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Section VI

G. Young, Section Chief

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A PILE WITH SPACE UNIFORM ACTIVITY

G. Goertzel

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March 25, 1947

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A Pile with Space Uniform Activity

G. Goertzel

The present note considers a pile-breeding arrangement which exhibits some desireable features and for which a reasonably exact solution may be obtained by elementary methods. A simple example of such a pile is presented here for illustration.

The system is considered to be divided into two regions: 1) a spherical core containing fissionable material, moderator, and breeding material and 2) a blanket containing moderator and breeding material. The moderating properties of the system are considered to be independent of position, both in the spherical core and the blanket. The density of fissionable material in the core is also assumed to be constant. In addition, it is required that the thermal neutron flux in the core be constant throughout the core. This means that the number of fissions per unit volume and time will be constant throughout the core, so that the heat liberation in the core is essentially independent of position.

In order to accomplish these aims, it is apparent that one must distribute the breeding material in the core in a manner determined from the condition that the neutron flux is to be constant throughout the core. The problem in the blanket is by no means so definite.

The example considered in this report is a system with 23 as the fissionable material, thorium as the breeding material, and heavy water as the moderator. It is assumed that thorium is

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available in large quantities so that, for the primitive discussion presented here, no effort need be made to keep the amount of thorium required to a minimum. However, an attempt is made to indicate the relation between the amount of heavy water and amount of 23 required to make the pile go.

Since the amount of thorium used is of no interest for the present, it is seen that the blanket may be subdivided into two regions, 1) a spherical shell reflector surrounding the core and consisting of heavy water, and 2) a thorium--heavy water mixture in a blanket surrounding the reflector such that the thermal neutron flux density at the reflector-blanket interface is essentially zero and also such that all neutrons entering the external breeding blanket may be assumed to be captured by thorium.

The requirements that the pile considered have

1. a spherical core containing a uniform density of 23 and a variable density of thorium in a heavy water moderator
2. a spherical shell reflector containing only heavy water
3. an external breeding blanket consisting of pure thorium or a thorium heavy water mixture
4. constant thermal flux throughout the core

is sufficient to define a two parameter set of piles. These parameters may be, for example, the radius of the core and the thickness of the reflector. However, for the present purpose one parameter will be taken as the 23 density in the core and the other parameter (which need not be specified in detail) will be fixed by the condition that the thorium density go to zero as the edge of the core is approached.*

* This assumption is made principally to simplify the discussion; in some uses it may also give the smallest actual mass of 23.

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We have thus defined a single parameter family of piles. Three such have been considered in some detail and their characteristics are summarised in Table I.

Pile	$\frac{N_{D_2O}\,\sigma_{a,D_2O}}{N_{23}\,\sigma_{23}}$	Mass of 23 (kg)	radius of core (cm)	radius of outside reflector surface (cm)	Neutron absorption per neutron absorbed in 23			
					Inside core		Outside core	
					in D_2O	in Th.	in D_2O	in Th.
A	0.028	3.7	104		0.028	0.89	0.42	
B	0.02	4.4	99	209	0.020	0.88	0.027	0.40
C	0.01	7.2	92	135	0.010	0.86	0.010	0.46

Table I Summary of characteristics of three piles with space uniform activity.

Pile A is the limiting case in which the mass of 23 is as low as is consistent with the hypothesis of a 23, heavy water pile with space uniform activity. Piles B and C are of more practical interest since they each involve sufficiently small moderator absorption that the breeding ratio may be greater than unity, providing other losses (eg. fission products, reprocessing etc) can be kept likewise small. Pile B will contain 3.9 kg of 23 and approximately 47 tons of heavy water whereas Pile C has 6.4 kg of 23 and about 13 tons of heavy water, assuming no heavy water in the breeding blanket. By increasing the mass of 23 still further, one can reduce the heavy water required, but the fact that in this case an appreciable portion of fast neutrons

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would leak out the reflector would probably make the specific arrangement considered here impractical.

In Figure 1 the neutron flux and thorium density in the core, are plotted against the radius for Pile B. Figure 2 is a plot of thorium dencity in which the abscissa is the relative volume rather than the radius.

