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SCATTERING AND ABSORPTION OF NEUTRONS BY POLARIZED NUCLEI

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SCATTERING AND ABSORPTION OF NEUTRONS BY POLARIZED NUCLEI

M. E. Rose

It is the purpose of the following to examine, from the theoretical point of view, certain experiments which one might be able to perform with polarized neutron beams incident on polarized nuclei. In this note we consider (a) coherent and incoherent scattering in a crystal taking into account elastic scattering only, (b) scattering and absorption at higher energies where diffraction effects are not present and only a single nucleus need be considered, (c) the determination of angular momenta of levels of compound nuclei, and (d) the process of polarization of resonance neutrons by polarized absorbers.

An attempt has been made to determine what nuclear information could be obtained in a practical way by determining in what manner various effects depend on the polarization of nuclei and neutrons. It may turn out that certain suggestions involve virtually impossible measurements. However, the discussion given below is based on the feeling that at present it is premature to estimate what nuclear polarizations will eventually be attainable.

1. Scattering by Crystals

The general formalism has been given by W. G. Pollard*. The

* Forthcoming report entitled "Effect of Spin Dependent Interactions in Diffraction."

procedure is the usual one employed in neutron scattering problems of using the Born approximation with a δ-function interaction between neutron and nuclei. If the νth nucleus is located at the position $\underline{r}_\nu$ the interaction energy at the neutron position $\underline{r}$ is

$$V(\underline{r}) = -\frac{\hbar^2}{2m}\sum_\nu (a_\nu + b_\nu\, \underline{i}_\nu \cdot \underline{s})\,\delta(\underline{r} - \underline{r}_\nu) \tag{1}$$

where i_ν is the nuclear spin, s the neutron spin and a_ν, b_ν are the scattering amplitudes giving the coherent and incoherent scattering in the unpolarized case.

$$a_\nu = \frac{4\pi}{k}\;\frac{i_\nu\,\eta_0 + (i_\nu + 1)\eta_1}{2i_\nu + 1}$$

$$b_\nu = 2\,\frac{4\pi}{k}\;\frac{\eta_1 - \eta_0}{2i_\nu + 1} \tag{2}$$

where η_0 and η_1 are the phase shifts for total angular momentum $j = i + \frac{1}{2}$ and $j = i - \frac{1}{2}$ respectively. In (2) k is the magnitude of the wave vector of the incident neutrons.

In terms of this notation Pollard's result for the scattering per unit solid angle, wherein the neutron wave vector changes from $\underline{k}_1$ to $\underline{k}_s$, $|\underline{k}_1| = |\underline{k}_s|$, is

$$\sigma(\vartheta) = \frac{1}{(4\pi)^2}\sum_{\nu\nu'}\Big\{a_\nu^* a_{\nu'} + \tfrac{1}{4}|b_\nu|^2 i_\nu(i_\nu+1)\delta_{\nu\nu'} + \tfrac{1}{4}b_\nu^* b_{\nu'} m_\nu m_{\nu'}(1-\delta_{\nu\nu'})$$
$$+ \tfrac{1}{4}(|c+|^2 - |c-|^2)\Big[2a_\nu^* b_{\nu'} m_{\nu'} + 2a_{\nu'} b_\nu^* m_\nu - |b_\nu|^2 m_\nu \delta_{\nu\nu'}\Big]\Big\} e^{i\xi_{\nu\nu'}} \tag{3}$$

Here

$$\xi_{\nu\nu'} = (\underline{k}_1 - \underline{k}_s)\cdot(\underline{r}_\nu - \underline{r}_{\nu'})$$

Also m_ν is the z component of spin of the νth nucleus and $|c+|^2$, $|c-|^2$ are the probabilities that the z component of the spin of the incident neutron is $\frac{1}{2}$, $-\frac{1}{2}$ respectively. Of course, $|c+|^2 + |c-|^2 = 1$. The neutron polarization f_n is the average value of the s_z divided by the maximum value $\frac{1}{2}$. Evidently,

$$f_n = |c+|^2 - |c-|^2 \qquad (4)$$

It is clear that the first two terms of (3) give the scattering for unpolarized neutrons and unpolarized nuclei. The coherent scattering is

$$(\sigma_o(\vartheta))_c = \frac{1}{(4\pi)^2}\left|\sum_\nu a_\nu\, e^{-i\xi_\nu}\right|^2 \qquad (5)$$

and the (isotropic) incoherent scattering is

$$(\sigma_o(\vartheta))_i = \frac{1}{4}\,\frac{1}{(4\pi)^2}\sum_\nu |b_\nu|^2\, i_\nu\,(i_\nu + 1) \qquad (6)$$

