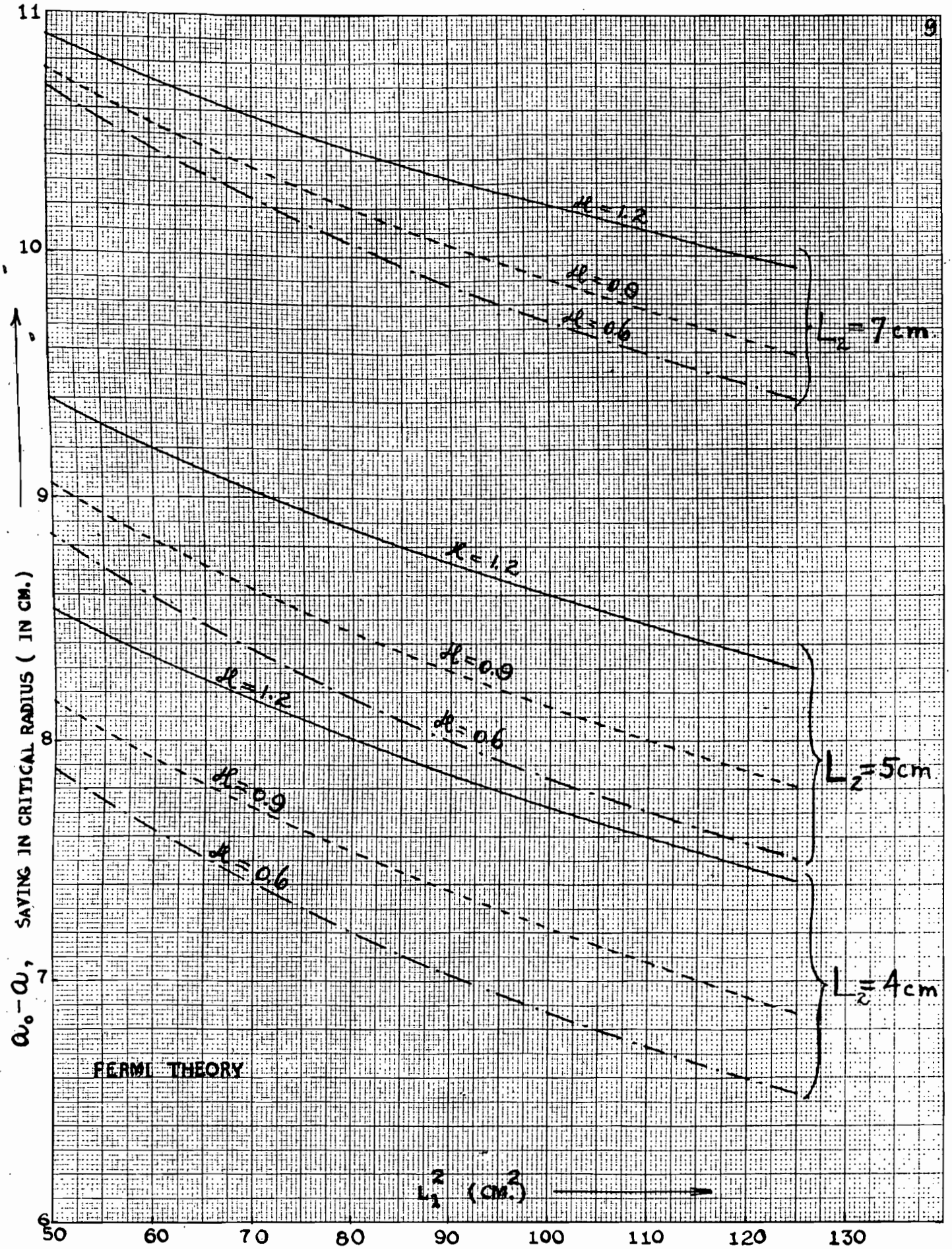
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy
MADE IN U.S.A.



