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MoNP-424

U. S. ATOMIC ENERGY COMMISSION

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	TO	INITIALS	DATE
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2	Lexington, Mass. Attn: Prof. W. Whitman		
3			
4			Inv. 76
5	AEC RESEARCH AND DEVELOPMENT REPORT		
6			
7			
8	SPECIAL REREVIEW FINAL DETERMINATION		
9	CLASS. U AUTH.		
10	REVIEWERS/VERIFIERS CLASS. DATE (1) P.S. Baker U 9-11-80		
11	(2) A.T. Hrasky U 8-21-80		
12			
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PLUTONIUM THERMAL PILES

R. C. Mason

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CLASSIFICATION CHANGED TO: DECLASSIFIED
BY AUTHORITY OF: TID-14002
BY: DRS/W. Brown 11/15/77

Date Rec'd.: 12/17/47 Date Issued: 1/2/47
Date Rec'd Figures: 12/31/47

[redacted]

[redacted]

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PLUTONIUM THERMAL PILES

R. C. Mason

Summary

Through the fission of Pu^{241}, the higher isotopes of plutonium increase the breeding ratio in a pile which uses Pu^{239} as the major fuel. In this report, curves show the build up of the isotopes and other important quantities in a model pile. Probable values of breeding ratio in a graphite moderated pile of the Hanford type are worked out, and the factors affecting them discussed.

A pile starting up on natural uranium will run some time on the plutonium produced, without addition of new fissionable material; when new fuel is required, uranium with a low content of U^{235} may be used. Though definite figures involve η_{41}, which is at present unknown, there is hope of such piles showing a breeding ratio above .85.

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INDEX

		Page
I.	Introduction	1
II.	Simple Model Pile	1
	A. Cross Sections	1
	B. Isotope Build Up	5
	C. Denaturing	7
	D. Radioactivity	8
	E. Neutron Production	9
	F. Excess Neutron Production	12
	G. Breeding Ratio	14
	H. Effect of Additional Neutron Losses	17
	1. Fission Product	17
	2. Control Absorbers	19
	3. Chemical Processing	20
	4. Temperature Effect on α	24
	I. Effect of Additional Neutron Gains	24
	1. Breeding Blanket	24
	2. Fast Fission	24
	J. Definition of g	25
	K. Probable g_∞	28
	L. Comparison of Plutonium with Other Fuels	30
	M. Operating Questions	31
	1. Initial Decrease in Σ	31
	2. Power Production	32
	3. Presence of U^{235}	33
III.	Pile of Hanford Type, with U^{235}	33
	A. Excess Neutron Production	34

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	Page
B. Critical Concentration of U^{235} and Breeding Ratio	34
C. Withdrawal of Plutonium	36
D. Condition for $g_{\infty}=1$	38
E. Effect of Losses	41
IV. Conclusions	42

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FOREWORD

This report, together with Mon P-287 by Mason and Mon C-374 by Ott, presents the work done by them while at the Clinton Training School on piles which begin operation on natural U. A more complete bibliography on the subject is given in a recent note bearing Central File No. 47-12-71.

The considerations on Pu isotopes are relevant for Pu thermal piles generally, and one can consider Pu piles with no U in their cores just as Ott in CL-RWS-31 and CL-RWS-41 considered U units with no Th in the cores. While this possibility is implicit in the present references, the authors here have mostly considered examples having fertile material in the cores.

Mr. Mason was called back to Westinghouse sooner than had been expected, which interrupted revision of the manuscript for the present report. Thus it is perhaps not in the final form he would have given it had there been more time available.

Gale Young

PLUTONIUM THERMAL PILES

I. Introduction

The probability of Pu^{241} possessing a large fission cross section, brought out in MonP-287, has raised a new interest in plutonium thermal piles. Such piles would be fed natural or perhaps uranium depleted in U^{235}, but they would run primarily on Pu^{239} and its higher isotopes. H. C. Ott, in MonC-374, has considered a heavy water moderated uranium-plutonium pile, while this report will investigate the general possibilities of graphite moderated piles of the Hanford type.

II. Simple Model Pile

As a first step, in order to fix ideas, suppose a very simple model be considered: one in which only Pu^{239} and U^{238} exist originally, and to which pure U^{238} is added in proper amount to maintain the pile critical as the isotopes of plutonium build up. Before the isotopic constitution as a function of time can be determined, some choice of cross sections must be made.

A. Cross Sections.

As the plutonium isotopes grow, the rates of neutron production and absorption change, so that the energy distribution of pile neutrons will vary with time. Consequently, because of resonance absorption, the effective cross sections, at least Pu^{239} and U^{238}, and possibly of the other isotopes as well, will not be constant.

Let

$\mathcal{N}$ = number of neutrons produced per unit volume, per second, per unit thermal flux

φ = thermal flux

$p(E)$ = probability of fission neutron reaching energy E

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For a homogeneous pile

$$(1)\quad p(E) = \mathcal{E}^{-\int_E^{E_f}} \frac{\sum_j N_j \sigma_{aj}}{\sum_j N_j(\sigma_{sj} + \sigma_{aj})} \frac{1}{\bar{\xi}} \frac{dE}{E}$$

with

$$\bar{\xi} = \frac{\sum_j N_j \sigma_{sj} \xi_j}{\sum_j N_j \sigma_{sj}}$$

We suppose the criticality of the pile is determined by a characteristic quation of the Fermi type.

$$(2)\quad -\lambda B^2 \varphi - \sum_j N_j \sigma_{ajth} \varphi + \mathcal{N} p(E_{th}) \mathcal{E}^{-B^2 \tau(E_{th})} \varphi = 0$$

We may define an average resonance absorption cross section, σ_{ari}, for the ith species of atom by the following:

$$(3)\quad N_i \bar{\sigma}_{ari} \varphi = \mathcal{N} \varphi \int_{E_{th}}^{E_f} \mathcal{E}^{-B^2 \tau(E)} \frac{\partial p(E)}{\partial E} dE \frac{N_i \sigma_{ai}}{\sum_j N_j \sigma_{aj}}$$

Where the limits of integration are the thermal and fission energies.

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From (1)

$$(4)\quad \frac{\partial P}{\partial E} = P(E)\,\frac{\sum_j N_j\,\sigma_{aj}}{\sum_j N_j(\sigma_{sj}+\sigma_{aj})}\,\frac{1}{\xi}\,\frac{1}{E}$$

so eq. (3) transforms to

$$(5)\quad \overline{\sigma_{ari}} = \mathcal{N}\int_{E_{th}}^{E_f} \frac{P(E)\cdot \epsilon^{-B^2\tau(E)}}{\xi\sum_j N_j(\sigma_{sj}+\sigma_{aj})}\,\sigma_{ai}\,\frac{dE}{E}$$

In the same way, an average resonance fission cross section $\bar{\sigma}_{fri}$ can be defined by merely replacing σ_{ai} in the above integral by σ_{fi}.

For simplicity, we denote the integral in eq. (5) by I_{ai}, so

$$(6)\quad \begin{aligned} \bar{\sigma}_{ari} &= \mathcal{N} I_{ai} \\ \bar{\sigma}_{fri} &= \mathcal{N} I_{fi} \end{aligned}$$

If the thermal cross sections are indicated by the subscript th, then the <u>total</u> cross section

$$(7)\quad \begin{aligned} \bar{\sigma}_{ai} &= \bar{\sigma}_{athi} + \bar{\sigma}_{ari} \\ \bar{\sigma}_{fi} &= \bar{\sigma}_{fthi} + \mathcal{N} I_{fi} \end{aligned}$$

athi I_{ai}

In order to examine a little more closely the average resonance cross section, suppose these simplifying assumptions are made:

[redacted]

[redacted]

1. $\bar{\xi} \sum_j N_j(\sigma_{sj}+\sigma_{aj}) \sim \xi_M N_M \sigma_{SM}$, the subscript M denoting the properties of the moderation alone.

2. Resonance levels of all Pu isotopes are below those of U^{238}.

3. All resonance levels are low enough that $\epsilon^{-B^2 \tau(E)}$ may be taken as constant in eq (5), with the value $\epsilon^{-B^2 \tau(E_{th})}$.

4. The dilution of the plutonium is large enough so that the p for Pu alone approaches one.

Then, if P_{28} is the value of $P(E_{th})$ for U^{238} alone, eq (5) reduces to

$$(8) \quad \bar{\sigma}_{ar1} = \mathcal{N}\epsilon^{-B^2 (E_{th})} P_{28} \cdot \frac{1}{\xi_M N_M \sigma_{SM}} \int_{E_{th}}^{E_f} \sigma_a \frac{dE}{E}$$

for other than 28 atoms; while for the latter

$$(9) \quad \bar{\sigma}_{ar\ 28} = \mathcal{N}\epsilon^{-B^2 \tau(E_{th})} (1-P_{28})$$

In MonP-287, pp. 22 ff.,

$$\frac{\mathcal{N}}{\xi_M N_M \sigma_{SM}} \int \sigma_a \frac{dE}{E} \text{ for 49 was}$$

evaluated, using the $\mathcal{N}$ appropriate for the Hanford pile. The coefficient

[redacted]

[redacted]

$P_{28}\varepsilon^{-B^2\tau}$, which amounts to about .86 in the Hanford pile, was neglected. Since in that particular case, the Pu sample being irradiated was pure, unmixed with 28, there was justification for the omission. For a homogeneous mixture, however, the coefficient should be included. For a heterogeneous assembly, a space as well as an energy average must be used. Thus, the $\bar{\sigma}_{a49}$ depends on the geometrical distribution and the concentration of 28, as well as on $\mathcal{N}$. Likewise for $\bar{\sigma}_{a40}$, $\bar{\sigma}_{a41}$, except that the experiments of Mon P-287 gave the ratio of $\bar{\sigma}_{a40}$ and $\bar{\sigma}_{t41}$ to $\bar{\sigma}_{c49}$; so for other arrangements one may expect these ratios to be more accurate than the separate cross sections.

Since only orienting calculations are intended here, it will be assumed that the cross sections of the higher isotopes of Pu are those of Mon P-287, and that their ratio to σ_{a49} is also the same as in Mon P-287.

B. <u>Isotope Build Up.</u>

We shall suppose the concentration of Pu^{239} to be varied in such a way as $\mathcal{N}$ and P change as to maintain always

(10) $$N_{49}\ \bar{\sigma}_{a49} = \text{constant}$$

This requirement keeps $N_{49}\ \bar{\sigma}_{c49}$ also substantially constant, since by MonP-287 a 50% change in $\mathcal{N}$ would only change $\bar{\sigma}_{c49}$ by $1\frac{1}{2}\%$; so further assume,

(11) $$N_{49}\ \bar{\sigma}_{c49} = \text{constant}.$$

The cross sections and decay constants which will be used are,

[Ref: MonP-287; CL-MS-20]

[redacted]

[redacted]

$\bar{\sigma}_{c49} = 515$ b

$\bar{\sigma}_{f49} = 1005$

$\bar{\sigma}_{a49} = 1520$

$\lambda_{49} = 8.95 \times 10^{-13}$

$\sigma_{c40} = \sigma_{a40} = 2000$

$\lambda_{40} = 3.6 \times 10^{-12}$

$\sigma_{f41} = 25{,}000$

$\sigma_{c41} = 500$

$\lambda_{41} = 1.56 \times 10^{-9}$

$\left.\begin{array}{l} \sigma_{c42} = 600 \\ \sigma_{a42} = 600 \end{array}\right\}$ Purely arbitrary; chain stopped and end product neglected.

$\sigma_{c51} = 550$

$\lambda_{51} = 4.35 \times 10^{-11}$

$\lambda_{52} = 1.13 \times 10^{-5}$

$\lambda_{62} = 5.3 \times 10^{-8}$

From these the equilibrium concentrations which build up may be readily calculated as:

$\frac{N_{40}}{N_{49}} = .258$

$\frac{N_{41}}{N_{49}} = .0198$

$\frac{N_{42}}{N_{49}} = .0165$

[redacted]

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$$\frac{N_{51}}{N_{49}} = .00056 \text{ for } \varphi = 10^{14}$$

$$= .0056 \text{ for } \varphi = 10^{13}$$

$$\frac{N_{51}}{N_{49}} = 2.74 \times 10^{-6}$$

$$\frac{N_{62}}{N_{49}} = .00058$$

The concentrations as functions of time are shown in Fig. 1, where per cent of equilibrium concentration is plotted against time* in 10^{14} years. It is seen that Pu^{240}, Pu^{241} would reach substantial equilibrium in about half a year at a flux of 10^{14}, or in five years at a flux of 10^{13}. The higher isotopes would take about 5 times as long.

If the higher isotopes have a large resonance absorption, then the time scale of Fig. 1 would be considerably in error; as already mentioned, the equilibrium concentration ratios are less likely to be far wrong.

