Reactor Theory

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Date May 15, 1946

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Subject Re: The Control Problem and the Critical Size of an Enriched, BeO Moderated Pile

Copy # 10

By A. V. Martin

Gale Young

To R. G. Sachs

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young

METALLURGICAL LABORATORY

P. O. BOX 5207

May 15, 1946

Chicago 80, Illinois

TO: R. G. Sachs

FROM: A. V. Martin

RE: The Control Problem and the Critical Size of an Enriched, BeO Moderated Pile.

New calculations have been made concerning the control problem and the critical size of an enriched, BeO moderated pile. We let x denote the ratio of the weight of UO_2 to the weight of BeO in such a pile. We have calculated various quantities as functions of x, including the weight of the pile, the weight of pure U^{25} in the pile, the number of control rods needed, the maximum theoretical conversion ratio, etc. The results of these calculations are given in the tables and graphs at the end of this letter.

The values used for the various constants are those which appear in MUC-KW-60 and MUC-KW-61. The calculations were first made assuming the density of the BeO was 2.7 gms/cc, and the results obtained were then adjusted to allow for 20% voids, and BeO density 2.9; so that the final effective density is 2.32.

In computing the control problem the theory of CP-1461 was followed. A cylindrical control rod whose axis is parallel to the axis of the pile is assumed to affect the neutron flux in such a way that there is a cylindrical surface, of radius r_s, concentric with the surface of the control rod, through which the net flux of neutrons of any energy is zero. This cylinder then acts as a perfect reflector. We place n such reflectors, each of radius r_s and each controlled by a poison rod along its central axis, in the central section of the pile in such a way as to form a compact pattern of right circular cylinders, each of which is tangent to or slightly overlaps one or more others. The total volume of these n cylinders is now regarded as one large central reflector, which we idealize as a right circular cylindrical reflector whose effective radius r is approximated by assuming that $\pi r^2 = n \pi r_s^2$.

Let $\phi(\rho)$ denote the radial variation of the neutron density in a right circular cylindrical pile of radius R_o and height h. Then $B = \frac{\pi^2}{h^2} + \frac{(2.4048)^2}{R_o^2}$. Define $\beta = \frac{2.4048}{R_o}$. If a concentric right circular cylindrical reflector of radius r is placed in the pile, and the outside radius of the pile is increased from R_o to R in order to keep the pile of critical size, then we have, in the pile with reflector,

[redacted]

[redacted]

[redacted]

[redacted]

$$\phi(\rho) = J_0(\beta\rho) + AN_0(\beta\rho),$$

$$\frac{d\phi(r)}{d\rho} = -\beta\left[J_1(\beta r) + AN_1(\beta r)\right] = 0,$$

and

$$\phi(R) = 0 .$$

The second of these equations gives $A = -\frac{J_1(\beta r)}{N_1(\beta r)}$, and combining this with the other two, we obtain

$$\frac{J_0(\beta R)}{N_0(\beta R)} = \frac{J_1(\beta r)}{N_1(\beta r)} .$$

This relation permits us to plot r/R_0 as a function of R/R_0, which has been done in Figure I. We now indicate how to use this result in order to compute the radius r of the reflector necessary to control the pile.

We find, in the usual manner, the radius R and height h of the optimum cylindrical shape ($h/R \approx 1.85$) of a hot pile of critical size in which the Xe poisoning has reached its steady state, using the characteristic equation $L^2B_{hot} + 1 = ke^{-\tau B_{hot}}$. We next compute the buckling B_{cold} for the cold pile in which there is no poison. Then from the relation

$$B_{cold} = \frac{\pi^2}{h^2} + \frac{2.4048^2}{R_0^2} \qquad (1)$$

we find the critical radius R_0 which this pile would have if there were no reflector in it. We now have R and R_0, which enables us by means of Figure I to find r. Our approximation of the number of control rods needed is then given by $(r/r_s)^2$.

We now consider a similar problem—that of determining a measure of the safety afforded by a given number of control rods. Suppose then that we are given the dimensions R and h of the hot pile containing Xe, and are allowed a given number n (in our case, we choose n = 7) of control rods, and we wish to determine how large a value, B_{max}, of the buckling can be controlled by these n rods. We find r_s by the theory of CP-1461, and set $r = r_s\sqrt{n}$. We then find R_0 from Figure I, and B_{max} is determined by the relation

$$B_{max} = \frac{\pi^2}{h^2} + \frac{2.4048^2}{R_0^2} .$$

If B_{cold} denotes as before our calculated value for B in the cold pile containing no Xe, then B_{max}/B_{cold} is a measure of the margin of safety given by these n control rods.