The third term, as will be shown below makes a contribution to the cross section provided that the nuclei are partially polarized, even though the neutrons are not. The origin of this contribution can be understood from the fact that it expresses the correlation in spin directions of pairs of nuclear scattering centers in the lattice. The remaining terms arise only when both neutrons and nuclei are polarized.

We must now average (3) over all magnetic substates of the nuclei. Each nucleus orients itself in the external field practically independently of the others. Denoting the probability that $i_{\nu z}$ have the value m_ν by $p(m_\nu)$ we have

$$\overline{m_\nu} = \sum_{m_\nu} m_\nu\, p(m_\nu) = f_N\, i_\nu \qquad (7)$$

and

$$\overline{m_\nu\, m_{\nu'}} = \sum_{m_\nu} m_\nu \ p(m_\nu) \sum_{m_{\nu'}} m_{\nu'}\, p(m_{\nu'}) = f_N\, f_{N'}\, i_\nu\, i_{\nu'} \qquad (8)$$

where f_N is the polarization of the nuclei. We provide for the possibility of non-mono-nuclear lattices by the prime on $f_{N'}$. Both f_n and f_N vary in the range -1 to 1 the extreme values corresponding to complete polarization. When f_n and f_N have the same sign we speak of parallel polarization, otherwise of anti-parallel polarization.

Writing the cross section in the form

$$\sigma = \sigma_0 + \sigma_1$$

where σ_0 is the cross section given by the sum of (5) and (6), and dropping the argument ϑ, we have

$$\sigma_1 = \frac{1}{4}\,\frac{1}{(4\pi)^2}\sum_{\nu\nu'} e^{i\xi_{\nu\nu'}} \Big\{ b_\nu^* \, b_{\nu'}\, i_\nu\, i_{\nu'}\, f_N\, f_{N'}\,(1-\delta_{\nu\nu'})$$

$$+ f_n\left[2a_\nu^*\, b_{\nu'}\, i_{\nu'}\, f_{N'} + 2a_{\nu'}\, b_\nu^*\, i_\nu\, f_N - |b_\nu|^2\, i_\nu\, f_N\, \delta_{\nu\nu'}\right]\Big\}$$

$$= \frac{1}{4}\,\frac{1}{(4\pi)^2}\Big\{\Big|\sum_\nu e^{-i\xi_\nu}\, b_\nu\, f_N\, i_\nu\Big|^2 - \sum_\nu \big|b_\nu\, f_N\, i_\nu\big|^2$$

$$+ 2f_n\left[\Big|\sum_\nu e^{-i\xi_\nu}\,(a_\nu + i_\nu\, b_\nu\, f_N)\Big|^2 - \Big|\sum_\nu e^{-i\xi_\nu}\, b_\nu\, f_N\, i_\nu\Big|^2\right.$$

$$\left. - \Big|\sum e^{-i\xi_\nu}\, a_\nu\Big|^2 - \tfrac{1}{2}\sum_\nu |b_\nu|^2\, f_N\, i_\nu\right]\Big\}$$

Here $\xi_\nu = (k_s - k_i) \cdot r_\nu$. These additional terms due to polarization are evidently made up of contributions to both coherent and incoherent scattering. These are respectively,

$$(\sigma_1)_c = \frac{1}{4} \frac{1}{(4\pi)^2} \left\{ \left| \sum_\nu e^{-i\xi_\nu} b_\nu i_\nu f_N \right|^2 + 2f_n \left[\left| \sum_\nu e^{-i\xi_\nu} (a_\nu + i_\nu b_\nu f_N) \right|^2 - \left| \sum_\nu e^{-i\xi_\nu} b_\nu i_\nu \right|^2 - \left| \sum e^{-i\xi_\nu} a_\nu \right|^2 \right] \right\} \tag{9}$$

and

$$(\sigma_1)_i = -\frac{1}{4} \frac{1}{(4\pi)^2} \left\{ \sum_\nu \left| b_\nu \right|^2 f_N^2 i_\nu^2 + f_n \sum \left| b_\nu \right|^2 f_N i_\nu \right\} \tag{10}$$