C. <u>Denaturing</u>.

One of the features of a plutonium pile which offers some attraction is the "denaturing" effect of Pu^{240}. Presumably, any international control scheme might require less stringent inspection and control safeguards for piles in which the fissionable material is militarily ineffective.

---

* actually, flux-time.

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In Fig. 2, the ratio of concentration of Pu^{240} to Pu^{239} is plotted against time, with the assumptions as to cross sections of the previous section. The ratio at which the material is effectively denatured is not known to us. Hanford product has about $1\frac{1}{2}\%$ Pu^{240}. If one guesses a pile starting on pure Pu^{239} might be "safe" when the relative concentration of Pu^{240} reaches 10%, then Fig. 2 gives the time for denaturing on this basis as .075 years at a flux of 10^{14}, or about 20 weeks at a flux of 2×10^{13}.

D. Radioactivity.

In the elements which build up from Pu^{239}, either by neutron capture or radioactive decay, there are four α-emitters (Pu^{239}, Pu^{240}, Am^{241}, Cm^{242}) and two β emitters (Pu^{241}, Am^{242}) which need to be considered in determining the activity of the pile material. Pu^{242} is neglected solely because its properties are unknown; and the activities of the elements which result from the decay of the above are negligible.

If all the above products are left in the pile until equilibrium concentrations are established, then examination of the data of the previous section shows the short life of Cm^{242} makes it the only important α emitter, despite its low concentration. The β emitters are of equal importance, since they occur in sequence, and for the same reason, the activity of each is equal the α activity of Cm^{242}. In equilibrium, then, calculation gives the following activities:

α activity $\sim$ 500 curies/mole 49

β activity $\sim$ 1000 curies/mole 49

The above is an upper limit to the activity of the pile material. A

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lower limit may be found by supposing chemical processing removes all elements except plutonium, so their activities never become appreciable. Then the α activity of Pu^{239} and Pu^{240} will be of the same magnitude in equilibrium, and β emission will arise only from Pu^{241}. Calculation then gives:

α activity $\sim$ 30/curies mole 49

β activity $\sim$ 500/curies mole 49

Since U^{238} is presumed present, the β emission of U^{239} and Np^{239} would also have to be considered. If the breeding ratio of the pile approaches 1, then in equilibrium the activities of these two would be several hundred times that of the Pu above. Whether or not the equilibrium activity would be allowed to obtain would depend on the construction and processing schedule of the pile. In any event, the disintegration rates of U^{239} and Np^{239} are so large, a moderate "cooling period" could reduce their activity to much less than that of the plutonium. (The same remarks apply to the U^{237} which might be formed from any U^{235} present in an actual pile.) In contrast, the half life of Pu^{241} is 14 years or greater (references previously cited), so no practical cooling period is going to have much effect on the activity of the plutonium when equilibrium attains.

E. Neutron Production.

The rate of production of neutrons, $\mathcal{N}$, in the model pile will be a function of time through the growth of the fissionable isotope Pu^{241}:

(12) $$\mathcal{N} = N_{49}\,\bar{\sigma}_{a49}\,\bar{\eta}_{49} + \left(\frac{1+\alpha_{41}}{1+\alpha_{41}+\alpha'_{41}}\right)\eta_{41}\,\bar{\sigma}_{f41}\,N_{41}$$

[redacted]

where $\alpha = \frac{\sigma_c}{\sigma_f}$, $\alpha' = \frac{\lambda/\varphi}{\sigma_f}$

and $\sigma_t = \sigma_a + \frac{\lambda}{\varphi}$.

We have supposed that N_{49} will be adjusted as $\mathcal{N}$ and $\bar{\sigma}_{a49}$ varies, in such a way that $N_{49}\ \bar{\sigma}_{a49}$ = constant. Dividing eq (12) by this constant

$$(13) \quad \frac{\mathcal{N}}{N_{49}\ \bar{\sigma}_{a49}} = \bar{\eta}_{49} + \frac{1+\alpha_{41}}{1+\alpha_{41}+\alpha'_{41}}\ \eta_{41}\ \frac{N_{41}\ \bar{\sigma}_{t41}}{N_{49}\ \bar{\sigma}_{a49}} .$$

Since both α_{41}, α'_{41} are certainly small

$$\frac{1+\alpha_{41}}{1+\alpha_{41}+\alpha'_{41}} \rightarrow 1 - \alpha'_{41}$$

$$\rightarrow 1$$

As η_{41} itself is unknown, the value used for it in future will be taken as actually including the $1 - \alpha'_{41}$. $\bar{\eta}_{49}$, since it is the average for thermal and resonance fissions, will be a function of $\mathcal{N}$. Substitution of cross sections previously used show however, that for $\mathcal{N}$ changing by 50% from the $\mathcal{N}$ applying in the origin of those sections, $\bar{\eta}_{49}$ would vary by less than 1%.

If ${}_\infty N_{41}$ denotes the steady state value of N_{41}, then

$${}_\infty N_{41}\ \sigma_{t41} = N_{49}\ \bar{\sigma}_{c49}$$

since the radioactive decay rate of the intermediate member Pu^{240} is negligible compared to its neutron capture rate. Thus eq. (13) becomes

$$(14) \quad \frac{\mathcal{N}}{N_{49}\ \bar{\sigma}_{a49}} = \bar{\eta}_{49} + \eta_{41}\ \frac{\bar{\sigma}_{c49}}{\bar{\sigma}_{a49}}\ \frac{N_{41}}{{}_\infty N_{41}} .$$

Just as in the case of $\bar{\eta}_{49}$, $\frac{\bar{\sigma}_{c49}}{\bar{\sigma}_{a49}}$ will be a function of $\mathcal{N}$;

[redacted]

[redacted]

also as for $\bar{\eta}_{49}$, $\frac{\bar{\sigma}_{a49}}{\bar{\sigma}_{a49}}$ will change only slightly for a large variation in $\mathcal{N}$, so the ratio may be taken as constant at

$$\frac{\bar{\sigma}_{a49}}{\bar{\sigma}_{a49}} = \frac{\alpha_{49}}{1+\alpha_{49}}$$

with α_{49} the value corresponding to the σ's of section B. Thus

$$(15) \quad \frac{\mathcal{N}}{N_{49}\,\bar{\sigma}_{a49}} = \bar{\eta}_{49} + \frac{\alpha_{49}}{1+\alpha_{49}}\ \eta_{41}\ \frac{N_{41}}{\infty N_{41}}$$

If the value of $\mathcal{N}$ at time zero, when $N_{41} = 0$, is called $\mathcal{N}_0$, then

$$\frac{\mathcal{N}_0}{N_{49}\,\bar{\sigma}_{a49}} = \bar{\eta}_{49}$$

and

$$(16) \quad \frac{\mathcal{N}}{\mathcal{N}_0} = 1 + \frac{\alpha_{49}}{1+\alpha_{49}}\ \frac{\eta_{41}}{\bar{\eta}_{49}}\ \frac{N_{41}}{\infty N_{41}}\ .$$

The time variation of $\mathcal{N}$ follows that of $\frac{N_{41}}{\infty N_{41}}$ since $\frac{\mathcal{N}-\mathcal{N}_0}{\mathcal{N}_0}$ is directly proportional to $\frac{N_{41}}{\infty N_{41}}$. The latter was computed in Fig. 1, and is repeated in Fig. 3. Again, the calculation used constant cross sections, and so may be considerably in error in time scale (less than a factor of two though).

When equilibrium attains, and $\frac{N_{41}}{\infty N_{41}} = 1$, very closely

$$(17) \quad \frac{\mathcal{N}}{\mathcal{N}_0} = 1 + \frac{\alpha_{49}}{1+\alpha_{49}}\ \frac{\eta_{41}}{\bar{\eta}_{49}}\ .$$

[redacted]

[redacted]

Putting in $\alpha_{49} = .512$, $\nu_{49} = 2.95$, $\eta_{49} = 1.95$. (Ref. CLM-KW-1, Maxwell averages, also Section A: L. W. Marshall in Memo 7-2733 uses $\eta_{49} = 1.85$, $\alpha_{49} = .59$; it is not known that these are any more reliable than the above.)

$$\frac{\mathcal{N}}{\mathcal{N}_0} = 1.434 \text{ for } \eta_{41} = 2.5$$
$$= 1.52 \text{ for } \eta_{41} = 3.0$$

F. <u>Excess Neutrons Production</u>

Another quantity which is valuable in following the time behavior of the model pile is the excess of neutrons produced over the absorption in Pu isotopes. We denote this by $\mathcal{E}$, in

(18) $$\mathcal{E} = \mathcal{N} - \sum_{Pu} N \bar{\sigma}_{aPu}$$

$\mathcal{E}$ represents the <u>maximum</u> number of neutrons which might be absorbed in U^{238}; and the change of $\mathcal{E}$ with time will give the change in maximum conversion of U^{238}. The actual change in conversion will not attain this latter figure, however, because of fission product absorption, alteration in control absorbers, and change in fast neutron leakage, which will be proportional to $\mathcal{N}$.

$$\mathcal{E} = \mathcal{N} - N_{49}\,\bar{\sigma}_{a49} - N_{40}\,\bar{\sigma}_{a40} - N_{41}\,\bar{\sigma}_{a41}$$
$$- \text{absorption in higher isotopes}$$

The last term will be neglected, since the concentration of higher isotopes is so small their contribution will not be over 1% of $\mathcal{E}$. Then

(19) $$\frac{\mathcal{E}}{N_{49}\,\bar{\sigma}_{a49}} = \frac{\mathcal{N}}{N_{49}\,\bar{\sigma}_{a49}} - 1 - \frac{N_{40}}{N_{49}}\,\frac{\bar{\sigma}_{a40}}{\bar{\sigma}_{a49}} - \frac{N_{41}}{N_{49}}\,\frac{\bar{\sigma}_{a41}}{\bar{\sigma}_{a49}}$$

[redacted]

[redacted]

Since

$$\infty N_{40} \bar{\sigma}_{a40} = N_{49} \bar{\sigma}_{c49}$$

substituting from previous equations (19) becomes

$$(20) \qquad \frac{\mathcal{E}}{N_{49} \bar{\sigma}_{a49}} = \eta_{49} - 1 + (\eta_{41} - 1) \frac{\bar{\sigma}_{c49}}{\bar{\sigma}_{a49}} \frac{N_{41}}{\infty N_{41}} - \frac{\bar{\sigma}_{c49}}{\bar{\sigma}_{a49}} \frac{N_{40}}{\infty N_{40}}$$

With the approximations already considered,

$$(21) \qquad \frac{\mathcal{E}}{N_{49} \bar{\sigma}_{a49}} = (\eta_{49} - 1) + \frac{\alpha_{49}}{1 + \alpha_{49}} \left[(\eta_{41} - 1) \frac{N_{41}}{\infty N_{41}} - \frac{N_{40}}{\infty N_{40}} \right]$$

The quantity in the bracket in eq. (21) we call β. β varies from zero when the pile starts on pure Pu^{239}, to $\beta_\infty = \eta_{41} - 2$ at equilibrium. Thus $\mathcal{E} - \mathcal{E}_o$ is directly proportional to β. From the $\frac{N_{41}}{\infty N_{41}}$ and $\frac{N_{40}}{\infty N_{40}}$ already found, $\frac{\beta}{\beta_\infty}$ has been plotted in Fig. 3 for two values of η_{41}. In Fig. 4, β for three values of η_{41} is plotted on an extended time scale.