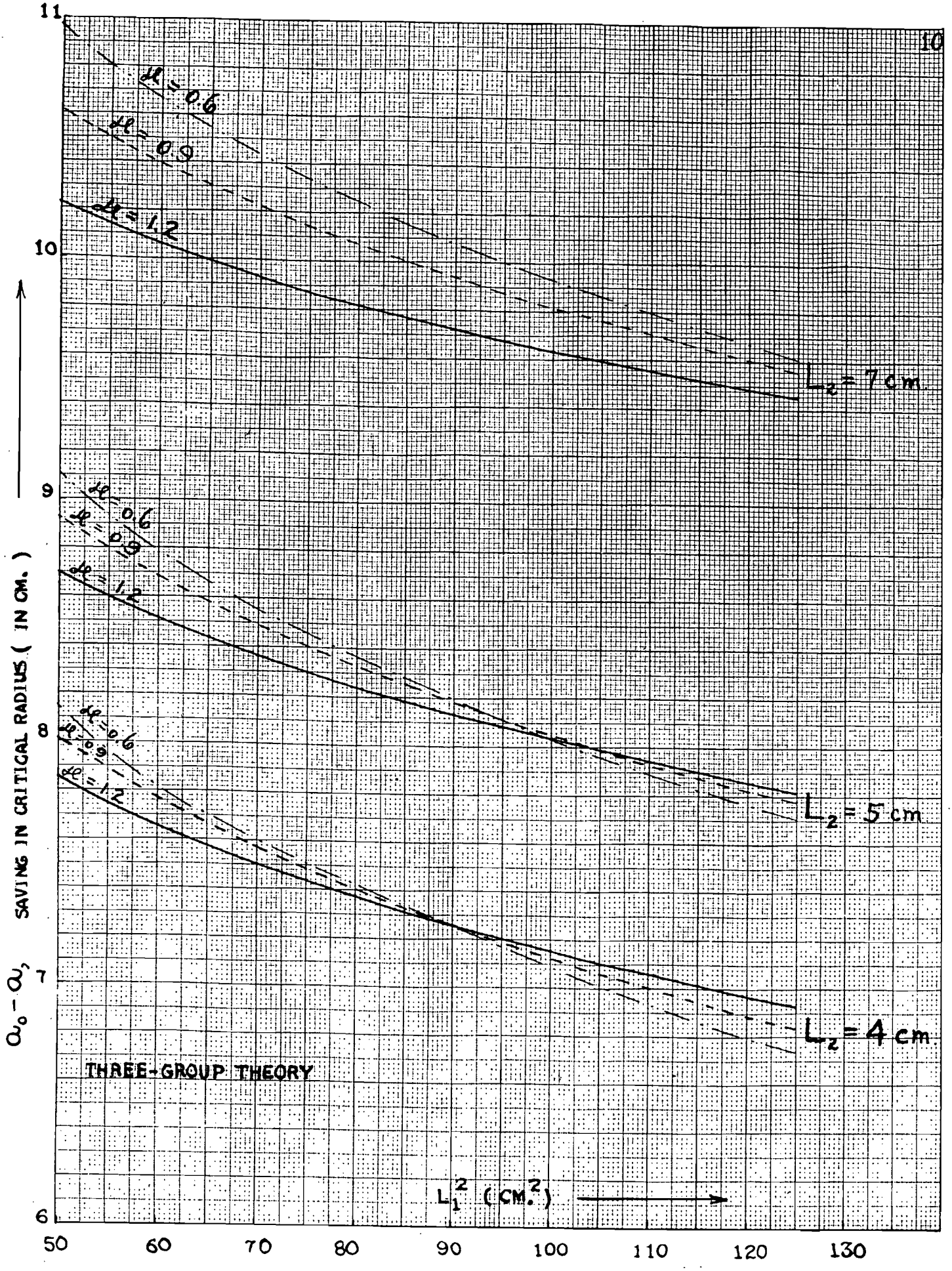
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 6 mm. lines accented, cm. lines heavy.
MADE IN U.S.A.