We first consider a monatomic, mono-isotopic lattice. Then all a_ν, b_ν, i_ν, f_N are the same and for no absorption a and b are real quantities.

$$(\sigma_1)_c = \frac{1}{4} \frac{1}{(4\pi)^2} (f_N^2 b^2 i^2 + 4f_n f_N i a b) \left| \sum_\nu e^{i\xi_\nu} \right|^2 \tag{11}$$

and the extra incoherent scattering per atom is

$$(\sigma_1)_i = -\frac{1}{4} \frac{1}{(4\pi)^2} (f_N^2 b^2 i^2 + f_N f_n b^2 i) \tag{12}$$

Expressing the total differential cross sections in terms of the cross sections for the unpolarized case as follows

$$(\sigma_1)_c = (\sigma_0)_c \ (1 + \rho_c)$$

$$(\sigma_1)_i = (\sigma_0)_i \ (1 + \rho_i)$$

so that ρ_c and ρ_i are the relative increase in cross sections due to polarization, we have

$$\rho_c = \frac{f_N^2\, b^2\, i^2}{4a^2}\left(1 + 4\,\frac{f_n}{i\, f_N}\,\frac{a}{b}\right) \qquad (13)$$

and

$$\rho_i = -f_N\,\frac{f_N\, i + f_n}{i + 1} \qquad (14)$$

Equation (14) applies to the incoherent scattering provided that this is practically entirely due to spin-dependence of the nuclear forces. If there are other sources of incoherent scattering these will not be affected by the polarization. If the contribution to the incoherent scattering due to spin effects is σ_{si} and the additional incoherent scattering is σ_i' then the total incoherent scattering with and without polarization would be

$$(\sigma_0)_i = \sigma_{si} + \sigma_i'$$

and

$$(\sigma_0 + \sigma_1)_i = (1 + \rho_i)\,\sigma_{si} + \sigma_i'$$

If one knows the polarization factors f_N, f_n and the nuclear spin a measurement of incoherent scattering with and without polarization would, in principle, determine separately the spin and non-spin part of the incoherent scattering.

For both neutrons and nuclei completely polarized in the parallel sense $f_N = f_n = \pm 1$, we have $\rho_i = -1$ and the incoherent scattering disappears, as was to be expected. The minimum value of ρ_c is $-f_n^2$ so the coherent scattering can vanish only if $f_n = \pm 1$ and $f_N = \pm 2a/ib$. We also check that reversing both polarizations together has no effect but reversing either one changes the scattering and in a way which depends on the relative neutron and nuclear polarizations; that is, it is not true that ρ_c or ρ_i or the sum of polarization-dependent cross sections changes sign when the polarizations of neutrons (or nuclei) are changed in direction. This effect which is due to spin correlation of nuclear pairs is, of course, absent in the case of scattering or absorption by single nuclei, see Section 2. It is also observed the entire polarization effect is present only if the nuclear forces are spin-dependent, $b \neq 0$.

It is seen that the coherent scattering coefficient ρ_c depends only on the ratio a/b. This can be evaluated, within a sign, from the scattering by a single nucleus

$$\frac{a}{b} = \pm \frac{1}{2} \sqrt{\frac{i(i+1)\sigma_c}{\sigma_i}} \tag{15}$$

where σ_c and σ_i are the "coherent" and "incoherent" cross sections for the single atom

$$\sigma_c = \frac{4}{k^2} \left(\frac{i\,\eta_0 + (i+1)\,\eta_1}{2i+1} \right)^2 \tag{16a}$$