In computing r_s (for definition, see above) after CP-1461 [redacted] group model is used for which the value of τ given in MUC-KW-[redacted]

[redacted]

appropriate. Instead we used a value τ' obtained from the equation

$$B^2 + \left(\frac{1}{L^2} + \frac{1}{\tau'}\right)B - \frac{k-1}{L^2\tau'} = 0,$$

in which L^2, k and B are obtained in the usual manner, using a one-group theory in the cold pile without poison, and using the constants of MUC-KW-60 and MUC-KW-61.

In CP-1461 it is assumed that the active material of the pile comes right up to the surface of the control rod. In our case it is assumed that the control rod, of radius r_1, is surrounded by an annular region, of outside radius r_2, through which the coolant flows. To compute the effective radius r_o of the rod, taking into account the existence of this gap, we proceed as follows. We first compute the linear extrapolation length ℓ which we would use if there were no annular region for the flow of coolant around the control rod, assuming that the solid BeO, with no voids, comes right up to the surface of the rod. This expression for ℓ is the one obtained by taking the first three terms of an expression given in MT-135, namely

$$\ell = \lambda(.7104 + \frac{.2524}{a} + \frac{.0949}{a^2}),$$

where λ is the transport mean free path in the solid BeO, and $a = \frac{r_1}{\lambda}$. We now take as the effective radius r_o of the control rod the value

$r_o = \frac{r_1 - \ell}{r_1 - .7104\,\lambda}(r_2 - .7104\,\lambda)$. This gives only a first approximation for the effective radius of the rod, and it may be worth while eventually to investigate this problem more carefully. We note in support of the present approximation that if we increase the radius of the rod indefinitely, keeping $r_2 - r_1$ fixed, then the effective radius approaches $r_2 - .7104\,\lambda$, the correct value for $r_2 = \infty$; and if r_1 approaches r_2, remaining large compared with λ, we again get the correct expression for r_o. In our case we use $r_1 = 3.175$ cm, $r_2 = 3.731$ cm, $\lambda = 1.433$ cm, which give $r_o = 2.47$ cm. We then use this value of r_o in conjunction with values of B, R, h, L^2, τ', etc., which are based on an average density of 2.32 gm/cc, to compute r_s.

In finding the resonance escape probability, p, one uses for $A = \int \frac{\sigma\, dE}{E}$ the graph of Figure 7, p. 31a, Chapter IV, section E of the Project Handbook. However, for our pile this graph is not extended to sufficiently large values of σ_s. In Figure II of this memorandum, this graph has been extrapolated to its asymptotic value of 240. It should be emphasized that this extrapolation was made entirely for the purpose of having standard values so that comparisons may be made between results obtained by various computers, and that it is based on no experimental evidence other than that of the graph in the Handbook and the assumption that the asymptotic value is 240.

[redacted]

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We define the conversion ratio C as the ratio of the total number of neutrons leaking from the pile per second to the total number of neutrons absorbed in U^{235} per second. In computing C we assume for simplicity that all resonance absorption in U^{238} occurs at 6.6 ev, and that all absorption in U^{235} occurs at .09 ev. The formula for C which we used is then

$$C = \frac{B\lambda}{3N_{25}\sigma_a^{25}}\left[1 + \frac{\eta f_{25}}{L^2B}\left\{1 - (1-p)e^{-B\tau(6.6)} - pe^{-B\tau(.09)}\right\}\right]$$

where $\tau(a)$ denotes the age of a neutron from fission energy to energy a.

In Figure III we have multiplied R by 10^{-1} and the total weight by 10^{-3} before plotting, in order to be able to exhibit the various quantities on the same graph, for convenience in making comparisons.

E. H. Ostrow and Mildred Goldberger performed some of the calculations of this report.

jjp

A. V. Martin

cc-Willard, Daniels, Robertson, Hutchison, Zinn, Ostrow, Hughes, McCullough, G. Young, Clinton File, Technical File

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HOT PILE

x wt. of UO_2 / wt. of BeO	Maximum theoretical conversion ratio	L^2	k	f_{25}	f_{BeO}	f_{28}	f_{Xe}	$B \times 10^3$
.003		389.8	1.57	.7685	.1902	.0101	.0312	.908
.004	.796	306.9	1.63	.8068	.1498	.0106	.0328	1.122
.006	.836	215.2	1.69	.8493	.1053	.0112	.0345	1.455
.008	.839	166.1	1.71	.8721	.0810	.0115	.0354	1.661
.015	.812	91.8	1.68	.9064	.0448	.0120	.0368	1.993
.0275	.767	51.2	1.62	.9252	.0250	.0122	.0376	2.163