It will be noted that β first is negative, reaching a minimum and finally becoming positive again as time goes on. The negative dip in β is caused by the Pu^{240} growing in first and absorbing neutrons, before the Pu^{241} builds up from the Pu^{240}; later the gain of neutrons from the fission of Pu^{241} overcomes the absorption in both isotopes. The duration

[redacted]

[redacted]

of negative β is rather short, being between a few days and a few weeks, depending on η_{41} and φ. After this dip, $\frac{\beta}{\beta_\infty}$ follows closely the curve of $\frac{N_{41}}{{}_\infty N_{41}}$

Denoting by subscripts o and ∞ the values of $\mathcal{E}$ at startup and after attainment of equilibrium

$$(22) \qquad \frac{\mathcal{E}_\infty - \mathcal{E}_o}{\mathcal{E}_o} = \frac{\alpha_{49}}{1+\alpha_{49}} \; \frac{\eta_{41} - 2}{\eta_{49} - 1}$$

$$\frac{\mathcal{E}_o}{N_{49}\,\sigma_{a49}} = .95$$

$$\frac{\mathcal{E}_\infty}{N_{49}\,\sigma_{a49}} = 1.119 \quad \text{for } \eta_{41} = 2.5$$

$$= 1.288 \quad \text{for } \eta_{41} = 3$$

and

$$\frac{\mathcal{E} - \mathcal{E}_o}{\mathcal{E}_o} = .178 \quad \text{for } \eta_{41} = 2.5$$

$$= .356 \quad \text{for } \eta_{41} = 3$$

G. <u>Breeding Ratio</u>

If we define (see further discussion of the definition in a later section) the breeding ratio, g, as the number of fissionable atoms created per second per fissionable atom destroyed, then in this hypothetical pile which contains only pure U^{238} (i.e. no U^{235}),

[redacted]

[redacted]

$$(23) \quad g = \frac{N_{28}\,\bar{\sigma}_{c28} + N_{40}\,\bar{\sigma}_{c40}}{N_{49}\,\bar{\sigma}_{a49} + N_{41}\,\bar{\sigma}_{t41}}$$

or

$$(24) \quad g = \frac{\dfrac{N_{28}\,\bar{\sigma}_{c28}}{N_{49}\,\bar{\sigma}_{a49}} \cdot \dfrac{N_{40}\,\bar{\sigma}_{c40}}{N_{49}\,\bar{\sigma}_{a49}}}{1 + \dfrac{N_{41}\,\bar{\sigma}_{t41}}{N_{49}\,\bar{\sigma}_{a49}}}$$

Now we are presuming N_{28} will be varied in such a way as to maintain the pile critical always. We can get an upper limit to $N_{28}\,\bar{\sigma}_{c28}$, and to g, by supposing the <u>change</u> in $N_{28}\,\bar{\sigma}_{a28}$ as the higher Pu isotopes build up in the pile is just the <u>change</u> in $\mathcal{E}$. That is, we neglect for the present the fission product absorbers, and any alteration in control absorbers or fast leakage, as mentioned in the previous section. Further, rather than fix numerical values, we introduce g_0 for $\dfrac{N_{28}\,\bar{\sigma}_{c28}}{N_{49}\,\bar{\sigma}_{a49}}$ at start up when $N_{40} = N_{41} = 0$. Then, at any other time,

$$(25) \quad \frac{N_{28}\,\bar{\sigma}_{c28}}{N_{49}\,\bar{\sigma}_{a49}} = g_0 + \frac{\mathcal{E} - \mathcal{E}_0}{N_{49}\,\bar{\sigma}_{a49}}$$

Substituting from previous equation

$$(26) \quad g = \frac{g_0 + (\eta_{41} - 1)\dfrac{N_{41}\,\bar{\sigma}_{t41}}{N_{49}\,\bar{\sigma}_{a49}} - \dfrac{N_{40}\,\bar{\sigma}_{a40}}{N_{49}\,\bar{\sigma}_{a49}} + \dfrac{N_{40}\,\bar{\sigma}_{c40}}{N_{49}\,\bar{\sigma}_{a49}}}{1 + \dfrac{N_{41}\,\bar{\sigma}_{t41}}{N_{49}\,\bar{\sigma}_{a49}}}$$

Since $\sigma_{c40} = \sigma_{a40}$, eq. (26) becomes

$$(27) \quad g = \frac{g_0 + \dfrac{\alpha_{49}}{1+\alpha_{49}}\,(\eta_{41} - 1)\,\dfrac{N_{41}}{\infty\,N_{41}}}{1 + \dfrac{\alpha_{49}}{1+\alpha_{49}}\,\dfrac{N_{41}}{\infty\,N_{41}}}$$

[redacted]

[redacted]

with

$$(28) \qquad g_\infty = \frac{g_0 + \frac{\alpha_{49}}{1+\alpha_{49}} (\eta_{41} - 1)}{1 + \frac{\alpha_{49}}{1+\alpha_{49}}}$$

The time variation of g is easiest shown through

$$(29) \qquad \frac{g - g_0}{g_\infty - g_0} = \frac{\left(1 + \frac{\alpha_{49}}{1+\alpha_{49}}\right) \frac{N_{41}}{{}_\infty N_{41}}}{1 + \frac{\alpha_{49}}{1+\alpha_{49}} \frac{N_{41}}{{}_\infty N_{41}}}$$

This quantity, calculated from the $\frac{N_{41}}{{}_\infty N_{41}}$ already found, is plotted as the dotted line in Fig. 3.

In order to afford a quick estimate of the maximum g which could be expected in a pile of this type, g_∞ from eq (28) has been plotted as a function of g_0 for different η_{41} in Fig. 5.

It will be noted that ε gives the excess neutrons, which aside from other losses might be captured in the breeding material, U^{238}. The presence of Pu^{240} thus <u>reduces</u> ε, but <u>increases</u> the numerator of g, since capture of a neutron by Pu^{240} leads directly to a fissionable atom. Because of these opposing effects, the terms in N_{40} in eq (26) cancel, and leave only terms in N_{41} in g.

We are now interested in what g_0 may be. For a Hanford pile on start up, g_0 is about .82. (Ref: Private Communication, K. Way; also from

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data in Memo 7-2733, L. W. Marshall) Suppose the U^{235} in that pile were replaced by Pu^{239}, and the concentrations of Pu^{239}, U^{238} were adjusted so the pile would run under the new conditions identically as under the old. Then with the Pu, g_0 would be changed just by $\bar{\eta}_{49} - \eta_{25}$. Since $\bar{\eta}_{25} = 2.12$ (CLM-KW-1) while $\bar{\eta}_{49} = 1.95$; the new g_0 would be approximately $.82 - .17 = .65$. By Fig. 5, when equilibrium isotope concentrations had built up, g_∞ would be .99 for $\eta_{41} = 3$, or .865 for $\eta_{41} = 2.5$, or so on.

Whether or not g_0 could be increased we leave for later discussion, and proceed now to consider some factors which will certainly decrease g_∞ below the maximum heretofore depicted.

H. Effect of Additional Neutron Losses

1. Fission Products

Because of neutron absorbing fission products, the amount of U^{238} which can be put in the pile will be reduced. If neutron capture is the only important cause of decay, then in steady state

$$N_p \sigma_{ap} = y_p (N_{49} \bar{\sigma}_{f49} + N_{41} \bar{\sigma}_{f41})$$

where N_p is the concentration of product p, σ_{ap} is cross section, and y_p the fission yield, assumed the same for Pu239 and Pu241.

Then
$$\frac{\sum_p N_p \sigma_{ap}}{N_{49} \bar{\sigma}_{a49}} = y\left(\frac{\bar{\sigma}_{f49}}{\bar{\sigma}_{a49}} + \frac{\bar{\sigma}_{f41}}{\bar{\sigma}_{a49}} \frac{N_{41}}{N_{49}}\right)$$
$$\sim y\left(\frac{1}{1+\alpha_{49}} + \frac{\alpha_{49}}{1+\alpha_{49}}\right)$$
$$= y$$

[redacted]

[redacted]

where y is the sum of the yields of all such fission products.

In calculating the effect on g_∞ of the fission product absorbers, $\frac{\sum_p N_p \bar{\sigma}_{ap}}{N_{49}\bar{\sigma}_{a49}}$ must be subtracted from $\frac{N_{28}\bar{\sigma}_{a28}}{N_{49}\bar{\sigma}_{a49}}$; by eqs (25) and (28) this is exactly equivalent to reducing by y the g_0 figured for a pile with neglect of fission products.

Undoubtedly in the type of pile here considered, Xe^{135} would be present in equilibrium amount; we shall assume Sm^{139} will also. With a yield of 5.5% for the former and 1.4% for the latter (CC-3420) y would be .069, or say .07. Thus in the example considered in the previous section, g_0 would now be reduced to .65 - .07 = .58, from Fig. 5, g_∞ becomes .94 for $\eta_{41} = 3$, or .81 for $\eta_{41} = 2.5$.

Reference to eqs (27), (28) (30) shows that if <u>only</u> Pu^{239} were present

$$\delta g_\infty = -\frac{y}{1+\alpha_{49}}$$

while if the higher isotope is in equilibrium

$$\delta g_\infty = \frac{-y}{1+\frac{\alpha_{49}}{1+\alpha_{49}}}$$

Substitution of numerical values shows the reduction of g_∞ because of fission product build up is 13% greater with Pu^{241} in equilibrium than if Pu^{239} alone is present. The increased effect on g_∞ occurs because of the increase in effective fissions per fissionable atom destroyed, α_{41} being very small compared to α_{49}.

If the fission products are limited by processing to a definite concentration, then

$$\delta g_\infty(\text{with } 41+49) = \delta g_\infty(\text{with 49 alone}) \times \frac{1}{1+\frac{\alpha_{49}}{1+\alpha_{49}}}$$

$$= .75\ \delta g_\infty(\text{with 49 alone})$$

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Because of the greater number of neutrons formed when Pu^{241} is present, the effect on g_∞ of a fixed loss is less in this case than for Pu^{239} alone.

2. Control Absorbers

If neutron absorbers, in the form of necessary control rods, or perhaps structural changes, are added, beyond those incorporated when g_0 is figured, then in order to maintain the same criticality conditions U^{238} will have to be reduced and g will decrease. The effect can be seen by decreasing g_0 by $\frac{S_+}{N_{49}\bar{\sigma}_{a49}}$, where S_+ is the total cross section of the added absorbers.

Frequently, such absorbers are specified in terms of $\frac{\delta k_{eff}}{k}$.

If $$k_{eff} = \frac{\nu}{\text{Neutrons absorbed} + \text{neutrons leaking from pile}}$$

then a departure from a steady state in which $k_{eff} = 1$ will be given by

$$\delta k_{eff} = \frac{-\text{ change in neutrons absorbed}}{\nu}$$

Since k will be near 1, approximately

$$\delta g_0 = \frac{\nu}{N_{49}\bar{\sigma}_{a49}}\left(\frac{\delta k_{eff}}{k}\right)$$

From eq (15) $\frac{\nu}{N_{49}\bar{\sigma}_{a49}}$ will be between 2 and 3, probably nearly 3; so, very roughly,

$$\delta g_0 \approx 3\frac{\delta k_{eff}}{k}$$

Thus for $\frac{\delta k_{eff}}{k} = 3$ to 4%, $\delta g_0 \approx .1$.

Again Fig. 5 shows the change in g_∞; if in the example previously carried through, g_0 is decreased from .58 to .48, $g_\infty = .865$ for $\eta_{41} = 3$; $g_\infty = .74$ for $\eta_{41} = 2.5$.

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3. Chemical Processing

In the chemical processing of pile materials necessarily some loss of fissionable atoms, or potentially fissionable atoms, will occur. The effect is the same as a reduction in breeding ratio. We suppose the processing takes place in discrete steps - the material is supposed to remain in the pile for a time T_c, then be processed and returned again to the pile- but we average the loss over all time. The loss of 28 atoms which have not been converted into 49 is not counted.

Let f = fraction of total fissionable atoms which fission in T_c

c_1 = fraction of atoms of any species which are lost in one processing

$C = \frac{c_1}{T_c}$ = average fraction of atoms lost per second.

Then

$$\varphi (N_{49} \bar{\sigma}_{f49} + N_{41} \bar{\sigma}_{f41}) \; T_c = f \, (N_{49} + N_{41})$$

so

$$(31) \quad \frac{c}{\varphi} = \frac{c_1}{f} \left[\frac{\bar{\sigma}_{f49} + \frac{N_{41}}{N_{49}} \bar{\sigma}_{f41}}{1 + \frac{N_{41}}{N_{49}}} \right]$$

Thus $\frac{c}{\varphi}$ has the dimensions of a cross section: the cross section which gives the average processing loss. To every σ_a, then, $\frac{c}{\varphi}$ must be added.

It is convenient to call $\frac{c_1}{f} = z$, and to introduce $\gamma = \frac{c}{\varphi \sigma_a}$.

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From eq (31)

$$(32) \qquad \gamma_{49} = z\left[\frac{\frac{\bar{\sigma}_{f49}}{\bar{\sigma}_{a49}} + \frac{N_{41}}{N_{49}}\frac{\bar{\sigma}_{f41}}{\bar{\sigma}_{a49}}}{1 + N_{41}/N_{49}}\right]$$

We shall consider only equilibrium values, so

$$_{\infty}N_{41}\ \bar{\sigma}_{t41}(1+\gamma_{41}) = {}_{\infty}N_{40}\,\bar{\sigma}_{c40}$$

and

$$_{\infty}N_{40}\ \bar{\sigma}_{c40}(1+\gamma_{40}) = N_{49}\,\bar{\sigma}_{c49} \quad , \text{ so}$$

$$(33) \qquad {}_{\infty}N_{41}\,\bar{\sigma}_{t41}\ (1+\gamma_{41}) = N_{49}\,\bar{\sigma}_{a49}\left(\frac{\alpha_{49}}{1+\alpha_{49}}\right)\left(\frac{1}{1+\gamma_{40}}\right)$$

Thus, by substitution, since $\bar{\sigma}_{f41} \sim \bar{\sigma}_{t41}$

$$\gamma_{49} = z\ \frac{1 + \frac{\alpha_{49}}{(1+\gamma_{41})(1+\gamma_{40})}}{1+\alpha_{49}\left[1 + \frac{\gamma_{41}}{\gamma_{49}}\left(\frac{1}{1+\gamma_{41}}\right)\left(\frac{1}{1+\gamma_{40}}\right)\right]}$$

$$(34) \qquad \gamma_{40} = \gamma_{49}\ \frac{\bar{\sigma}_{a49}}{\bar{\sigma}_{c40}} = .76\,\gamma_{49}$$

$$\gamma_{41} = \gamma_{49}\ \frac{\bar{\sigma}_{a49}}{\bar{\sigma}_{t41}} = .058\,\gamma_{49}$$

Since we are interested only in small values of z, certainly below .1, we can get limits of γ_{49} for these values:

For $z \to 0$, $\gamma_{49} \to .982z$

$z \to .1$, $\gamma_{49} \to .96z$

We shall assume that for any z, γ_{49} has the average value of .97z.

