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PHYSICS DIVISION

REVISED DRAFT OF GLOSSARY IN TERMS OF REACTOR THEORY

NICHOLAS M. SMITH, JR.

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REVISED DRAFT OF GLOSSARY IN TERMS OF REACTOR THEORY

Nicholas M. Smith, Jr.

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REVISED DRAFT OF GLOSSARY IN TERMS OF

REACTOR THEORY

Nicholas M. Smith, Jr.

The attached glossary of terms used in reactor theory has been compiled as part of a larger glossary of terms in Nuclear Energy and Engineering, the drafting of which has been sponsored by the Association of Mechanical Engineers and the work carried out by a Glossary Committee of which Professor John Dunning of Columbia University is chairman.

The terms have been placed in seven groups for the purpose of compiling and reviewing the definitions:

I. General Physics

II. Reactor Theory

III. Nuclear Reactor Engineering

IV. General Chemistry

V. Chemical Engineering

VI. Biological and Medical

VII. Instrumentation

Similar work by other working groups, notably by I.R.E. and A.I.E.E. on instrumentation will be incorporated.

[redacted]

[redacted]

After declassification, where necessary, the various sections will be submitted to referees elected by the National Research Council, edited, combined in alphabetical form, and then submitted to a Glossary Review Committee appointed by the National Research Council and having representation by all of the interested professional societies (including the A.E.C.). This committee will endorse the glossary on behalf of the societies represented, and then turn it over to the American Standards Association for publication.

It is felt that in addition to filling an important need at the present time, the glossary is significant in that it is an initial step in the direction of declassified handbook compilation in this field.

The style chosen in this section is more for the trained engineer who is first approaching a study of reactor theory. However, whenever possible, the discussion is started with a general word definition which should serve the less technical reader.

An attempt has been made to link closely related concepts and in this sense the work is of a slightly encyclopedic nature. In particular a few terms like "Boltzmann equation" and "perturbation theory," etc. have been described in more detail.

There is little room for originality in a work of this sort and the author has drawn heavily from all available sources: the project

[redacted]

reports, standard texts, and particularly the lecture notes on Pile Physics I by Soodak and Campbell and on Pile Physics II by A. M. Weinberg. In a few instances it was found that it was necessary to generalize some concepts which were in use in the days of large, thermal, almost bare piles.

This printing is a first revised draft and is limited to a few copies for declassification purposes and for submission to committee members and referees.

[redacted]

NUCLEAR ENGINEERING GLOSSARY

Reactor Theory

compiled by

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Oak Ridge National Laboratory
Oak Ridge, Tennessee

"A" neutrons

Neutrons of such energy that they are strongly absorbable by silver. Usage is obsolete. See "neutron energy group."

absorption

A nuclear process in which a particle (i.e. a neutron, proton, alpha particle, mesotron, electron, or gamma ray, etc.) losses its identity by combining with another particle or nucleus. A common neutron absorption occurs when a neutron enters into a nucleus and a gamma ray is emitted (capture). Whenever a neutron enters a nucleus (is absorbed by) and another neutron is emitted the process is called "inelastic scattering" although it is a true absorption process since the identity of the original neutron is lost.

[redacted]

Glossary, Reactor Theory

absorption coefficient

The negative of the logarithmic change in the intensity of radiation due to absorption processes (poor geometry) in penetrating a thin lamina of material:

$$\tau_x = -\frac{d \log I}{dx} = -\frac{1}{I}\frac{dI}{dx}$$

where τ_x is the linear absorption coefficient I the intensity and dx the thickness of the lamina. If instead of the distance one uses the mass per unit area, dm, the mass absorption coefficient, τ_m is given by

$$\tau_m = -\frac{1}{I}\frac{dI}{dm}$$

If the absorption coefficient is constant the exponential absorption law results:

$$I/I_o = e^{-\tau_x x} \quad e^{-\tau_m m}$$

Many different symbols are used for the absorption coefficients.

See also: cross-section, absorption

absorption cross-section

See: cross-section, absorption.

[redacted]

Glossary, Reactor Theory

activation, A

By activation is meant a measure of the radioactivity produced in a detector (such as indium or gold) by neutron bombardment. The saturated activity is the value of the activity under constant bombardment for an infinite time. For practical purposes the saturated activity is reached in a few (five or six) mean lives. The activity, A, of a thin foil having N atoms placed for a time t in a flux ϕ, measured at a time T after removal from the flux bath will be

$$A = n\lambda = N\sigma_a \phi (1 - e^{-\lambda t}) e^{-\lambda T} ,$$

where σ_a is the microscopic absorption cross-section, λ is the radioactive decay constant, and n is the number of atoms present which are subject to radioactive decay. The saturated activity is reached whenever

$$e^{-\lambda t} << 1$$

say, when $e^{-\lambda t} = .01$, or when $t = \frac{4.6}{\lambda}$ i.e. when $t = 6.7\tau$; where τ is the half-life. Since waiting so long is an inefficient use of bombardment time, one is usually satisfied with

$$e^{-\lambda t} = 0.1 \quad \text{or} \quad t = 3.3\tau .$$

[redacted]

Glossary, Reactor Theory

active lattice

See: lattice, array.

adjoint equation

See: perturbation theory.

adjoint function

A solution of the adjoint equation. See: perturbation theory.

adjoint operator

Properly speaking, adjointness is a relation held between two operators. An operator P^+ is adjoint to an operator P if

$$\int_\tau u^+ \circ P v \, d\tau = \int_\tau v \cdot P^+ u^+ \, d\tau$$

where u is a solution of $P^+ u^+ = \lambda u^+$

and v is a solution of $P v = \lambda v$.

See: perturbation theory.

[redacted]

Glossary, Reactor Theory

<u>age (Fermi), τ</u>

One sixth the mean square radial distance between birth at energy E_o and the point where the neutron crosses energy E. τ is a function of energy and has units of length squared.

$$\tau(E) = \int_E^{E_o} \frac{\lambda_t}{3\xi\Sigma_s} \frac{dE}{E}$$

where E is the energy

λ_t is the transport mean-free-path

ξ is the logarithmic slowing-down decrement

and Σ_s the macroscopic scattering cross-section.

The Fermi Age gets its name since it has a spacial distribution of the slowing down density, q(E), a similar relation as that of time in the diffusion of heat.

Soodak defines the age such that $d\tau$ (Soodak) = $-d\tau$ (above);

$$\tau(E)_{Soodak} = \int_{E_{th}}^{E} \frac{\lambda_t}{3\xi\Sigma_s} \frac{dE}{E}$$

where

E_{th} is the thermal energy (1/40 ev.)

[redacted]

Glossary, Reactor Theory

<u>albedo</u> A (Soodak)
β (Weinberg)

The ratio of the back current of radiation across a boundary between two media to the forward or incident current; the reflection coefficient of a boundary. The albedo is a function of the geometry of the boundary surface and the angular distribution of the incident neutrons as well as a function of the scattering and absorption properties of the two media.

The albedo of an infinite block of a substance with a plane face to an incident current of neutrons in the approximation of elementary diffusion theory is given by:

$$\beta = \frac{1 - 2/3\, \lambda_t \mathcal{H}}{1 + 2/3\, \lambda_t \mathcal{H}}.$$

where λ_t is the transport mean-free-path and $\mathcal{H} = L^{-1}$ is the reciprocal of the diffusion length.

[redacted]

Glossary, Reactor Theory

<u>associated Legendre functions, $P_\ell^m(x)$</u> also known as <u>associated spherical harmonics</u>

The associated Legendre functions are particular solutions of the differential equation:

$$(1 - x)y'' + 2xy' + \left[\ell(\ell + 1) - \frac{m^2}{1 - x^2}\right] y = 0.$$

The particular solution referred to is

$$y = (1 - x^2)^{m/2} \quad \frac{d^m P_\ell(x)}{d x^m} = P_\ell^m(x),$$

where $P_\ell(x)$ is Legendre Polynomial (known also as surface zonal harmonic, or Legendre coefficients) and is given by the finite series obtained from:

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \quad \frac{d^\ell \left[(x^2 - 1)^\ell\right]}{d x^\ell}$$

The functions $\frac{d^m P_\ell(x)}{d x^m}$ are denoted by some authors as $P_\ell^{(m)}(x)$ and are called "Helmholtz functions".

<u>attenuation</u>

The reduction of intensity of radiation (neutrons, gamma rays, other particles, sound waves, radio waves, electrical signals, etc.) by means of an interposed medium or circuit.

[redacted]

Glossary, Reactor Theory

attenuation factor

The ratio of the incident intensity to the transmitted intensity. The attenuation factor is always greater than unity.

augmentation distance

See: extrapolated boundary.

Bessel functions

Solutions of the differential equation

$$\frac{d^2y}{dz^2} + \frac{1}{z} \frac{dy}{dz} + \left(1 - \frac{n^2}{z^2}\right) y = 0.$$

See, for instance, Whitaker and Watson, "A Course of Modern Analysis," Chapter XVII, p. 355 ff. Macmillan Company, N. Y. C. Also Watson, "Bessel Functions" Macmillan Company, N. Y. C., 1945.

See also discussion under wave equation.

biological shield

The material placed around a source of radiation to reduce the intensity of radiation leaking out to an amount permissible physiologically.

[redacted]

Glossary, Reactor Theory

Boltzmann equation of neutron diffusion

a. vector flux

b. scalar flux

c. Legendre polynomials expansion

d. s, p, d, f, g, h etc. scattering

e. one dimensional Boltzmann equation

f. transport theory

g. diffusion mean-free-path

h. generalized Fick's law

i. transport mean-free-path

j. elementary diffusion theory

An integro-differential equation of diffusion which takes into account the velocity and angular distribution of the neutrons in the (isotropic) medium, one form of which is:

$$\triangle \cdot \underline{F}(\underline{r},v,\underline{\Omega}) + N\sigma F(\underline{r},v,\underline{\Omega}) =$$

$$\int_{v'}\int_{\Omega'} F(\underline{r},v',\underline{\Omega}')\, N\sigma_s(v,v',\underline{\Omega},\underline{\Omega}')dv'd\underline{\Omega}' + S(\underline{r},v,\underline{\Omega}).$$

where $\underline{F}(\underline{r},v,\underline{\Omega})$ is the vector flux of neutrons $= n(r,v,\underline{\Omega})v\underline{\Omega}$ i.e. = neutron density in phase space: $n(\underline{r},v,\underline{\Omega})v\underline{\Omega}$ d(volume) = the number of neutrons/cm whose velocities lie in unit range around the vector $v\underline{\Omega}$; v = speed of neutrons; and $\underline{\Omega}$ = unit vector in arbitrary direction.

(continued)

[redacted]

Glossary, Reactor Theory

Boltzmann equation (continued)

$\underline{r}$ = position vector from origin

N = number of atoms/cm^3 of medium

$F(\underline{r}, v, \underline{\Omega})$ = scalar part of vector flux (it's magnitude)

σ = total (atomic) cross-section of atoms of medium with respect to neutron collisions

$\sigma_s(v, v', \underline{\Omega}, \underline{\Omega}')dvdv'd\Omega d\Omega'$ = differential scattering cross-section per atom of the medium for process of scattering a neutron from its original velocity between $\underline{v}'$ and $\underline{v}' + d\underline{v}'$ (speed v' and direction $\underline{\Omega}'$) to its final velocity between $\underline{v}$ and $\underline{v} + d\underline{v}$, and

$S(\underline{r}, v, \underline{\Omega})d(\text{volume})dvd\underline{\Omega}$ = source strength at $\underline{r}$ in number of neutrons per cm per second produced in the volume element with speed between v and $v + dv$ with direction between $\underline{\Omega}$ and $\underline{\Omega} + d\underline{\Omega}$.

When the neutron distribution depends only on one coordinate, say the x coordinate in the three-dimensional medium the angular distribution of the neutrons depends only on the cosine of the angle between $\underline{\Omega}$ and the $\underline{\Omega}'$ the directions before and after scattering, designated by μ_0. If the differential scattering cross-section is expressed in terms of μ_0 rather than $\underline{\Omega}$ there results:

$$\sigma_s(v, v', \mu_0) = 2\pi\sigma_s(v, v', \underline{\Omega}, \underline{\Omega}'),$$

(continued)

[redacted]

Glossary, Reactor Theory

Boltzmann equation (continued)

where $\sigma_s(v,v',\mu_o)d\mu$ is the v' into v cross-section for scattering neutrons from their original direction into an elementary ring on the unit sphere which subtends an angle between $\cos^{-1}\mu_o$ and $\cos^{-1}(\mu_o + d\mu_o)$ with the original direction.

If this latter cross-section is expressed as a sum of Legendre polynomials

$$\sigma_s(v,v',\mu_o) = \sum_{\ell} \frac{2\ell + 1}{2} \sigma_{s\ell}(v,v') P_{\ell}(\mu_o)$$

we have by the orthogonality theorem

$$\sigma_{s\ell}(v,v') = \int \sigma_s(v,v',\mu_o) P_{\ell}(\mu_o) d\mu_o .$$

The coefficient $\sigma_{s\ell}(v,v')$ of the ℓ-th harmonic gives the contribution to the total scattering from velocity v' into v of that particular harmonic. It is customary to use spectroscopic terminology and refer to σ_{so} as the cross-section for s (spherically symmetric) scattering, σ_{s1} the cross section for p-scattering, etc., where the letters s,p,d,f,g,h etc. are used for values of 0,1,2,3,4,5 etc. respectively.

It can be easily shown that $\sigma_{so}(v,v')$ is the total cross-section for scattering from v' into v, and that the cross-section for p-scattering, σ_{s1} is equal to the total scattering cross-

(continued)

[redacted]

Glossary, Reactor Theory

Boltzmann equation (continued)

section times the average cosine of the scattering angle, i.e.

$$\sigma_{s1}(v,v') = \mu_o \, \sigma_{so}(v,v').$$

If the Boltzmann equation is integrated with respect to the azimuthal angle around the x-axis by making use of the addition theorem of Legendre functions in order to express μ_o in terms of μ the cosine of the angle between $\underline{r}$ and the x-axis, and if the further restriction is made that $\sigma_{s\ell}(v,v')$ is zero unless $v = v'$ (i.e. no energy loss) there results the one-dimensional Boltzmann equation:

$$\frac{dF}{dx}(x,v,\mu) + N\sigma F(x,v,\mu) = 1/2 \sum_{\ell} (2\ell+1) \, N\sigma_s(v) P_\ell(\mu) \int_{-1}^{1} P_\ell(\mu')$$

$$F(x,v,\mu')d\mu' + S(x,v,\mu);$$

The central problem of transport theory is the calculation of the angular distribution of neutron velocity vectors in a medium.

Integrating the source free transport equation with respect to μ, and defining

$$\overline{\mu^2} = \frac{\int_{-1}^{1} \mu^2 F(x,v,\mu)d\mu}{\int_{-1}^{1} F(x,v,\mu)d\mu}$$

(continued)

[redacted]

Glossary, Reactor Theory

Boltzmann equation (continued)

and the diffusion mean-free-path, $\lambda_d(v)$ by

$$\lambda_d(v) = \frac{1}{N\sigma_{so}(1 - \overline{\mu_o}) + N\sigma_a}$$

there results the generalized Fick's law which states that the diffusion current is proportional to the gradient of the flux; (provided $\overline{\mu^2}$ is not a function of position):

$$j(x,v) = -\lambda_d \overline{\mu^3} \frac{dF_o}{dx}$$

where $F_o = \int_{-1}^{1} F(\underline{x},v,\mu)d\mu$.

Thus the diffusion coefficient of speed v is given by

$$D_o(v) = \lambda_d \overline{\mu^2}.$$

If the absorption is negligible, the diffusion mean-free-path becomes the transport mean-free-path, $\lambda_t(v)$:

$$\lambda_t(v) = \frac{1}{N\sigma_{so}(1 - \overline{\mu_o})} = \frac{\lambda_s}{1 - \overline{\mu_o}}$$

Fick's law, the basic equation of elementary diffusion theory, holds rigorously only for the case of monoergic neutrons and if furthermore, in the one dimensional case, $\overline{\mu^2}$ is independent of position. A more general restriction must be applied in the three dimensional case.

[redacted]

Glossary, Reactor Theory

Boltzmann equation of neutron diffusion, asymptotic solution of

Asymptotic solutions of the Boltzmann equation are solutions assuming the constancy of $\overline{\mu^2}$, the mean square of the cosine, averaged with respect to the angular distribution of the neutron flux at a point. It may be shown (for one diffusion in one direction) that

$$\overline{\mu^2}\,(x,v) = \frac{1}{3}\left[1 + 2\,\frac{F_2(x,v)}{F_o(x,v)}\right]$$

where $F_o(x,v)$ and $F_2(x,v)$ are the zeroth and second Legendre component of the flux $F(x,v,\mu)$. Thus the constancy of $\overline{\mu^2}$ depends on the constancy of the ratio $F_2(x,v)/F_o(x,v)$. There are two important cases in which this ratio is constant:

I. $$F(x,v,\mu) = \frac{1}{2}\,F_o(x,v) + \frac{3}{2}\,F_1(x,v)\,P_1(\mu)$$

In this first case it may be shown that the medium must be non-capturing, i.e., for this case simple diffusion theory applies only to non-capturing medium.

II. $F(x,v,\mu)$ is a product of a function of x (of spacial coordinates) and a function of μ. This case applies to a capturing medium, and the elementary diffusion solution

$$F(x,v,\mu) = f(\mu)\,e^{\pm\gamma x}$$

(continued)

[redacted]

Glossary, Reactor Theory

Boltzmann equation of neutron diffusion, asymptotic solution of (continued)

applies only if $\varkappa$ is a solution of a characteristic equation which is a function of the scattering cross-sections for s,p,d,f etc. scattering, and of σ_a. The approximation containing the first two Legendre scattering cross-section components (σ_{s0}, σ_{s1}) is:

$$\frac{N\sigma_{s0}}{2\varkappa}\log\frac{N\sigma+\varkappa}{N\sigma-\varkappa} = \frac{1+\dfrac{3N\sigma_{s0}\overline{\mu_0}N\sigma_a}{\varkappa^2}}{1+\dfrac{3N^2\sigma\sigma_a\overline{\mu_0}}{\varkappa^2}} \qquad (1)$$

where the relation $\sigma_{s1} = \overline{\mu_0}\,\sigma_{s0}$ has been used to simplify the expression on the right.

More commonly isotropic scattering is assumed ($\overline{\mu_0} = 0$), yielding

$$\frac{N\sigma_{s0}}{2\varkappa}\log\frac{N\sigma+\varkappa}{N\sigma-\varkappa} = 1. \qquad (2)$$

These equations have one pair of real roots ($\pm\varkappa$) which are less than $N\sigma$ in absolute value, and an infinite set of imaginary roots.