$$\sigma_i = \frac{4}{k^2} \frac{i(i+1)}{2i+1} (\eta_1 - \eta_0)^2 \tag{16b}$$

The sign of a/b can also be determined from studies of diffraction in the polarized case by observing whether the coherent scattering increases or decreases when the neutron polarization is increased. For parallel polarization an increase in coherent scattering with increase of f_n means $a/b > 0$ and a decrease means $a/b < 0$. In practise it would be simpler, to reverse the direction of neutron polarization. Then if the scattering with parallel and anti-parallel neutron polarization are σ_+ and σ_- respectively the magnitude and sign of ab is determined from a measurement of $\sigma_+ - \sigma_-$. In addition, if one measures the scattering with and without nuclear polarization, but with no neutron polarization in both cases, the difference in these two measurements gives b^2, that is, the true (spin-dependent) incoherent scattering without polarization. Even more directly, a measurement of coherent scattering without polarization gives a^2 and with ab determined one gets the spin-incoherent scattering from $b^2 = (ab)^2/a^2$. These measurements would be practical even with small nuclear polarization if the incoherent scattering amplitude $b \gg a$, the coherent scattering amplitude. The example given below for H, where this condition is fairly well fulfilled, amply illustrates the point.

It is also important to emphasize that here as in all cross section measurements one can obtain information only about products of scattering amplitudes. In the present case one obtains a^2, ab and b^2. Thus, one can change the sign of both a and b simultaneously without contradicting any experimental results. Therefore, at best, one can determine only the absolute magnitudes of the phase shifts $|\eta_0|$ and $|\eta_1|$ and their relative sign. This information is already more than one can obtain from diffraction with no polarization.

A numerical application will serve to indicate the magnitude of the expected effects. For hydrogen we have

$$\sigma_i = 18 \text{ barns}, \; \sigma_c = 2 \text{ barns}, \; i = \tfrac{1}{2}, \; a/b < 0$$

then

$$\rho_c = 3f_N^2 \, (1 - 1.15 \, f_n/f_N)$$

$$\rho_i = -\frac{2}{3} \, f_N \, (f_n + \frac{1}{2} \, f_N)$$

Neutron polarizations $|f_n| = 0.3$ have already been obtained. Assuming a nuclear polarization of 0.2 we have

Relative Changes in Scattering

Due to Polarization

f_n	0.3	-0.3	0
ρ_c	-.088	.328	.120
ρ_i	-.080	.040	-.013

the 41% increase in coherent scattering on reversal of the neutron polarization is an effect easily observable although the changes in other cases are not particularly large. These results would also indicate that diffraction of neutrons of known degree of polarization might be an accurate method of measuring small nuclear polarizations.

Considering a lattice composed of two types of nuclei with polarizations f_1 and f_2 we can easily obtain the relative changes in coherent and incoherent scattering in the case that the structure factors $\sum_\nu e^{i\xi_\nu}$ for the two nuclei have the same or opposite numerical value.

We find

$$\rho_c = \frac{1}{4}\,\frac{1}{(a_1 \pm a_2)^2}\left\{(b_1\, i_1\, f_1 \pm b_2\, i_2\, f_2) + 4f_n(a_1 \pm a_2)(b_1\, i_1\, f_1 \pm b_2 i_2\, f_2)\right\} \tag{17}$$

and the $\pm$ sign goes with ratio of structure factors $= \pm 1$, respectively. The incoherent scattering gives a coefficient

$$\rho_i = -\frac{b_1^2\, i_1^2\, f_1^2 + b_2^2\, i_2^2\, f_2^2 + f_n(b_1^2\, i_1\, f_1 + b_2^2\, i_2\, f_2)}{b_1^2\, i_1\,(i_1 + 1) + b_2^2\, i_2\,(i_2 + 1)} \tag{18}$$

which is, of course, independent of the values of the structure factors. It is easily checked that when $b_1 = b_2$, $a_1 = a_2$, $i_1 = i_2$, $f_1 = f_2$ these results reduce to (13) and (14) respectively, wherein (17) must be taken with $+$ sign. Again we check that for complete parallel polarization there is no incoherent scattering for the bi-nuclear lattice. In fact, from (10) this is seen to be generally true.