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Then

(35) $\gamma_{49} = .97z,\ \gamma_{40} = .735z,\ \gamma_{41} = .056z$

In calculating the breeding ratio, now eq (23) changes to

$$g = \frac{N_{28}\,\bar{\sigma}_{c28} + N_{40}\,\bar{\sigma}_{c40}}{N_{49}\,\bar{\sigma}_{a49}\,(1+\gamma_{49}) + N_{41}\,\bar{\sigma}_{t41}\,(1+\gamma_{41})}$$

or

(36) $$g = \frac{g_0 + (\eta_{41} - 1)\dfrac{N_{41}\,\bar{\sigma}_{t41}}{N_{49}\,\bar{\sigma}_{a49}}}{1 + \gamma_{49} + \dfrac{N_{41}\,\bar{\sigma}_{t41}\,(1+\gamma_{41})}{N_{49}\,\bar{\sigma}_{a49}}}$$

Again considering the steady state value, and substituting from above

(37) $$g_\infty = \frac{g_0 + \dfrac{\alpha_{49}}{1+\alpha_{49}}(\eta_{41} - 1)\left(\dfrac{1}{1+\gamma_{41}}\right)\left(\dfrac{1}{1+\gamma_{40}}\right)}{1 + \gamma_{49} + \dfrac{\alpha_{49}}{1+\alpha_{49}}\left(\dfrac{1}{1+\gamma_{40}}\right)}$$

Since the γ's are small, to a fair approximation

(38) $$g_\infty \approx \frac{g_0 + \dfrac{\alpha_{49}}{1+\alpha_{49}}(\eta_{41} - 1)(1 - \gamma_{41} - \gamma_{40})}{1 + \gamma_{49} + \dfrac{\alpha_{49}}{1+\alpha_{49}}(1 - \gamma_{40})}$$

From eq (38), it is seen that the major effect on g is through the γ_{49} in the denominator, representing the processing loss of Pu^{239}. The γ_{41}, γ_{40} in the numerator account for the effective reduction in neutrons available for breeding because of the waste of 41, 40 in processing. The γ_{40} in the denominator shows the loss of 40 which might have been converted into 41 has the same effect as a greater destruction of 41; γ_{41} does not enter here because the greater loss of

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41 directly is exactly balanced by a reduction in N_{41} under the conditions of chemical processing loss.

Substituting from eq (35)

$$(39) \quad g_\infty \approx \frac{g_0 + .338(\eta_{41} - 1)(1 - .79z)}{1.338 + .722z}$$

As z is small, g_∞ will be nearly linear in z, or approximately

$$(39a) \quad g_\infty \approx \frac{g_0 + .338(\eta_{41} - 1)}{1.338} - z\left[.336(\eta_{41} - 1) + .405\, g_0\right]$$

Thus

$$(39b) \quad \frac{\partial g_\infty}{\partial z} = -.336(\eta_{41} - 1) - .405\, g_0$$

For the range of η_{41} and g_0 here considered, $\frac{\partial g_\infty}{\partial z}$ would be between .75 and 1. Since the z to be expected can only be estimated, it will be sufficient to take $\frac{\partial g_\infty}{\partial z}$ as 1, and thus to say $\delta g_\infty = -z$.

As a lower limit to be expected, c_1 might be .001 to .002; possibly f might be greater than .1. On the optimistic side, then, z might be as low as .01, and thus the reduction in g_∞ due to processing losses only about .01 - a relatively minor effect. On the other hand, if c_1 were as great as .01, and if f had to be as small as .05, on the pessimistic side z could be greater than .2. In the latter case, the reduction in g_∞ would be very serious.

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4. Temperature Effect on α.

The calculations in this report are carried out for room temperature, since the data on cross sections apply only at about that temperature. Actually, it is expected the piles discussed herein would produce power, so the operating temperature might be quite high. How the elevated temperature might affect the cross sections of the Pu^{240}, Pu^{241} is not known, but it is certain the α_{49} will be increased. Thus g will be reduced, though the fissioning of Pu^{241} will ameliorate the effect to some extent.

I. Effect of Additional Neutron Gains

1. Breeding Blanket

Now consideration can be given to some ways in which increases might come to g_∞. One of these is through the capture of leakage neutrons in U^{238} placed around the pile. If a fraction ℓ of the neutrons produced leak from the pile, and if of these again a fraction b could be captured in 28, then the effective increase in g_0 would be

$$(40) \qquad \delta g_0 = \frac{b \ell \nu}{N_{49} \bar{\sigma}_{a49}}$$

From eq (15) $\frac{\nu}{N_{49} \bar{\sigma}_{a49}}$ in equilibrium is $1.95 + .338\,\eta_{41}$, which as an upper limit might approach 3. If the Hanford pile is a guide, ℓ might be about .04; so if b could be as large as .5, δg_0 would amount to .06. Significant gains could result in this way.

2. Fast Fission.

Another way in which g_∞ might increase is through fast fission. Usually, the effect of fast fission is represented by a coefficient E multiplying η.

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From eq (28)

$$\delta g_\infty = \frac{\left(\eta_{49} + \frac{\alpha_{49}}{1+\alpha_{49}}\,\eta_{41}\right)}{1 + \frac{\alpha_{49}}{1+\ {}_{49}}}\,(E-1) = (1.46 + .253\,\eta_{41})(E-1)$$

If the effect of fast fission has already been considered in the g_0 used, then the first term should be omitted.

As an example, suppose $\eta_{41} = 2.75$, $E = 1.035$, then $\delta g_\infty = .075$ Again, a significant gain results, which shows deliberate encouragement of fast fission might be worthwhile.

J. Definition of g

In piles which produce some new fissionable material as they run, some figure of merit is needed to assay the performance of the pile as a manufacturer, and to compare one pile with another.

The criterion has been called the "breeding ratio", g; and for a pile which contains only one fissionable species, a definition in common usage is

(I) g = number atoms of A produced in pile per atom of A destroyed

When a pile contains, and produces, more than one fissionable species, an extension of the definition of g becomes necessary. The direction the extension takes will depend upon primary purpose of the pile.

In Section II G, and also in MONC-374 by H. C. Ott, the definition chosen is the logical extension of the above to

(II) g = number of fissionable atoms, of all species, produced in the pile per fissionable atom, of all species, destroyed.

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Then $\frac{1}{1-g}$ = number of fissionable atoms used in the pile per fissionable atom supplied.

On the basis of definition II, if $g_\infty < 1$,

Number atoms 49 added to pile per 49 consumed in pile $= (1 - g_\infty)\left[1 + \frac{\alpha_{49}}{1+\alpha_{49}}\right]$

and

Fissions per atom 49 added to pile $= \frac{1}{(1 - g_\infty)\left[1 + \frac{\alpha_{49}}{1+\alpha_{49}}\right]}$

In this particular model pile, in which only Pu isotopes are fissionable, simpler expressions for the above would result if definition I had been used, with A denoting the major isotope 49. Then

Number atoms 49 added to pile per 49 consumed $= 1 - g_\infty$

Fission per atom 49 added to pile $= \frac{1}{1 - g_\infty}$

The latter relation results since $\alpha_{41} \to 0$. In a pile in which U^{235} is also present, as considered in a later section, definition I does not yield any simpler expressions.

Other definitions of g appropriate for particular purposes, might be thought of; for example, if a pile is run chiefly to produce atom C when fed fissionable species B, then g might be defined as

(III) g = number of atoms of C produced per atom of B consumed

Or if power production is the chief aim, then a simple criterion for performance would be given by

(IV) g = number of fissions from atoms created in pile per fission of all kinds in pile.

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$$\frac{1}{1-g} = \frac{\text{energy produced in pile}}{\text{energy produced only by fission of material fed to pile}}$$

g by IV differs from g by II because in general α will not be the same for all fissionable species present in the pile.

For the simple case of definition I, one can write

$$g = \eta - 1 - \text{neutron losses}$$

where each term is referred to the destruction of one atom of A. For definition II, a similar relation can be used, except the reference basis needs to be the destruction of one fissionable atom of all kinds. Thus, we need an average $\bar{\eta}$, defined as

$$\bar{\eta} = \frac{\bar{\eta}_{49} N_{49} \sigma_{a49} + \eta_{41} N_{41} \sigma_{a41}}{N_{49} \sigma_{a49} + N_{41} \sigma_{t41}}$$

$$\sim \frac{\bar{\eta}_{49} + \frac{\alpha_{49}}{1+\alpha_{49}} \eta_{41}}{1 + \frac{\alpha_{49}}{1+\alpha_{49}}}$$

Numerically

$\eta_{41} = 3$,	$\bar{\eta} = 2.22$
$= 2.75$,	$= 2.16$
$= 2.5$,	$= 2.09$

In the previous section, it was simpler to compute losses on the basis of atoms of 49 destroyed, since $N_{49} \sigma_{a49}$ was assumed constant. The reduction was conveniently accomplished by subtracting these neutron losses from g_0

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in the numerator of the expression for g. This was identically equivalent to expressing them in terms of total fissionable atoms destroyed, by division by $1+\frac{\alpha_{49}}{1+\alpha_{49}}$, the ratio of total atoms destroyed per 49 destruction, and subtracting then from g_∞.

K. Probable g_∞

The results discussed in detail in previous sections are recapitulated in Table I below. As a starting point, it is assumed the pile is duplicate of Hanford, except that Pu^{239} is substituted for U^{235}, and the concentrations of Pu^{239}, U^{238} are adjusted so the pile is critical. The substitution reduces g_0 from the .82 of Hanford at start up to .65.

In the second line, the effect of fission products is subtracted. Processing might occur frequently enough so equilibrium Sm did not build up; on the other hand, products other than Xe, Sm would appear to increase the reduction.

The .1 neutron per 49 atom destroyed subtracted for controls and possible structural changes perhaps is on the high side. Likewise, the .1 for chemical processing may be greater than the figure to which chemists may ultimately reduce losses.

The gain in a breeding blanket is speculative. The fast fission effect is added for the 41 fissions only, the effect for 49 presumably already being taken into account in the original g_0.

As the figures for g_∞ are possibly rather conservative, one might hope for an η_{41} high enough to give $g_\infty = .85$. Thus, the existence of the higher isotopes of Pu has been enough, to counterbalance the losses introduced, and to raise g from .65 to perhaps .85. The net effect of the neutron losses and gains is seem to be about - .09 on g_∞.

At the end of Table I is given the value that the initial g_0 would have to have, in order to come out at the end with $g_\infty = 1$. g_0 is so high, the impossibility of obtaining a true breeder with a pile of this type is indicated. How high g_0 might become will be considered in a later section.

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TABLE I

Breeding Ratio in Pu Model Pile

Based on Hanford initial $g_0 = .82$

Cause	Change in g	Resultant g_0	g_∞	
			$\eta_{41} = 3$	$\eta_{41} = 2.5$
Initial		.82		
Substitution of 49 for 26	$\delta g_0 = -.17$	.65	.99	.865
Fission Products (Xe + Sm)	$\delta g_0 = -.07$	.58	.94	.81
Controls, Structure	$\delta g_0 = -.1$	.48	.865	.74
Chemical Processing	$\delta g_\infty = -.1$		.765	.64
Breeding Blanket	$\delta g_\infty = +.06$		.825	.70
Fast Fission	$\delta g_\infty = .027,$ $.022$		.852	.722

Value of initial g_0 for final $g_\infty = 1$

$\eta_{41} = 3$ $\quad g_0 = 1.02$

$= 2.5$ $\quad 1.19$

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L. Comparison of Plutonium with Other Fuels

Ott, in CL-HWS-31, has considered the effect of higher isotopes of uranium on the operation of a U^{233}-Th breeder. Comparison of his results with those obtained here for the Pu-U^{238} pile show three striking differences: (1) the time scale for the build up of the higher isotopes from U^{233} is very much longer than for the plutonium case, because of the much larger cross sections of all Pu isotopes. Depending on the unknown σ_c for U^{234}, the difference in time of attaining equilibrium might be as great as a factor of 10. (2) The excess of neutrons produced over those absorbed in U isotopes (analogous to the quantity ε of this report) first decreases because of the formation of U^{234}; then increases again, with the buildup of U^{235} which is fissionable; then decreases again as U^{236} comes in. The Pu shows the first decrease, but not the second; the time scale is much less for Pu. (3) The net loss (counting in the recovery from fission of U^{235}) of neutrons to higher isotopes, per neutron absorbed in U^{233}, is only .00274. The similar quantity for the plutonium model pile, per neutron absorbed in Pu^{239}, is - .169 for $\eta_{41} = 2.15$, or - .338 for $\eta_{41} = 3$; the - indicates that the loss is a gain. Thus, in the U^{233}-Th, the higher isotopes have practically no effect on the breeding, while in the Pu^{239}-U^{238} they contribute materially to increase the breeding ratio.