The solution of the characteristic equation yields the transport theory expression for the $\varkappa$ and the diffusion length L, and the diffusion constant D_0, since

(continued)

[redacted]

Glossary, Reactor Theory

Boltzmann equation of neutron diffusion, asymptotic solution of (continued)

$$L = \varkappa^{-1}$$

$$D_o = \frac{N\sigma_a}{\varkappa^2}.$$

If the capture cross-section is small compared to the scattering cross-section, the transcendental equation (1) can be solved by series expansion, yielding, correct to terms of the first degree in σ_a/σ:

$$\varkappa^2 = 3N\sigma(1 - \overline{\mu_o})\, N\sigma_a \left\{ 1 - \frac{4}{5}\,\frac{\sigma_a}{\sigma} + \frac{\sigma_a \overline{\mu_o}}{(1 - \overline{\mu_o})} + \cdots\cdots\cdots \right\}$$

Since the angular distributions in these two cases are very special ones, involving only a simple multiplicative constant, they are not sufficiently general to enable one to satisfy the boundary conditions near a source or a boundary. Hence, elementary diffusion theory, based on Fick's Law, holds strictly only for the asymptotic solutions of the Boltzmann equation.

[redacted]

Glossary, Reactor Theory

bombardment damage

The effect of radiation (neutrons, fission fragments, a particle, etc.) on a substance, by implication undesirable.

boundary conditions

Natural or artificial constraints to a physical situation made at the discontinuities (i.e., the boundaries or interfaces between media). Thus a piano string clamped at two points by bridges impose the boundary condition that the displacement of the string be always zero at these points. For a bare pile the boundary condition at the surface is that the return current of neutrons is zero.

breeder pile (mother, mature)

A nuclear reactor whose net breeding gain is greater than one, i.e., a reactor which produces more new fissionable atoms than are consumed in the chain reaction.

breeding gain, g

The ratio of the fissionable atoms produced from fertile material to fissionable atoms destroyed.

$$g = \eta - 1 - \text{losses}$$

where η = neutrons per thermal neutron absorbed in fissionable material. The losses include the neutron captures not leading to fission, or leading to the formation of a fissionable atom, as well as leakage neutrons not absorbed in fertile material. Thus $\eta - 1$ is the absolute maximum for a breeding gain.

[redacted]

Glossary, Reactor Theory

<u>breeding rate</u>

The rate at which fertile material is transformed into fissionable material in, say, grams per second.

<u>buckling, geometrical</u>, B_g^2 (some authors use B instead of B^2)

See discussion under wave equation. The geometric buckling is an eigenvalue resulting from a solution of the space form of the wave equation (the scalar Helmholtz equation) with boundary conditions. The condition of criticality of a nuclear reactor is that the geometrical and the material buckling shall be equal, (Soodak uses the name "Laplacian" and writes the scalar Helmholtz wave equation as $\Delta\phi - \nabla\phi = 0$).
where the symbol ∇ is the Laplacian and is the negative of the buckling B_g^2. Since the name Laplacian leads to confusion with the Laplacian operator ∇^2 (or Δ) this usage is not popular.

[redacted]

Glossary, Reactor Theory

<u>buckling, material</u>, B_m^2 (Weinberg), B_m, or $-\Delta_m$ (Soodak)

The value of B^2 in the characteristic equation

$$-L^2B^2 - 1 = \frac{k}{p}\,\bar{P}\,(E_s, B^2)$$

where L = diffusion length

k = multiplication constant

p = resonance escape probability

$\bar{P}_{oo}(E,B^2)$ = the three dimensional Fourier transform of the appropriate infinite slowing down kernel

The critical condition is that the material buckling shall equal the geometrical buckling.

<u>buckling vector, B^2</u>

The vector formed by considering as the ith component the buckling associated with the ith dimension. In Chartesian coordinates

$$\underline{B}^2 = \underline{i}B_x^2 + \underline{j}B_y^2 + \underline{k}B_z^2 \ .$$

<u>"C" neutrons</u>

By "C" neutrons is meant the neutrons of such energy that they are strongly absorbable in cadmium. "C" neutrons have energies from thermal up to 0.3 electron volts.

See: <u>neutron energy groups</u>.

[redacted]

Glossary, Reactor Theory

cadmium ratio

By cadmium ratio is meant the ratio $= \frac{A_z}{A_{Cd\ z\ Cd}}$, i.e., the ratio of the neutron-induced saturated activity, A_z, in an unshielded (element z) foil to the saturated activity of the same foil when it is covered with cadmium; $A_{Cd\ z\ Cd}$. It is used to determine the ratio of the total flux (thermal and epi-thermal) to that of epi-thermal neutrons alone, or in general to differentiate the effects of the total flux of neutrons and the total flux less the thermal neutrons. A high cadmium ratio at a point in a medium means that the ratio of thermal to fast flux is high there.

capture cross-section

See: cross-section, radiative capture.

cell

The smallest volume in a heterogeneous pile having symmetry is called the cell, usually taken with the uranium lump as the center.

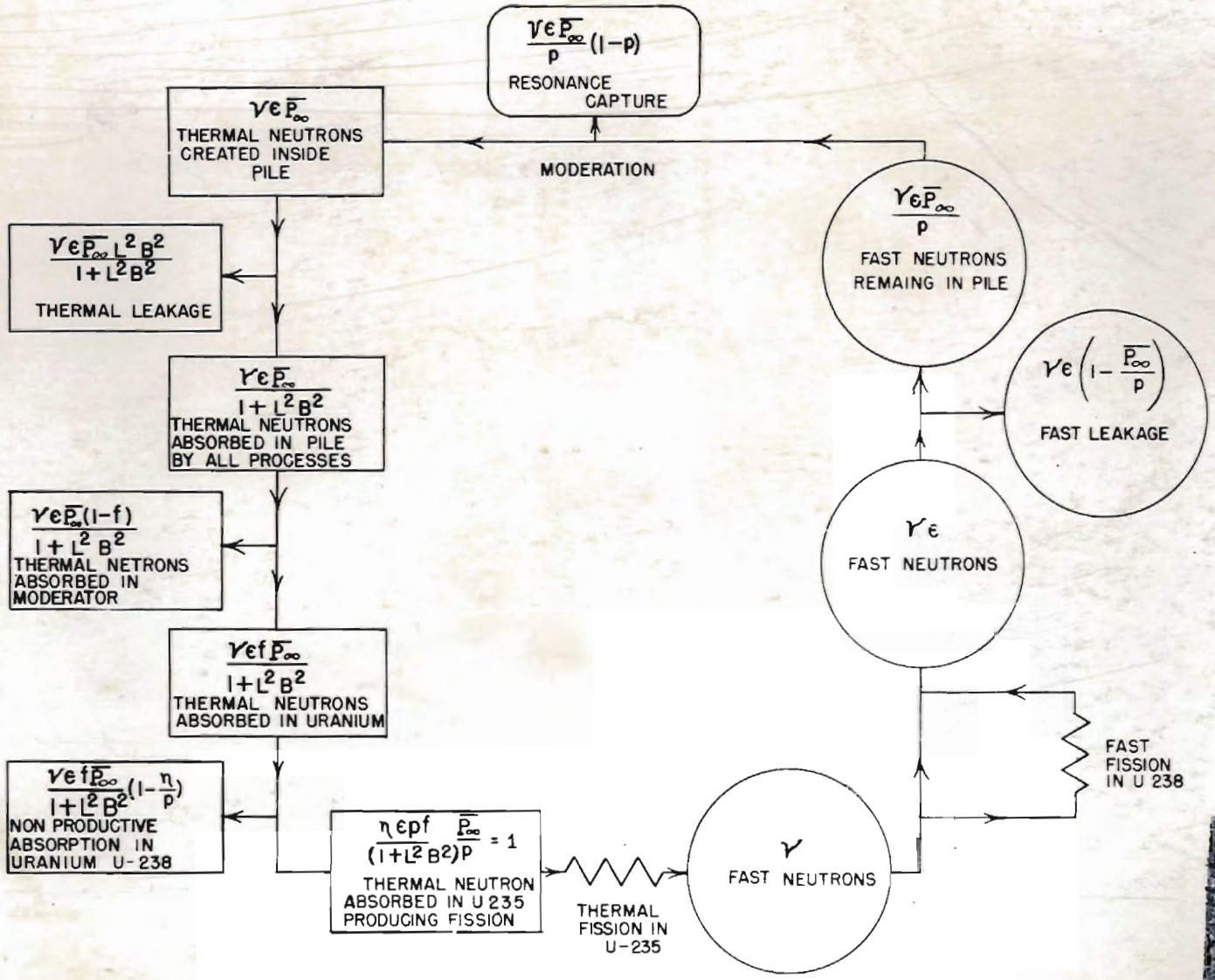
[redacted]

Glossary, Reactor Theory

chain reaction

A reaction in which one of the agents entering into the reaction is itself produced by the reaction. A chain reaction can be convergent, critical, or divergent. Referring to the neutron-fission chain reaction, a thermal neutron plus a fissionable atom, say uranium 235, produces a fission which yields two fission fragment atoms plus a number of fast neutrons plus energy. The fast neutrons can then be slowed down to form thermal neutrons. The reaction is critical if the average number of neutrons produced is equal to the average number consumed (by fissionable material, other material, or by leakage). If the number produced is less than the number consumed, the reaction is convergent; if more are produced than consumed the reaction is divergent.

See Figure.



THE THERMAL CHAIN - REACTION IN A BARE PILE
IN TERMS OF REACTION THEORY

[redacted]

Glossary, Reactor Theory

Definitions:

ν = neutrons/fission

ε = fast multiplication constant

p = resonance escape probability

$\bar{P}_{\infty} = \bar{P}_{\infty}(E_s, B^2)$, i.e., a function of the energy and the buckling, B^2, $\bar{P}_{\infty}$ is the three-dimensional Fourier transform of the slowing down kernel, $P_{\infty}(E_s, \underline{r}, \underline{r}')$

$P_{\infty}(E_s, \underline{r}, \bar{\underline{r}})$ = the probability that a fast neutron born at $\underline{r}'$ reaches energy E_s at $\underline{r}$ (for an infinite medium).

B^2 = the buckling, the lowest eigenvalue of the equation $\nabla^2\varphi + B^2\varphi = 0$, where ϕ is the flux.

L^2 = the square of the diffusion length.

f = thermal utilization, the ratio of the absorption in uranium to the total absorption of thermal neutrons.

η = fission neutrons per thermal neutron absorbed in uranium.

$\eta \varepsilon pf = k$ = the multiplication constant.

The equation

$$\frac{k\,\bar{P}_{\infty}}{(1 + L^2B^2)p} = 1$$ is called the "characteristic" equation.

[redacted]

Glossary, Reactor Theory

channelling effect

In a medium having many voided volumes inside it the transmission of radiation is greater than that of a homogeneous medium of the same average density and microscopic properties. This increase in transmission over that of the equivalent homogeneous medium is called the channelling effect.

channelling effect factor

The ratio of the thickness of a shielding medium having voided volumes inside it to the thickness of a homogeneous medium having the same average density, the same microscopic properties and the same attenuation. The channelling effect factor is always greater than unity.

characteristic equation

The equation

$$- L^2 B^2 - 1 = \frac{k}{p} \bar{P}_{\infty} (E_s, B^2) \quad ,$$

is known as characteristic equation, where

L is the diffusion length

B^2 a constant called the material buckling

k the multiplication constant

p the resonance escape probability

$\bar{P}_{\infty}(E_s, B^2)$ the Fourier transform of the appropriate infinite slowing down kernel

and E_s the energy of the slow neutron.

See: pile equation, asymptotic solution

[redacted]

Glossary, Reactor Theory

completeness (of a set of functions)

See: orthogonality and completeness

concentration factor, C

A term used by Christy to denote the ratio of the macroscopic absorption in uranium to that in the moderator.

control, absorber

By absorber control is meant the control of a nuclear reactor by means of a substance which absorbs neutrons. The substance is commonly cadmium or boron, in the form of a rod containing this substance inserted into the reactor. When the rod has such great absorption that all the neutrons (thermal) striking it are absorbed then in a sense the control is by configuration also, since the volume occupied by the rod is denied to the thermal neutrons. Since materials have high absorption cross-sections only for thermal or semi-fast neutrons, control by absorption is applicable to only thermal and intermediate piles.

control, configuration

Configuration control is called the control of the reactivity of a nuclear reactor through changes in the disposition of part of the active volume or part of the reflector volume.

[redacted]

Glossary, Reactor Theory

control rod

Control rods are given names corresponding to three different essential functions, namely:

a. Power control rod: a control rod affecting only a small change in reactivity which is a fine control of reactivity. The power control rod is normally designed so that it is impossible to go from delayed to prompt critical through motion through its entire range.

b. Safety rod: a control rod held in such a position that its motion will reduce the reactivity of a reactor to below critical. The activation of the rod is caused by some emergency condition, such as power level too high, etc.

c. Shim rod: a control rod affecting a relatively large change is reactivity of a reactor. It is used for coarse control of reactivity.

conversion

A term used by some authors to denote the breeding of fissionable material from fertile material by a nuclear reactor having a breeding gain less than unity.

[redacted]

Glossary, Reactor Theory

convolution

The convolution of a function g(x) with respect to a function h(x), is defined by

$$f(x) = \int_0^x g(t)\, h(x - t)\, dt$$

The convolution is also known under the name "resultant", "Faltung" by German and American authors, or "Composition" by the French. A convolution (or folding) is independent of the order in which the folding occurs, i.e.

$$f(x) = \int_0^x h(t)\, g(x - t)\, dt = \int_0^x g(t)\, h(x - t)\, dt$$

convolution of diffusion and Gaussian infinite slowing down kernels

See: kernels, etc.

convolution of diffusion and transport infinite slowing down kernels

See: kernels, etc.

convolution of Gaussian and transport infinite slowing down kernels

See: kernels, etc.

[redacted]

Glossary, Reactor Theory

core

The core of a nuclear reactor is the volume containing the fissionable material. In a heterogeneous pile the core is the volume of the active cells. The concept of the "core" loses its meaning in a pile where continuous variation of fissionable material is permitted.

cosine average

Average cosine of individual scattering; $\bar{\mu}_0$

On colliding with an atom a neutron will be deflected (in the laboratory system of reference) through an angle equal to $\cos^{-1} \mu_0$. The average value of μ_0 for all possible collisions weighted by their frequency of occurance is the average cosine of individual scattering. For scattering isotropic in the center of mass system

$$\bar{\mu}_0 = 2/3\ A \quad ;$$

where A is the atomic number of the atom with which the collision is made.

cosine average

Average of cosine squared of angular distribution of neutrons.

See: discussion of Boltzmann equation.

[redacted]

Glossary, Reactor Theory

critical equation

The equation (for the bare pile)

$$B_g^2 = B_m^2$$

is known as the critical equation where B_g^2 is the geometrical buckling (eigenvalues of the wave equation) and B_m^2 is the material buckling (solution of the characteristic equation).

Normally B_g^2 for a specific geometry is used, and B_m^2 for a particular slowing down kernel is used to form a specific critical equation.

In general a "critical equation" is one linking the dimensions of a pile (the core if the reflector is given) or the mass of fissionable material with the microscopic constants of the material of which the pile is composed.

critical experiment

The operation of an (usually unshielded) experimental nuclear reactor at very low energy levels for purposes of studying its characteristics; determination of critical mass and/or dimensions, flux distributions, etc. is called a critical experiment.

 [redacted]

Glossary, Reactor Theory

critical size

The physical dimensions of the core of a nuclear reactor when the k excess is zero is known as the critical size, the material and structure of the core, and the material and thickness of the reflector having been specified.

cross section

The probability of occurence of a nuclear reaction per atom per incident particle per square centimeter is called the cross section. The unit is the cm^2. Since nuclear radii are of the order of 1.5 to 8 times 10^{-12} cm, neutron reaction cross sections are of the order of 10^{-24} cm^2; a unit often referred to as the barn.

cross-section, absorption σ_a

The absorption cross-section is the sum of the cross-sections for all neutron reactions with an atom except $\sigma_{elastic}$ and $\sigma_{inelastic}$ (i.e., the sum of σ's for all reactions in which the end product is not a neutron and the original atom).

cross-section, capture

See: cross section, radiative capture

[redacted]

Glossary, Reactor Theory

cross-section corrected for kT, σ_{kT}

The cross-section corrected for kT is the cross-section for thermal neutrons corrected to that for a monoenergetic group having energy = kT where k is the Boltzmann constant per molecule, and T the absolute temperature Kelvin of the medium, that is, for neutrons having velocity v given by $v = \left(\frac{2kT}{m}\right)^{\frac{1}{2}} = 2200$ meters/sec. for $T = 293^{\circ}$ A and where $E(kT) = 0.025$ electron volts.

If an absorber obeying the 1/v law is placed in a non-absorbing medium of neutrons in thermal equilibrium and the average cross section $\overline{\sigma_a}$ measured, then σ_{kT} is given by

$$\sigma_a(kT) = \frac{2}{\sqrt{\pi}}\,\overline{\sigma_a} = 1.128\,\overline{\sigma_a} \quad .$$

When absorption measurements are made with thermal neutrons in equilibrium with paraffin and the cadmium absorbable component taken, called "C" neutrons, the average energy corresponding to this measurement is around 0.04 ev., then

$$\sigma_a(kT) = 1.3\,\sigma_{a(C)}$$

Absorption cross-sections measured in a graphite or heavy-water moderated thermal pile are made with neutrons having higher energy than 0.025 ev since uranium preferentially absorbs slow neutrons, here

$$\sigma_a(kT) = 1.2\,\sigma_{a(Pile)} \quad .$$

[redacted]

Glossary, Reactor Theory

cross-section, differential scattering, $\sigma(v', v, \underline{r}', \underline{r})$

By differential scattering cross-section is meant the v' to v and $\underline{r}'$ to $\underline{r}$ scattering cross-section per unit solid angle, where v' and v are the speed before and after collision, $\underline{r}'$ and $\underline{r}$ the direction unit vector before and after collision.

See discussion under Boltzmann equation.

cross-section, fission, σ_f

The cross-section for the reaction of neutron absorption followed by fission, symbolized the (n,f) reaction.

cross-section, macroscopic vs. microscopic

The microscopic cross-section is the cross section per atom of isotope, the macroscopic cross-section is the product of the microscopic cross-section by the number of atoms per unit volume i.e., it is the cross-section per cm^3. All cross-sections may be expressed in either the microscopic or macroscopic form.

[redacted]

Glossary, Reactor Theory

cross-section, per atom of element

(called also just "cross-section per atom")

The microscopic cross-section for a given nuclear reaction referred to the natural element, although the reaction involves only one of the natural isotopes. If A_i is the relative abundance ($\sum_i A_i = 1$) of the ith isotope, the (isotopic) cross-section, σ_i, is related to the cross-section per atom σ_{Ai} by

$$\sigma_{Ai} = A_i \, \sigma_i \quad .$$

cross-section, radiative capture

(also called just "capture cross-section.")