It is also seen that in such cases for which a diffraction peak is absent due to accidental cancellation of the coherent scattering in the unpolarized case, $a_1 \pm a_2 = 0$, the peak may be present with some intensity if appreciable nuclear polarization is achieved, since in this case

$$\sigma_{coh} \sim (b_1\, i_1\, f_1 \pm b_2\, i_2\, f_2)^2$$

From a measurement of the coherent scattering without polarization one can determine $(a_1 + a_2)^2$ and $(a_1 - a_2)^2$ if both peaks are of sufficient intensity. This gives the relative sign of a_1 and a_2. With nuclei polarized but neutrons unpolarized one can obtain, at least in principle, $(i_1 b_1 f_1 + i_2 b_2 f_2)^2$ and $(i_1 b_1 f_1 - i_2 b_2 f_2)^2$ and hence the relative sign of b_1 and b_2. With neutrons also polarized one gets the additional data $(a_1 + a_2)(i_1 b_1 f_1 + i_2 b_2 f_2)$ and $(a_1 - a_2)(i_1 b_1 f_1 - i_2 b_2 f_2)$. From this the relative signs of $(a_1 \pm a_2)$ and $(i_1 b_1 f_1 \pm i_2 b_2 f_2)$ are fixed for both ratios of the structure factors.

2. Scattering by a Single Nucleus

The foregoing can readily be specialized to the case of a single nucleus. Here the scattering is isotropic so that it will be more convenient to introduce the total cross section. This will again be denoted by σ. For the unpolarized case the cross section is simply $\sigma_o = \sigma_c + \sigma_i$ where σ_c and σ_i are given by (16). When both neutrons and nuclear are polarized we have

$$\begin{aligned}\sigma &= \sigma_o(1+\rho) \\ &= \frac{1}{4\pi}\left[a^2 + \frac{1}{4}\, i(i+1)b^2 + i\, f_n f_N\left(ab - \frac{1}{4}b^2\right)\right] \qquad (19)\end{aligned}$$

and

$$\rho = i\, f_n f_N \frac{\frac{b}{a}\left(1 - \frac{1}{4}\frac{b}{a}\right)}{1 + \frac{i(i+1)}{4}\frac{b^2}{a^2}} \qquad (20)$$

For H this gives

$$\rho_H = -\,0.95\, f_n f_N$$

and for $f_n = 0.3$, $f_N = 0.2$, we get a 5.7% decrease in total cross section.

This is a rather small effect. It may be noted that the primary information to be gained from scattering of polarized neutrons by polarized nuclei is a separate measurement of the cross sections for the two orientations of nuclear and neutron spin, i.e., from a comparison of the cross sections with and without polarization (either f_n or $f_N = 0$ and both f_n, $f_N \neq 0$) one obtains η_o^2 and η_1^2 separately. While such information can be obtained for slow neutrons from crystal diffraction data, and in the case of hydrogen (and possibly deuterium) from scattering in ortho- and para-molecules, it would be very desirable to know both phase shifts at all energies. From (20) it can be seen that the relative change in scattering cannot be greater, in order of magnitude, than $f_n\, f_N$ irrespective of the magnitude of b/a*. Therefore, it would be advantageous to attain large polarizations of neutrons and/or nuclei. There is also involved the question of obtaining polarized neutrons of higher than thermal energy. A possible method will be discussed in Section 4.

The total cross section with polarization can be written in terms of the phase shifts. Using (19) and (2) we find

$$\sigma = \frac{4\pi}{k^2}\left\{\frac{i\,\eta_o^2 + (i+1)\,\eta_1^2}{2i+1} + i\, f_n\, f_N\, \frac{\eta_1^2 - \eta_o^2}{2i+1}\right\} \qquad (21)$$

From this it can be seen that only the squares of the phase shifts enter and hence their sign is not determinable from a measurement of the total

* In this respect the scattering by a single nucleus differs from diffraction by a crystal because in the latter case one can measure the coherent scattering separately without polarizing nuclei or neutrons.