It should be pointed out that the above results are based on a primitive calculation in which no consideration has been given to the engineering features of the pile, so that the estimates of mass of critical material required and losses do not take into account holdup, absorption of structural and cooling system, poisoning, etc. A Gaussian slowing down function using the experimentally determined age of thermal neutrons in heavy water was used throughout the calculations. It was also assumed that elementary diffusion theory was applicable in the reflector. A brief summary of the method of calculation is given below. The various material properties used are listed in Table II.

A. Heavy Water.

Density 1.107 gm cm^{-3}

Diffusion Length 171 cm

age of thermal neutrons 125 cm^2

$\lambda_{tr} = 2.5$ cm

B. U^{233}

$\sigma_a = 500$ barns $\eta = 2.34$

Table II. Material Properties Used in Calculations.

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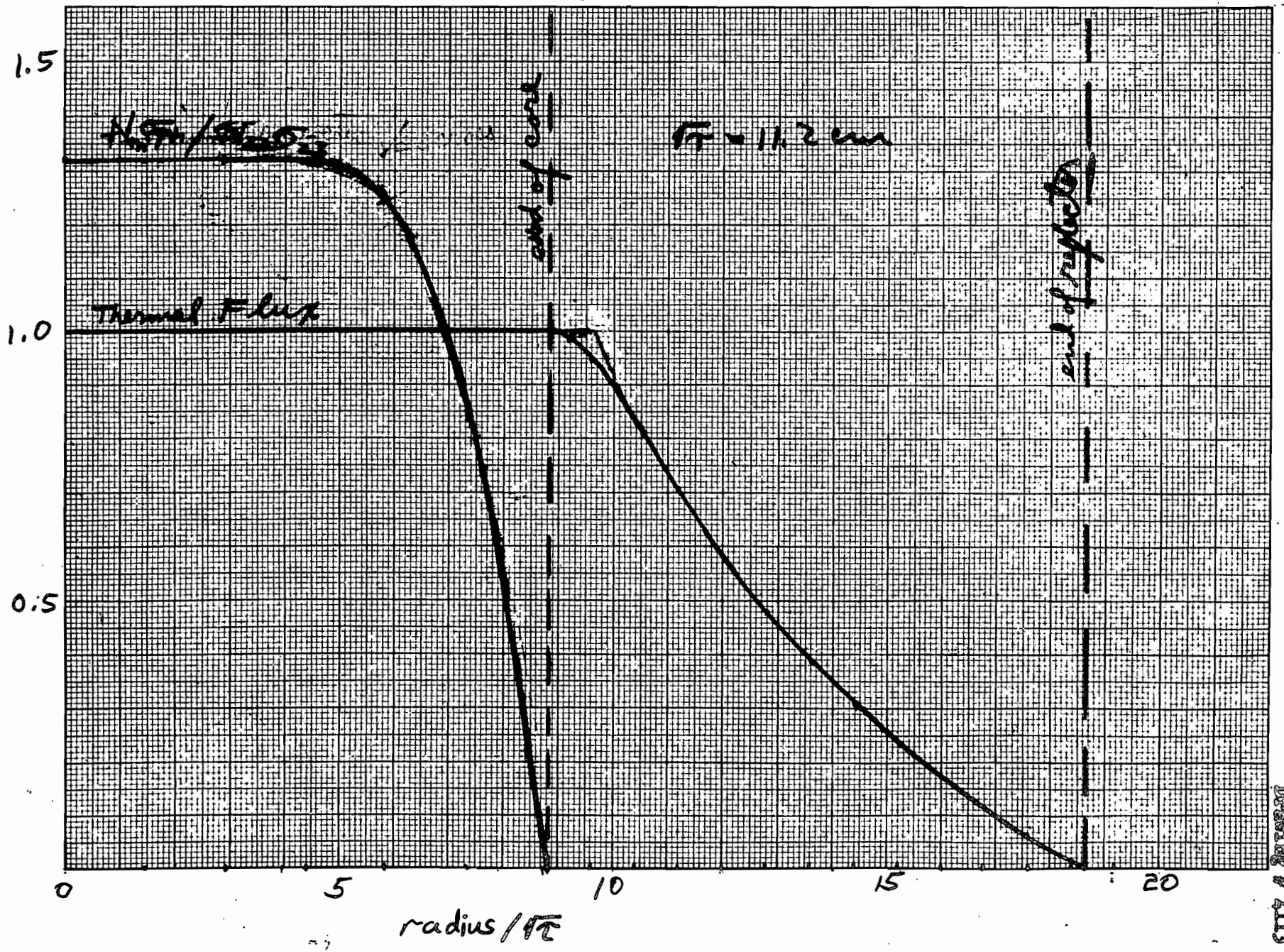