In a pile running on U^{235}, the build up of U^{236}, U^{237} would also be slow, the σ_{c25} being 85b, and σ_{c26}, 15b. Since U^{237} decays to Np^{237}, if chemical processing removes rapidly all elements save uranium, then for each neutron absorbed in U^{235}, $\frac{\alpha_{25}}{1+\alpha_{25}} = 0.15$ neutrons will be lost to the higher isotopes. On the other hand, if all isotopes are left in until equilibrium attains after two neutron captures and one β decay, Pu^{239} will be reached. Using $\eta_{41} = 3$, the $\bar{\eta}_{Pu} = 2.22$, so now the net loss of

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neutrons would be $\frac{\alpha_{25}}{1+\alpha_{25}}\left[3-(2.22-1)\right]=.27$ neutrons lost to higher isotopes per neutron absorbed in 25.

Wigner has noted (MUC-EPW-134) that a certain percent loss P of neutrons causes a reduction in breeding ratio which is $p\eta$, so the loss is relatively more important, the higher η. This appears to be true of a fixed loss of neutrons <u>only</u> for otherwise identical piles containing material of different η. For general comparison of different types of piles, one would not expect a constant per cent loss of neutrons in structure, controls, or by leakage.

As already noted in the 49-41 system, the presence of a fissionable higher isotope <u>increases</u> the relative fission product loss over that for 49 alone. In the 23-25 system the higher isotope has practically no effect, because α_{23} and α_{25} are nearly alike; since the α's are small compared to α_{49}, the fission product loss is more important in this system than in the plutonium system.

The chemical processing loss is relatively greater in those systems in which a high member fissions, since its neutron yield is reduced by disappearance of isotopes along the line. Such loss is more important in the 49-41 chain than in the 23-25, because likely $\eta_{41}>\eta_{25}$, and also because $\alpha_{49}>\alpha_{23}$.

M. <u>Operating Questions</u>

1. <u>Initial Decrease in ε</u>

Fig. 4 shows that initially β, and $\varepsilon-\varepsilon_o$, decrease but quickly recover. As U^{238} would have to be withdrawn, if changes in its concentration are made

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to maintain criticality, and then soon returned, one may ask if the dip in β may not be counterbalanced by controls, and the handling of large amounts of 28 avoided. The change of N_{28} will be just

$$\frac{\Delta N_{28}}{N_{28}} = \frac{\varepsilon - \varepsilon_0}{\varepsilon_0}$$

$$= \frac{\frac{\alpha_{49}}{1+\alpha_{49}}}{\eta_{49} - 1}\beta = .356\beta$$

since ν is small in this range and $\bar{\sigma}_{a28}$ will be practically constant. Fig. 4 allows the value of $\frac{\Delta N_{28}}{N_{28}}$ at β min to be computed; one sees it probably will be in the neighborhood of 1%. Numerically, $\frac{\Delta N_{28}}{N_{49}}\frac{\bar{\sigma}_{a28}}{\bar{\sigma}_{a49}}$ is about the same value, thus much smaller than the fission product absorbers. The appearance of Xenon will probably be more rapid than the change in β, but the Samarium will likely fall in the range of negative β. Both of these suggested variations in N_{28} can be handled by controls with N_{28} constant, without much loss in breeding.

2. <u>Power Production</u>

The power produced in the hypothetical pile considered here would be proportional at constant flux to the number of fissions per second, or, since in this model $N_{49}\,\bar{\sigma}_{a49}$ is supposed to be constant

$$P \propto \frac{N_{49}\bar{\sigma}_{f49}}{N_{49}\bar{\sigma}_{a49}} + \frac{N_{41}\,\bar{\sigma}_{f41}}{N_{49}\,\bar{\sigma}_{a49}}$$

or the ratio of the power at any time, to the initial power when $N_{41} = 0$,

$$\frac{P}{P_0} = 1 + \alpha_{49}\,\frac{N_{41}}{{}_{\infty}N_{41}}$$

At steady state, then, the power would have increased 51%. If this change were objectionable, the flux could be varied, and all the previous relations would go through just the same, since only φ_t is involved; with variable φ, the <u>time</u> scale would be altered.

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3. Presence of U^{235}

When the pile is going in steady state, it would make little difference whether pure U^{238} or natural uranium was used as fuel. Suppose for example, the pile had a g_∞ as low as .75; then 3 atoms of 28 would be supplied per atom of 49. If each 28 were accompanied by .007 atom 25, the change in 49 required would be slight.

On start up, however, if natural uranium were used, the effect would be great. In the next section, a pile starting on natural uranium with no plutonium initially present will be considered.

III. Pile of Hanford Type, with U^{235} Present

In a Hanford pile starting on natural uranium, the fission σ of the Pu^{239} which is created more than balances the reduction of σ_f through destruction of U^{235}, so the reactivity of the pile increases with time. We wish to follow this as the higher isotopes of Pu^{239} build up.

In a practical pile, one might imagine the excess reactivity reduced at frequent intervals by removal of a fraction of all the plutonium present. (Ref: Unpublished Memo* dated June 4, 1946 - "Power Piles Operating on Natural Uranium"). The mathematical analysis of such a problem is difficult: either a laborious step by step calculation, or, in the case of the approximation of a continuous removal, the solution of non-homogeneous differential equations would be required. Here, it will be supposed all isotopes remain in the pile, and as a further simplification, that the cross sections are constant, even though actually the resonance absorption will increase as the neutron production increases, as already discussed. With such an approximation, the concentration of each species can be calculated readily; the time variation will follow the same pattern as in the previous hypothetical pile, since the same cross sections are involved.

*By G. Young

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A. Excess Neutron Production

An interesting quantity is $\mathcal{E} - \mathcal{E}_o$ as already defined: the increase in the number of neutrons, which would have to be absorbed in controls if the pile remains just critical. Similar to previous equations, we have

$$(41) \quad \frac{\mathcal{E} - \mathcal{E}_o}{N_{28}\,\bar{\sigma}_{c28}} = (\bar{\eta}_{49} - 1)\,\frac{N_{49}\,\bar{\sigma}_{a49}}{N_{28}\,\bar{\sigma}_{c28}} + (\eta_{41} - 2)\,\frac{N_{41}\,\bar{\sigma}_{a41}}{N_{28}\,\bar{\sigma}_{c28}}$$

$$+ (\eta_{25} - 1)\left[\frac{N_{25}\,\bar{\sigma}_{a25}}{N_{28}\,\bar{\sigma}_{c28}} - \frac{{}_oN_{25}\,\bar{\sigma}_{a25}}{(N_{28}\,\bar{\sigma}_{c28})_o}\right]$$

Here we have assumed the Pu^{241} will always be in equilibrium with Pu^{240}, because of the much larger cross section of the former. ${}_oN_{25}$ denotes the concentration of U^{235} in natural uranium at start up; no addition is presumed to be made. It is further assumed N_{28} will be substantially constant during the period of interest.

From eq (41), $\frac{\mathcal{E} - \mathcal{E}_o}{N_{28}\,\bar{\sigma}_{c28}}$ has been computed, and is plotted in Fig. 6 for two values of η_{41}. In steady state, $\frac{N_{49}\,\bar{\sigma}_{a49}}{N_{28}\,\bar{\sigma}_{c28}} = 1$, and $\frac{N_{41}\,\bar{\sigma}_{a41}}{N_{49}\,\bar{\sigma}_{a49}} = \frac{\alpha_{49}}{1+\alpha_{49}}$, since the plutonium chain is derived only from the destruction of U^{238}; and the U^{235} concentration is zero. $\mathcal{E} - \mathcal{E}_o$ has a negative value, indicating the same pile which started out will not run. At ϕt between 1. and 1.5 - - that is, after 1 to 1.5 years at a flux of 10^{14}, or 5 to 7.5 years at a flux of 2×10^{13} - - $\mathcal{E} - \mathcal{E}_o$ becomes zero.

B. Critical Concentration of U^{235} and Breeding Ratio

In order to keep this identical pile (identical as to size, structure, absorbers, etc.) running, then, N_{25} could not be allowed to go to zero. Eq. (41) gives the requirement at steady state:

$$(42) \quad \frac{(\bar{\eta}_{49} - 1)}{\eta_{25} - 1} + \frac{(\eta_{41} - 2)}{(\eta_{25} - 1)}\,\frac{\alpha_{49}}{1+\alpha_{49}} + \frac{N_{25}\,\bar{\sigma}_{a25}}{N_{28}\,\bar{\sigma}_{c28}} = \frac{{}_oN_{25}\,\bar{\sigma}_{a25}}{(N_{28}\,\bar{\sigma}_{c28})_o} = 1.22$$

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(43) $$\frac{N_{25}\,\sigma_{a25}}{N_{28}\,\bar{\sigma}_{c28}} = .372 - .302\,(\eta_{41} - 2)$$

If one adjusts N_{28} to match the change in σ_{c28} so as to keep $N_{28}\,\sigma_{c28}$ <u>constant</u> (this means decreasing N_{28}), then

(44) $$\frac{N_{25}}{{}_{o}N_{25}} = .305 - .248(\eta_{41} - 2)$$

The breeding ratio,

$$g = \frac{N_{28}\,\bar{\sigma}_{c28} + N_{40}\,\bar{\sigma}_{c40}}{N_{49}\,\bar{\sigma}_{a49} + N_{41}\,\bar{\sigma}_{a41} + N_{25}\,\bar{\sigma}_{a25}}$$

(45) $$g_{\infty} = \frac{1 + \dfrac{\alpha_{49}}{1+\alpha_{49}}}{1 + \dfrac{\alpha_{49}}{1+\alpha_{49}} + \dfrac{N_{25}\,\bar{\sigma}_{a25}}{N_{28}\,\bar{\sigma}_{c28}}} = \frac{1.338}{1.338 + .372 - .302(\eta_{41} - 2)}$$

Values of $\frac{N_{25}}{{}_{o}N_{25}}$ and g_{∞} for a few η_{41} are tabulated in Table II.

TABLE II

η_{41}	$\frac{N_{25}}{{}_{o}N_{25}}$	g_{∞}
3	.057	.95
2.75	.119	.90
2.5	.181	.86
2.25	.243	.82

In contrast to the former example, which assumed Pu^{239} was added to the piles for which g_{∞} was less than one, this case envisages U^{235} as being the

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fissionable material supplied, along with the U^{238} for conversion into plutonium. Since ${}_oN_{25}$ is the density in natural uranium, the above table shows that the pile will run in equilibrium with quite a low concentration of U^{235}. Material of the low concentration of U^{235} would be continually replaced; the replacement would not have to be natural uranium, but need only have a higher concentration of U^{235} than the pile material. It would be perfectly feasible, for example, to make use of K-25 discard, which has about 2/3 the 25 concentration of natural uranium; in this pile, it would be further degraded.

C. Withdrawal of Plutonium

Instead of the laborious exact solution of required withdrawal rate of Pu to maintain always the pile critical during isotope build up, an idea of the results may be obtained from the analytically simple solution based on (1) assumption of Pu^{239} alone present, or (2) assumption of all higher isotopes in equilibrium with Pu^{239} at all times [Ref: Unpublished Memo dated June 4, 1946 - "Power Piles Operating on Natural Uranium"; here we extend this work slightly by using actual η's for 25 and 49, and by assumption (2).]

