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A. M. Weinberg, Section Chief

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VARIATIONAL METHOD FOR CALCULATING PILE PROPERTIES

By

H. C. Schweinler

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VARIATIONAL METHOD FOR CALCULATING PILE PROPERTIES

Continued operation of a pile at fluxes now contemplated will destroy an appreciable amount of fissionable material in a relatively short time. For example, 25 is destroyed at the rate of 2.2% per day in a slow-neutron flux of 4×10^{14} cm^{-2} sec^{-1} (the maximum flux anticipated in the enriched pile now being designed). After a week or two of operation under such conditions the local multiplication constant and the neutron distribution will have changed considerably; in fact, the changes are so large that first-order perturbation theory, the mathematical tool most frequently used for detailed pile calculations, is no longer applicable. Therefore a variational method has been applied to calculations of (i) the effective change in multiplication constant (δk_{eff}) and (ii) the importance of higher harmonics in the neutron density.

The pile equation with variable buckling

$$\nabla^2 \varphi + B \varphi = 0 \qquad (1)$$

can be written in the variational form

$$\delta \int \left[(\nabla \varphi)^2 - B \varphi^2 \right] dV = 0 \qquad (2)$$

provided that the buckling B does not depend upon φ and its derivatives. The function φ, defined more carefully later, is

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almost proportional to the slow-neutron density, and the variation δ is performed upon the coefficients a_i of the harmonics φ_i in the expansion of φ.

Upon the introduction of a change in multiplication constant δk_{eff} which is uniform throughout the pile and of such magnitude that the slow neutron density does not change appreciably during many neutron lifetimes, the two-group pile equation may be written

$$-\nabla^4 n_s + \nabla^2 \left\{ \left(\frac{1}{L^2_f} + \frac{1}{L^2_s} \right) n_s \right\} + \frac{k + \delta k_{eff} - 1}{L^2_f \, L^2_s} n_s = 0 \quad (3)$$

Here n_s, L^2_f, L_s, and k are the thermal neutron density, age, thermal diffusion length, and multiplication constant respectively. The last two quantities are related to their values L_{so} and k_o at time $t = 0$ by the equations

$$\frac{k}{k_o} = \frac{L^2_s}{L^2_{so}} e^{-\int_0^t n_s v \sigma_a^{25} dt'} \quad (4a)$$

and

$$\frac{L^2_{so}}{L^2_s} = 1 - .933 f_o \left\{ 1 - e^{-\int_0^t n_s v \sigma_a^{25} dt'} \right\} + \frac{N^{Xe} \sigma_a^{Xe} + N^{Sm} \sigma_a^{Sm}}{(N \sigma_a)_o} \quad (4b)$$

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where k_o, f_o and $(N\sigma_a)_o$ are the inital multiplication constant, thermal utilization, and macroscopic absorption cross-section per unit volume, respectively, and the terms with superscripts Xe and Sm refer to xenon and samarium poisoning.

The poisoning by fission products other than these two is assumed to be fifty barns per fission (private communication, L. W. Marshall to A. M. Weinberg). It is taken into account in the numerical coefficient .933. Finally, it should be noted that at the high neutron fluxes now contemplated the term $(N^{Xe}\sigma_a^{Xe} + N^{Sm}\sigma_a^{Sm}) / (N\sigma_a)_o$ in (4b) is nearly constant after steady-state operational conditions are reached; its value is approximately $f_o Y / (1 + q)$, in which Y is the sum of the fission yields of xenon and samarium (.058 + .013) and q is the "branching ratio" $\sigma_r^{25}/\sigma_f^{25}$ (0.18).

Following a suggestion of E. Greuling, we can replace $\nabla^4 n_s$ in (3) by $\Delta_1 \nabla^2 n_s$, in which Δ_1 is the eigenvalue of the fundamental neutron distribution for the Laplacian operator. This approximation leads to the equation

$$\nabla^2 \left\{ \left(\frac{1}{L^2_f} + \frac{1}{L^2_s} - \Delta_1\right) n_s \right\} + \frac{k + \delta k_{eff} - 1}{L^2_f + L^2_s - L^2_f L^2_s \Delta_1}$$

$$\left\{ \left(\frac{1}{L^2_f} + \frac{1}{L^2_s} - \Delta_1\right) n_s \right\} = 0 \qquad (5)$$

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The function φ and the buckling B in equation (1) above can now be defined more carefully by comparison with (5). Furthermore, before actually performing the variation indicated in (2) it is advisable to multiply by L^2_f (a constant) and to use the relationship

$$L^2_f B = \delta k_{eff} - 1 + \frac{\nu^{25} \varepsilon p}{.933} - \frac{\frac{\nu^{25} \varepsilon p}{.933} m^2 - L^2_f k_o \left(1 - \frac{Y}{.933}\right) + L^2_{so}(1 - L^2_f \Delta_1)(\delta k_{eff} - 1)}{m^2 + L^2_f \left(\frac{L^2_{so}}{L^2_s} - 1\right)} \tag{6}$$

in which ν^{25}, ε and p are the average number of neutrons per 25 fission, fast effect, and resonance escape probability respectively (the last two being very close to unity in this case), and $m^2 = L^2_f + L^2_{so} - L^2_f L^2_{so} \Delta_1$. The quantity $L^2_{so}/L^2_s - 1$ must be calculated from equation (4b) using a suitabily chosen average flux $n_s v$ in the exponential term for the variation is not to be performed upon the buckling itself. Equation (6) remains valid over a wider range of operating conditions if Y is replaced by $(1 - q)\,(N^{Xe} \sigma_a^{Xe} + N^{Sm} \sigma_a^{Sm}) / f_o\,(N \sigma_a)_o$.

To illustrate the method, calculations will be made for a spherical pile of radius R. Equation (2) then becomes

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$$\delta \int_0^R \left\{ L^2_f \, (\nabla\varphi)^2 - L^2_f \, B \, \varphi^2 \right\} 4\pi \, r^2 \, dr = 0 \qquad (2a)$$

with

$$\varphi = \sum_{j=1}^{n} a_j \, \frac{\sin j\pi \, r/R}{j\pi \, r/R} \qquad (7)$$

Equation (2a) can be transformed into

$$\delta \left\{ L^2_f \, \Delta_1 \sum a_j^2 \left(1 - \frac{1}{j^2}\right) + \sum\sum a_j \, a_k \, Q_{jk} \right\} = 0 \qquad (8)$$

in which $Q_{jk} = Q_{kj} = \frac{2}{\pi_{jk}} \int_0^{\pi} \sin j\,y \left[L^2_f \, B + L^2_f \, \Delta_1 \right] \sin ky \, dy$

The single variational equation (8) is equivalent to a set of homogeneous linear equations

$$L^2_f \, \Delta_1 \left(1 - \frac{1}{j^2}\right) a_j + \sum_{k=1}^{n} Q_{jk} \, a_k = 0, \; j = 1, \ldots, n \qquad (9)$$

The solution in first approximation is

$$a_j / a_1 = Q_{j1} \, / \, L^2_f \, \Delta_1 \left(1 - \frac{1}{j^2}\right) \quad \text{and}$$

$$Q_{11} = Q_{11} \, (\delta k_{eff}, T) = 0.$$

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Numerical results for the case $f_o = 0.65$, central flux 4×10^{14} cm^{-2} sec^{-1}, are

$$a_2/a_1 = -0.014\ T,\ a_3/a_1 \cong +0.0005\ T;$$

$$\delta k_{eff}/k = .0092\ T + 6.3 \times 10^{-5}\ T^2$$

where T is the time of operation in days.

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