COLD PILE

x wt. of UO_2 / wt. of BeO	L^2	k	f_{25}	$B \times 10^3$
.004	124.3	1.723	.8480	1.831
.008	75.8	1.803	.9141	2.319
.0275	23.3	1.717	.9676	2.610

x wt. of UO_2 / wt. of BeO	n=no. of control rods	$B_{max} \times 10^3$ for 7 control rods	B_{cold}/B_{hot}	B_{max}/B_{cold}	Total Weight kg.	Weight of pure U^{25} kg.	Radius of pile R	Height of pile h	r_s	R_o
.003					12,575	9.98	97.7	181		
.004	4.17	2.727	1.632	1.489	9,158	9.69	87.9	162	19.68	63.0
.006					6,204	9.84	77.2	143		
.008	2.72	4.314	1.396	1.860	5,096	10.8	72.3	134	16.67	57.2
.015					3,877	15.4	66.0	122		
.0275	1.90	4.563	1.207	1.748	3,420	24.9	63.3	117	13.15	55.3

[redacted]

[redacted]

[redacted]

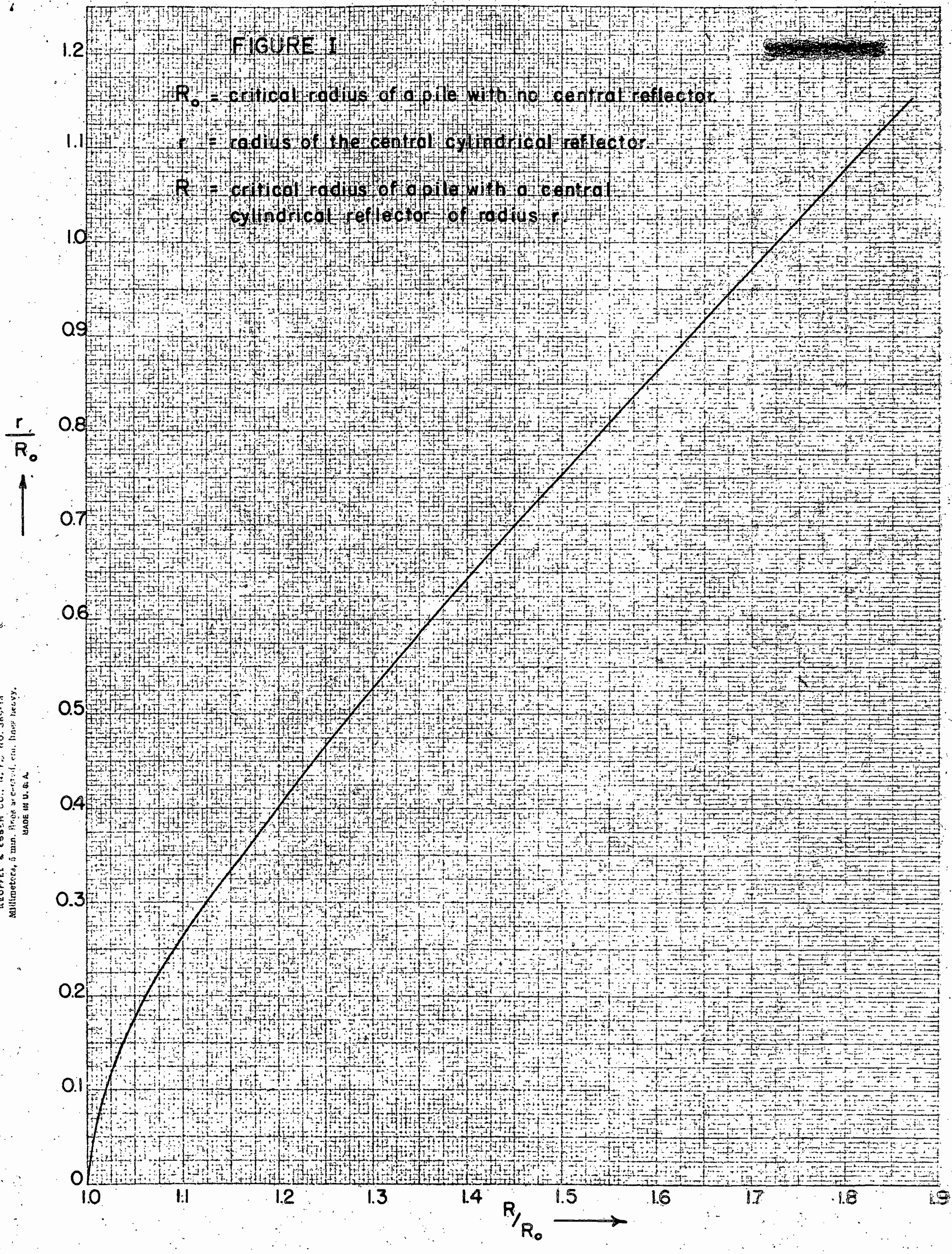
[redacted]
R_o = critical radius of a pile with no central reflector.
r = radius of the central cylindrical reflector.
R = critical radius of a pile with a central cylindrical reflector of radius r.
$\frac{r}{R_o}$
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
R/R_o