The cross section for the reaction $n + Z^A \longrightarrow Z^{A+1} + h\nu$, is called the cross-section for radiative capture, abbreviated as the (n, γ) process.

cross-section, scattering, σ_s, σ_{el}, σ_{inel}

The scattering cross-section, σ_s, is the sum of the cross-section for elastic scattering, σ_{el}, (energy conserved) plus that for inelastic scattering, σ_{inel}, (total absorption followed by emission of a neutron of lower energy).

[redacted]

Glossary, Reactor Theory

cross-section, scattering, elastic

See: cross-section, scattering

cross-section, scattering, inelastic

See: cross-section, scattering

cross-section, total

The total neutron cross-section is the sum of the cross-sections of all possible reactions which may take place; that is

$$\sigma = \sum_j \sigma_j$$

$$= \sigma_{el} + \sigma_{inel} + \sigma(n,\gamma) + \sigma(n,p) + \sigma(n,\alpha) + \sigma(n, 2n) + \text{etc.}$$

cross-section, transport scattering, σ_t

The product

$$\sigma_t = \sigma_s(1 - \bar{\mu}_o)$$

where σ_s is the scattering cross-section and $\bar{\mu}_o$ the average cosine of the scattering distribution is called the transport scattering cross-section. The factor $(1 - \bar{\mu}_o)$ is arbitrarily attached to the scattering cross-section in order that elementary diffusion theory may be applied.

[redacted]

Glossary, Reactor Theory

cross-section for s,p,d,f,g, etc scattering, $\sigma_{s\ell}$

The cross-section for s,p,d,f,g, etc. scattering, $\sigma_{s\ell}$ is the coefficient of the ℓth spherical harmonic in the expansion of the differential scattering cross-section, where "s" is used when $\ell = 0$, "p" when $\ell = 1$, "d" when $\ell = 2$, "f" when $\ell = 3$ and so on through the alphabet for higher values of ℓ.

It can be shown that the cross-section for s-scattering, σ_{so}, is the total scattering cross-section; and the cross-section for p scattering, σ_{s1}, the average cosine times σ_{so}, etc.,

$$\sigma_{s1} = \bar{\mu}_o \quad so \cdot$$

See discussion under Boltzmann equation.

"p" neutrons

Neutrons of such energy that they are strongly absorbable by rhodium. This term is now obsolete although it appears in the literature.

See neutron energy groups.

danger coefficient, k_d

The danger coefficient, k_d, is the per cent loss in thermal utilization for one per cent (by weight) impurity uniformly distributed in fissionable material; if original thermal utilization is unity.

del

See: gradient operator.

[redacted]

Glossary, Reactor Theory

delayed neutrons

See: prompt and delayed neutrons.

delta function, $\delta(x)$

The delta-function is defined to be zero everywhere except where its argument is zero, where it tends of infinity in such a manner that the integral

$$\int_{-\infty}^{+\infty} \delta(s)\, dx = 1,$$

so that

$$\int_{-\infty}^{+\infty} f(x)\, \delta(x - a)\, dx = f(a).$$

where f(x) is an arbitrary function of x. The delta function is not to be confused with the Kronecker delta, δ_{ij}.

The δ-function, named by Dirac, has been in use many years before the time of Dirac. It has sometimes been called an impulse function, or a jag-function.

[redacted]

Glossary, Reactor Theory

delta, Kronecker, δ_{ij}

The Kronecker delta, symbolized δ_{ij}, is a discontinuous factor which is taken to be unity when $i = j$ and zero when $i \neq j$, i.e.

$$\delta_{ij} = 1, \qquad i = j$$
$$= 0 \qquad i \neq j \quad .$$

It is not to be confused with the Dirac delta-function.

differential scattering cross-section

See: cross-section, differential scattering.

diffusion

The spreading out or dispersing of a quantity, (molecules, neutrons, electrons, light, etc.) in a medium through the action of random scattering.

diffusion coefficient of neutron density: D

The diffusion coefficient of neutron density, D, is the product of the neutron velocity and the diffusion coefficient of the neutron flux; the negative of the ratio of the net neutron diffusion current at a point and the gradient of the neutron density.

$D = v\, D_o$

v = neutron velocity

D_o = diffusion coefficient of neutron flux.

[redacted]

Glossary, Reactor Theory

diffusion coefficient; D_e

The constant of proportionality in Fick's law; the diffusion coefficient of flux is the negative of the net diffusion current at a point divided by the gradient of the flux. It is related to the microscopic constants of the medium to a fairly good approximation by the relation:

$$D_0 = \frac{N\sigma_a}{\varkappa^2} = \frac{\lambda_t}{3}$$

where $N\sigma_a$ = the macroscopic absorption cross-section of the medium, and

$\varkappa$ = the reciprocal of the diffusion length,

$\varkappa^2 = 3\,N\sigma_t N\sigma_a$.

See: Boltzmann equation, asymptotic solution of.

diffusion length, L (reciprocal $\varkappa = 1/L$)

One sixth the mean-square distance between the birth of a thermal neutron and the position of its absorption. Far from a plane neutron source in a sufficiently extended medium the neutron intensity decreases exponentially with relaxation length = L.

diffusion kernel

See: kernel, diffusion.

[redacted]

Glossary, Reactor Theory

diffusion mean-free-path

See: mean-free-path, diffusion and see also Boltzmann equation.

disadvantage factor, d

In a lattice, the ratio of the average neutron density in the moderator to the average density in the uranium lump, it depends on the energy of the neutrons as well as on the geometry and in a given lattice, its value is different for the thermal and epi-thermal neutrons.

displacement kernel

A kernel which is a function of the distance between two points in a medium. The slowing down kernels in general are displacement kernels in media of infinite extent, but not necessarily so in finite media. However, whenever, the finite and infinite slowing down kernels satisfy the same linear equation and by means of image piles, the slowing down kernel in a finite system may be taken to be the infinite displacement kernel.

See: kernel.

eigenfunction

The solution of a differential equation with boundary conditions having an eigenvalue.

See also wave equation.

[redacted]

Glossary, Reactor Theory

eigenvalue

In order to get a solution of a differential equation with boundary conditions it is sometimes necessary that an adjustable parameter can have only discrete values. These discrete values are called "eigenvalues."

See discussion under wave equation.

elementary diffusion theory

The diffusion theory based on Fick's Law, i.e., the proportionality between the diffusion current and the gradient of the flux. It is strictly valid only for monoenergetic neutrons and in a one-dimensional system in media where $\overline{\mu^2}$ (angular distribution) is not a function of position.

enrichment

Any process which changes the isotopic ratio; in reference to uranium, is a process which increases the ratio of U^{235} to U^{238} in uranium by separation of isotopes. Enrichment processes referred to are thermal diffusion, gaseous barrier diffusion, centrifugal and electromagnetic separation.

enrichment factor

The ratio of the isotopic ratio after enrichment to that before enrichment.

[redacted]

Glossary, Reactor Theory

epi-thermal neutrons

Neutrons having energies above the thermal range; also the term is sometimes used to mean neutrons having energies above the principle indium resonance (1.4 ev.) as measured by the cadmium ratio method.

Euler's equation

See: variation (Ritz) method

exponential pile

A pile of size less than critical which is used to measure the material buckling of a given material (uranium plus moderator or diluent) and thus to determine the critical size. With the usual placement of the source, and shape of pile, the decline in neutron density in one direction is exponential; hence, the name.

extrapolated boundary (augmentation distance)

The surface outside a nuclear reactor (active volume plus reflector) on which the neutron flux would be zero if extrapolated linearly from the true surface of the pile. The distance between the extrapolated boundary and the true boundary is the augmentation distance. The augmentation distance for a semi-infinite medium having a plane surface is .7104 λ_t where λ_t is the transport mean free path. The augmentation distance is in general a function of the curvature of the true surface and approaches 4/3 λ_t for a surface of infinite curvature (zero radius).

[redacted]

Glossary, Reactor Theory

fast effect

The increase in the number of neutrons in a thermal pile produced by fission with fast neutrons is called the fast effect.

fast multiplication factor

The number of neutrons slowed down to resonance energy (in absence of leakage) per neutron produced by thermal fission; it is greater than unity because of fissions caused by fast neutrons.

fast neutron

See: neutron, fast.

Fermi age

See: age.

[redacted]

Glossary, Reactor Theory

Fermi age model

A model for the slowing down of neutrons in a medium by elastic collisions in which the slowing down is assumed to follow a continuous function. If q_o is the slowing down density divided by the resonance escape probability ($= q(E)/p(E)$) then it is assumed that

$$q_o = \phi(E) \; \Sigma_s(E) \; \xi E$$

where ϕ (E) is the flux of neutrons having energy between E and E + dE

$\Sigma_s(E)$ is the macroscopic scattering cross section

ξ is the average logarithmic energy loss per collision, and

E is the energy.

There results that the spacial distribution of neutrons of energy E from a point source of neutrons of energy E_o is given by a three-dimensional Gaussian distribution. The term "age" arises from the fact that the differential equation for the slowing down density in terms of the "age" τ, where

$$\tau(E) = \int_E^{E_o} \frac{\lambda_t}{3 \xi \Sigma_s} \frac{dE}{E}$$

is

$$\nabla^2 q_o - \frac{\partial q_o}{\partial \tau} = 0;$$

(continued)

[redacted]

Glossary, Reactor Theory

Fermi age model (continued)

that is, the "age" τ takes a similar position to time in the differential equation for the flow of heat which is identical with the differential age equation.

See: kernel, Gaussian.

fertile material

A material which can be converted into fissionable material by radiative capture (n, γ) and subsequent disintegration. Uranium 238 is an example:

$$U^{238} + n \longrightarrow U^{239} \xrightarrow[\beta]{23\ \text{min}} Np^{239} \xrightarrow[\beta]{2.31\ \text{days}} Pu^{239}$$

with the quasi-stable end product being plutonium.

[redacted]

Glossary, Reactor Theory

Fick's law

The statement that the net diffusion current is proportional to the negative of the gradient of the neutron density is known as Fick's law, and is the basis of elementary diffusion theory. It is written:

$$\underline{j} = -D \text{ grad } n_o$$

where $\underline{j}$ is the vector net diffusion current, D the diffusion coefficient of neutron density, and n_o the neutron density. In neutron diffusion theory it is more convenient to write the law in terms of neutron flux, getting the generalized Fick's law:

$$\underline{j} = -D_o(v) \text{ grad } \phi \ ,$$

where $D_o(v)$ is the diffusion coefficient of neutron flux of speed v, and ϕ is the neutron flux.

finite life-time

See: life-time of a neutron in a finite medium.

fissile material

See: fissionable material.

[redacted]

Glossary, Reactor Theory

fission cross-section

See: cross-section, fission.

fission poisons

Fission fragments which have appreciable parasitic capture of neutrons are called fission poisons.

fission spectrum (of fission neutrons)

Because the fission neutrons are "evaporated" off of the excited fission fragments, they are not produced with discrete energy, but have energies ranging over a wide energy interval. The probability function, f(E) dE, giving the probability that a fission neutron once produced has energy between E and E + dE is known as the fission spectrum. f(E) may be normalized to unity; to η (the fission neutrons produced per thermal neutron absorbed); or to ν (the neutrons produced per fission).

fission yield

The fission yield of the ith fission product is defined as the number of atoms of the ith product formed per fission. The direct fission fragments forms the parent of a chain of radioactive transformations. Parent atoms may be formed at several places in the same chain. The sum of the yields of the parent fission fragments is within experimental error equal to 2.00: indicating binary fission.

[redacted]

Glossary, Reactor Theory

fissionable material (fissile material)

The term "fissionable material" is usually employed to refer to an element which will undergo fission on the absorption of a slow neutron. The word "fissile" is preferred by some authors who reserve "fissionable" to refer to spontaneous fission.

[redacted]

Glossary, Reactor Theory

Fourier transformation

A Fourier transformation may be regarded as a special case if a Laplace transformation in which the transform variable is a pure imaginary:

Provided that f(t) is piecewise continuous and that the integral $\int_{-\infty}^{+\infty} f(t)\,dt$ exists, we have

$$o(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t)\, e^{-i\omega t}\, dt$$

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} o(\omega)\, e^{-i\omega t}\, dt.$$

The Fourier integral theorem results from combining the two expressions:

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-i\omega(t-x)}\, f(x)\, d\omega dr$$

Tables of Fourier transform mates may be found in: George A. Campbell and R. M. Foster, "Fourier Integrals for Practical Applications", Bell Telephone System Technical Publications, Monograph B-584, September 1931; now published in book form by D. Van Nostrand Co., Inc., New York.

See: Laplace transformation.

[redacted]

Glossary, Reactor Theory

fundamental mode (of a nuclear reactor)

See: neutron distribution, fundamental and higher modes.

Gaussian distribution (normal error curve)

The Gaussian distribution is the probability distribution given for one dimension by:

$$p(u) = \sqrt{\frac{a}{\pi}}\ e^{-au^2} \quad ; \quad \int_{-\infty}^{+\infty} p(u)du = 1$$

where p(u) is the probability that the quantity under consideration shall have values between u and u + du. The parameter a is a measure of both the maximum and the width of the distribution. When u = 0, the maximum value of $p(u) = \sqrt{\frac{a}{\pi}}$ occurs. When $u = \frac{1}{\sqrt{a}}$, $p(u) = \frac{1}{e}\sqrt{\frac{a}{\pi}}$, that is 1/e of the maximum. The probable error is defined as that range of u for which the total probability is 1/2. Setting $\int_{-P.E.}^{+P.E.} p(u)\ du = 1/2$, we find from the tables that

$$P.E. = \frac{.4769}{\sqrt{a}}$$

If the transformation $x = u\sqrt{a}$ is made, the Gaussian distribution takes the form:

(continued)

[redacted]

Glossary, Reactor Theory

Gaussian distribution (normal error curve) (continued)

$$p(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}$$

Tables of $2 \int_0^t p(x)\, dx = \int_0^t \frac{2}{\pi} e^{-x}\, dx$ may be found in

Pierce "A Short Table of Integrals," Ginn and Company, New York, 1929.

The Gaussian distribution has been exploited widely in the representation of fluctuations of physical measurements although experience has shown that very few sets of experimental data appear to follow the law.

The distribution of velocities of the molecules in a gas at equilibrium was shown by Maxwell to be a three-dimensional Gaussian distribution.

The Random Walk in the limit of infinitesimally small steps also approaches the Gaussian distribution.

Gaussian slowing down kernel

See: kernel, slowing down-special case: Gaussian

Gaussian kernel with fission spectrum spread

See: kernel, slowing down - special case: Gaussian with fission spectrum spread.

[redacted]

Glossary, Reactor Theory

generation time

The generation time is a measure of the average time between successive generations of a neutron fission cycle. The generation time is essentially a property of the whole reactor, i.e. a macroscopic property. It may, however, be defined as a microscopic property by taking a small volume of material from around a given point in a pile and building a homogeneous pile out of similar material. The generation time of the large pile then becomes the generation time corresponding to the point. Let N(t) dt be the probability that a neutron exists for t seconds, then undergoes an absorption resulting in fission in the interval dt. Since there are other types of absorptions or losses, the integral of N(t) dt from zero to infinity will be less than one, say a value F. The generation time is then defined by:

$$\ell_o = \frac{1}{F} \int_0^\infty t\, N(t)\, dt.$$

Note that the generation time is not the average life-time of the neutron, since those neutrons absorbed or lost in a manner other than that producing fission have been discarded.

The time variable may be transformed into a space variable by the relation $x = \bar{v}t$. For the thermal (one group) case there results:

(continued)

[redacted]

Glossary, Reactor Theory

<u>generation time</u> (continued)

$$\ell_o = \frac{1}{N\sigma_f \bar{v}}$$

where $N\sigma_f$ is the macroscopic fission cross-section, and $\bar{v}$ the average velocity of the neutrons.

By means of the critical relation for the stationary state of a large thermal pile (one group case) one can show that the generation time; the mean life in an infinite lattice, ℓ; and the mean life in a finite reactor, ℓ^*, called the finite lifetime, are related as follows:

$$\ell^* = \frac{\ell_o}{\eta \varepsilon} = \frac{\ell}{1 + \ell^2 B^2}$$

where η is the number of thermal neutrons per neutron absorbed, ε is the fast multiplication factor, L the diffusion length, and B^2 the buckling in the finite pile.

[redacted]

Glossary, Reactor Theory

gradient (operator): (called "del")

The gradient operator is the vector differential operator defined by the relation (in cartesian coordinates):

$$\nabla = \underline{i}\,\frac{\partial}{\partial x} + \underline{j}\,\frac{\partial}{\partial y} + \underline{k}\,\frac{\partial}{\partial z}\,.$$

The gradient can operate only on a scalar function of the coordinates. Appropriate formulae for the gradient in other coordinate sets may be obtained from any book on vector analysis. In general, the gradient of a scalar function evaluated at a point is the rate of increase of the function taken in such a direction that this rate of increase is a maximum.

Green functions

See: kernels.

[redacted]

Glossary, Reactor Theory

group and phase velocity

The group velocity is the velocity with which a wave packet as a whole is transmitted through a medium. The phase velocity is the velocity with which a monochromatic wave is transmitted through the medium. The phase velocity is given by the product of the frequency, ν, and the wave length, λ. If dispersion exists in a medium through which waves are passing, that is, if the phase velocity is a function of the wave length, then the group velocity is different (in general) from any particular phase velocity. The group velocity, U, is related to phase velocity, V, to the wave length, or the wave number $k = 2\pi/\lambda$ by the relation:

$$U = V - \lambda \frac{dV}{d\lambda} = 2\pi \frac{d\nu}{dk} .$$

Hankel functions

Hankel functions are solutions of Bessel's equation.

See: discussion under wave equation.

Helmholtz equation

See: wave equation.

[redacted]

Glossary, Reactor Theory

homogeneous differential equation

See: linear and homogeneous differential equation.

hydraulic radius of a hole

The area of a hole divided by its wetted perimeter is called the hydraulic radius.

"I" neutrons

Neutrons of such energy that they are strongly absorbable by indium, that is, of energy in the neighborhood of 1.44 ev. This term is now obsolete, although it may be found in the literature. See: neutron energy groups.