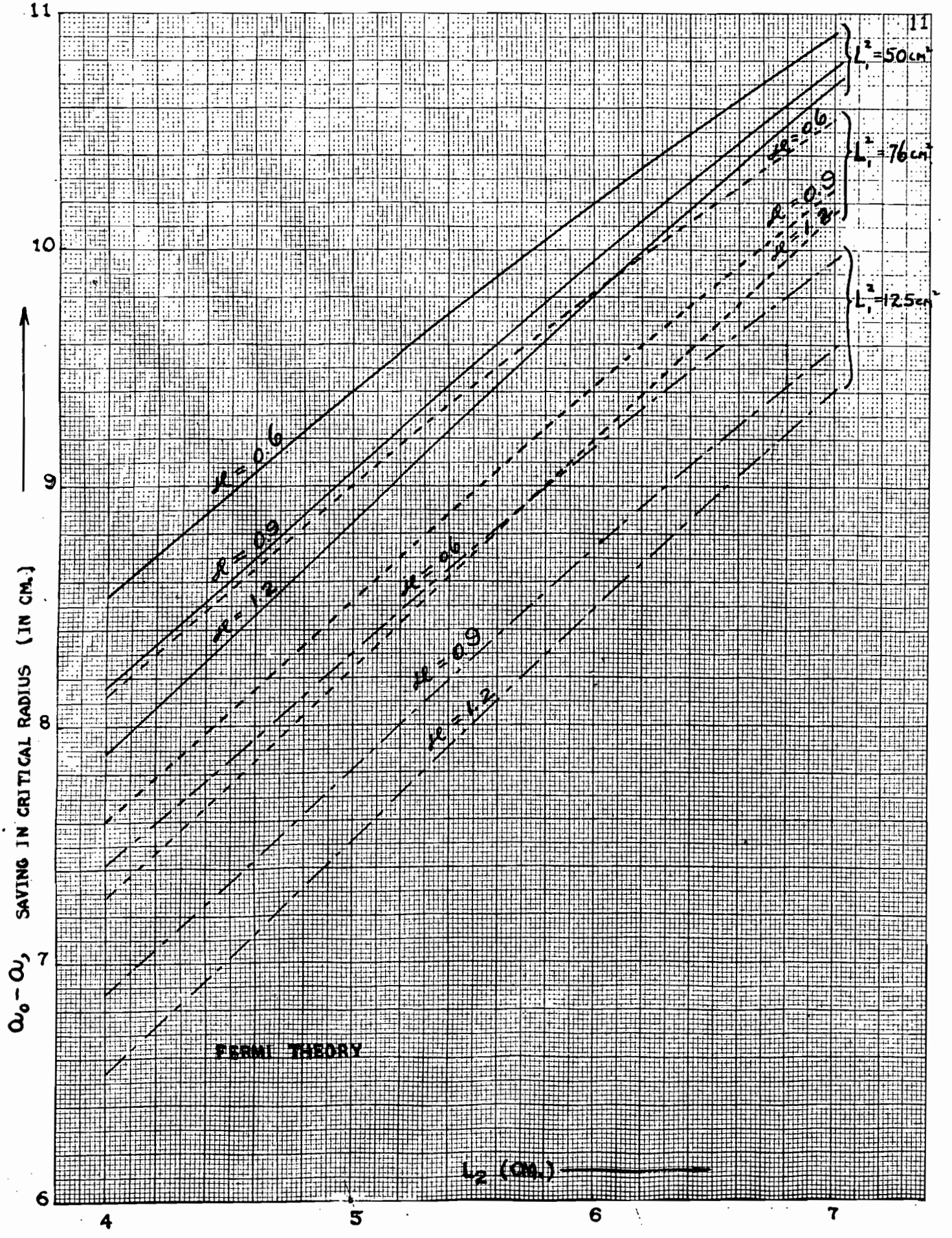
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy
MADE IN U.S.A.