Fig. 1. Flux and Thorium Distribution, Pile B

Drawing # 413

Mathematical Formulation [redacted]

The notation used is summarized in Table III. It is assumed that a value of $(N\sigma)_u/(N\sigma)_m$ is specified. Then one has for the slowing down density

$$q(r) = 4\pi(N\sigma)_u \eta f_0 \int_0^a \rho^2 d\rho \left[\frac{1}{4\pi\tau^{3/2}} \frac{\tau}{4\rho r} \left\{ \Phi_1\left(\frac{\rho - r}{2\sqrt{\tau}}\right) - \Phi_1\left(\frac{\rho + r}{2\sqrt{\tau}}\right) \right\} \right]$$

where the [ ] is the slowing down density at a point ρ from the origin for a spherical shell of radius r with unit strength (1 fast neutron/sec. from the entire shelf). Carrying out the integrations one has

$$Q(x) = \frac{q(x\sqrt{\tau})}{f_0 (N\sigma)_u \eta} = \tfrac{1}{2}\Phi\left(\frac{x+b}{2}\right) - \tfrac{1}{2}\Phi\left(\frac{x-b}{2}\right) + \frac{1}{2x}\Phi_1\left(\frac{x+b}{2}\right) - \frac{1}{2x}\Phi_1\left(\frac{x-b}{2}\right) . \qquad (1)$$

Table III Notation

$(N\sigma)_u$	absorption cross-section of fissionable material,	cm^{-1}
$(N\sigma)_m$	absorption cross-section of moderator	cm^{-1}
$(N\sigma)_b$	absorption cross-section of breeding material	cm^{-1}
λ_t	transport mean free path of moderator	cm
L	diffusion length $\left(\frac{\lambda_t}{3(N\sigma)_m}\right)^{\frac{1}{2}}$ in moderator	cm
τ	age of thermal neutrons in moderator	cm^2
η	fission neutrons produced per neutron absorbed in fissionable material.	
x =	$L/\sqrt{\tau}$	
f =	nv; thermal neutron flux	$cm^{-2}sec^{-1}$
fo =	thermal neutron flux in core	$cm^{-2}sec^{-1}$
a =	$b\sqrt{\tau}$; radius of core	cm
$\beta\sqrt{\tau}$	outer radius of reflector.	cm

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$r = x\sqrt{\tau}$ radius of any point from system orgin cm.

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2}\, dx$$

$$\Phi_1(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} = \frac{d\Phi}{dx}$$

$q(r)$ slowing down density for thermal neutrons $cm^{-3}sec^{-1}$

$$Q\left(\frac{r}{\sqrt{\tau}}\right) = q(r) / \left[f_0 (N\sigma)_u \eta\right]$$

$$F\left(\frac{r}{\sqrt{\tau}}\right) = \frac{f(r)}{f_0}$$

It is well to note that if $b \geqslant 6$, as is true for all piles of the type under consideration, one has very closely

$$Q(x) = \tfrac{1}{2} - \tfrac{1}{2}\Phi\left(\frac{x-b}{2}\right) - \frac{1}{2x}\Phi_1\left(\frac{x-b}{2}\right) \qquad (1a)$$

Eq. (1a) is used throughout the work below.