[redacted]

Glossary, Reactor Theory

image and negative piles

Under certain restrictions stated, namely, that the augmentation distance is independent of the energy of the neutron and the finite and infinite slowing down kernels obey the same linear equation, the solution of the asymptotic pile equations is also the solution of the finite pile equations. Quoting Weinberg: "Physically, the sense of this theorem is somewhat as follows: the neutron distribution in a finite pile can be calculated under the restrictions stated by extending the pile out to infinity and finding the asymptotic neutron distribution in this infinite system. This solution oscillates, positive neutron densities alternating with negative ones ad infinitum. The positive and negative densities are so distributed that on the pile boundary their superposed effect satisfies the boundary conditions."

The regions in which the neutron density is negative may be called a "negative" pile. The positive and negative piles are very similar to the system of images which are commonly used to solve ordinary boundary value problems. Hence, the use of the term "image" pile as synonymous with "negative and/or positive pile outside the physical boundary."

[redacted]

Glossary, Reactor Theory

importance function, $I(\underline{r})$

The importance function, $I(\underline{r})$, is defined as the total increase in the neutron inventory caused by placing one neutron at position $\underline{r}$ in a pile.

See: perturbation theory.

inelastic scattering (of a neutron)

Inelastic scattering is a nuclear process by which the neutron is absorbed into the nucleus, thus loosing its identity, followed by the re-emission of a neutron of energy less than that of the incident neutron. Inelastic scattering is an important process in the slowing down of very energetic neutrons by heavy elements. For sufficiently high energy of the incident neutron, a heavy element (such as iron) may have much better slowing down properties through inelastic scattering than had by a light element like hydrogen through elastic scattering.

See: scattering, inelastic.

"infinite" life-time.

See: life-time of a neutron in an infinite medium.

[redacted]

Glossary, Reactor Theory

inhour, ih,

The inhour is a unit of the reactivity (i.e. a measure of the relative excess k) of a pile. One inhour is the reactivity which will give the pile a period of one hour. For a pile which is very nearly critical, the reactivity in inhours is the inverse of the pile period measured in hours. For a pile which is far from critical the relation between the reactivity in inhours and the period is more complicated. The "inhour formula" gives reactivity in inhours (Weinberg).

$$\rho_{ih} = \frac{\dfrac{\ell^*}{k_{eff}\,T} + \sum_i \dfrac{\beta_i\, t_i}{T + t_i}\,\dfrac{\overline{P}^{i}_{\infty}(E_s, B_g^2)}{\overline{P}_{\infty}(E_s, B_g^2)}}{\dfrac{\ell^*}{3600\, k_{eff}} + \sum_i \dfrac{\beta_i\, t_i}{3600 + t_i}\,\dfrac{\overline{P}^{i}_{\infty}(E_s, B_g^2)}{\overline{P}_{\infty}(E_s, B_g^2)}}$$

where $\ell^* = \dfrac{1}{N\sigma_a v}\,\dfrac{1}{1 + L^2 B^2}$ is the "finite life time"

$k_{eff} = \dfrac{k\,\overline{P}_{\infty}(E_s, B_g^2)}{\overline{P}\,(1 + L^2 B^2)}$ is the "effective multiplication constant."

(continued)

[redacted]

Glossary, Reactor Theory

<u>inhour, ih,</u> (continued)

T is the pile period in hours.

β_i is the yield of the ith delayed neutron per prompt neutron.

t_i is the decay constant in hours of the ith delayed neutron.

$\overline{P^i_{\infty}}(E_s, B_g^2)$ is the Fourier transform of the slowing down kernel of the ith delayed neutrons.

$\overline{P}_{\infty}(E_s, B_g^2)$ is the Fourier transform of the slowing down kernel of the prompt fission neutrons.

E_s is the energy of the thermal neutrons, and

B_g^2 is the geometrical buckling.

<u>inhour formula</u>

See: <u>inhour</u>.

<u>inventory, neutron</u>

The neutron inventory is the total number of neutrons in a reactor at any one time. It is a convenient quantity for use in pile kinetics.

[redacted]

Glossary, Reactor Theory

k

(a) k delayed

(b) k effective

(c) k excess

(d) k finite

(e) k infinite

(f) k prompt

See: multiplication constant.

kappa, κ,

The reciprocal of the diffusion length.

See: diffusion length.

kernels - Green's functions (as applied to random walk of neutrons)

A Green's function is the solution of a linear differential equation and its boundary conditions in which the source term is taken to be the delta function in one or more dimensions (time may be included when studying neutron waves). Since the differential equation is linear then a superposition of solutions is a solution, and, therefore, any arbitrary source function may be taken as a superposition of delta functions.

If $G(x, x')$ is the Green's function solution for the delta function source at x'; then the solution for an arbitrary source

(continued)

[redacted]

Glossary, Reactor Theory

<u>kernels, Green's functions</u> (continued)

$f(x')$ defined between a and b will be

$$\phi(x) = \int_a^b f(x') \; G(x, x') \, dx' \quad .$$

If in addition it so happens in the physical problem that $f(x')$ is proportional to $\phi(x)$ - as it is in a homogeneous nuclear reactor - then the above equation becomes an integral equation in which the Green's function is the kernel. Hence, in neutron physics the Green's functions are called kernels.

Whenever the Green's functions are functions of $x - x'$ they are called displacement kernels. A Green's function is always symmetrical with respect to interchange of x and x'. Green's functions have the property that they have a finite discontinuity in their slope when $x = x'$ of value $\left[p(x)\right]^{-1}$, where the function $p(x)$ is the coefficient of $\frac{d\phi^2}{dx^2}$, the second derivative of the dependent variable. In many equations $p(x)$ is unity.

The Green's functions obviously depend on the differential equation and on the geometry of the delta function source.

A particular Green's function is called by the name of the differential equation and the geometry of the source.

(continued)

[redacted]

Glossary, Reactor Theory

kernels, Green's functions (continued)

The Green's functions of any one differential equation may be converted from any one geometry to another by simple integration or differentiation. Letting the subscript p refer to point geometry (unit point source); pl to plane; ss to spherical shell; ℓ to line we have

$$G_{pl}(\xi) = 2\pi \int_{\xi}^{\infty} \rho\, G_p(\rho)\, d\rho \quad ,$$

where ξ is the perpendicular distance from the plane to the field point, ρ is the distance from the field point to a general point in the plane. Similarly

$$G_{ss}(r,r') = \frac{2\pi}{4\pi\, r\, r'} \int_{r-r'}^{r+r'} \rho\, G_p(\rho)\, d\rho \ , \ (r > r')$$

where r' is the radius of the spherical shell and r the distance from the center of the shell to the field point.

The line kernel in terms of the point kernel is

$$G_{\ell}(\xi) = 2 \int_{\xi}^{\infty} \frac{\rho}{(\rho^2 - \xi^2)}\, G_p(\rho)\, d\rho$$

etc.

If no geometry is stated, it is understood that it is the point source kernel to which is referred.

[redacted]

Glossary, Reactor Theory

kernel, Christy-Wheeler

See: kernel, slowing down, infinite, convolution of: special case: convolution of Gaussian and transport kernels.

kernels, diffusion (Yukawa)

The diffusion or Yukawa kernel results from unit δ-function source solutions of the diffusion equation, i.e. the equation of elementary diffusion theory. Diffusion kernels for various geometries are given in the table below.

(continued)

[redacted]

Glossary, Reactor Theory

Table - Diffusion Kernels

Name	Symbol	Source Strength	The Kernel
Point	$G(r - r')$	1 neutron	$\dfrac{e^{-\varkappa\lvert \underline{r} - \underline{r}'\rvert}}{4\pi D_o \lvert \underline{r} - \underline{r}'\rvert}$
	r = position of field point; r' = position of source point.		
Plane	$G_{pl}(r, r')$	1 neut/cm^2	$\dfrac{e^{-\varkappa(x - x')}}{2k\, D_o}$
	x = position of field point; x' = position of source plane $\perp$ to axis.		
Spherical Shell	$G_s(r, r')$	1 neutron per shell of radius r'	$\dfrac{1}{8\pi r r' \varkappa D_o}\left[e^{-\varkappa\lvert r - r'\rvert} - e^{-\varkappa\lvert r \quad r'\rvert}\right]$
	r = distance of field point from center of shell; r' = radius of shell		
Line	$G(r, r')$	1 neutron per unit length	$\dfrac{1}{2\pi d D_o} K_o(\varkappa \rho)$ $\rho^2 = r^2 + r'^2 - 2rr' \cos\phi$
	r = position of field point; r' = vector $\perp$ to line.		
Cylindrical Shell	$G_c(r, r')$	1 neutron per shell of radius r' and unit length.	$\dfrac{1}{2\pi D_o} \cdot \begin{cases} K_o(\varkappa r)\, I_o(\varkappa r') & ; r > r' \\ K_o(\varkappa r')\, I_o(\varkappa r) & ; r < r' \end{cases}$

D_o = Diffusion coefficient of flux

$\varkappa$ = Reciprocal of diffusion length

$K_o(r)$ = Bessel function of second kind of imaginary argument

$I_o(r)$ = Bessel function of a first kind of imaginary argument

ρ = $\underline{r} - \underline{r}'$

ϕ = Angle between $\underline{r}$ and $\underline{r}'$

[redacted]

Glossary, Reactor Theory

kernel, displacement

See: **displacement kernel**

kernel -- slowing down, finite

The finite slowing down kernel is defined as the probability that a fission neutron created in unit volume at $\underline{r}'$ inside the finite pile, becomes a slow neutron in unit volume at $\underline{r}$. It is denoted by Weinberg as $P(E_s, \underline{r}, \underline{r}')$ where E_s is the energy of a thermal neutron. The finite slowing down kernel is not in general a displacement kernel.

[redacted]

Glossary, Reactor Theory

kernel, slowing down, infinite

The infinite slowing down kernel is defined as the probability that a fission neutron created in unit volume at $\underline{r}'$ in an infinite homogeneous medium becomes a slow neutron in unit volume at $\underline{r}$. It is denoted by Weinberg as $P_{\infty}\left[E_s, (\underline{r} - \underline{r}')\right]$. The infinite slowing down kernel is normalized so the integral of the kernel over all space is equal to the resonance escape probability, p.

All slowing down kernels are point kernels according to the above definition, any point kernel may be converted to any other geometry by suitable integration.

See: kernels - Green's functions.

kernel - slowing down, infinite, convolution of

In some media the experimental slowing down data can be fairly accurately fitted by a kernel resulting from the assumption that neutrons, born at energy E_o, slow down to an intermediate energy, E'', according to one kernel, $P_1(E_o, E'', \underline{r}' - \underline{r}'')$; then slow from this intermediate energy to thermal energy, E_s according to a second kernel, $P_2(E'', E_s, \underline{r}'' - \underline{r})$. As the point in space, $\underline{r}''$, occupied by the neutron when it reaches the intermediate energy E'' may be any point in the medium one must take a convolution of the kernels P_1 and P_2 over all space. There results

(continued)

[redacted]

Glossary, Reactor Theory

<u>kernel - slowing down, infinite, convolution of</u> (continued).

$$P(E_o,E_s,\underline{r}' - \underline{r}) = \int_{\text{all space}} P_1(E_o,E'',\underline{r}' - \underline{r}'') \cdot P(E'',E_s,\underline{r}''-\underline{r})d\underline{r}''.$$

That is, the convolution is over the space coordinates only, with E'' taken as an adjustable parameter.

The convolution of three kernels over the space coordinates, with intermediate energies of E'' and E''' is given by

$$P(E_o,E_s,\underline{r}' - \underline{r}) =$$

$$\int\int_{\substack{\text{all}\\\text{space}}} P_1(E_o,E'',\underline{r}'-\underline{r}'')\cdot P_2(E'',E''',\underline{r}''-\underline{r}''')\cdot P_3(E''',E_s,\underline{r}'''-\underline{r})d\underline{r}''d\underline{r}'''$$

Since the Fourier transform of the convolution of functions is the product of the separate Fourier transforms of the functions, for an n-fold convolution, the Fourier transform is

$$\bar{P}(E_o,E_s,B^2) = \bar{P}_1(E_o,E'',B^2)\cdot\bar{P}_2(E'',E''',B^2)\cdot\bar{P}_3(E''',E^{IV},B^2) \ldots \bar{P}_n(E^n,E_o,B^2).$$

From the three fundamental kernels - the Gaussian, the transport, and the diffusion - three important two-fold convolution kernels can be constructed. In addition, an n-fold convolution of the diffusion kernel will result in the multi-group kernel.

[redacted]

Glossary, Reactor Theory

<u>kernel - slowing down, infinite: convolution of, special case: convolution of diffusion and Gaussian kernels</u>

The convolution of a diffusion and a Gaussian kernel is:

$$\bar{P}_{\infty}(E_s, |\underline{r}-\underline{r}'|) = p \int_{\substack{\text{all}\\ \text{space}}} \frac{e^{-\frac{(\underline{r}-\underline{r}'')^2}{4\tau}}}{(4\pi\tau)^{3/2}} \; \frac{e^{-\frac{|\underline{r}''-\underline{r}'|}{L_1}}}{4\pi|\underline{r}''-\underline{r}'|L_1^2} \, d\underline{r}''.$$

where τ, the age, and L_1 the diffusion length are to be regarded as arbitrary parameters determined by experiment, E_s is the energy of the slow neutron, p the resonance escape probability, $\underline{r}$ the vector position of the field point, $\underline{r}'$ the vector position of the source point, and $\underline{r}''$ the vector position of the integration variable. The Fourier transform of this kernel is:

$$\bar{P}_{\infty}(E_s, B^2) = \frac{p\, e^{-\tau B^2}}{1 + L_1^2 B^2}$$

where B^2 is the buckling.

[redacted]

Glossary, Reactor Theory

kernel slowing down, infinite, convolution of: special case: convolution of diffusion and transport kernels

The convolution of the diffusion and the transport kernel is given by:

$$P_{\infty}(E_s, |\underline{r}-\underline{r}'|) = p \int_{\substack{\text{all}\\ \text{space}}} \frac{e^{-\frac{\underline{r}-\underline{r}''}{L_1}}}{4\pi|\underline{r}''-\underline{r}|L_1^2} \; \frac{e^{-N\sigma|\underline{r}''-\underline{r}'|}}{4\pi|\underline{r}''-\underline{r}'|^2} \, d\underline{r}'' ,$$

where E_s is the energy of the slow neutron, p the resonance escape probability, L_1 the diffusion length and $1/N\sigma$ the mean-free-path which are to be considered arbitrary parameters to be evaluated experimentally, $\underline{r}$ is the vector position of the field point, $\underline{r}'$ the vector position of the source point, and $\underline{r}''$ the vector position of the point of integration. The Fourier transform of the above is

$$\bar{P}_{\infty}(E_s, B^2) = \frac{p}{1 + L_1^2 B^2} \; \frac{N\sigma}{B} \tan^{-1}\left(\frac{B}{N\sigma}\right) ,$$

where B^2 is the buckling.

[redacted]

Glossary, Reactor Theory

<u>kernel - slowing down, infinite, convolution of: special case: convolution of Gaussian and transport kernels</u>

(<u>The Christy-Wheeler kernel</u>)

The Christy-Wheeler kernel is applicable to cases where the slowing down can be analyzed into a long first flight, during which the neutron suffers no collisions, followed by a Gaussian diffusion away from the point of first collision. The Christy-Wheeler kernel is

$$P_{\infty}(E_s, |\underline{r}-\underline{r}'|) = p \int_{\substack{\text{all}\\ \text{space}}} \frac{e^{-\frac{|\underline{r}-\underline{r}''|^2}{4\tau}}}{(4\pi\tau)^{3/2}} \; \frac{N\sigma\, e^{-N\sigma|\underline{r}''-\underline{r}'|}}{4\pi|\underline{r}''-\underline{r}''|^2} \, d\underline{r}'' ,$$

where E_s is the energy of the slow neutron, p the resonance escape probability, τ the Fermi age - to be considered as an arbitrary parameter to be chosen so as best fit the observed slowing down distribution, similarly the mean-free-path ($1/N\sigma$) is to be considered arbitrary and determined experimentally, $\underline{r}$ is the vector position of the field point, $\underline{r}'$ is the vector position of the source, and $\underline{r}''$ the vector position of the integration point.

The Fourier transform of the Christy-Wheeler kernel is:

$$\bar{P}_{\infty}(E_s, B^2) = p\, e^{-\tau B^2} \frac{N\sigma}{B} \tan^{-1}\left(\frac{B}{N\sigma}\right) ,$$

where B^2 is the buckling.

[redacted]

Glossary, Reactor Theory

<u>kernel - slowing down, infinite, convolution of; special case; multi-group kernels; n-fold convolution of the diffusion kernel</u>

The kernel which is formed by the convolution of several diffusion kernels leads to the multi-group kernels. The expression for the convolution is obvious (see discussion of convolution kernel). The Fourier transform of the multi-group of n groups is:

$$\bar{P}_{\infty}(E_s, B^2) = \frac{p}{\prod_{i=1}^{n-1} (1 + L_i B^2)} ,$$

where E_s is the energy of the slow neutron, p the resonance escape probability, L_i the diffusion length of the i + 1st group which may be taken to be the change of Fermi age appropriate to this group, the symbol $\prod$ denotes the multiplication operator. In the limit as n goes to infinity it may be shown that the multi-group slowing-down kernel becomes the Fermi (Gaussian) kernel.

[redacted]

Glossary, Reactor Theory

kernel, slowing down, infinite, Fourier transform of

The three dimensional Fourier transform of the slowing-down kernel appears in the characteristic equation:

$$L^2B^2 - 1 + \frac{k}{p}\,\bar{P}_\infty(E_s, B^2) = 0$$

where $\bar{P}_\infty(E_s, B^2)$ is the Fourier transform of the slowing down kernel, L is the diffusion length, k is the multiplication constant, p is the resonance escape probability, B^2 is the buckling and E is the energy of the slow neutron. The Fourier transform of the kernel $P_\infty(E_s, \underline{r})$ is given by:

$$\bar{P}\,(E_s, B^2) = \int_{-\infty}^{\infty} e^{i\underline{B}\cdot\underline{r}}\, P_\infty(E_s, |\underline{r}|)\, d\underline{r}$$

$$= 4\pi \int_0^{\infty} \frac{\sin Br}{Br}\, P_\infty(E_s, r)\, r^2\, dr \quad ,$$

where $\underline{B} = \underline{i}\, B_x + \underline{j}\, B_y + \underline{k}\, B_z$, and r is the distance between the source and the field point. The Fourier transform of the various kernels are given under the heading pertaining to that special kernel.

The Fourier transform of the infinite slowing down kernel may be shown to be the probability that a fast neutron becomes thermal inside the pile.