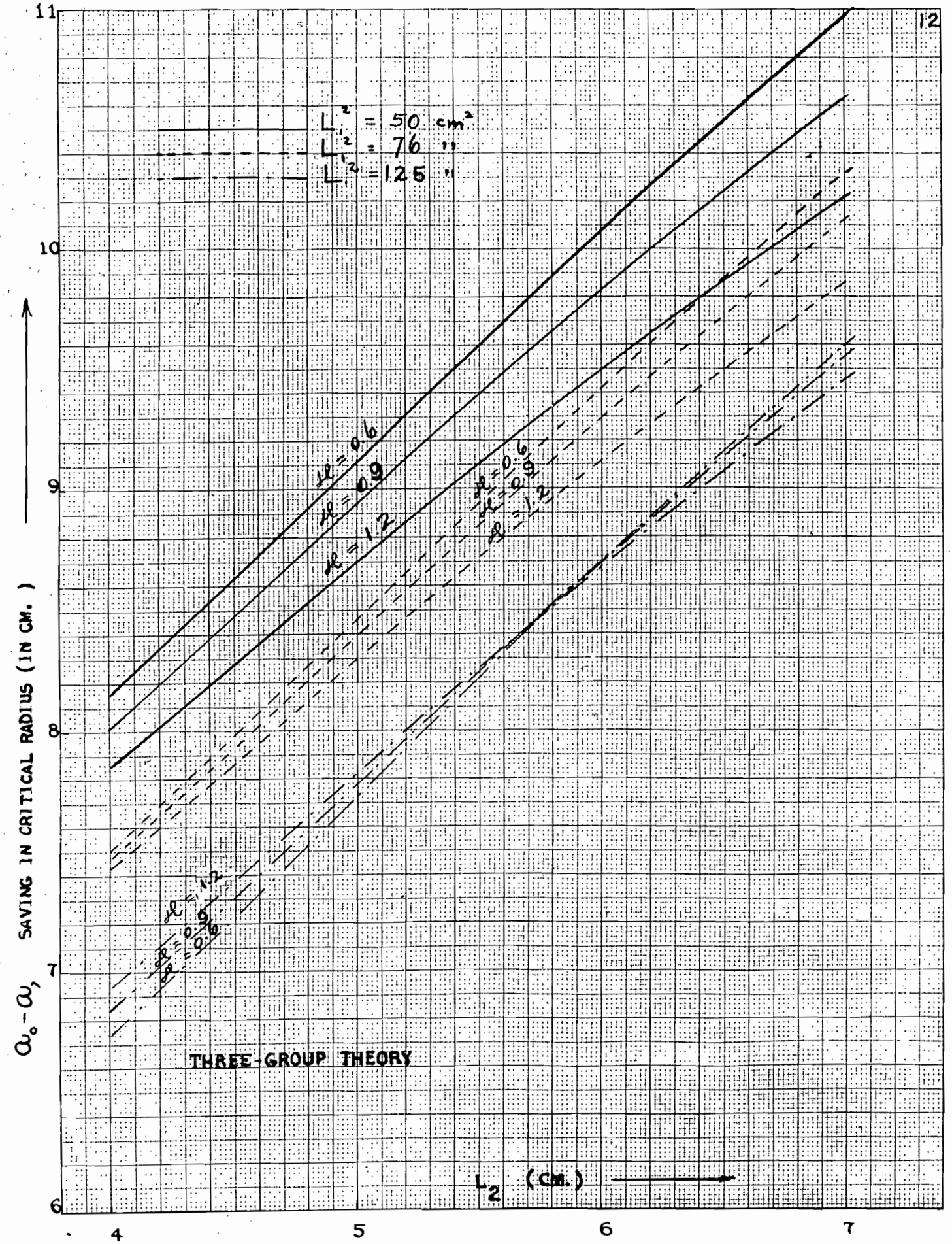
KEUFFEL & ESSER CO., N. Y. NO. 359-12
Millimeters, 5 mm. lines accented, cm. lines heavy
MADE IN U.S.A.