From Section IIA and eq (41) under assumption (1), the required concentration of Pu^{239} is given by

$$(46)\quad (\eta_{49} - 1)\frac{N_{49}\,\bar{\sigma}_{a49}}{N_{28}\,\bar{\sigma}_{c28}} = (\eta_{25} - 1)\frac{{}_oN_{25}\,\bar{\sigma}_{a25}}{N_{28}\,\bar{\sigma}_{c28}}\left[1 - e^{-\bar{\sigma}_{a25}\varphi t}\right]$$

Steady state operation will begin at a t for which $\frac{N_{49}\,\bar{\sigma}_{a49}}{N_{28}\,\bar{\sigma}_{c28}} = 1$

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The fractional withdrawal rate, w, is given by

$$(47) \quad \frac{w\,N_{49}}{\varphi N_{28}\,\bar{\sigma}_{c28}} = 1 - \frac{N_{49}\,\bar{\sigma}_{a49}}{N_{28}\,\bar{\sigma}_{c28}} - \frac{1}{\varphi}\,\frac{d}{dt}\,\frac{N_{49}}{N_{28}\bar{\sigma}_{c28}}$$

and the total amount withdrawn per unit volume of the pile, is

$$(48) \qquad S_{49} = \int_0^t w N_{49}\,dt$$

Eqs (47) and (48), with the aid of eq (46), reduce to

$$(49) \quad \frac{w}{\varphi\bar{\sigma}_{a49}} = \frac{N_{28}\,\bar{\sigma}_{c28}}{N_{49}\,\bar{\sigma}_{a49}} - 1 - \frac{\bar{\sigma}_{a25}}{\bar{\sigma}_{a49}}\left[\frac{\epsilon^{-\bar{\sigma}_{a25}\varphi t}}{1-\epsilon^{-\bar{\sigma}_{a25}\varphi t}}\right]$$

$$(50) \quad \frac{S_{49}}{\varphi t N_{28}\,\bar{\sigma}_{c28}} = \frac{N_{49}\,\bar{\sigma}_{a49}}{N_{28}\,\bar{\sigma}_{c28}} \cdot \frac{1}{\varphi t\,\bar{\sigma}_{a25}}\left(1 - \frac{\bar{\sigma}_{a25}}{\bar{\sigma}_{a49}}\right)$$

$$+ \left[1 - \left(\frac{\eta_{25}-1}{\eta_{49}-1}\right)\frac{N_{25}\,\bar{\sigma}_{a25}}{N_{28}\,\bar{\sigma}_{c28}}\right]$$

Eq (49) shows that w is first positive (material being withdrawn) then zero, and later negative (material being added, from store S_{49}). When the pile reaches steady state, $\frac{N_{49}\,\bar{\sigma}_{a49}}{N_{28}\,\bar{\sigma}_{c28}} = 1$, w is negative. At that time, by eq (46),

$\epsilon^{-\bar{\sigma}_{a25}\varphi t} = .305$, $\bar{\sigma}_{a25}\varphi t = 1.19$, so from eq (50) $\frac{S_{49}}{\varphi t \cdot N_{28}\,\bar{\sigma}_{c28}} = .085$

That is, 8.5% all the Pu^{239} made in the pile is retained outside the pile. The time involved ($\bar{\sigma}_{a25}\varphi t = 1.19$) is 3.3 years at $\varphi = 2\times10^{13}$.

Under assumption (2) which would imply long steps between withdrawal or addition of material, $\eta_{49} - 1$ will be increased to $\eta_{49} - 1 + \frac{\alpha_{49}}{1+\alpha_{49}}(\eta_{41} - 2)$. We calculate only for $\eta_{41} = 3$; then $\eta_{49} - 1$ in the above eqs (46) to (50) is multiplied by 1.356. Steady state sets in at $\epsilon^{-\bar{\sigma}_{a25}\varphi t} = .06$, $\bar{\sigma}_{a25}\varphi t = 2.81$, and

$$\frac{S_{49}}{\varphi t \cdot N_{28}\,\bar{\sigma}_{c28}} = .162$$

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In this case, the excess 49 is about doubled, and the time for steady state is 7.8 years at $\varphi = 2\times10^{13}$. We would expect the correct solution for an actual pile to lie between these two extremes.

D. Condition for $g_\infty = 1$.

While the assumption of $N_{28}\bar{\sigma}_{c28}$ constant was the simplest mathematically in computing concentrations, let us examine the consequences on the steady state g_∞ of increasing this quantity, and let us ask, what would $N_{28}\bar{\sigma}_{c28}$ have to be in order to achieve a g_∞ of one? It is clear that in this case, the concentration of all plutonium atoms will be directly proportional to $N_{28}\bar{\sigma}_{c28}$, while absorption in controls and permanent absorbers, as well as leakage in the first approximation (the fast leakage actually is proportional to $\mathcal{N}$) will be independent of N_{28} for constant pile size. Thus it might be practical to increase N_{28} enough for the relative losses of neutrons to go down so $g_\infty \to 1$.

In this case, let the 28 absorption be included in $\mathcal{E}$, so

$$(51)\quad \mathcal{E} = (\eta_{49} - 1)\, N_{49}\,\bar{\sigma}_{a49} + (\eta_{41} - 2)\, N_{41}\,\bar{\sigma}_{a41} - N_{28}\,\bar{\sigma}_{c28}$$

We suppose $g_\infty = 1$, so $N_{25} = 0$. If the pile runs on the original N_{28}, N_{25}, then also

$$\mathcal{E} = (\eta_{25} - 1)\,{}_oN_{25}\,\bar{\sigma}_{a25} - (N_{28}\,\bar{\sigma}_{c28})_o$$

so

$$(\eta_{49} - 1)\, N_{49}\,\bar{\sigma}_{a49} + (\eta_{41} - 2)\, N_{41}\,\bar{\sigma}_{a41} - N_{28}\,\bar{\sigma}_{c28}$$
$$= (\eta_{25} - 1)\,{}_oN_{25}\,\bar{\sigma}_{a25} - (N_{28}\,\bar{\sigma}_{c28})_o$$

or

$$(52)\quad (\eta_{49} - 1) + (\eta_{41} - 2)\,\frac{\alpha_{49}}{1+\alpha_{49}} - 1 = \left[(\eta_{25} - 1)\,\frac{{}_oN_{25}\,\bar{\sigma}_{a25}}{(N_{28}\,\bar{\sigma}_{c28})_o} - 1\right]\frac{(N_{28}\,\bar{\sigma}_{c28})_o}{N_{28}\,\bar{\sigma}_{c28}}$$

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Putting in numerical values,

$$(53) \quad \frac{(N_{28}\,\bar{\sigma}_{c28})_o}{N_{28}\,\bar{\sigma}_{c28}} = .927(\eta_{41} - 2) - .137$$

Now writing down $\frac{\mathcal{N}}{\mathcal{N}_o}$ for this case

$$(54) \quad \frac{\mathcal{N}}{\mathcal{N}_o} = \frac{\eta_{49} + \frac{\alpha_{49}}{1+\alpha_{49}}\,\eta_{41}}{\eta_{25}\;\frac{{}_oN_{25}\,\bar{\sigma}_{a25}}{(N_{28}\,\bar{\sigma}_{c28})_o} \cdot \frac{(N_{28}\,\bar{\sigma}_{c28})_o}{N_{28}\,\bar{\sigma}_{c28}}}$$

Putting in numbers

$$(55) \quad \frac{\mathcal{N}}{\mathcal{N}_o} \quad \frac{.754 + .131\,\eta_{41}}{\frac{(N_{28}\,\bar{\sigma}_{c28})_o}{N_{28}\,\bar{\sigma}_{c28}}}$$

and substituting from eq (48)

$$(56) \quad \frac{\mathcal{N}}{\mathcal{N}_o} = \frac{.754 + .131\,\eta_{41}}{.927(\eta_{41} - 2) - .137}$$

Values computed from eqs.(53) and (56), for the case where $g_{\infty}=1$, are given in columns two and three of Table III.

TABLE III

η_{41}	$\frac{N_{28}\,\bar{\sigma}_{c28}}{(N_{28}\,\bar{\sigma}_{c28})_o}$	$\frac{\mathcal{N}}{\mathcal{N}_o}$	$\frac{\bar{\sigma}_{c28}}{(\bar{\sigma}_{c28})_o}$	$\frac{N_{28}}{(N_{28})_o}$
3	1.265	1.45	1.17	1.08
2.75	1.79	1.95	1.45	1.23
2.5	3.06	3.32	2.18	1.14

[redacted]

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In order to separate the change required in N_{28} from the total $N_{28}\bar{\sigma}_{c28}$, the alteration in average effective capture cross section must be evaluated. For a homogeneous mixture, log p_{28} would be proportional to N_{28}; for the Hanford structure the relation will still be approximately true. If any actual increase in average density of 28 atoms is secured for example, by increase in slug radius, the relative surface absorption of resonance neutrons would decrease; further the volume occupied by graphite would probably have to decrease. The two effects, somewhat offset each other, so it will be assumed that log $P_{28} \propto N_{28}$, and further that P_{28} is near enough one for

$$(1 - P_{28}) \propto N_{28}$$

Thus, by eq (9) the resonance capture part of $\bar{\sigma}_{c28}$ will be proportional to $\mathcal{N} N_{28}$.

We may write

$$(57) \quad \bar{\sigma}_{c28} = \bar{\sigma}_{cth28} + \bar{\sigma}_{rc28} \frac{\mathcal{N}}{\mathcal{N}_0} \frac{N_{28}}{(N_{28})_0}$$

where $\bar{\sigma}_{rc28}$ is evaluated for the present Hanford slug. Directly from (52) we obtain

$$(58) \quad \left[\frac{\bar{\sigma}_{c28}}{(\bar{\sigma}_{c28})_0}\right]^2 = \frac{\bar{\sigma}_{cth28}}{(\bar{\sigma}_{c28})_0}\left[\frac{\bar{\sigma}_{c28}}{(\bar{\sigma}_{c28})_0}\right] + \frac{\bar{\sigma}_{rc28}}{(\bar{\sigma}_{c28})_0} \frac{\mathcal{N}}{\mathcal{N}_0} \cdot \frac{N_{28}\,\bar{\sigma}_{c28}}{(N_{28}\,\bar{\sigma}_{c28})_0}$$

where $(\bar{\sigma}_{c28})_0$ is the value by (57) for $\frac{\mathcal{N}}{\mathcal{N}_0} = 1 = \frac{N_{28}}{(N_{28})_0}$

For the Hanford pile, from data given in Memo 7-2733, L. W. Marshall, one can compute $(\bar{\sigma}_{c28})_0 = 3.26\,b$; $\bar{\sigma}_{cth28} = 2.21$; and $\bar{\sigma}_{rc28} = 1.05$. From other data (K. Way, private communication), one can calculate, $\bar{\sigma}_{c28} = 3.32\,b$; $\bar{\sigma}_{cth28} = 2.26$; and $\bar{\sigma}_{rc28} = 1.06$. In each case, the $\bar{\sigma}_{cth28}$ refers to a Maxwellian average at room temperature. We choose the latter set of figures, and from the $\frac{\mathcal{N}}{\mathcal{N}_0}$, $\frac{N_{28}\,\bar{\sigma}_{c28}}{(N_{28}\,\bar{\sigma}_{c28})_0}$ of Table III compute by eq (53) the $\frac{\bar{\sigma}_{c28}}{(\bar{\sigma}_{c28})_0}$ given in the

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fourth column of that table. Then from columns two and four, $\frac{N_{28}}{(N_{28})_o}$ of the fifth column follows.

If no other losses entered, the increase in concentration of U^{238} necessary to achieve $g_\infty=1$ is not large if η_{41} is high, and would perhaps be attainable.

E. Effect of Losses.

In Sections III B and D, the additional losses and gains of neutrons, over those present in the natural uranium pile on start up, discussed in Sections II H and I, were not counted. In Table I it is seen that the effect of all of these on g_∞ is a reduction of about .09. Since the concentration of U^{235} is not very high, these added losses and gains in the present pile would be nearly the same. In order to take them into account in Table II, g_∞ should be decreased roughly, by .09.

In setting up the requirements for $g_\infty=1$ in Section III D, the added losses can be brought in exactly, by referring them to one neutron absorbed in Pu^{239} by subtracting $.09(1+\frac{\alpha_{49}}{1+\alpha_{49}})$ from eq. 51. The .137 in eqs. (53) and (56) becomes changed to .461. The figures of Table III may then be revised to those in Table IV below

TABLE IV

η_{41}	$\frac{N_{28}\,\bar{\sigma}_{c28}}{(N_{28}\,\bar{\sigma}_{c28})_o}$	$\frac{\nu}{\nu_o}$	$\frac{\bar{\sigma}_{c28}}{(\bar{\sigma}_{c28})_o}$	$\frac{N_{28}}{(N_{28})_o}$
3	2.12	2.44	1.62	1.31
2.75	4.27	4.77	2.92	11.46
2.5	3.33	3.60	1.96	1.7

An increase of 30% to 50% in average concentration of U^{238} might be

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difficult to obtain (10% to 15% increase in slug diameter, for instance). Perhaps the geometrical arrangement of the U^{238} could be changed so as to increase $\bar{\sigma}_{c28}$ further. In any event, if the pile were to start on natural uranium, and were expected to run finally with $g_\infty = 1$, provision would have to be made in the design for changing uranium concentration.