cross section. We can also write (21) in the form

$$\sigma = \frac{4\pi}{k^2}\left\{\frac{i\,\eta_o^2}{2i+1}(1 - f_n f_N) + \frac{(i+1)\,\eta_1^2}{2i+1}\left(1 + \frac{i}{i+1} f_n f_N\right)\right\}$$

$$= \frac{i(1 - f_n f_N)\,\sigma_{i-\frac{1}{2}} + (i+1)\left(1 + \frac{i}{i+1} f_n f_N\right)\sigma_{i+\frac{1}{2}}}{2i+1} \tag{22}$$

which represents a weighted average of the cross sections for the two j-values. From (22) we see that we can also write

$$\rho = \frac{i\, f_n f_N\,(\sigma_{i+\frac{1}{2}} - \sigma_{i-\frac{1}{2}})}{i\,\sigma_{i-\frac{1}{2}} + (i+1)\,\sigma_{i+\frac{1}{2}}} \tag{23}$$

The sign of ρ for given $f_N f_n$ is again seen to be an indicator of the comparative strength of nuclear forces for the two spin orientations corresponding to $j = i \pm \frac{1}{2}$.

From (23) and (16) we can express $\sigma_{i \pm \frac{1}{2}}$ in terms of the quantities capable of being measured:

$$\sigma_{i-\frac{1}{2}} = \sigma_o\left(1 - \frac{i+1}{i}\,\frac{\rho}{f_n f_N}\right) \tag{24a}$$

$$\sigma_{i+\frac{1}{2}} = \sigma_o\left(1 + \frac{\rho}{f_n f_N}\right) \tag{24b}$$

It is instructive to consider the limiting case of complete polarization even though this is a situation which one hardly expects to approach in practise. For the parallel allignment of spins, $f_n = f_N = \pm 1$ we have from (22)

$$\sigma = \sigma_{+} = \sigma_{i+\frac{1}{2}}$$

This is to be expected since the total z component of spin is $\pm(i+\frac{1}{2})$ which is a substate belonging to $j = i + \frac{1}{2}$ only. However, for the anti-parallel allignment, $f_n = - f_N = \pm 1$,

$$\sigma = \sigma_{-} = \frac{2i\ \sigma_{i-\frac{1}{2}} + \sigma_{i+\frac{1}{2}}}{2i+1}$$

and both cross sections $\sigma_{i\pm\frac{1}{2}}$ enter. The z component of total spin is $i - \frac{1}{2}$ which is a substate of both values of j. Only in the classical limit of large spin i does $\sigma_{-} = \sigma_{i-\frac{1}{2}}$.

3. Angular Momentum of Levels of Compound Nucleus

Perhaps the most readily obtainable information once one can produce polarized neutrons of resonance energies, is the determination of the angular momentum of levels of compound nuclei. Actually this information can be obtained from scattering as well as absorption although in the former case it may be necessary to take into account the effects of multiple scattering and, in particular the depolarization effects resulting therefrom. It will be considered for the present that these effects are negligible; i.e., the target thickness is small compared to the scattering mean free path. This assumption is, of course, not restrictive when the absorption cross section is much larger than the scattering cross section.

Considering a beam of slow neutrons incident on a nucleus with spin i, the total angular momentum of the combined system is $J = i \pm \frac{1}{2}$.

Thus, levels of the compound nucleus with both spin values can be involved in the scattering or absorption. In the case of well-separated levels only one of these will be involved for resonance energy neutrons. Of course, the existence of the resonance implies well-separated levels. The problem is to determine which value of J refers to a particular resonance. As is shown below, this can be determined at once from the qualitative observation as to the direction in which the cross section changes when the polarization (of either neutrons or nuclei) is "turned on."

While the results obtained in Section 2 are applicable here this is not readily apparent since it was assumed that the phase shifts are real whereas absorption corresponds to complex phase shifts. Actually this difference is irrelevant. Only the properties of angular momentum operators are involved; that this is so is brought out more clearly by the following alternative procedure.

Denoting the wave functions of neutron, initial nucleus and compound nucleus by χ, ψ and Φ respectively we would write the cross section in the form*

$$\sigma = \text{const.} \sum_{m_i m} p(m_i\, m) \left| (\Phi_{JM}, \chi_{\frac{1}{2},m}\, \psi_{i,m_i}) \right|^2 \qquad (24)$$

where M, m_i and m are the z-components of the spins J, i and $\frac{1}{2}$ of compound nucleus, initial nucleus and neutron respectively; $p(m_i\, m)$ is the probability that $i_z = m_i$ and $s_z = m = \pm \frac{1}{2}$. The matrix element in (24) actually involves

*cf. Bethe and Placzek, Phys. Rev. 51, 481 (1937).

only the part of the respective wave functions which refer to the angular momenta. All other factors, widths, resonance energy, etc., are in the constant factor in front. It has also been assumed that only one level is in resonance.