FIGURE I

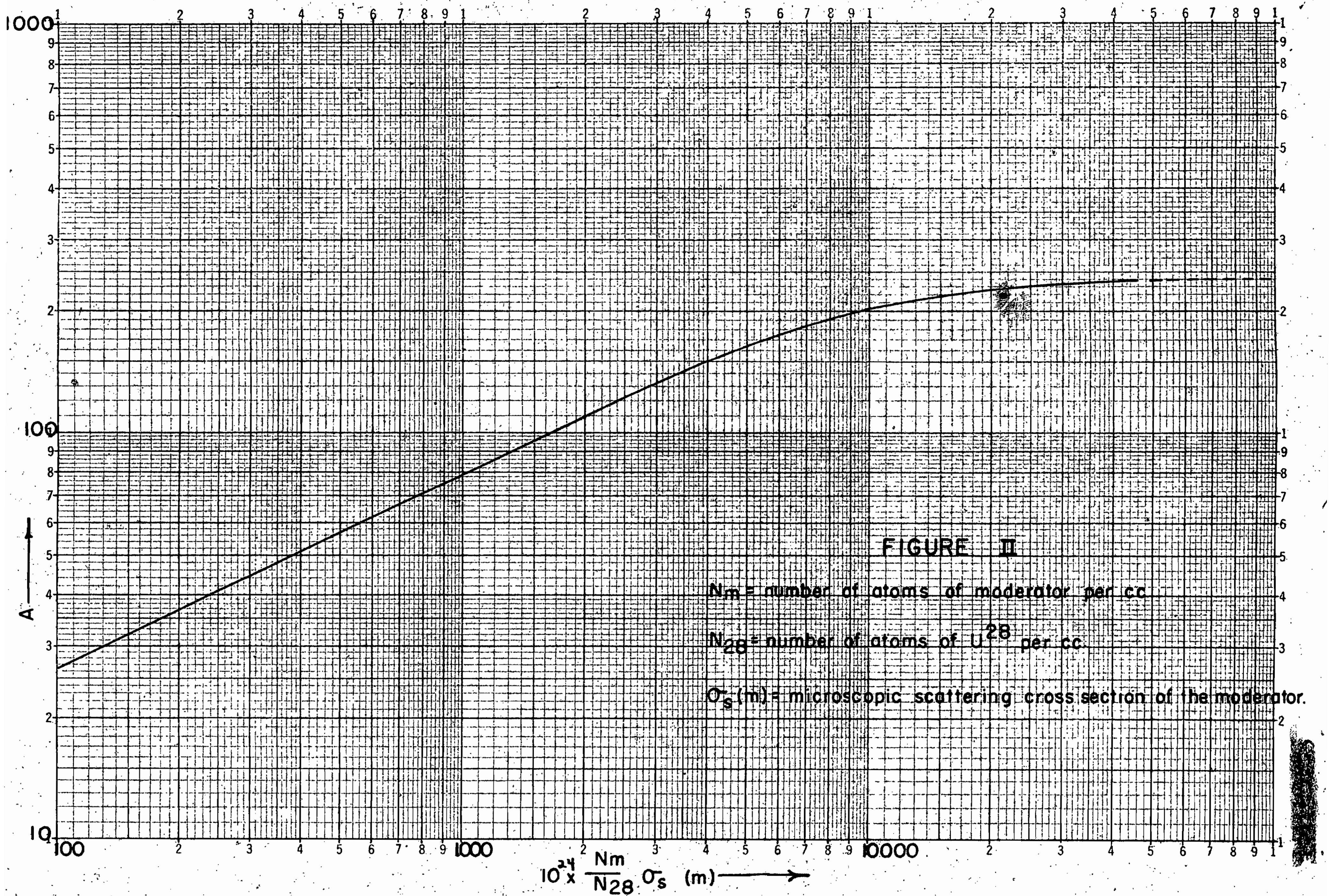

FIGURE II

N_m = number of atoms of moderator per cc.