[redacted]

Glossary, Reactor Theory

kernel - slowing down, infinite Fourier transform of, moment form of

The moment form of the Fourier transform of a slowing down kernel is obtained by expanding

$$\frac{\sin B_g r}{B_g r}$$

appearing in the three dimensional Fourier transform and integrating term by term. There results

$$\bar{P}_\infty(E, B^2) = p \sum_{n=0}^{\infty} \frac{(-)^n}{(2n+1)!} B^{2n} \overline{r^{2n}(E_s)}$$

where

$$\overline{r^{2n}(E_s)} = \frac{4 \int r^{2n} P_\infty(E_s, r)\, r^2 dr}{4 \int P_\infty(E_s, r)\, r^2 dr}$$ that is, the

$2n^{th}$ moment of the slowing down kernel, $P_\infty(E_s, r)$; E_s is the energy of the slow neutron. The first two terms of the moment form, when put into the characteristic equation yield

$$B^2 = \frac{k-1}{M^2}$$

where

$$M^2 = L^2 + \frac{\overline{r^2}}{6}$$

M^2 is called the migration area and

$\frac{\overline{r^2}}{6}$ is equal to the age, τ, in the Fermi theory.

The above characteristic equation is applicable if k is very nearly one, and/or if the pile is very large.

[redacted]

Glossary, Reactor Theory

kernel - slowing down, infinite, special case: Gaussian (Fermi) slowing down kernel

In the cases where the Fermi age slowing down theory is applicable the slowing down kernel in the infinite system is given by:

$$P_{\infty}(E, r-r') = \frac{p(E)}{\left[4\pi\tau(E)\right]^{3/2}} \, e^{-\frac{(r-r')^2}{4\tau(E)}}$$

where E is the energy of the slowing neutron, $\underline{r}'$ the position of birth, $\underline{r}$ the field point, $\tau(E)$ the Fermi age of a fission neutron slowed to energy E, and p(E) is the resonance escape probability. The above kernel is the point-source solution of the age equation:

$$\nabla^2 q - \partial q/\partial\tau = 0.$$

The Fourier transform of the Fermi kernel is

$$\bar{P}_{\infty}(E,B^2) = p(E)\, e^{-B^2\tau(E)}$$

where B^2 is the buckling.

[redacted]

Glossary, Reactor Theory

<u>kernel - slowing down, infinite, special case: Gaussian kernel with fission spectrum spread</u>

A more accurate representation of the Gaussian kernel is a super-position of the simple Gaussian kernel integrated over the fission spectrum. Let the probability that a fission neutron is produced having energy lying between E' and E' + dE' be represented by

$$f(E')\,dE'$$

then the Gaussian kernel with fission spectrum spread is

$$P_\infty(E,r) = p(E) \int_0^\infty \frac{e^{-\frac{r^2}{4\tau(E,E')}}}{4\pi\tau(E,E')^{3/2}} f(E')\,dE' ,$$

where $\tau(E,E')$ is the Fermi age of a neutron born at energy E' and slowed to energy E, and r is (r - r') the scalar distance between the source point and the field point. The Fourier transform is:

$$\bar{P}_\infty(E,B) = p(E) \int_0^\infty e^{-B^2\tau(E,E')} f(E')\,dE'$$

[redacted]

Glossary, Reactor Theory

<u>kernel - slowing down, infinite, special case: one group kernel</u>

In the case of one group theory, the neutron is assumed to have been born slow and the slowing down kernel is a delta function

$$P_{oo}(E,r) = p(E)\,\delta(\underline{r})$$

where p(E) is the resonance escape probability. It has the Fourier transform:

$$\bar{P}_{oo}(E,B^2) = 1 \cdot p(E)$$

[redacted]

Glossary, Reactor Theory

kernel - slowing down, infinite, special case: two group kernel

In the two group model neutrons are either "fast" or "slow" and moderation is described as a diffusion process. A fast neutron is assumed to diffuse without loss of energy until it has travelled a mean square distance equal to its actual slowing down length, at which time it disappears by becoming a slow neutron. The two group slowing down kernel is then:

$$P_{\infty}(E_s, r) = \frac{p\, e^{-r/L_1}}{4\pi r L_1^2}$$

where p is the resonance escape probability, r is the scalar distance between the source point and the field point, E_s is the energy of the slow neutron and L_1 is the slowing down length given by

$$L_1^2 = \frac{\overline{r^2}(E_s)}{6}$$

That is, the two group slowing down kernel is a diffusion kernel.

The Fourier transform of this group kernel is

$$\overline{P}_{\infty}(E_s, B^2) = \frac{p}{1 + L_1^2 B^2}$$

where B^2 is the buckling.

[redacted]

Glossary, Reactor Theory

kernel, transport

The transport kernels result from delta function source solutions of the transport equation. It can be shown that the transport kernels may be obtained from the diffusion kernels by the equation

$$K_g(x,x') = D_o \int_{N\sigma}^{\infty} G_g(\lambda, x, x')\, d\lambda \quad .$$

That is, by integration over λ from $N\sigma$ to ∞, where $N\sigma$ is the total macroscopic cross section and g represents any particular geometry.

The point transport kernel is

$$K_p(r,r') = \frac{1}{4\pi} \frac{e^{-N\sigma|r-r'|}}{|r-r'|^2}$$

and the plane transport kernel

$$K_{pl}(x,x') = \frac{1}{2} E_o(N\sigma|x-x'|)$$

where E_o is the exponential integral $E_o(y) = \int_y^{\infty} e^{-u} \frac{du}{u}$

The Fourier transform of the point transport kernel is:

$$\bar{P}_{\infty}(E_s, B^2) = \frac{N\sigma}{B} \tan^{-1} \frac{B}{N\sigma} \quad ,$$

where B^2 is the buckling.

[redacted]

Glossary, Reactor Theory

kernel, Yukawa

See: kernel, diffusion.

Kronecker delta

See: delta, Kronecker.

[redacted]

Glossary, Reactor Theory

LaPlace transformations

If a function F(t) is defined for all positive values of the variable t and of the order of e^{pt}, is multiplied by e^{-pt} and integrated with respect to t from zero to infinity, a new function f(p) of the complex variable p is obtained; that is

$$f(p) = \int_0^\infty e^{-pt} F(t)dt.$$

This operation is called the LaPlace transformation of F(t). F(t) may be obtained from its LaPlace transform by the inverse LaPlace transformation:

$$F(t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{pt} f(p)\, dp$$

Here the path of integration is paralled to the imaginary axis on the complex plane, and the value of γ, a real number, is so chosen that the path is to the right of all poles.

In many problems the mathematical operations required are more easily accomplished in transform space than in ordinary space. Many types of problems, particularly those involving convolution recursion formulae, ordinary and partial differential equations with boundary conditions, etc., are solved by means

(continued)

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Glossary, Reactor Theory

LaPlace transformations (continued)

of the LaPlace transformation method by purely operational technique without resort to intuition or induction.

Good elementary texts on the LaPlace transform operational method are:

1. Ruel V. Churchill, "Modern Operational Mathematics in Engineering," McGraw-Hill Book Company, New York, 1944.

2. H. S. Carslaw and J. C. Jaeger, "Operational Methods in Applied Mathematics," Oxford University Press, 1941.

LaPlacian, geometrical

See: buckling, geometrical. See also wave equation.

LaPlacian, material

See: buckling, material. See also wave equation.

LaPlacian operator,

(a) cartesian coordinates

(b) cylindrical coordinates

(c) spherical coordinates

See: wave equation.

[redacted]

Glossary, Reactor Theory

lattice

By lattice is meant a heterogeneous combination of parts in such a manner that a two or three dimensional repeating symmetry is developed. The smallest volume having symmetry is called the lattice cell.

Legendre coefficient

See: wave equation.

Legendre component of differential scattering cross-section

See: Boltzmann equation and cross-section, differential scattering.

Legendre functions

See: wave equation.

Legendre function, associated

See: wave equation.

Legendre polynomials

See: wave equation.

[redacted]

Glossary, Reactor Theory

<u>lethargy</u>

Lethargy is the name given by M. E. Rose to the variable u, where

$$u = \log_e \frac{E_o}{E} ,$$

where u is the lethargy, E is the energy of the neutron, and E_o is its original energy. If $E = E_o$ the lethargy is zero; if E is zero, then the lethargy is infinite, hence its name.

Many problems involving the slowing down of neutrons are more conveniently handled when transformed in terms of the lethargy.

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Glossary, Reactor Theory

<u>life-time of a neutron in finite medium, ℓ^* (called also "finite life-time.")</u>

The life-time of a neutron in a finite medium is defined as the mean time a neutron exists in a pile, all types of loses being considered. It is given by the relation

$$\ell^* = \int_0^\infty t\, P(t)\, dt,$$

where ℓ^* is the finite life-time, P(t) dt is the probability that a neutron exists for t seconds and then is lost to the medium through either absorption or leakage in the next interval dt.

For a thermal pile it may be shown that the finite life-time is given by:

$$\ell^* = \frac{1}{N\sigma_a \bar{v}} \frac{1}{1 + L^2 B^2} = \frac{\ell}{1 + L^2 B^2}$$

where $N\sigma_a$ is the total macroscopic absorption cross-section, $\bar{v}$ is the average thermal velocity, L is the diffusion length, B^2 the buckling, and ℓ the mean life for the equivalent infinite lattice.

H. Soodak uses a slightly different definition:

$$\ell^* = \frac{\ell k}{\bar{P}\,(E_s, B^2)}$$

as the fundamental life time.

<u>life-time of a neutron in an infinite lattice</u>

(Sometimes called the "<u>infinite life-time</u>")

The life-time of the neutron in an infinite medium is defined as that mean life-time a neutron in a pile will have if subjected to absorption processes only. It is defined by the relation

$$\ell = \int_0^\infty t\, M(t)\, dt;$$

where ℓ is the life-time (i.e, the mean life); M(t) dt is the probability that a neutron exists for t seconds and then undergoes absorption in the next interval dt. The qualifying phase "in an infinite medium" simply implies that M(t) is normalized; that is, that there is no loss of neutrons through leakage. If H is the integral of M(t) dt from zero to infinity, then the life-time as defined above (considering only absorption) is defined by

$$\ell = \frac{1}{H} \int_0^\infty t\, M(t)\, dt$$

for any homogeneous medium, finite or infinite.

The time variable may be transformed into a space variable by the relation $x = vt$. For the thermal (one group) case there results:

$$\ell = \frac{1}{N\sigma_a \bar{v}},$$

where $N\sigma_a$ is the macroscopic (total) absorption cross-section, and $\bar{v}$ the average velocity of the neutrons.

For the relation between the various kinds of mean lives of neutrons see <u>Generation time</u>.

[redacted]

Glossary, Reactor Theory

<u>linear differential equation</u>

<u>linear</u> and <u>homogeneous differential equation</u>

A differential equation is <u>linear</u> when it would be of the first degree if the dependent variable and all its derivatives were regarded as algebraic unknown quantities.

If it is linear and contains no term which does not involve the dependent variable or one of its derivatives, it is said to be <u>linear</u> and <u>homogeneous</u>.

<u>logarthmic energy decrement, ξ</u>

The average decrease in the natural logarithm of the energy of a neutron per collision. Note that the antilog of ξ is <u>not</u> the average energy loss per collision. Where A is the atomic number of an atom, ξ is given for elastic collisions and isotopic scattering by:

$$\xi = 1 + \frac{r}{1 - r} \log r$$

where

$$r = \left(\frac{A - 1}{A \quad 1}\right)^2$$

$$\xi \doteq \frac{2}{A \quad 2/3} \; ; \quad A > 10.$$

[redacted]

Glossary, Reactor Theory

macroscopic cross-section

See: cross-section, macroscopic.

macroscopic and microscopic pile theory

The assumption is made that certain factors called the macroscopic properties of a pile can be treated independently of other factors, called microscopic properties of the pile.

Thus the buckling, wave equation solutions, $k_{effective}$, k_{excess}, reflector savings, etc. are macroscopic properties, whereas k_{∞}, the resonance escape probability p, the fast multiplication constant ε, thermal utilization, f; slowing down kernels, etc. are microscopic properties of the pile.

Rigorously speaking, this assumption of the independence of the macro and the microscopic properties is false, although it is a very good assumption for a large pile and breaks down more and more as the pile gets smaller.

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Glossary, Reactor Theory

Maxwellian distribution (of molecular velocities)

In a monatomic gas at thermal equilibrium the probability $\rho dv_x dv_y dv_z$, that a molecule shall have velocity components lying between v_x and $v_x + dv_x$, v_y and $v_y + dv_y$, and between v_z and $v_z + dv_z$ is

$$\rho \, dv \, dv \, dv = \frac{dN}{N} = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} \, dv_x dv_y dv_z \quad ,$$

or in one dimension, by the equal partition of energy

$$\rho \, dv_x = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-mv_x^2/2kT} \, dv_x \quad ,$$

where m is the molecular mass, k the Boltzmann gas constant, T the absolute temperature.

This probability distribution is known as the Maxwellian distribution. The Maxwellian distribution is a particular use of the Gaussian or normal distribution.

[redacted]

Glossary, Reactor Theory

mean-free-path, diffusion, λ_d

The diffusion mean-free-path is defined by the relation:

$$\lambda_d = \frac{1}{N\sigma_{so}(1 - \bar{\mu}) + N\sigma_a},$$

where $N\sigma_{so}$ is the macroscopic scattering cross-section, $N\sigma_a$ the total macroscopic absorption cross-section, and $\bar{\mu}_o$ the average cosine of the scattering angle.

See also "Boltzmann equation."

[redacted]

Glossary, Reactor Theory

mean-free-path, scattering, λ_s

The mean-free-path for scattering is the mean distance traveled between the scattering collisions, i.e., the first moment of the scattering distribution law:

$$\lambda_s = \frac{\int_0^\infty x\, e^{-N\sigma_{so}x}}{\int_0^\infty e^{-N\sigma_{so}x}} = \frac{1}{N\sigma_{so}}$$

where σ_{so} = the total atomic cross-section for scattering;

N = no/cm^3 of scattering centers.

The mean-free-path for scattering is equal to the mean distance between collisions only when the total scattering cross-section is approximately equal to the total cross-section, i.e., when the absorption cross-section can be neglected with respect to the scattering cross-section. This assumption is not valid for high energies of the neutron.

[redacted]

Glossary, Reactor Theory

mean-free-path, total, λ

The mean-free-path is the mean distance traveled between collisions. Since the probability of a collision is given by the exponential law, the total mean-free-path is given by the first moment of this distribution:

$$\lambda = \frac{\int_0^\infty x\, e^{-N\sigma x}}{\int_0^\infty e^{-N\sigma x}} = \frac{1}{N\sigma}$$

where N = no/cm^3 of atoms

σ = total atomic cross-section for all processes.

[redacted]

Glossary, Reactor Theory

mean-free-path, transport, λ_t

The transport mean-free-path is defined by

$$\lambda_t = \frac{1}{N\sigma_{so}(1 - \bar{\mu}_o)}$$

where σ_{so} is the total scattering atomic cross-section and $\bar{\mu}_o$ is the average cosine of the scattering angle. The factor $(1 - \bar{\mu}_o)$ is arbitrarily attached to the mean-free-path to produce the "transport mean-free-path." As a length it has significance only in the diffusion equations.

See also the discussion of Boltzmann equation.

microscopic cross-section

See "cross-section, macroscopic, microscopic"

[redacted]

migration area

The sum of the Fermi age and the square of the diffusion length is called the migration area. The Fermi age used is that obtained by averaging over the fission spectrum as a source. The migration area arises theoretically whenever the characteristic equation employing the first two terms of the moment form of the Fourier transform of the slowing down kernel is applicable, that is whenever the multiplication constant of the pile is near unity, or where the pile is very large:

$$B_g^2 = \frac{k - 1}{M^2} ,$$

where k is the multiplication constant and B_g^2 the geometrical buckling.

migration length

The migration length is the square root of the migration area, i.e., the square root of the sum of the square of the diffusion length and the Fermi age with source energy averaged over the fission spectrum.

[redacted]

Glossary, Reactor Theory

moderation

By moderation is meant the slowing down of neutrons. A substance so used is called a moderator. The substances of low atomic weight make the best moderators since the loss of energy per collision of a neutron increases as the atomic weight decreases. However, at neutron energies above ≈ 1 Mev, a substance having inelastic scattering cross-section (like Fe) is a desirable moderator.

moderator

A substance, such as graphite, used to slow down neutrons by means of elastic collisions is called a moderator.

[redacted]

Glossary, Reactor Theory

moment form of characteristic pile equation

The moment form of the characteristic pile equation is:

$$1 = \frac{k}{1 + L^2 B_g^2} \sum_{n=0}^{\infty} \frac{(-)^n}{(2n+1)!} B_g^{2n} \overline{r^{2n}(E_s)} ,$$

where

$\overline{r^{2n}(E_s)}$ is the $2n^{th}$ moment of the slowing down kernel;

$$\overline{r^{2n}(E_s)} = \frac{4\pi \int_0^{\infty} r^{2n} P_{\infty}(E_s,r)}{4\pi \int_0^{\infty} P_{\infty}(E_s,r)\, r^2 dr} = \frac{4\pi}{p} \int_0^{\infty} r^{2n} P_{\infty}(E_s,r) r^2 dr$$

L is the diffusion length

B_g^2 the geometrical buckling

k the multiplication constant

E_s the energy of the thermal neutron

$P_{\infty}(E_s,r)$ the slowing down kernel

p the resonance escape probability

See also "kernel, slowing down, moment form of Fourier transform".

moments of Fourier transform of slowing down kernel

See "kernel, slowing down, Fourier transform of, moment form of".

[redacted]

Glossary, Reactor Theory

Monte-Carlo method (stochastic and deterministic method)

The "Monte-Carlo" method, suggested by Ulam and von Neumann, is a method of solution of a group of physical problems by means of a series of statistical experiments. Inasmuch as the accuracy of the results depends on the number of trials, the method is practical only when set up to be performed by computing machines.

The method consists of playing a game whose rules are determined by the particular physical problem encountered. Frequently, the physical problem does not completely determine a quantity (the length of a particular flight of a neutron between collisions, for instance) but gives instead a probability distribution function for the quantity considered (in the above mentioned example the function $P(x)dx = \frac{1}{\lambda} e^{-x/\lambda} dx$ is the probability that the neutron goes without collision a distance x, then makes a collision between x and $x + dx$).

A choice of a specific value from this probability function is made in such a manner that an infinite number of such choices would result in the given probability function.

This choice is accomplished by choosing a number at random from the interval between (say) zero and unity; then by the "stochastic" transformation a one-to-one correspondence between the set of random numbers and the specific values of the physical quantities is set up.

(continued)

[redacted]

Glossary, Reactor Theory

Monte-Carlo method (continued)

The entire physical problem (for a single event) is followed in this manner. A large number of such single-event "experiments" are thus compiled, from which desired average values may be computed. The validity of the average values obviously depends on the number of trials.