Corresponding to the fact that the addition of angular momenta and $\frac{1}{2}$ gives both resultants $i \pm \frac{1}{2}$ the wave function ϕ_{JM} can be expressed as a mixture (linear combination) of all wave functions corresponding to angular momenta which can give the required resultant in the sense of quantum mechanical addition of vectors. Thus,

$$\phi_{JM} = \sum_{m_i m} (i \tfrac{1}{2} m_i m \mid i \tfrac{1}{2} J M) \chi_{\frac{1}{2}m} \psi_{i, m_i} \tag{25}$$

where we have used the notation of Condon and Shortley* for the coefficients (transformation amplitudes for vector addition). Inserting (25) in (24) and using the fact that $\chi_{\frac{1}{2}m} \psi_{im_i}$ form an ortho-normal set of functions we find

$$\sigma = \text{const} \sum_{m_i m} p(m_i m) \left| (i \tfrac{1}{2} m_i m \mid i \tfrac{1}{2} J M) \right|^2 \tag{26}$$

The coefficients which are needed here are listed below:

J	$m = \frac{1}{2}$	$m = -\frac{1}{2}$
$i + \frac{1}{2}$	$i + m_i + 1$	$i - m_i + 1$
$i - \frac{1}{2}$	$i - m_i$	$i + m_i$

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* Cf. Condon and Shortley, Theory of Atomic Spectra, pp 73-78

For $J = i + \frac{1}{2}$ we then have

$$\sigma = \text{const.} \sum_{m_i} p(m_i) \left\{ \left| c_+ \right|^2 (i + m_i + 1) + \left| c_- \right|^2 (i - m_i + 1) \right\}$$

where we have used

$$p(m_i, \pm \tfrac{1}{2}) = p(m_i) \left| c_\pm \right|^2$$

and $p(m_i)$ is the probability that $i_z = m_i$ which is, of course, independent of m. We now use

$$\sum p(m_i) m_i = f_N \, i$$

and evaluate the constant so that for unpolarized nuclei (or neutrons) $\sigma = \sigma_{i+\frac{1}{2}}$. Then with (4) we find

$$\sigma = \sigma_{i+\frac{1}{2}} \left(1 + f_n f_N \frac{i}{i+1}\right) \tag{27}$$

Similarly, if the resonance level has $J = I - \frac{1}{2}$ we find

$$\sigma = \sigma_{i-\frac{1}{2}} \left(1 - f_n f_N\right) \tag{28}$$

It will be seen that the total cross section (22) obtained in Section 2 is simply the sum of (27) and (28) weighted with the factors $i/2i + 1$ and $(i + 1)/2i + 1$ which are simply the relative statistical weights of the two resonance levels. In the resonance case considered here either $\sigma_{i+\frac{1}{2}} \gg \sigma_{i-\frac{1}{2}}$ or $\sigma_{i-\frac{1}{2}} \gg \sigma_{i+\frac{1}{2}}$ so that either (27) or (28) represents the total cross section.

From (27) and (28) it is clear that one needs only to observe whether the cross section increases or decreases when the neutrons polarization is changed from zero to a value corresponding to parallel spin allignment with the nuclei. In the former case one concludes unequivocally that $J = i + \frac{1}{2}$ and in the latter case one has $J = i - \frac{1}{2}$. The magnitude of the effect is of order $f_n f_N$ so that for the example considered above ($f_n = 0.3$, $f_N = 0.2$) the cross section changes by about 6 percent. Such a change should not be too difficult to observe at least qualitatively.