IV. Conclusions

This report has attempted to analyze the effect of the higher isotopes of plutonium on the breeding ratio in graphite moderated thermal piles, analogous or identical with the Hanford pile. The fissioning with a small α of Pu^{241} certainly increases the value of g obtainable. No very definite numbers can be given, but it seems extremely doubtful if a true breeder, with g considerably larger than 1, could exist. It is optimistic, but not without hope, to expect to attain a g = 1. It seems fairly certain a g of .85 to perhaps .9 might be gotten, if the hoped for high value of η_{41} really exists, and if operation at high temperatures does not seriously change the relative cross sections. The effect of the higher isotopes, then, can be said definitely to balance several extraneous neutron losses, and to increase the Hanford g over its value on starting up of a fresh pile.

The piles discussed here as "thermal", really involve resonance absorption to a considerable degree, in both Pu^{239} and U^{238}, and possibly to an unknown amount in other isotopes. Information of any kind on Pu^{240}, Pu^{241}, and better data for Pu^{239} are badly needed. There may be hope of increasing g through design which encourages resonance absorption in U^{238}, and perhaps discourages it in Pu^{239}; and which adds to fast fission in all cases.

Even though g may be less than one, the pile can be made to run a long time on the plutonium manufactured in the pile; and when new fuel is necessary

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uranium with quite a low concentration of U^{235} can be utilized. Thus, discarded material from isotope separation plants can be used to produce power. If Pu^{239}, manufactured in some other plant, were used as the fuel, then $\frac{1.13}{1 - g}$ times as much energy would be developed as in a pile which burned the Pu^{239} directly, with a $g = o$.

In a heavy water moderated pile, Ott [MonC 374; e.g. p-31] has shown how the build up of the higher isotopes of plutonium increases g from .86 to 1.05 (based on $\eta_{41} = 2.75$). He does not consider some neutron losses specifically, but believes in any event the pile can be made to run with a $g = 1$. Furthermore the heavy water pile also would use as fuel uranium depleted to some extent in U^{235}. From the standpoint of breeding ratio, the heavy water pile seems distinctly superior.

Before much better estimates of breeding can be made, it will be necessary to have more information about the Pu isotope cross sections, how they vary with neutron energy, and particularly the η for Pu^{241}.

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Drawing # 5038

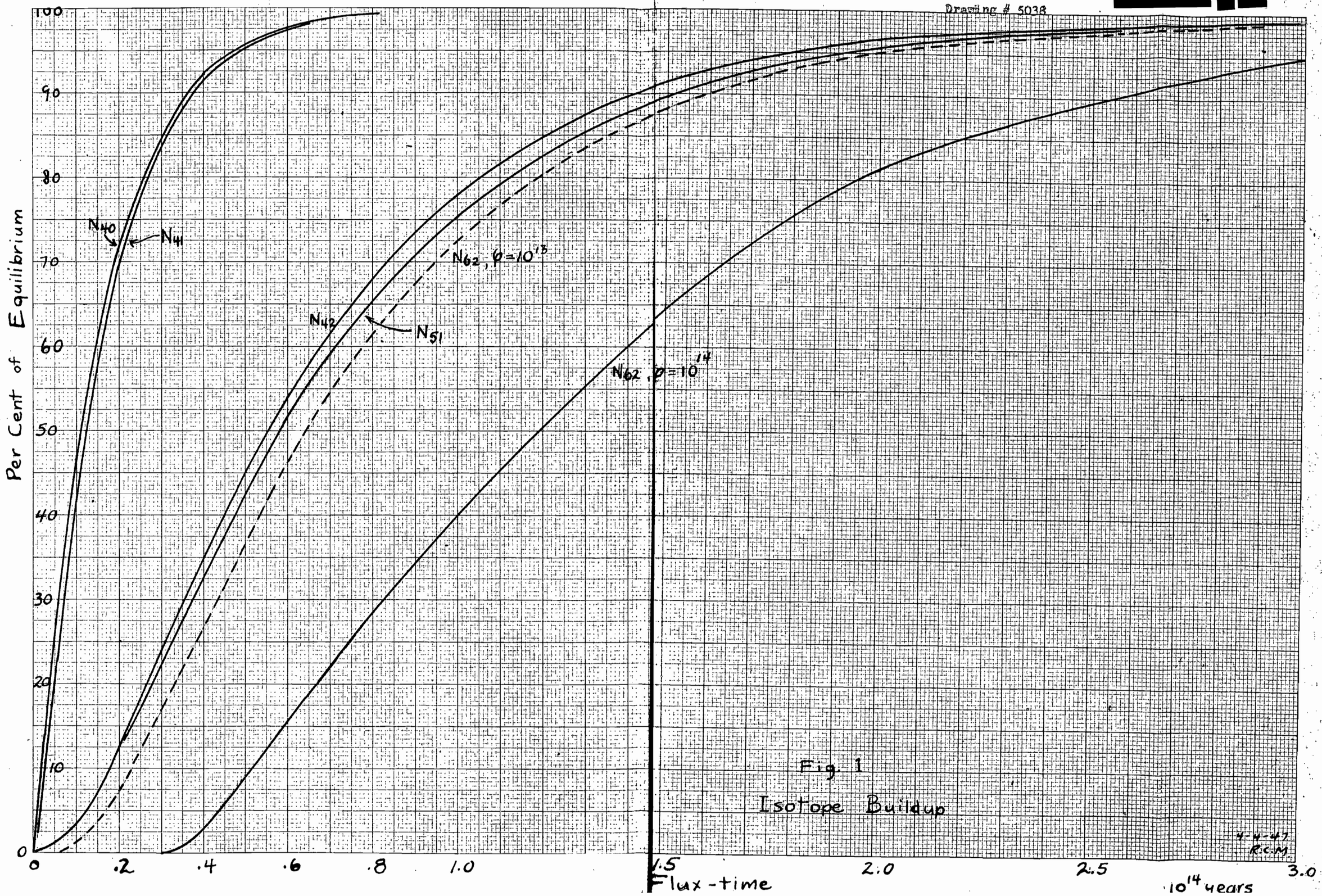

Fig. 1

Isotope Buildup

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Drawing # 503A

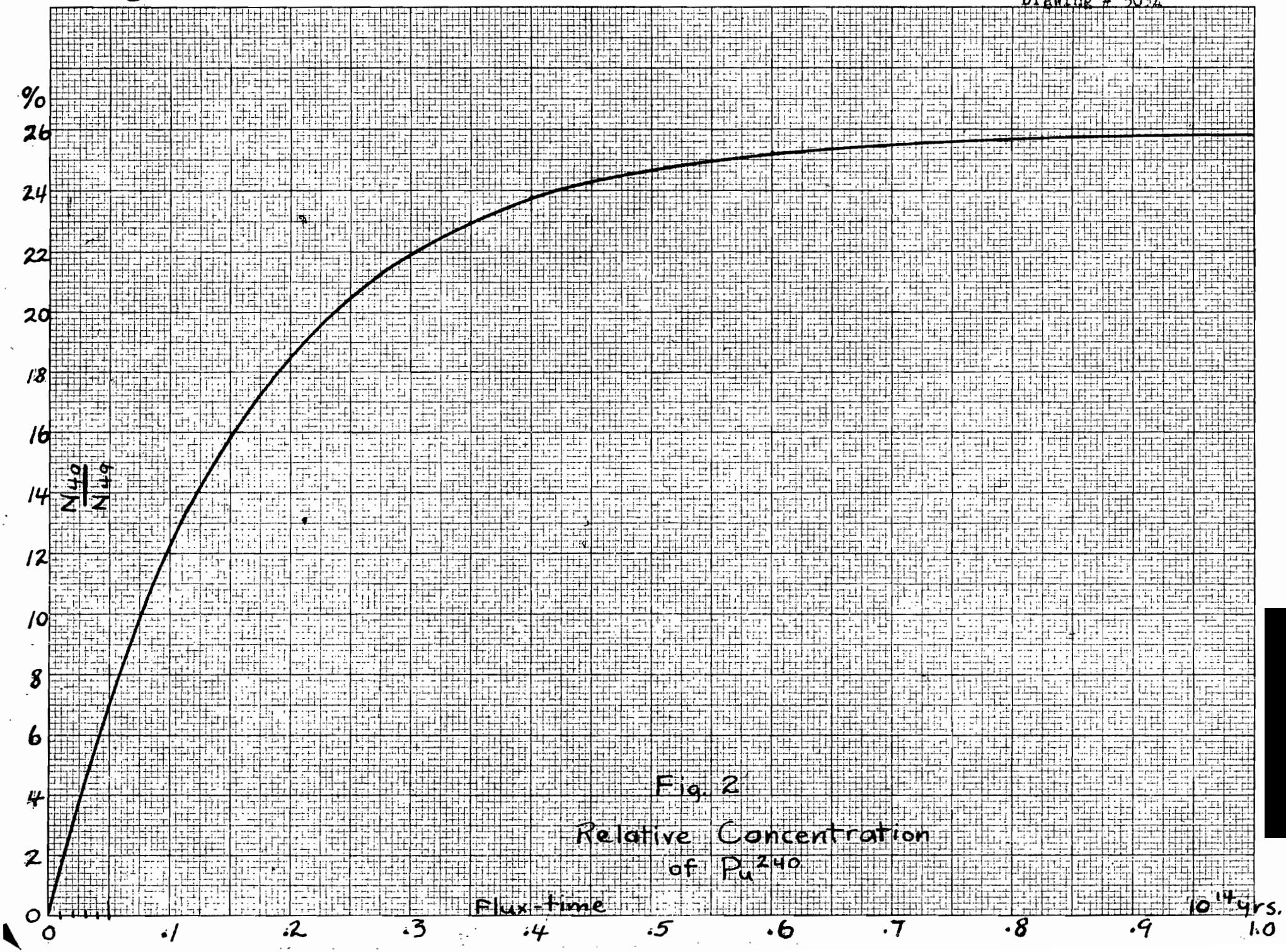

Fig. 2

Relative Concentration of Pu^{240}

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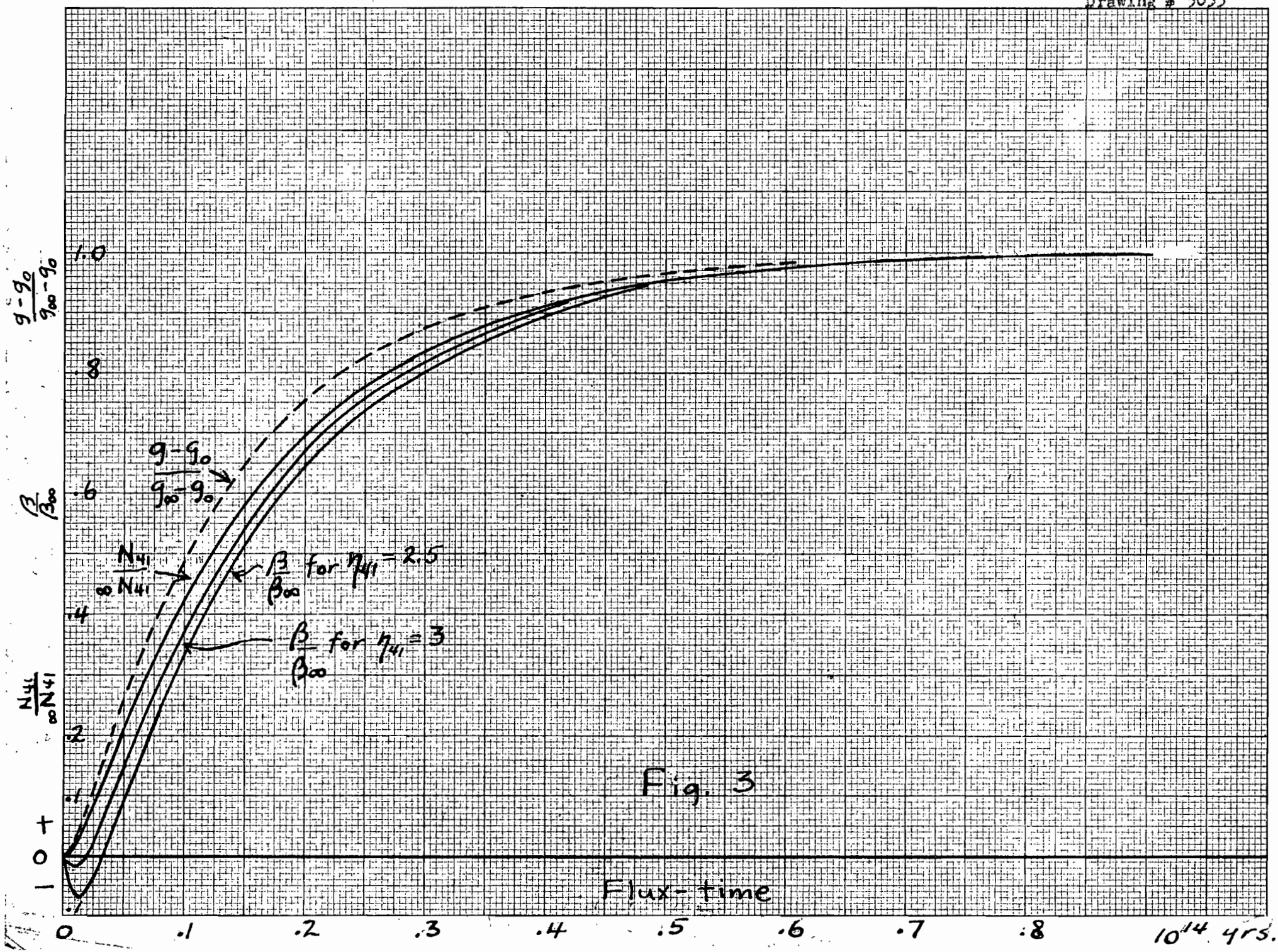