For instance, suppose one is following the history of a particular neutron through a medium. One first selects three random numbers to determine the coordinates of the point of origin of the neutron; two more random numbers determine the direction of its flight. If the neutron is born with a probability spread in its speed a random number is chosen and by suitable transformation a particular speed is selected from the probability distribution. Another random number selects its flight length from the probability distribution of path lengths; an eighth random number determines what happens at the end of the flight length (absorption in what element or scattering by what element and through what angle); if scattering, a ninth random number determines the aximuthal angle of scattering. The law of conservation of energy and momentum determines its new velocity, etc. The process is continued until the neutron is absorbed. From computation of many such experiments the age or slowing down kernel may be evaluated.

(continued)

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Glossary, Reactor Theory

Monte-Carlo method (continued)

The Stochastic Transformation

It is desired to select the number "at random", that is, with uniform probability, from the interval between 0 and 1, and for each value of the number so chosen there corresponds a particular value of the argument in the probability function.

The problem is to find the functional relation between the set of points on the uniform distribution between 0 and 1 from which the random selection is made, and the probability function. That is, a number ξ, is selected at random from the uniform probability distribution

$$p(\xi) = 1 \tag{1}$$

and by means of the transformation

$$x = A(\xi) \tag{2}$$

we have

$$d\xi = p(x)dx, \tag{3}$$

where $0 \leq \xi \leq 1$; $a \leq x \leq b$. Equation (3) may be solved

$$\xi(x) = \int_a^x p(x)dx. \tag{4}$$

Equation (1), having a normalization constant of unity insures that the range of definition ξ is a unit range.

(continued)

[redacted]

Glossary, Reactor Theory

Monte-Carlo method (continued)

The Stochastic Transformation (continued)

Equation (4) has the lower limit so chosen that this range is between 0 and 1, since p(x) is normalized:

$$\int_a^b p(x)dx = 1.$$

$x = A(\xi)$ is now obtained from (4), that is, the transformation desired is the inverse of the integral of the probability function.

In case the probability distribution is a function of more than one variable, the transformation is somewhat more complicated.

The method may also be applied to obtain a value for a definite integral. Suppose the definite integral involves integration over a large number of coordinates, (say N). To evaluate the integral by numerical methods, a prohibitive amount of labor is required. For instance, if N = 20, and the integral is summed for ten different values of each variable -- a rather coarse grid for a numerical integration -- a total of 10^{20} terms must be summed to obtain an answer.

In the Monte-Carlo method of evaluation one first transforms each of the N variables so that the N new variables are defined on unit range. One then selects N + 1 numbers at random. The first N are used to compute a value of $f(x_1, \ldots\ldots, x_N)$ which is then compared with the $(N+1)^{st}$ random number, r_{N+1}. If

 [redacted]

Glossary, Reactor Theory

Monte-Carlo method (continued)

$r_{N+1} < f(x_1, \ldots, x_N)$ the point is "inside" the integral, and vice versa. The ratio of the points "inside" to the total number of points in the limit as the number of points goes to infinity gives:

$$\lim_{\substack{\text{choices} \\ \to \infty}} \frac{\text{points inside}}{\text{all points}} = \sum_1 \cdots \sum_N f(x_1, \ldots x_N) \Delta x_1 \ldots \Delta x_N$$

Since the "random" numbers are limited to rational decimal fractions by the finite number of digits in the calculating machine the Δx_i are not infinitesimals. However, if the number of digits is reasonably large, the above evaluation becomes very close to the desired integral.

With the help of fast computing machines now under development, the Monte-Carlo method has promise of revolutionizing the whole field of applied mathematics.

Reference

Ulam and von Neumann, Bulletin of the American Mathematical Society, 53, 1120 (1947).

[redacted]

Glossary, Reactor Theory

multi-group, pile equations

The multi-group equations are approximations to the pile equations which are solvable for the pile with reflector. In the multi-group representation, a neutron of energy E_i is assumed to diffuse with constant energy until it undergoes an "absorption" which transfers it from energy E_i, that is, from the ith group to energy E_{i+1}, to the next lowest group. The group of lowest energy, the thermal group, has true absorptions in which the thermal neutrons are absorbed and k fast (group 1) neutrons are born. If there are M total groups, the multi-group equations take the form in the steady state:

$$D_i \Delta \phi_i - N\sigma_{ai} \phi_i + k\, N\sigma_{aM} \phi_M = 0$$

$$D_i \Delta \phi_i - N\sigma_{ai} \phi_i + N\sigma_{a(i-1)} \phi_{i-1} = 0$$

where

D_i is the diffusion constant of flux ϕ_i

ϕ_i is the ith flux (neutrons of energy E_i and speed V_i)

$N\sigma_{ai}$ is the fictitious macroscopic absorption cross-section of the ith group

k is the multiplication constant

M is the subscript of the lowest (thermal) group.

In setting up the equations for a particular case, the problem is that of choosing properly the diffusion constants and the "absorption" cross-sections.

(continued)

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Glossary, Reactor Theory

multi-group (continued)

Since $D_i = \frac{\lambda_{ti}}{3}$, where λ_{ti} is the transport mean-free-path of the ith group, and since the boundary conditions are taken to be

$$\left| \begin{array}{ll} \overline{\phi_i} & \text{continuous} \\ \overline{\lambda_{ti}\,\phi_i'} & \text{continuous} \end{array} \right| \text{ at the boundary}$$

one sets $D_i = \frac{1}{3}\,\overline{\lambda_{ti}}$,

where

$$\overline{\lambda_{ti}} = \int_{E_{i-1}}^{E_i} \lambda_t(E)\,\phi(E)\,dE$$

is the average mean-free-path weighted with the normalized flux-energy distribution, $\phi(E)$, and $\lambda_t(E)$ is the λ_t as a function of energy.

The "absorption" cross-section is determined so that the change in age is satisfied

$$\tau_i - \tau_{i-1} = \frac{1}{3N\sigma_{ai}\,N\sigma_{ti}} .$$

The multi-group equations are solved by straight forward solution, by matrix methods, or numerically.

In the limit as the number of groups goes to infinity the multi-group equations become the Fermi equations. Multi-group theory is useful in obtaining the fluxes of neutrons of various energies and the critical condition for a pile with reflector, for the effect of control rods, etc.

[redacted]

multi-group slowing down kernel

See "kernel, slowing down, infinite, convolution of, special case: convolution of diffusion kernels".

[redacted]

Glossary, Reactor Theory

multiplication constant of a nuclear reactor (k)

a. k delayed and k prompt

By k delayed is meant the usual definition of the multiplication constant k, in which the delayed as well as the prompt neutrons are considered.

By k prompt is meant the value the multiplication constant would have in the absence of the delayed neutrons (i.e. prompt neutrons only). The two concepts have utility in describing the kinetics of a nuclear reactor, since the fast transients are determined more or less by k prompt, while the "stable" period is determined by k delayed. We have

$$k_p = \frac{k_d}{1 + \beta}$$

where β is the total number of delayed neutrons produced per prompt neutron.

[redacted]

Glossary, Reactor Theory

multiplication constant (k) (continued)

b. (1) k effective, k_{eff} (Weinberg) (preferred definition)

The effective multiplication constant is defined by Weinberg as the ratio of the number of neutrons produced on the average in one generation to the number of neutrons which are absorbed and/or leak out (of the finite system) on the average in one generation.

$$k_{eff} = \frac{k\,\overline{P_\infty}(E_s, B_g^2)}{p(1 + L^2B_g^2)} \quad .$$

(2) Some Manhattan Project literature defined k_{eff} as:

$$k_{eff} = \frac{k}{p}\overline{P_\infty} - L^2B_g^2 \quad .$$

(3) Soodak used k_{eff} as

$$k_{eff} = k - (1 + L^2B^2)\frac{p}{\overline{P_\infty}(E_s, B_g^2)} = \rho$$

where ρ is called the "reactivity". Weinberg states that Soodak uses ρ as "k_{excess}," although in Soodak's notes ρ is synonymous with the term "k_{eff}." His term, ρ, is more properly an excess k, since it is zero when the pile is critical

c. k excess, k_{ex}

Weinberg uses the term k_{excess} to mean the k effective of a finite system minus one. The critical condition for a nuclear reactor is that k_{ex} shall be zero.

(continued)

[redacted]

Glossary, Reactor Theory

multiplication constant (k) (continued)

c. k excess, k_{ex} (continued)

Soodak used "$k_{effective}$" synonymously with "reactivity",

$$k_{eff}(\text{Soodak}) = \frac{k_{ex}(\text{Weinberg})}{k_{eff}(\text{Weinberg})} k = \rho_{\text{Soodak}} = \rho_{\text{Weinberg}} k$$

where ρ represents "reactivity".

d. k finite, k

The finite multiplication constant is defined as the ratio of the number of neutrons which become slow in one generation (on the average) to the number of slow neutrons captured in one generation (on the average) in a finite system. k(finite) is equal to k(infinite) if, and only if, the neutron energy spectrum in the finite system is everywhere equal to the energy spectrum in the corresponding infinite system. In the finite system the fast flux and fast leakage will not in general be proportional to the slow flux and leakage, that is, the slowing down kernel is a function of the geometry of the finite system. Hence there is a difference between k_{∞} and k.

(continued)

[redacted]

Glossary, Reactor Theory

multiplication constant (k) (continued)

e. k infinite, k_∞

The infinite multiplication constant is defined as the ratio of the number of neutrons which become slow (i.e. are "produced") in one generation to the number of slow neutrons captured in one generation in an infinite system.

It is given by

$k = \epsilon p f \eta$, where

ϵ = fast multiplication factor

p = resonance escape probability

f = thermal utilization

η = number of neutrons produced per thermal neutron absorbed.

negative pile

See "image and negative piles".

[redacted]

Glossary, Reactor Theory

neutron current

Neutron current is defined as the net number of neutrons which cross a square centimeter of surface of a given plane per second. Its units are the same as that of neutron flux; but is differentiated from the flux in that the current is the net flow across a boundary; those neutrons crossing from one side considered positive; those from the other considered negative, whereas the flux is the number per cm^2 all of which go in either direction and which penetrate a unit volume. Except for the vector flux, the neutron flux is a scalar quantity. The neutron current is therefore a vector quantity.

In elementary diffusion theory, the (vector) neutron current is proportional to the gradient of the neutron flux. Across a boundary the neutron current and flux are taken to be continuous.

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Glossary, Reactor Theory

neutron current (continued)

The vector net diffusion current of speed v is

$$\underline{j}(\underline{r},v) = \underline{i}_x\, j_x(\underline{r},v) + \underline{i}_y\, j_y(\underline{r},v) + \underline{i}_t\, j_t(\underline{r},v)$$

where $j_x(\underline{r},v)$ is the net current through a unit area perpendicular to the x axis; $\underline{i}$ is the unit vector in the x direction, etc; $\underline{r}$ is the position of the volume element considered.

The vector net total diffusion current (integrated with respect to v) is similarly:

$$\underline{j}(\underline{r}) = \underline{i}_x\, j_x(\underline{r}) + \underline{i}_y\, j_y(\underline{r}) + \underline{i}_t\, j_t(\underline{r}).$$

[redacted]

Glossary, Reactor Theory

<u>neutron current</u> (continued)

<u>a. of speed v</u>:

The current of neutrons, i.e. the number per second of neutrons of speed v which cross a square centimeter of area is given by the relation:

<u>positive current of speed v</u>:

$$j_+(x,v) = \int_0^1 F(x,v,\mu)\,\mu d\mu \qquad \text{(to right)}$$

<u>negative current of speed v</u>:

$$j_-(x,v) = \int_0^{-1} F(x,v,\mu)\,\mu d\mu \qquad \text{(to left)}$$

<u>b. net current of speed v, j(x,v)</u> by

$$j(x,v) = j_+(x,v) - j_-(x,v) = \int_{-1}^{1} F(x,v,\mu)\,\mu d\mu$$

where $F(x,v,\mu)$ is the neutron density in phase space.

[redacted]

Glossary, Reactor Theory

neutron current (continued)

c. analogous relations hold for the total neutron current (of all velocities)

positive total current:

$$j_+(x) = \int_0^1 \phi(x,\mu)\,\mu d\mu$$

negative total current:

$$j_-(x) = \int_0^{-1} \phi(x,\mu)\,\mu d\mu$$

d. the net total current

$$j(x) = \int_{-1}^{1} \phi(x,\mu)\,\mu d\mu$$

where $\phi(x,\mu)$ is the scalar sum of all fluxes of neutrons/cm^3 having direction cosines between μ and $\mu + du$.

$$\phi(x,\mu) = \int_v F(r,v,\mu)\,d\mu\ .$$

[redacted]

Glossary, Reactor Theory

neutron density, n

The number of neutrons of all velocities per cubic centimeter.

neutron density in phase space, $F(r,v,\mu)$

The number of neutrons per cubic centimeter of scalar speed between v and v + dv whose direction lies in the solid angle $d\Omega$ around the unit vector $\underline{\Omega}$ in the volume element dx dy dz located at x,y,z.

[redacted]

Glossary, Reactor Theory

neutron distribution: fundamental and higher modes (harmonics)

There are two ways in general use in which the neutron distribution in a pile may be divided into components, one of which may be called the fundamental distribution and the others called the higher modes. This duality has resulted from the fact that for a bare pile the two viewpoints become identical.

Consider a pile core with reflector, in general in a transient state (non-critical), with neutron distribution at some time t of $n(\underline{r},t)$:

A. In one viewpoint the neutron distribution is expanded in terms of a series of characteristic functions $Z_i(\underline{r})$ which are solutions of the Helmholtz equation:

$$\nabla^2 Z(\underline{r}) + B^2 Z(\underline{r}) = 0$$

for the boundary conditions that the $Z_i(\underline{r}) = 0$ on the extrapolated boundary. That is, the distribution is expanded in terms of the solutions of the Helmholtz equation for a bare pile of size and shape equal to the actual core plus reflector. The solutions of the wave equation yield eigenvalues B_i^2 and eigenfunctions $Z_i(\underline{r})$. The latter form a complete, orthogonal set of functions.

(continued)

[redacted]

Glossary, Reactor Theory

neutron distribution: fundamental and higher modes (harmonics) (continued)

The neutron distribution $n(\underline{r},t)$ may then be expressed

$$n(\underline{r},t) = \sum_{i=0}^{\infty} n_i(\underline{r},t)\, Z(\underline{r}).$$

By definition the first term $n_o(\underline{r},t)\, Z_o(\underline{r})$ is called the fundamental, and the general term $n_i(\underline{r},t)\, Z_i(\underline{r})$ is called the higher mode of order i. For a rectangular pile form a set bearing a harmonic relationship (the buckling eigenvalues, B_i^2, form a harmonic series: 1:1/2:1/3:1/4 etc.) and hence the higher modes are then called harmonics.

In the viewpoint the stationary state of a pile with reflector contains the fundamental and all higher mode components, whereas a bare pile has only the fundamental component. In the general pile (with reflector) the fundamental and higher modes bear no simple relationship to the time dependent behavior of a non-critical pile.

B. In the second viewpoint the neutron distribution $n(\underline{r},t)$ is expanded in terms of a series of time-independent functions $n_i(\underline{r})$:

$$n(\underline{r},t) = \sum_{i=0}^{\infty} n_i(\underline{r})\, e^{\lambda_i t}$$

where the $n_i(\underline{r})$ are eigenfunctions of the general pile equation (with negative neutron densities permitted). The effect of the

(continued)

[redacted]

neutron distribution: fundamental and higher modes (harmonics) (continued)

delayed neutrons is for the moment ignored. The time decay constants λ_i are ranked in increasing order; the smallest, λ_o, is the reciprocal of the stable period and the corresponding distribution $n_o(\underline{r})$ is called the fundamental.

In the stationary pile, $\lambda_o = 0$ and only the $n_o(\underline{r})$ distribution is present, since all the other λ_i are negative and the higher modes (the transients) die out.

While one would prefer this latter viewpoint, it is not always useful, since the distributions $n_i(\underline{r})$ and the λ_i for the general case cannot be obtained rigorously, and often cannot even be approximated analytically.

The fundamental and higher mode distributions are illustrated in the figure for the two cases:

(continued)

[redacted]

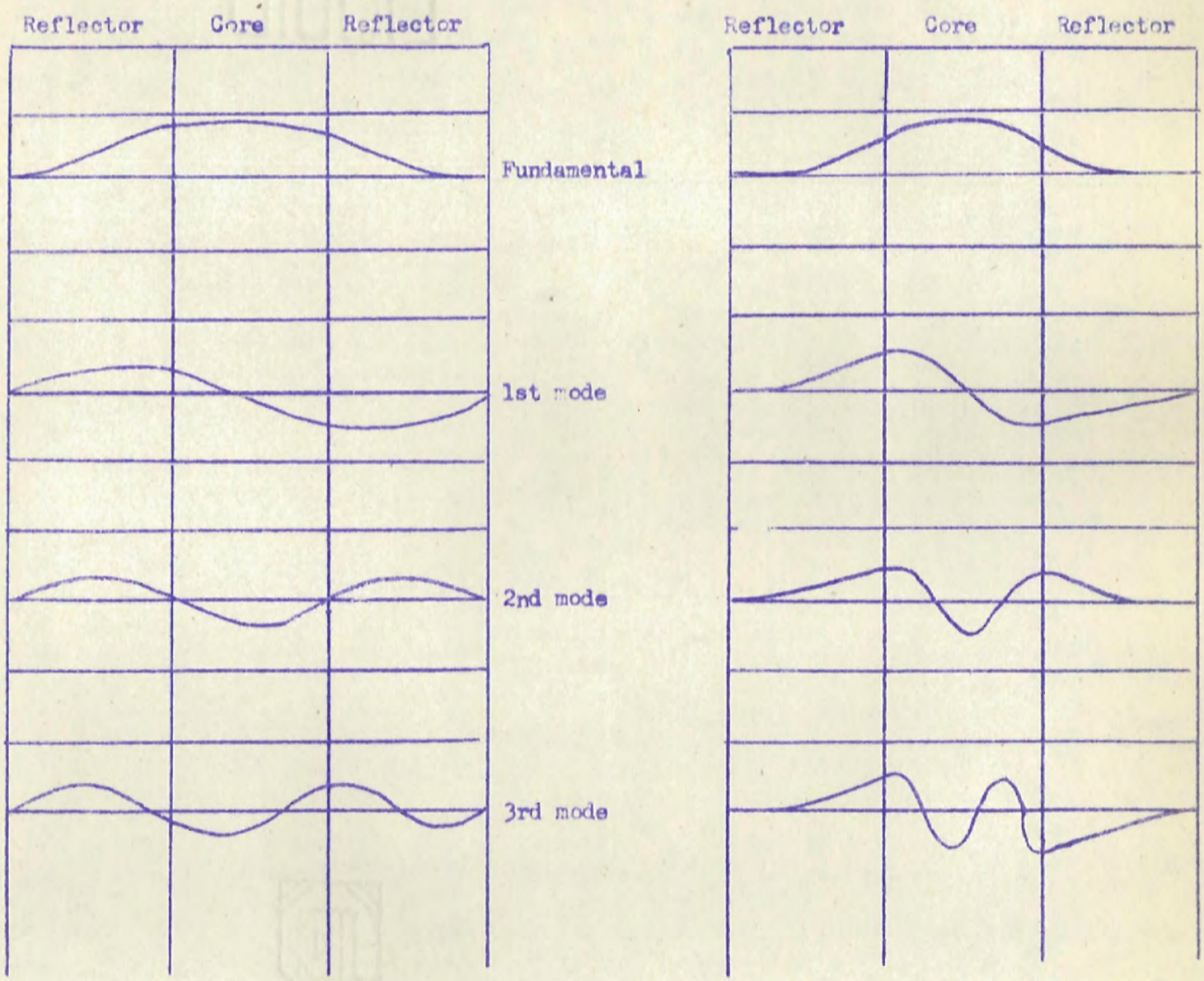

Fig. 2. Illustration of the Fundamental and Higher Modes of Neutron Distribution from Two Viewpoints.