Since the thermal flux f is constant in the core, one has, in the core,

$$\left[(N\sigma)_u + (N\sigma)_m + (N\sigma)_b\right] f_0 = q(r) \text{ or } \frac{(N\sigma)_b}{(N\sigma)_u} = \eta Q\left(\frac{r}{\sqrt{\tau}}\right) - \frac{(N\sigma)_m}{(N\sigma)_u} - 1 \qquad (2)$$

which gives the breeding material distribution in the core. Since $(N\sigma)_b/(N\sigma)_u$ decreases as r increases, one sees that it may go to zero at the edge of the core. The core size will be determined so that this happens.* Therefore, the equation to determine the core size is

$$0 = \eta Q(b) - 1 - \frac{(N\sigma)_m}{(N\sigma)_u} \text{ or } \tfrac{1}{2}\eta\left(1 - \frac{2}{\sqrt{\pi}\, b}\right) = 1 + \frac{(N\sigma)_m}{(N\sigma)_u} \qquad (3)$$

*See footnote on page 2.

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The thermal neutron flux f in the reflector is then the solution of

$$\frac{\lambda' t}{3}\left(f'' + \frac{2}{r} f'\right) - (N\sigma)_m f + q(r) = 0$$

which satisfies the boundary conditions

$$f(a) = f(b\sqrt{\tau}) = f_0$$

$$f'(a) = 0$$

riting $f(r) = f_0 F(r/\sqrt{\tau})$

one has

$$F(x) = \frac{1}{2x}\left[\frac{\gamma(N\sigma)_u}{\varkappa(N\sigma)_m}\int_0^{x-b} e^{y/\varkappa}\, Q(y+b)\,(y+b)\,dy - b - \varkappa\right] \exp\left(-\frac{x-b}{\varkappa}\right)$$

$$\frac{1}{2x}\left[\frac{\gamma(N\sigma)_u}{\varkappa(N\sigma)_m}\int_0^{x-b} e^{-y/\varkappa}\, Q(y+b)\,(y+b)\,dy + b - \varkappa\right] \exp\left(\frac{x-b}{\varkappa}\right) \qquad (4)$$

one uses (4) to determine the edge of the reflector by noting that F(x) goes to zero here, so that

$$F(\beta) = 0 \qquad (5)$$

defines β.

Using (3) to obtain b and (5) to obtain β, one thus has the pile dimensions for any specified $\frac{(N\sigma)_u}{(N\sigma)_m}$. If the value of $\frac{(N\sigma)_u}{(N\sigma)_M}$ is too small, Eqs. (3) and (5) will be incompatible. The limiting value of $(N\sigma)u/(N\sigma)m$ is found by setting $\beta = \infty$, i.e. by requiring the second [ ] in (4) to vanish. This condition together with (3) will determine both b and $(N\sigma)u/(N\sigma)m$ for the limiting case.

It is now feasible to determine the neutron economy. The neutrons absorbed in moderator inside the pile per neutron absorbed by fissionable material is

$$(N\sigma)m/(N\sigma)u \qquad (6)$$

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The neutrons lost to moderator in the reflector are given by

$$\frac{4\pi\,(N\sigma)_m \int_0^{\beta\sqrt{\tau}} f(r)\, r^2\, dr}{\frac{3}{4}\pi\tau^{3/2}\, b^3\, (N\sigma)_u\, f_o} = \frac{3}{b^3}\,\frac{(N\sigma)_m}{(N\sigma)_u}\int_0^{\beta-b} F(x+b)\,(x+b)^2\, dx \qquad (7)$$

The neutrons absorbed in breeding material inside the pile are given by

$$\frac{(N\sigma)_b}{(N\sigma)_u} = \frac{3}{b^3}\int_0^b \frac{(N\sigma)_b}{(N\sigma)_u}\, x^2\, dx \qquad (8)$$

The neutrons absorbed in the external blanket equals the outflow from the reflector*, i.e.

$$\frac{\left(-\frac{\lambda t}{3} f'(\beta)\right) 4\pi(\beta\sqrt{\tau})^2}{\frac{4}{3}\pi\tau^{3/2}\, b^3\, f_o\, (N\sigma)_u} = -\gamma^2\,\frac{3\beta^2}{b^3}\, F'(\beta)\,\frac{(N\sigma)_B}{(N\sigma)_u} \qquad (9)$$

Calculations presented previously were made with the above formulae and the constants of Table II.