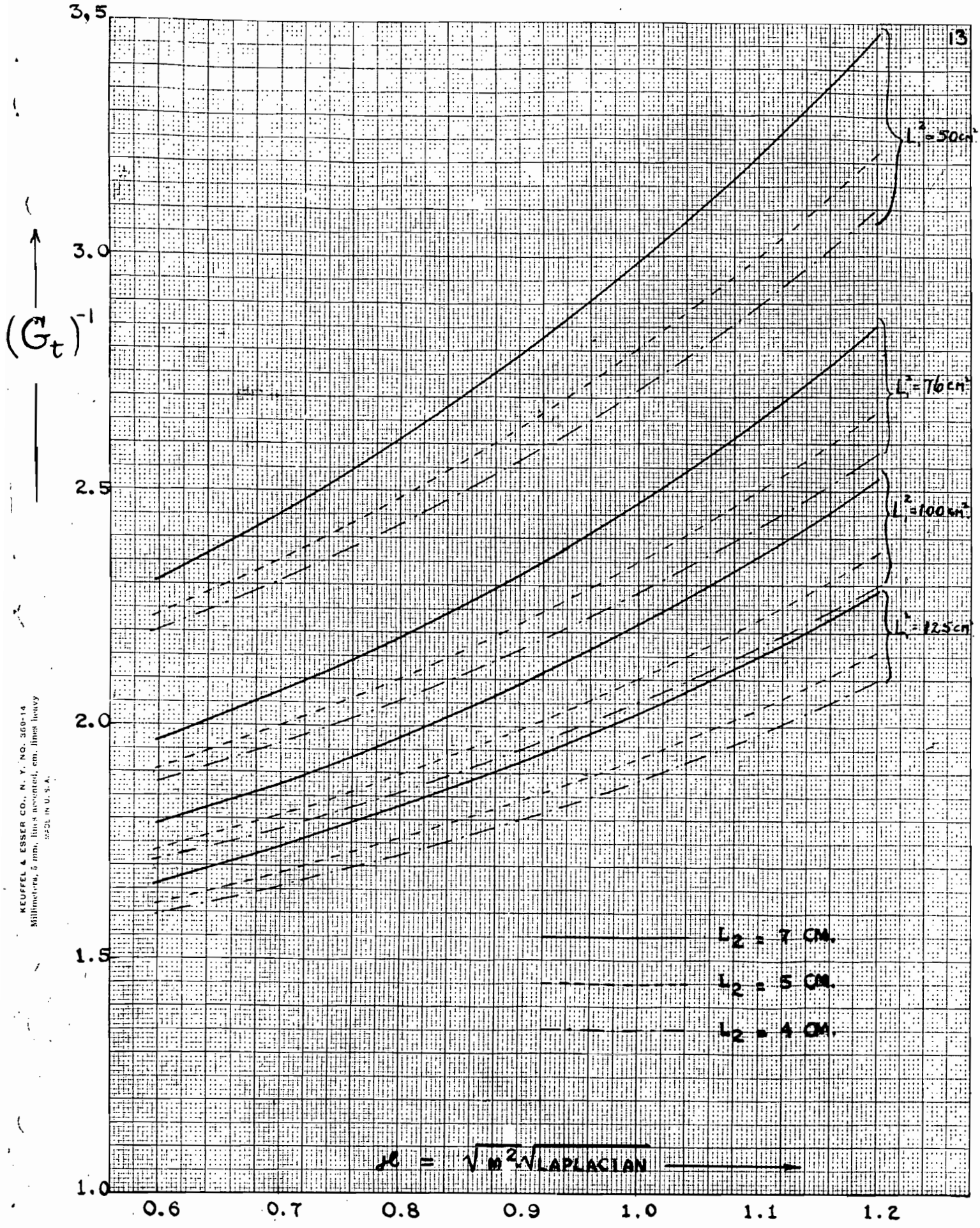


KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U.S.A.