4. Polarization of Resonance Neutrons

In the preceding section it was assumed that one can obtain polarized resonance neutrons. Actually, the known methods of polarizing neutrons are effective either for thermal neutrons (polarization in ferromagnetic materials) or for very fast neutrons (scattering in He where the polarization is due to spin-orbit coupling, or scattering in heavy nuclei where it is due to the coupling of the neutron moment with the contracted electric field of the nuclear charge). However, it is evident that polarized neutrons of resonance energies can also be produced by virtue of the large difference in cross section for the two angular momentum values $J = i \pm \frac{1}{2}$. For this it is necessary that the target nuclei be polarized. It will be seen that the neutron polarization produced can be made large if sufficient neutron flux is available.

The intensity of polarized neutrons as well as their polarization is easily calculated. Let I_+ and I_- be the transmitted intensities of neutrons with $m = +\frac{1}{2}$ and $-\frac{1}{2}$ respectively. Then the total intensity is

$$I = I_+ + I_- \tag{29a}$$

and the spin intensity in the transmitted beam is

$$\mathcal{J} = I_+ - I_- \tag{29b}$$

Denoting the distance of penetration into an absorbing foil by x and the resonance cross sections for $m = \pm\frac{1}{2}$ by $\sigma_\pm$ we have

$$\frac{dI_+}{dx} = -N\sigma_+ I_+ , \frac{dI_-}{dx} = -N\sigma_- I_- \tag{30}$$

where N is the number of absorbing nuclei per unit volume. Considering first the case that the resonance level corresponds to $J = I + \frac{1}{2}$ we have

$$\sigma_\pm = \sigma_o \left(1 \pm \frac{i\, f_N}{i + 1}\right) \tag{31}$$

where $\sigma_o = \sigma_{i+\frac{1}{2}}$ is the cross section for no polarization.

From (29) and (30) we have

$$-\frac{dI}{dx} = N\sigma_o \left(I + \frac{i\, f_N}{i + 1}\mathcal{J}\right)$$

$$-\frac{d\mathcal{J}}{dx} = N\sigma_o\left(\mathcal{J} + \frac{i\, f_N}{i + 1} I\right) \tag{32}$$

In (30) we have assumed no depolarization effects due to multiple scattering which will be fully justified for the case of large absorption. In any case, since the scattering is isotropic any neutrons which becomes depolarized by scattering are effectively removed from the transmitted beam. This is in contrast to the situation in which neutrons are polarized by the long range force fields such as are operative in the ferromagnetic or contracted Cunlomb field scattering. In these cases the scattered neutrons are, for the most part, deflected through small angles.

The solution of (32) subject to the initial conditions

$$I = 1, \qquad \mathcal{J} = 0 \qquad \text{at } x = 0$$

is

$$I = e^{-N\sigma_o x} \cosh \frac{i\, N\, \sigma_o\, f_N}{i+1}\, x \tag{33}$$

and

$$\mathcal{J} = -e^{-N\sigma_o x} \sinh \frac{i\, N\sigma_o\, f_N}{i+1}\, x \tag{34}$$

The polarization of the transmitted neutrons is

$$f_n = \mathcal{J}/I = -\tanh \frac{i\, N\sigma_o\, f_N}{i+1}\, x \tag{35}$$

For $J = i - \frac{1}{2}$ we obtain the corresponding results by replacing $i\, f_N/i+1$ by $-f_N$. Then

$$I = e^{-N\sigma_o x} \cosh N\, \sigma_o\, f_N\, x \tag{33a}$$

$$\mathcal{J} = e^{-N\sigma_o x} \sinh N\sigma_o\, f_N\, x \tag{34a}$$

and

$$p = \tanh N\sigma_o\, f_N\, x \tag{35a}$$

It will be noted that for $J = i + \frac{1}{2}$ the neutrons are polarized in the antiparallel sense. This is a consequence of the fact that the cross section in this case is larger when the neutron spin is parallel to the nuclear polarization. When $J = i - \frac{1}{2}$, the reverse is true and the neutrons are polarized parallel to the nuclear polarization. Using an analyzer to detect the neutron polarization one could obtain the J value again.

It also follows from these results that while increasing nuclear polarization produces larger neutron polarizations the latter is not necessarily limited by the former. To produce neutrons polarized to a much greater extent than the nuclear allignment it is only necessary to use a thick absorber so that the preferential absorption can eliminate more and more of the neutrons with one spin component. It follows, therefore, that the essential limitation is one of neutron flux and detection sensitivity.