*To this must be added the neutrons slowing down in the blanket, but this was negligible for the cases considered.

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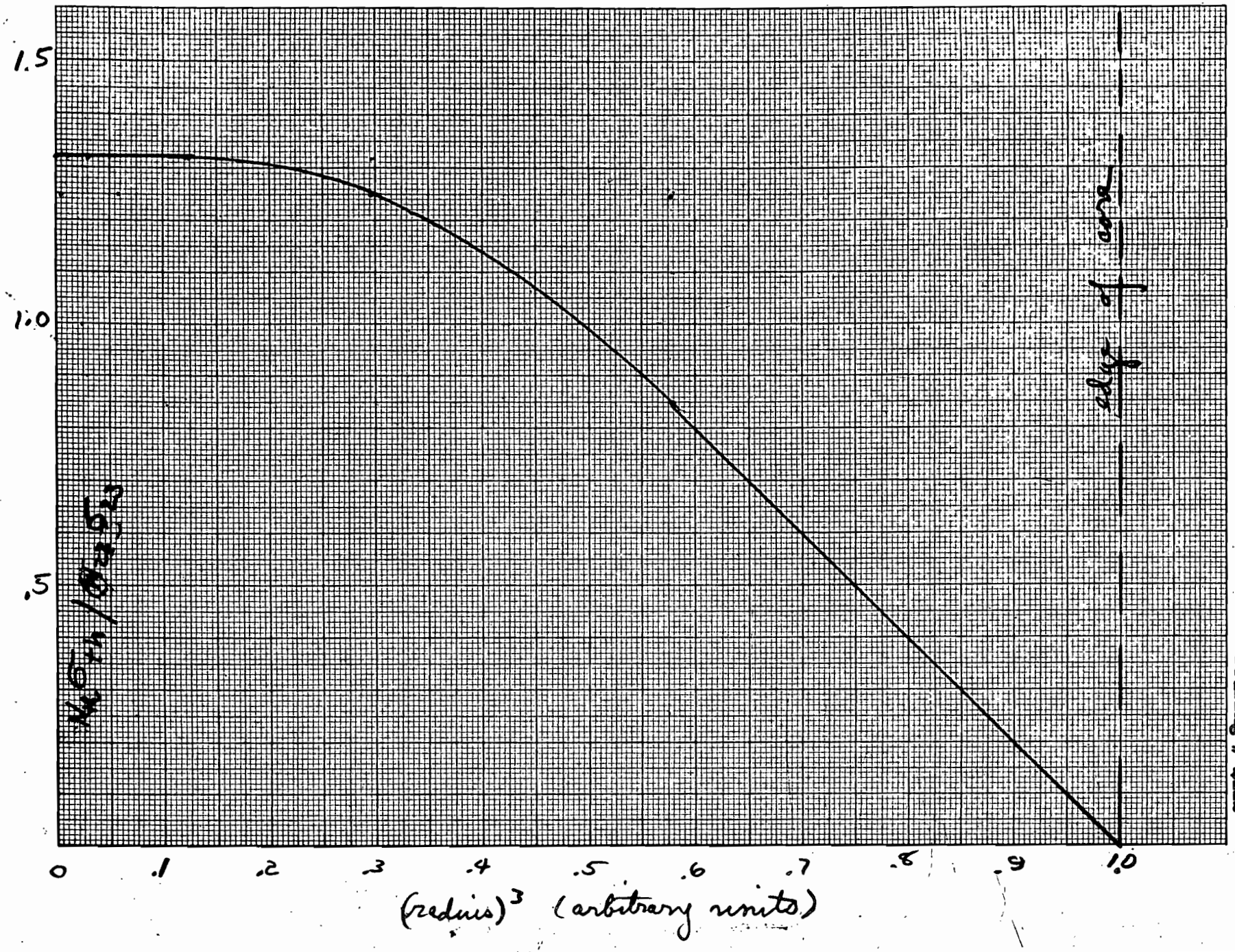

Drawing # 4112

Fig. 2. Thorium D, distribution, pile B.

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