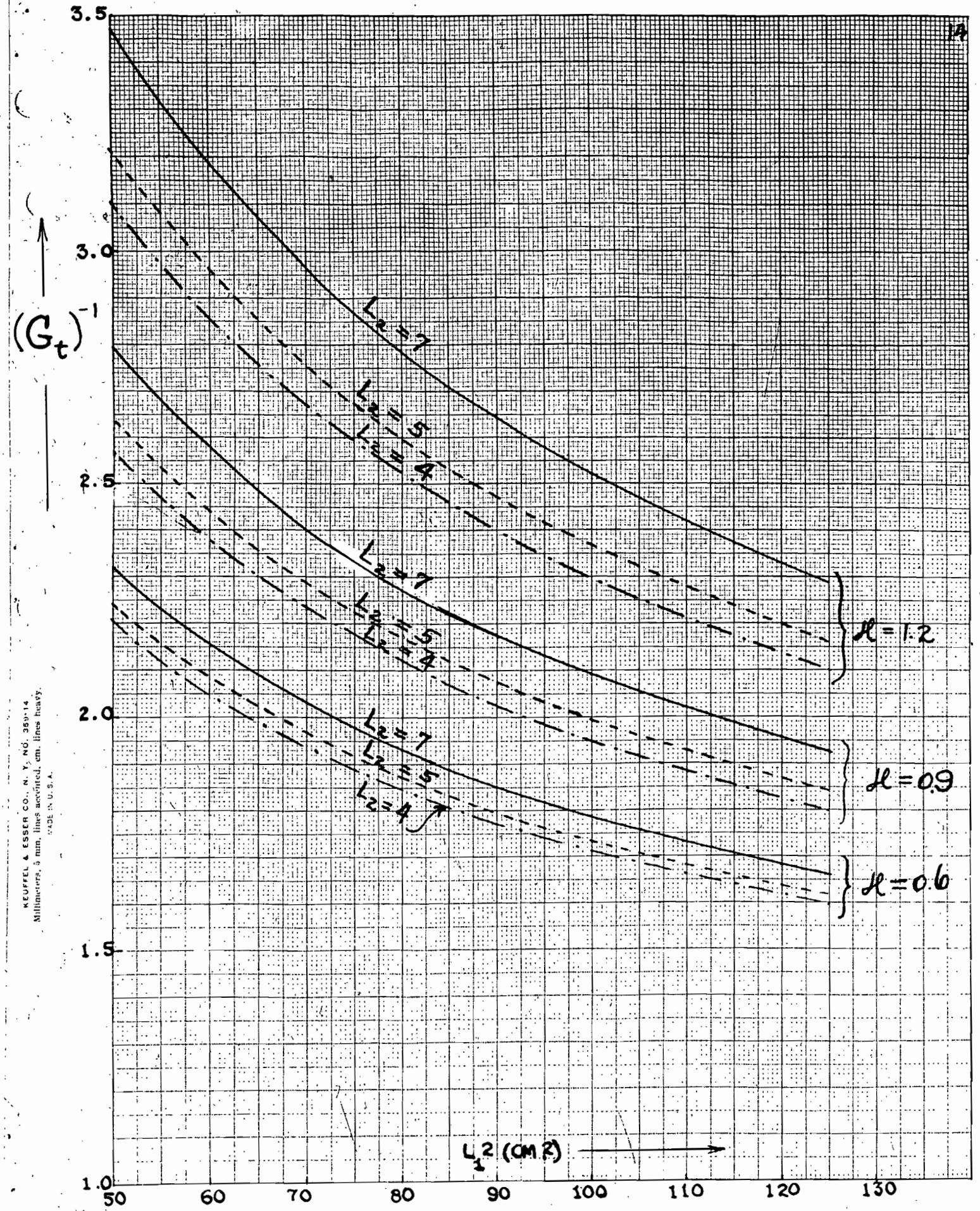


KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 6 mm. lines accented, cm. lines heavy
MADE IN U. S. A.