Fig. 3

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Drawing # 5035

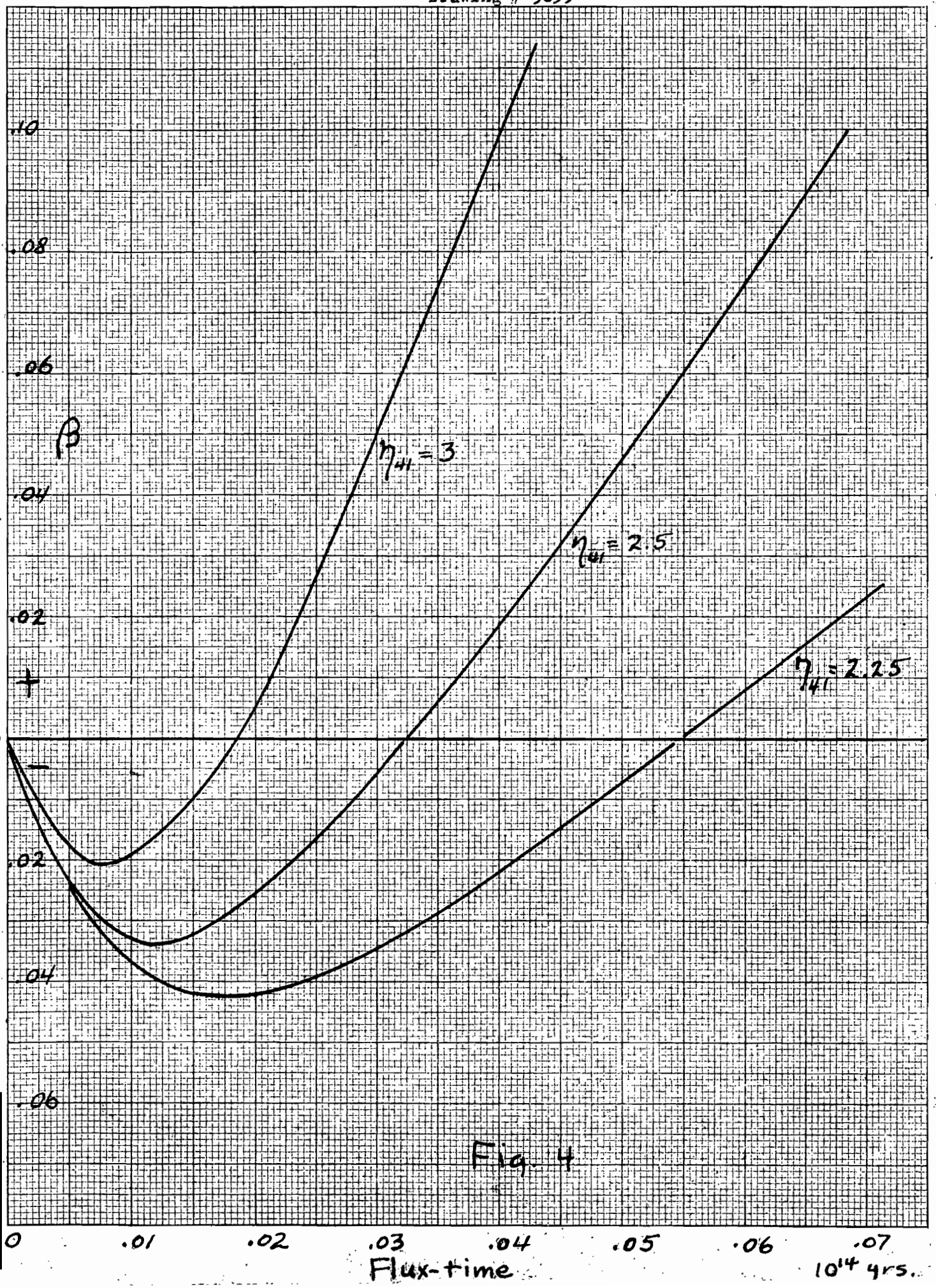

Fig. 4

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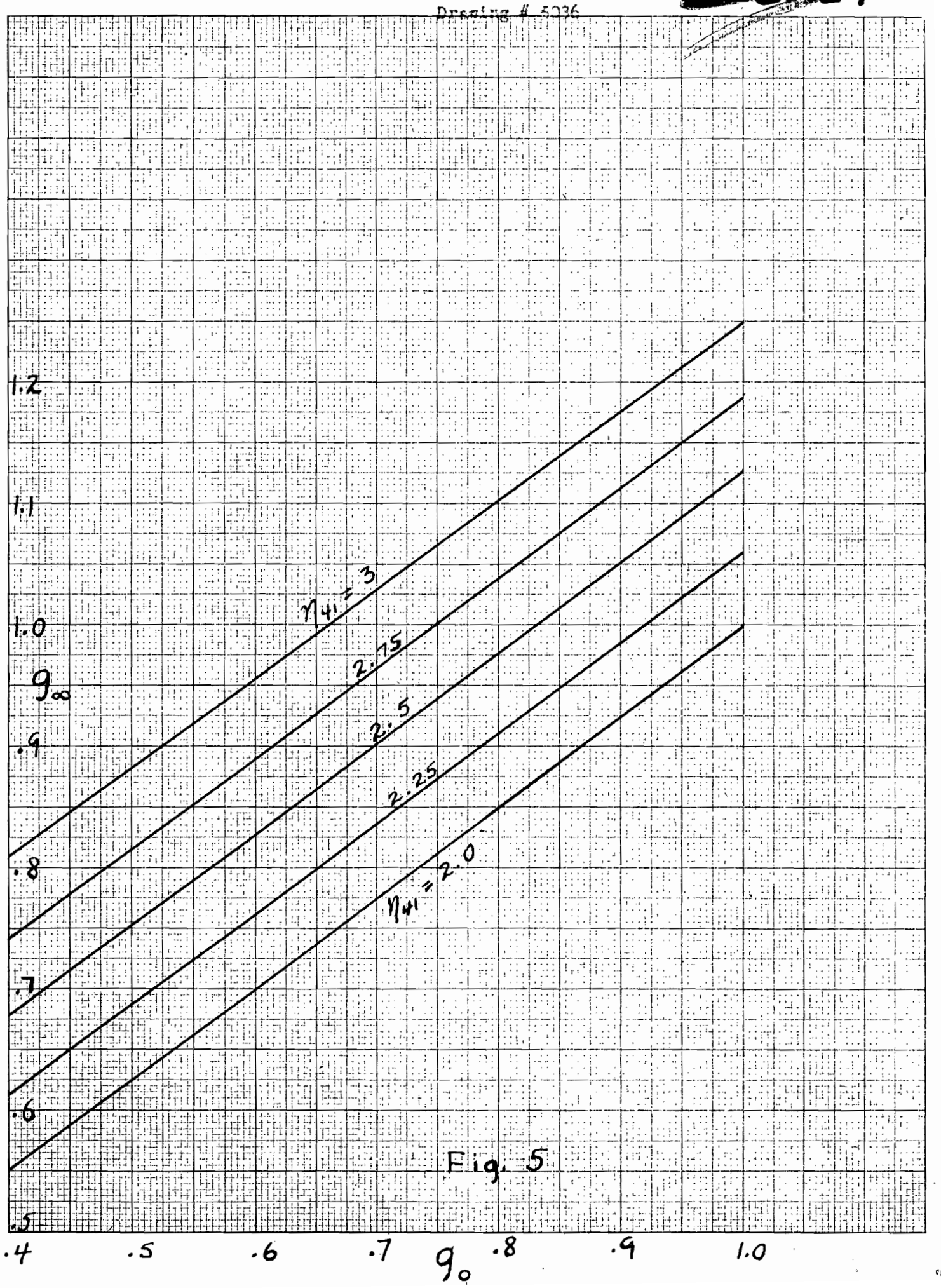

Fig. 5

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Drawing # 5037

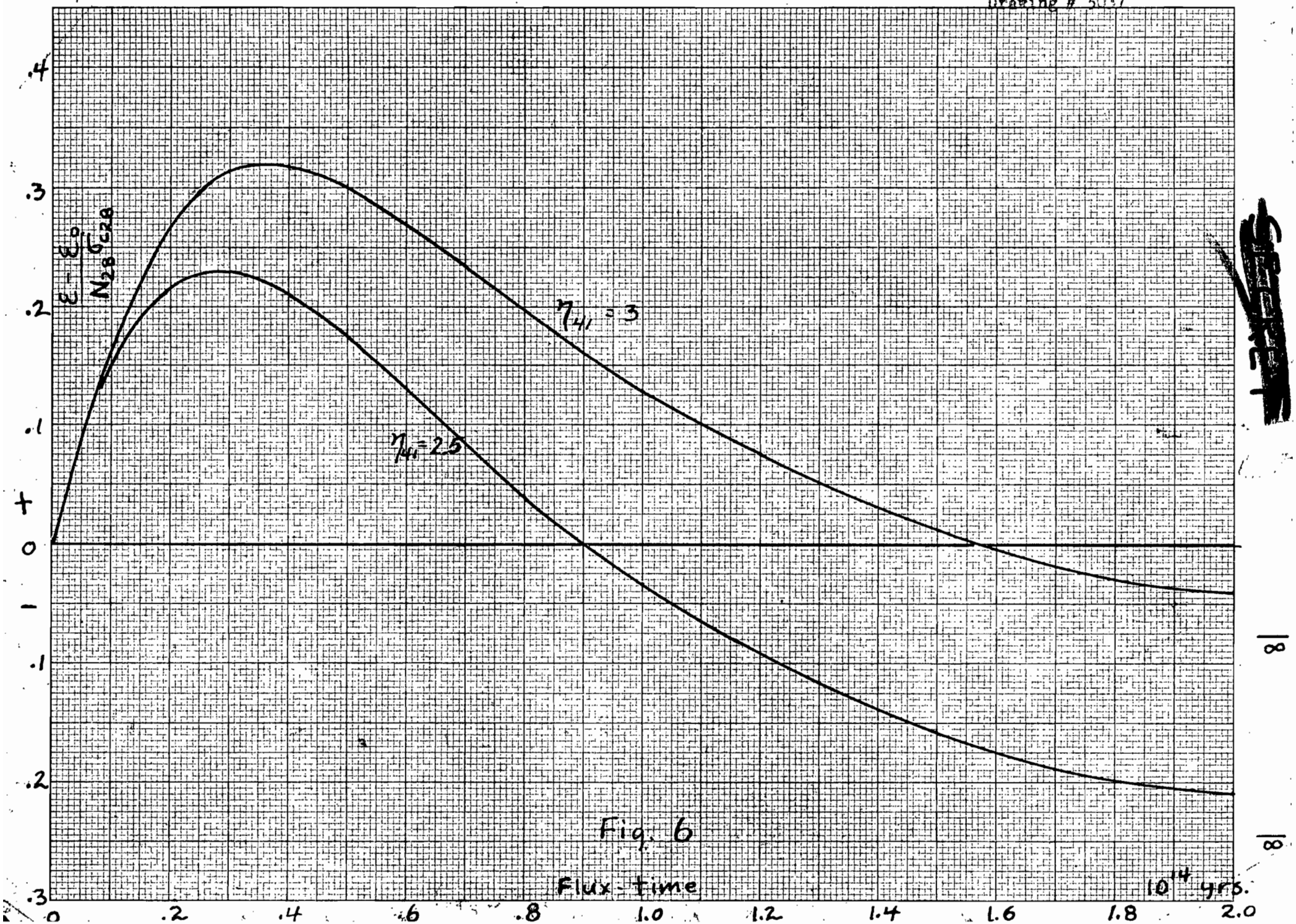

Fig. 6

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