[redacted]

Glossary, Reactor Theory

neutron energy groups

In the early days of neutron experimentation the neutron energy was specified by a material which had strong absorption for neutrons of that energy as follows:

Name of Group	Absorber	Approximate Energy	Linear Absorption Coefficient
A	Silver		210 cm^{-1}
C	Cadmium	0.3 volts - thermal	130 cm^{-1}
D	Rhodium		41 cm^{-1}
I	Indium	1.44 volts	

neutrons, fast

Usually means neutrons with energies higher than about 10^5 electron volts.

neutron flux, ϕ_o (total)

a. total

The scalar sum of all the fluxes in a cubic centimeter.

$$\phi_o(r) = \sum n_{oi}(\underline{r})\, v_i$$

where $n_{oi}(\underline{r})$ is the total number of neutrons per cm^3 having speed (scalar) v_i cm/sec, and $\underline{r}$ is the position vector.

(continued)

neutron flux (continued)

b. total flux of speed v. $F_o(r,v_i)$

The neutron flux of speed v_i is the product of v_i and $n_{oi}(r)$, the total number of neutrons per cm^3 having scalar speed v_i cm/sec.:

$$F_o(\underline{r},v_i) = n_{oi}(\underline{r})\ v_i$$

where $\underline{r}$ is the position vector.

c. vector flux of velocity v. $\underline{F}(\underline{r},v,\underline{\Omega})$

The vector flux of velocity v is the product of the density of the neutrons in phase space (i.e. in unit range around the vector v where v is the speed and $\underline{\Omega}$ the unit vector in the direction of the velocity) and the velocity $v\underline{\Omega}$:

$$\underline{F}(\underline{r},v,\underline{\Omega}) = n(\underline{r},v,\underline{\Omega})\ v\underline{\Omega}\ ,$$

where $\underline{r}$ is the position vector.

d. total vector flux $\underline{\Phi}(\underline{r},\underline{\Omega})$

The total vector flux is the integral of the vector flux with respect to the scalar speed v. It gives the total flux per cubic centimeter in unit range around the unit vector $\underline{\Omega}$.

$$\underline{\Phi}(\underline{r},\underline{\Omega}) = \int_0^\infty \underline{F}\ (\underline{r},v,\underline{\Omega})\ dv.$$

The integral of the total vector flux over all directions is the total flux $\Phi_o(\underline{r})$.

(continued)

[redacted]

Glossary, Reactor Theory

neutron flux (continued)

e. Legendre component flux of speed v.

If the scalar part of the vector flux is transformed in terms of the cosine of the angle with respect to the axis, μ, and the scalar flux is further a function x alone, the new function may be expanded in a series of Legendre functions:

$$F(x,v,\mu) = \sum_{\ell=0} \frac{2\ell+1}{2} F_\ell(x,v) P_\ell(\mu)$$

where the coefficients F(x,v) are given by

$$F_\ell(x,v) = \int_{-1}^{1} P_\ell(\mu) F(x,v,\mu) d\mu$$

since the $P_\ell(\mu)$ are orthogonal.

The coefficient, $F_\ell(x,v)$, is called the Legendre component of flux of speed v and order ℓ.

neutron flux distribution, fundamental and higher modes (harmonics)

See "neutron density distributions, fundamental and harmonic".

The same discussion as applied to the neutron density distributions apply to the neutron flux distributions with the word "flux" put in place of "density", and the corresponding symbols changed.

[redacted]

neutron hardening

The effect caused by the diffusion of thermal neutrons through an absorber obeying the 1/v absorption law. As the slower neutrons are preferentially absorbed, the average energy of the neutrons becomes greater as they progressively diffuse.

neutron inventory

See "inventory, neutron".

neutron, thermal

See "thermal neutron".

neutrons per fission, ν

A self-defining constant of the fission process.

neutrons per thermal neutron absorbed, η

A self-defining constant of the fission process. As all of the neutrons absorbed in uranium do not produce fission, η is less than ν, the neutrons per fission.

$$\eta = \frac{\Sigma_{U235}}{\Sigma_{U235} + \Sigma_{U238}} \nu$$

where ν is the neutrons per fission

Σ_{U235} is the macroscopic absorption cross-section of U235

Σ_{U238} is the macroscopic absorption cross-section of U238

[redacted]

Glossary, Reactor Theory

non-breeder pile (daughter, adolescence, infantile)(converter)

A pile in which the inventory of fissionable material in the pile decreases as it produces energy, sometimes called a "converter." A pile which makes no conversion of fertile material to fissionable material at all may be called an "infantile pile," although this term is not in general use.

[redacted]

Glossary, Reactor Theory

normal error curve

See "Gaussian distribution".

nuclear reactor

See "pile".

one-group kernel

See "kernels, slowing down, special case, one-group".

orthogonality and completeness

a. orthogonality

A set of functions $S_n(x)$ defined between $a \leq x \leq b$ are said to be orthogonal if

$$\int_a^b S_n(x)\, S_m(x)\, dx = c_{nm}\, \delta_{nm}$$

where δ_{nm} is the Kronecker delta.

If, furthermore, $cn = 1$, then the set of functions are said to be ortho-normal.

b. completeness

A set of functions is said to be complete if an arbitrary function, $f(x)$, at least piecewise continuous, satisfying the same boundary conditions as the functions of the set, can be expanded as follows:

$$f(x) = \sum_{n=1}^{\infty} a_n\, S_n(x)$$

the a_n being constant coefficients.

[redacted]

Glossary, Reactor Theory

orthogonality theorem

If a set of functions, $S_n(x)$, defined between $a \leq x \leq b$ is ortho-normal and complete, an arbitrary function $f(x)$ which is at least piecewise continuous may be expanded in a series

$$f(x) = \sum_{n=0}^{\infty} a_n S_n(x)$$

where the coefficients a_n are determined by

$$a_n = \int_a^b f(x) S_n(x) dx.$$

period of a nuclear reactor

a. stable period

The time required for the neutron inventory to change a factor e, assuming an exponential rise. Whenever the pile kinetics is such that the neutron inventory changes in a manner described by several exponential functions; the longest period (i.e. the reciprocal of the coefficient of t in the exponent of e) is called the stable period.

b. transient periods

All other periods are transient periods.

perturbation

A change in a known system.

See "perturbation theory".

[redacted]

Glossary, Reactor Theory

perturbation theory (as applied to reactor theory) including

perturbation

self-adjoint operator (Hermitian operator)

adjoint operator

adjoint equation

adjoint function

importance function

A perturbation means a change from a familiar situation, and this is the sense in which it is used in perturbation theory. Perturbation theory is designed to proceed from a known problem, such as the solution of the pile equations, where the eigenvalues and the eigenfunctions are known, to the computation of new eigenfunctions and eigenvalues brought about by a perturbation, that is, a known change, in the system. Because the operational complexity increases with the magnitude of the perturbation, perturbation theory is useful when the magnitude of the perturbation is reasonably small.

Thus if the neutron flux and the buckling is solved for a given pile, one may, by means of the perturbation theory, compute easily such things as the change in k_{eff} necessary to compensate for a small absorber placed in a localized position, or the change in k_{eff} necessary to compensate for a small change in the concentration of fissionable material at various points, etc.

(continued)

[redacted]

Glossary, Reactor Theory

perturbation theory (continued)

Consider the multi-group pile equations written in the form:

$$Mn = \frac{\partial n}{\partial t}, \qquad (1)$$

where M is the appropriate matrix operator and n is the vector $(n_1, n_2, n_3 \ldots\ldots n_m)$ and where m is the number of groups and n_i is the neutron flux distribution of the ith group. Equation (1) is simply a condensation of the m group equations, since the equation of the ith group is obtained by

$$\sum_{j=1}^{m} M_{ij}\, n_i = \frac{\partial n_i}{\partial t} .$$

Actually, all but two of the M_{ij} for constant i are zero.

Solutions of (1) are now considered which depend on the time exponentially, that is

$$n_i = n_{io}\, e^{\lambda t} ; \quad \frac{\partial n_i}{\partial t} = \lambda_i n .$$

Since the group equations are coupled by the chain reaction, all the λ_i for the various groups are equal, we have then

$$Mn = \lambda n . \qquad (2)$$

The solution of (2) results in general in the eigenvalue λ_k and eigenfunction n_k, but since we are interested usually in compensating changes which will keep the pile in a steady state, or in changes in the longest (stable) period, we shall henceforth consider only the lowest eigenvalue λ_o, and write it simply as λ.

(continued)

perturbation theory (continued)

There is a difficulty in going beyond these simple aims, since to use the perturbation theory to evaluate the n_k (perturbed eigenfunctions) in the perturbed pile, it is necessary that the n_k of the unperturbed pile form a complete set, and this has not been demonstrated.

Hermitian self-adjoint operator; adjoint operator; adjoint functions

Consider equation (2) with two particular eigenfunctions n_k, n_ℓ

$$M\, n_k = \lambda_k n_k$$

$$M\, n_\ell = \lambda_\ell n_\ell \, .$$

It is evident on cross-multiplication and integration that

$$\int n_\ell\, M\, n_k\, d\tau = \lambda_k \int n_k\, n_\ell\, d\tau$$

$$\int n_k\, M\, n_\ell\, d\tau = \lambda_\ell \int n_k\, n_\ell\, d\tau .$$

Subtracting, one gets

$$\int n_\ell\, M\, n_k\, d\tau - \int n_k\, M\, n_\ell\, d\tau = (\lambda_k - \lambda_\ell) \int n_k\, n_\ell\, d\tau .$$

If the eigenfunctions are orthogonal, that is, if

$$\int n_k\, n_\ell\, d\tau = \delta_{k\ell}\, \delta_{k\ell}$$

where $\delta_{k\ell}$ is the Kronecker delta, then it is evident that

$$\int n_\ell\, M\, n_k\, d\tau = \int n_k\, M\, n_\ell\, d\tau \, . \qquad (3)$$

(continued)

perturbation theory (continued)

Hermitian self-adjoint operator; adjoint operator; adjoint functions (continued)

An operator obeying equation (3) is said to be Hermitian or self-adjoint, and the eigenvalues of

$$Mn = \lambda_n$$

if M is self-adjoint and the eigenfunctions are orthogonal.

Suppose that the n_k do not form an orthogonal set as is generally true of the solution for an arbitrary pile; then we define the operator M^+ which is adjoint to M (calling it the adjoint operator, by the relation

$$\int n_k^+ M n_\ell \, d\tau = \int n_\ell M^+ n_k^+ \, d\tau ,$$

where, now, the n_k are the eigenfunctions of the adjoint equation,

$$M^+ n^+ = \lambda n^+ .$$

The functions n^+ are called the adjoint functions.

On going through a similar process as that leading to the definition of the Hermitian operator, i.e., cross-multiplication and integration, we get

$$\int n_\ell M^+ n_k^+ \, d\tau - \int n_k^+ M n_\ell \, d\tau = (\lambda_k - \lambda_\ell) \int n_k^+ n_\ell \, d\tau$$

that is, since M and M^+ are adjoint, the n_k^+ functions are orthogonal to the function n_k:

$$\int n_k^+ n_\ell \, d\tau = C_{k\ell}^+ \delta_{k\ell} . \qquad (4)$$

(continued)

[redacted]

Glossary, Reactor Theory

perturbation theory (continued)

pile perturbation theory

The pile equation operator M is in general not self-adjoint, except for a bare pile, where it does give eigenfunctions which form a complete orthogonal set. Where the pile has a reflector the eigenfunctions are not orthogonal. (We might, however, infer their completeness since we may consider the reflector a perturbation on the bare pile.) Therefore, we consider the adjoint equation of the known pile

$$M^{+} n^{+} = \lambda n^{+} \qquad (5)$$

and write down the equation of the perturbed pile as

$$(M + V)N = \lambda' N \qquad (6)$$

where V is a "small" addition to M. Multiplying equation (5) by N, equation (6) by n, integrating and subtracting the first from the second:

$$\int n^{+} MN d\tau - \int NM^{+} n d\tau + \int n^{+} VN d\tau = (\lambda' - \lambda) \int n^{+} N d\tau$$

or therefore

$$\lambda' - \lambda = \frac{\int n^{+} VN d\tau}{\int n^{+} N d\tau} .$$

We now assume V is small, and that we may replace, in the first order of approximation N, by n, getting the first order.

(continued)

[redacted]

Glossary, Reactor Theory

perturbation theory (continued)

pile perturbation theory (continued)

perturbation solution of the change in pile period:

$$\Delta\lambda = \frac{\int n_0 V n_0 d\tau}{\int n_0 n_0 d\tau} \quad . \qquad (7)$$

The zero subscripts are added to indicate we are interested in the change in the fundamental mode. V, the part of the operator added by the perturbation, may be a term added by a change in any of the pile constants, localized sinks or sources, etc., or by a combination of all these changes. If one of these is $\delta k_{effective}$, and we put $\Delta\lambda$ equal to zero, then equation (7) will give the change in k_{eff} necessary to compensate the other changes.

adjoint functions

We now look at the relation between the operator M, and the adjoint operator M^+. Texts on matrix algebra will show that the adjoint M^+ is formed by replacing each element of M, $M_{\ell k}$, by the adjoint of the element, and transposing (interchanging indices, i.e. rows and columns).

$$M^+ = \left\| M^+_{\ell k} \right\|$$

where

$$M = \left\| M_{k\ell} \right\|$$

(continued)

perturbation theory (continued)

adjoint functions (continued)

Since it can be shown that the elements of the pile equation matrix operator are all Hermitian, then one simply interchanges rows and columns in the m trix M to get M^+. When M^+ operates on the vector $n^+ = (n_0^+, \ldots\ldots\ldots, n_m^+)$ the resulting equations (the adjoint equations) represent an m-group chain cycle which is exactly reversed from the natural neutron chain reaction. That is, the fictitious "adjoint" neutrons get faster and faster, combine other neutrons with fission fragments to form a fissionable nucleus, then emit a thermal neutron which repeats the chain. The extra neutrons are supplied by leakage into the system.

importance function

We now define the importance function $I_i(\underline{r})$. The importance function represents the net number of neutrons added to the neutron inventory (i.e. the whole pile) brought about by placing one neutron of energy corresponding to the ith group at position $\underline{r}$. That is, $I_i(\underline{r})$ is a weighting function for the importance of a neutron of energy corresponding to the ith group at $\underline{r}$. Obviously, a neutron in the center of the pile has more importance than one near the edge.

It can be shown that the importance functions are the normalized adjoint functions n_i^+:

$$I_i(\underline{r}) = \frac{n_i^+(\underline{r})}{\int n_i^+(\underline{r})\, d\tau} .$$

(continued)

perturbation theory (continued)

importance function (continued)

The importance function for a bare pile, or for a large pile where the one-group theory applies, is therefore equal to the normalized flux.

phase velocity

See "group and phase velocity".

pile equation, asymptotic (uniform pile)

For a case of a uniform pile with transport mean-free-path independent of energy, the following "asymptotic pile equation" holds far (i.e. one diffusion length) from boundaries, as the slowing down kernel is assumed to be the infinite (displacement) slowing down kernel:

$$L^2 \Delta \Phi_s(\underline{r}) - \Phi_s(\underline{r}) + \frac{k}{p} \int_{\substack{\text{all}\\ \text{space}}} \Phi_s(r') \, P_\infty(E_s|r-r'|) \, d\underline{r}' = 0.$$

The fundamental theorem of the pile theory states that the asymptotic uniform pile equation is satisfied by a solution of

$$\Delta \Phi_s + B^2 \Phi_s = 0$$

(the space form of the wave equation) provided B^2 is a root of the characteristic equation

$$-L^2B^2 - 1 + \frac{k}{p} \overline{P_\infty}(E_s, B^2) = 0$$

(continued)

Glossary, Reactor Theory

pile equation, asymptotic (uniform pile) (continued)

and the slowing down density $q_\infty(E,r)$ is given by

$$q_\infty(E,r) = \frac{k}{p} N\sigma_a \bar{P}(E,B^2)\, \phi_s(\underline{r}),$$

i.e., $q_\infty(E,r)$ is proportional at every energy to $\phi_s(\underline{r})$.

In the above equations:

L is the diffusion length

Δ is the Laplacian operator

ϕ is the flux of slow neutrons

$\underline{r}$ is the position vector

k the multiplication constant

p the resonance escape probability

$\underline{r}'$ the integration position variable

$P_\infty(E_s, |\underline{r}-\underline{r}'|)$ the slowing down kernel

E_s the energy of the slow neutrons.

B^2 is a constant called the buckling; if a root of the characteristic equation, it is called the material buckling; if an eigenvalue of the space form of the wave equation, it is called geometric buckling, and $\bar{P}_\infty(E_s,B^2)$ is the three dimensional Fourier transform of $P_\infty(E_s,|r-r'|)$.

The asymptotic pile equation applies with good approximation to a large bare pile or to a large pile with reflector slowing down properties identical with slowing down properties of the active lattice. In the latter case the "fundamental theorem of pile theory" must be generalized to include all of the normal modes of the space form of the wave equation.

(continued)

[redacted]

pile equation, asymptotic (uniform pile) (continued)

Under the following restrictions the finite pile equations and the asymptotic pile equations have the same solution up to within a mean-free-path of the boundary:

If λ_t is independent of the neutron energy, and if the finite and infinite slowing down kernels satisfy the same linear equation, then the asymptotic solution holds everywhere (i.e., is identical with the solution of the finite equations) in a critical pile, except within a distance of the order of a mean-free-path from the boundary. The slowing down density in the pile is identical with the infinite slowing down density except within a distance of the order of a mean-free-path from the boundary. (Weinberg)

See "image and negative piles". See also

"critical equation"

"wave equation, space form of"

"Fourier transform"

"kernels, etc".