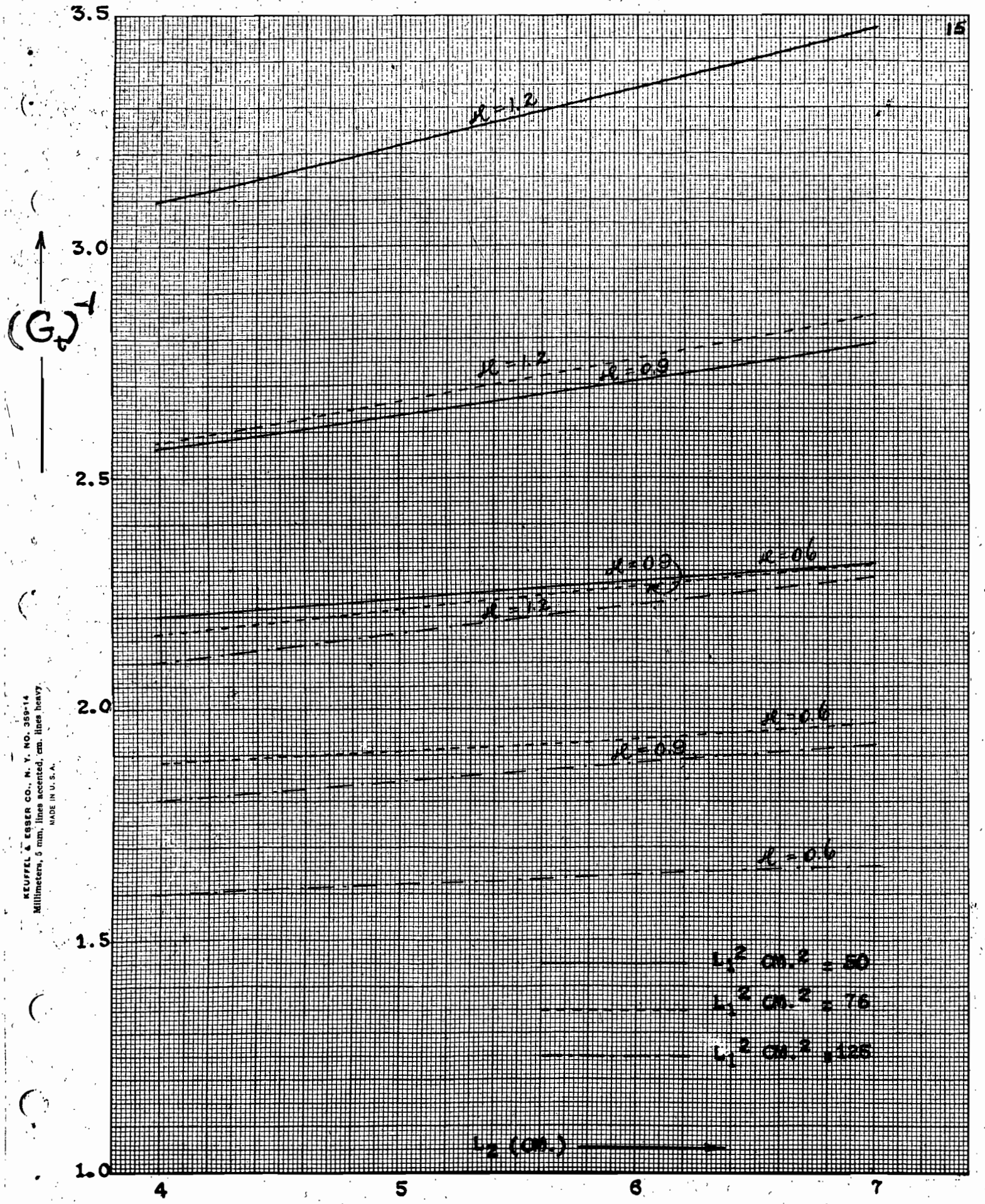

$L_1^2 = 50$ cm²
$L_1^2 = 76$ "
$L_1^2 = 125$ "
$\mu = 0.6$
$\mu = 0.9$
$\mu = 1.2$
$\mu = 0.6$
$\mu = 0.9$
$\mu = 1.2$
$\mu = 1.2$
$\mu = 0.9$
$\mu = 0.6$
THREE-GROUP THEORY
$a_0 - a_1$, SAVING IN CRITICAL RADIUS (IN CM.)
L_2 (CM.)
11
10
9
8
7
6
4
5
6
7
12



KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy
MADE IN U. S. A.



KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U.S.A.



KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U. S. A.

RECEIVED
MAY 27 1948
OAK RIDGE NATIONAL
LABORATORIES
CENTRAL FILES