N_{28} = number of atoms of U^{28} per cc.

$\sigma_s(m)$ = microscopic scattering cross section of the moderator.

[redacted]

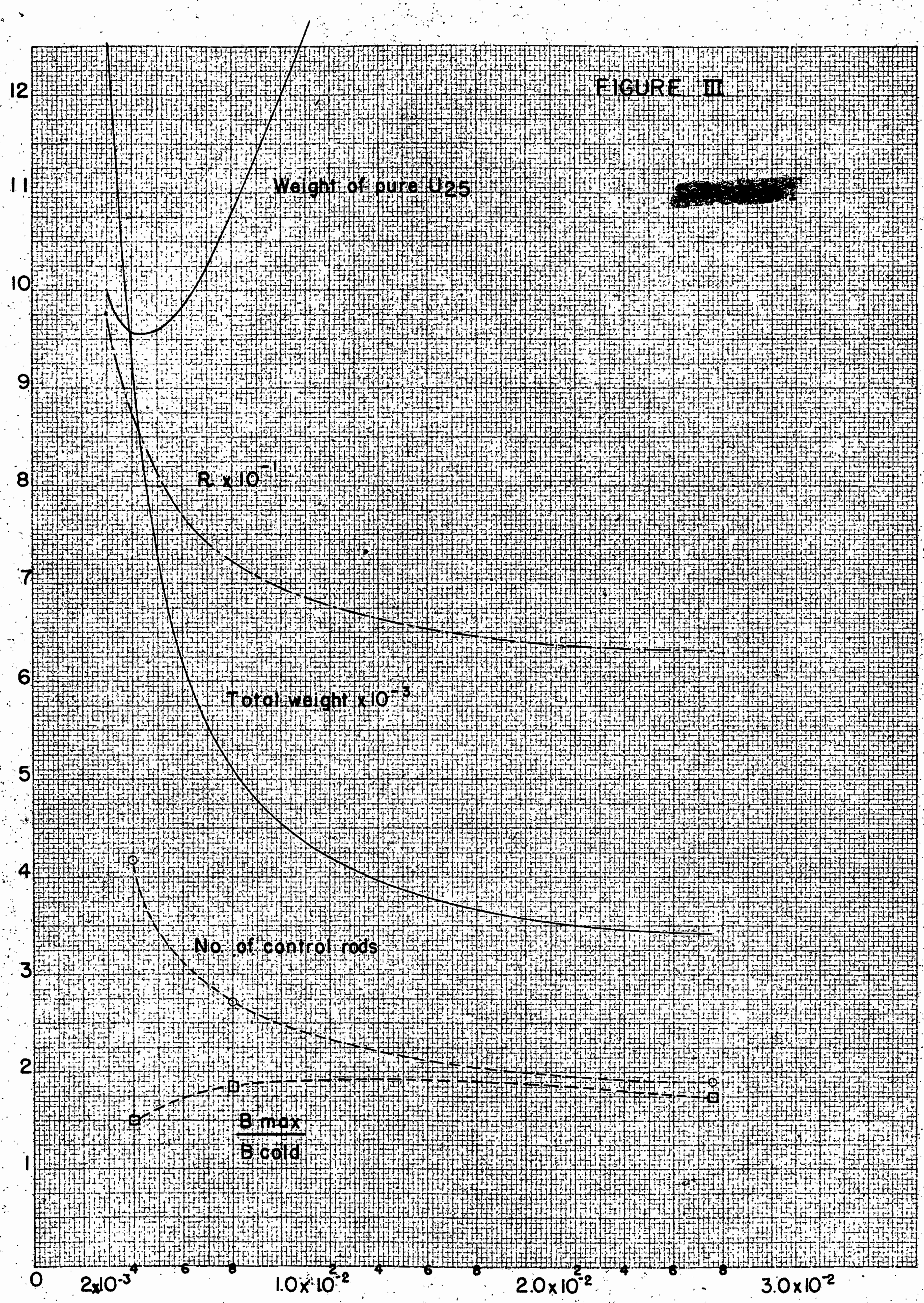
FIGURE III
Weight of pure U25
[redacted]
R x 10^-1
Total weight x 10^-3
No. of control rods
B max / B cold
12
11
10
9
8
7
6
5
4
3
2
1
0
2x10^-3
4
6
8
1.0x10^-2
2
4
6
8
2.0x10^-2
2
4
6
8
3.0x10^-2