[redacted]

Glossary, Reactor Theory

pile equation, finite

The following equations pertaining to a finite pile are called the "finite pile equations" by Weinberg:

$$\nabla \cdot D_o \nabla \Phi_s(\underline{r}) - N\sigma_a \Phi_s(\underline{r}) + q(E_s, \underline{r}) = 0$$

$$q(E, \underline{r}) = \int_{\text{pile}} \frac{k}{p} N\sigma_a \Phi_s(\underline{r}') \, P(E, \underline{r}, \underline{r}') d\underline{r}$$

$$\left.\frac{1}{\Phi_s}\frac{\partial \phi_s}{\partial \nu} = \frac{1}{.71\,\lambda_t(E_s)}\,; \quad \frac{1}{q}\frac{\partial q}{\partial \nu} = \frac{1}{.71\,\lambda_t(E)}\right\} \text{ on the physical boundary}$$

where

$\nabla \cdot$ is the divergence operator

∇ is the gradient operator

D_o is the diffusion coefficient of neutron flux

$N\sigma_a$ is the macroscopic absorption coefficient

$\Phi_s(\underline{r})$ is the thermal neutron flux at a unit volume at position $\underline{r}$.

$q(E_s, \underline{r})$ is the source term; the flux of neutrons becoming thermal per unit volume at $\underline{r}$.

E_s is the energy of thermal neutrons

k is the finite multiplication constant

p is the resonance escape probability

$\underline{r}'$ is the vector position variable of integration

$\underline{P}(E, \underline{r}, \underline{r}')$ is the finite slowing down kernel

ν is the coordinate normal to the physical boundary

λ_t is the transport mean-free-path.

See "pile equations, asymptotic".

Glossary, Reactor Theory

pile (nuclear reactor)

The pile got its name from the early days of the development of a nuclear chain reaction. Since the word adequately described the appearance and at the same time avoided a description of its purpose, "pile" served as a good camouflage name during its secret development. The word is used in various fashions to describe the nuclear reactor and its concomitant parts; the part (active portion containing fissionable material plus reflector) internal to the shield; or sometimes to the active portion only. The word "core" is coming into general use for the name of the active portion, reserving the word "pile" or "nuclear reactor" for the entire structure.

a. fast

A fast pile is a nuclear reactor in which the largest part of the fissions are produced by neutrons having energies above 10^5 ev.

b. heterogeneous

A heterogeneous pile is one in which the microscopic properties as a function of position are not constant. In this sense all real piles are heterogeneous. A more useful usage of the term refers to a pile in which the fissionable material is lumped and surrounded by the moderating material. The smallest volume having symmetry is called a "cell". The cells are placed in a regular array or lattice to build up the reactor.

c. homogeneous

1) a homogeneous pile is one having constant microscopic properties. This restrictive definition restricts the term for

(continued)

[redacted]

Glossary, Reactor Theory

pile (nuclear reactor) (continued)

c. homogeneous (continued)

description of bare piles at constant temperature which in addition have the material composing it mixed homogeneously.

2) The term is used with less restriction, referring to piles in which the material is mixed homogeneously (it must then have no reflector).

3) The term refers to a pile in which the core has its material mixed homogeneously (it can then have a reflector of different material).

The most useful definition is probably the second.

d. intermediate

A nuclear reactor in which a large number of fissions are caused by neutrons having energies in the intermediate region (1 to 10^5, ev.).

e. pebble

A pebble pile is a reactor made by stacking (regularly or at random) pebbles made of moderating material and of fissionable material, or a homogeneous mixture of both. A coolant is circulated through the pores between the pebbles to cool the pile.

f. thermal (slow)

A nuclear reactor in which practically all of the fissions occur from the absorption of thermal neutrons.

[redacted]

Glossary, Reactor Theory

pile period

See "period, pile".

poisoning

Decreases in multiplication constant caused by impurities, i.e., by parasitic absorption of neutrons.

power control rod

See "control rod"

power level (heat)

The power level of a pile is the number of watts (or kilowatts) of heat produced by the pile.

prompt and delayed neutrons

More than 99% of the neutrons produced in the fission process are "boiled off" of the moving fission fragment, and hence are produced in a time immeasurably short (by present techniques) after the fission occurred. These are called "prompt" neutrons. The remaining one per cent or so are delayed a measurable time, appearing in several groups with half lives from fractions of a second to almost 100 seconds. These latter are called "delayed" neutrons.

During the successive beta-decays of the fission fragments that follow fission it is probable that some nuclei are formed in very highly excited states. If the excitation energy exceeds the binding

(continued)

prompt and delayed neutrons (continued)

energy of the neutron, then the nucleus will preferentially decay by neutron emission instead of by beta-particle emission. The neutron emission follows immediately upon the formation of the highly excited nucleus, being a daughter of a chain of previous beta-decays. Hence it is delayed by the appreciable time required for these previous delays. The delayed neutrons from the fission of U^{235} are given in the (declassified) Soodak notes (Soodak and Campbell "Elementary Pile Theory").

Half Period	% of Fission Neutrons, β_i	Identification of Delayed Emitter
55.6 sec	.026	Kr^{87}
22.0	.17	Xe^{137}
4.51	.21	
1.52	.24	
.43	.084	
.05	.029	
Total β =	.76%	

radiative capture cross-section

See "cross-section, radiative capture".

random walk ρ

The path followed by a particle as it makes random scattering collisions in a medium. The word "random" implies that a scattering in any direction is uniformly as probable as a scattering any other direction.

reactivity

The term reactivity is used with slightly different definitions by different authors.

Weinberg: $\rho(B_g^2)$ The relative excess multiplication constant:

$$\rho(B_g^2) = \frac{k_{eff} - 1}{k_{eff}} = 1 - \frac{p(1 + L^2B_g^2)}{k\,\bar{P}_\infty(E_s, B_g^2)} .$$

Soodak reactivity:

$$\rho = k - \frac{p(1 + L^2B_g^2)}{\bar{P}_\infty(E_s, B_g^2)} .$$

The Weinberg definition is convenient in that the kinetics of a pile not too far from critical may be expressed universally, i.e.

$$\rho(B_g^2) = \sum_i \frac{\beta_i\,\tau_i}{T_i\,\tau_i} \qquad \text{(See inhour formula)}$$

where

β_i is the yield of the ith delayed neutron per prompt neutron

T_i is the decay constant in hours of the ith delayed neutron

T is the pile period in hours.

See: inhour.

reflector (tamper)

A volume of scattering material placed around the active volume of the pile (around the core). The reflector serves to scatter back into the active volume some of the neutrons which would otherwise be lost to the chain reacting system. Because of the back scattering from the reflector the critical size of the active volume can be reduced over that of the bare pile.

reflector savings

The reflector savings is the amount by which the half-thickness of the active volume of a nuclear reactor is decreased due to the reflector from what it would be for a bare pile. Evidently, the reflector savings depends on the reflector thickness in particular, and in general, on the geometry of the pile.

The reflector savings as a function of the thickness of reflector rises with constantly decreasing slope until it becomes asymptotic to a constant value. The initial slope for ordinary water is much greater than that of graphite, although the asymptotic value reached is just the reverse. Thus a thin reflector of ordinary water is better than a thin reflector of graphite; whereas a thick reflector of graphite is better than a thick reflector of water.

resonance absorption

Absorption of neutrons in a more or less narrow energy level. This is the primary process for capture of neutrons in the energy range between thermal and 10^5 electron volts.

resonance escape probability, p(E)

The fraction of the originally fast neutrons (produced by fission or otherwise) in an infinite pile which reaches energy E, i.e. it is the slowing down density for a source strength of one fast neutron per cm^3 per second.

Lumping of the uranium in a pile results in an increase in the resonance escape probability by the screening of the interior of the lump from neutrons of resonance energy by the surface layer,

(continued)

[redacted]

resonance escape probability (continued)

and by decreasing the number of resonance neutrons. These advantages are gained at the expense of thermal utilization, which is decreased by lumping. The net effect, for optimum configuration, is to increase k, the multiplication constant, however.

resonance integral

The resonance integral is part of the exponent appearing in the formula for the resonance escape probability:

$$p = \exp\left(-\int \frac{\sigma_a}{\sigma_a + \sigma_s}\,\frac{dE}{E}\right).$$

The integral

$$\sigma_s \int \frac{\sigma_a}{\sigma_a \quad \sigma_s}\,\frac{dE}{E} \qquad \text{is the resonance integral;}$$

if $\sigma_s >> \sigma_a$, and σ_s = a constant. Then

$$\sigma_s \int \frac{\sigma_a}{\sigma_a + \sigma_s}\,\frac{dE}{E} = \int \sigma_a\,\frac{dE}{E}\ .$$

The integral on the right is usually given as the resonance integral, although in calculations the long form is used.

Ritz variation method

See "variation (Ritz) method".

[redacted]

Glossary, Reactor Theory

safety control rod

See "control rod".

scalar flux

See "neutron flux, scalar".

scattering cross-section

See "cross-section, scattering".

scattering cross-section, elastic

See "cross-section, scattering".

scattering cross-section, inelastic

See "cross-section, scattering".

scattering, elastic

A process in which the identity of the colliding particles is preserved and the total kinetic energy and momentum is conserved.

scattering, inelastic

A process in which the colliding neutron is absorbed in the nucleus and a similar particle emitted. Thus the identity of the incident particle is lost, and kinetic energy is not conserved.

[redacted]

Glossary, Reactor Theory

self-adjoint operator

Let u and v be two "acceptable" functions, i.e. solutions of a given problem, defined over a certain range of configuration space. The operator P is said to be self-adjoint or Hermitian if

$$\int_{\tau} u^* \cdot Pv d \quad = \int_{\tau} v \cdot P^* u^* d \quad ,$$

where the asterisk (*) indicates the complex conjugate.

See also "perturbation theory".

shield

The material placed around a nuclear reactor to reduce the radiation escaping to a permissible level.

shield, thermal

An inner portion of the shield designed to remove a large fraction of the primary (heat producing) gamma-rays. Iron has been employed as a thermal shield.

shim control rod

See "control rod".

sigma pile

A parallelopiped composed of some slowing down material with one long dimension. It is used to measure slowing down ranges and thermal diffusion lengths, and thus indirectly for the calculation of absorption cross-sections.

[redacted]

Glossary, Reactor Theory

slowing down density, q(E)

The number of neutrons slowed down below a given energy per cm^3 per second. In a non-absorbing medium of infinite extent (no leakage) the slowing down density is obviously a constant. For the case of continuous slowing down (Fermi theory) the slowing down density is related to the flux of neutrons of energy E and speed v by the relation

$$n(E)v = \frac{q(E)}{\xi \Sigma_s} \cdot \frac{1}{E} ,$$

where ξ is the logarithmic slowing down decrement, Σ_s is the macroscopic scattering cross-section and n(E) is the number of neutrons in energy range dE about E per cm^3.

When absorption is present in the media, and for the case of continuous slowing down in a homogeneous mixture, the slowing down density is given by:

$$\frac{q(E)}{q(E_o)} = e^{-\frac{1}{\xi}\int_E^{E_o} \frac{\Sigma_a}{\Sigma_s + \Sigma_a} \cdot \frac{dE}{E}} = p(E),$$

where E_o is the initial energy, and Σ_a is the macroscopic absorption cross-section. The ratio of q(E) to $q(E_o)$ is by definition the resonance escape probability at energy E.

[redacted]

Glossary, Reactor Theory

slowing down index, ξ (average loss in the logarithm of the energy in one collision)

also known as the logarithmic energy decrement per collision.

$$\xi = 1 + \frac{r}{1-r} \log r \doteq \frac{2}{A + 2/3} \quad (A > 10)$$

where

$$r = \left(\frac{A-1}{A+1}\right)^2$$

and A = atomic weight.

slowing down kernel

See "kernel, slowing down".

slowing down power, $\Sigma_s \xi$

The product of the macroscopic scattering cross-section and the logarithmic energy decrement per collision. It is a measure of the moderating ability of an atomic species per atom. The slowing down power may be regarded as the average loss in log E per centimeter of travel.

specific power (pile)

The specific power of a pile is defined as the number of kilowatts of heat produced divided by the inventory of fissionable material (in kilograms) in the pile.

[redacted]

spherical coordinates (Laplacian operator expressed in)

See "wave equation".

spherical harmonics

See "wave equation".

stable period

See "period, pile"

statistical weight (of a region of a pile)

The statistical weight of a volume R of a pile is defined as follows:

$$W = \frac{\int_R \underline{\phi}\,\underline{\phi}\,d\tau}{\int_{\substack{\text{whole}\\ \text{pile}}} \underline{\phi}\,\underline{\phi}\,d\tau},$$

where $\underline{\phi}$ is the vector neutron flux:

$$\underline{\phi} = \left\{\phi_1, \phi_2, \phi_3 \ldots\ldots\ldots \phi_m\right\}$$

and $\underline{\phi}$ the vector adjoint flux

$$\underline{\phi} = \left\{\phi_1, \phi_2, \phi_3 \ldots\ldots\ldots \phi_m\right\}$$

as determined from the matrix multi-group equation, and the matrix multi-group adjoint equation.

(continued)

[redacted]

Glossary, Reactor Theory

statistical weight (of a region of a pile) (continued)

For a large or bare pile (whose multiplication constant is necessarily near unity) the statistical weight reduces to that for one group theory where the adjoint and normal neutron fluxes are identical.

$$W = \frac{\int_R \phi^2 \, d\tau}{\int_{\substack{\text{whole}\\ \text{pile}}} \phi^2 \, d\tau} .$$

In this last case the statistical weight may be regarded as the relative contribution to the buckling by the region R to that of the entire reaction.

See "perturbation theory."

Stochastic transformation

See "Monte-Carlo method"

tamper

See "reflector"

[redacted]

Glossary, Reactor Theory

tesseral harmonics

Functions formed by the product of cos nØ or sin nØ and the associated Legendre functions. A tesseral harmonic is a special case of a spherical harmonic:

tesseral harmonic

$$T_m^n = \cos n\phi \, P_m^n(\mu)$$

$$\text{or} \quad \sin n\phi \, P_m^n(\mu)$$

spherical harmonic

$$Y_m(\theta,\phi) = \sum_{n=-m}^{m} A_n \cos n\phi \, P_m^n(\mu) \quad B_n \sin n\phi \, P_m^n(\mu) \, .$$

thermal column

A column of moderating material extending through the shield into the reflector of a pile. Its purpose is to slow down to thermal energies the neutrons which diffuse through it.

thermal neutron

Neutrons which are in thermal equilibrium with the medium. Strictly, this term should apply only to a large number of neutrons with a Maxwellian distribution of velocities which corresponds to the temperature of the medium; but it is also used more loosely to designate single neutron groups whose kinetic energy is of the order of magnitude of kT.

[redacted]

Glossary, Reactor Theory

thermal shield

See "shield, thermal".

thermal utilization, f

In the active material, or in a lattice cell of a heterogeneous pile, the ratio of the thermal neutrons absorbed in uranium to the total absorbed (in the cell). The maximum value of the thermal utilization occurs (for a given ratio of uranium to moderator) for a homogeneous mixture. The lumping of the uranium always decreases the thermal utilization.

total cross-section

See "cross-section, total".

transient period

See "period, pile".

transport diffusion equation

See "Boltzmann equation".

transport kernel

See "kernel, transport".

transport scattering cross-section

See "cross-section, transport scattering".

[redacted]

two-group slowing down kernel

See "kernel, slowing down, infinite, convolution of, special case: two-group, convolution of diffusion kernels".

variation (Ritz) method (as applied to nuclear reactor theory) for solution of the wave equation

Euler's equation

The Ritz variation method is a means for obtaining the approximate eigenvalues and eigenfunction solutions of the wave equation. It is a useful method for obtaining the buckling and slow neutron distribution in a bare pile of complicated shape, etc., and on generalization, is useful for obtaining the same values for a slab and spherical pile with a thick reflector having slowing-down properties similar to that in the active portion of the pile.

Consider the integral

$$\int I(x,\phi,\phi_x)\, d\tau = \int \left[(\nabla \phi)^2 - (B^2\phi)^2 \right] d\tau \quad , \qquad (1)$$

where $\nabla\phi$ is the gradient of the slow neutron flux

B is the buckling

ϕ is the slow neutron flux

$d\tau$ is the volume element

x is one of the coordinates

ϕ_x is the partial derivative of ϕ with respect to x.

(continued)

<u>variation (Ritz) method</u> (continued)

If a function ϕ is selected which is not the actual solution, the integral (1) computed, then compared with a similar integral obtained with the functions ϕ_2, ϕ_3, etc., the lowest value will correspond to the nearest correct solution for B^2, but greater in magnitude and the eigenfunction will usually approximate the correct eigenfunction.

Instead of trying many different ϕ_i, however, it is easier to select a finite set ϕ_i, which satisfy the boundary conditions, and let

$$\phi = \sum_i a_i \phi_i .$$

One evaluates equation (1), then varies a_i in such a manner that the integral is an external:

$$\frac{\delta I}{\delta a_i} = 0 ,$$

getting a finite set of simultaneous equations which can be solved for the a_i.

From the calculus of variation, it can be shown that the necessary and sufficient condition that the integral be an extremal:

$$\delta \int I(r,\phi,\phi_x)\, d\tau = 0$$

is given by <u>Euler's equation</u>:

$$\frac{\partial I}{\partial \phi} - \frac{d}{dx}\frac{\partial I}{\partial \phi_x} = 0 \qquad (2)$$

(continued)

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variation (Ritz) method (continued)

where it is understood that the differentiation is taken over all of the coordinates. Application of Euler's equation to the integral, (1), it yields:

$$\nabla^2 \phi + B^2 \phi = 0 ,$$

i.e. the wave equation. Thus if one can discover a function ϕ which satisfies the boundary conditions and makes equation (1) an extremal (actually a minimum, which can be demonstrated) then it is a solution of the wave equation and B^2 is the lowest eigenvalue.

For the set of simultaneous equations to be independent, the determinant of the coefficients of the a_i must vanish. The solution of this determinant gives values of B^2, the lowest of which is an approximation to B_o^2, the buckling of the fundamental mode.

By operation of Green's theorem

$$\int_\tau \nabla\phi \cdot \nabla\psi \, d\tau = \int_s \phi \nabla\psi \cdot d\underline{s} - \int_\tau \phi \nabla^2 \psi \, d\tau$$

to the normalized equation (1):

$$\frac{\delta \int_\tau \left[\nabla\phi \cdot \nabla\phi - (B^2\phi)^2\right] d\tau}{\int_\tau \phi^2 \, d\tau}$$

(continued)

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variation (Ritz) method (continued)

one gets (since the surface integral vanishes and since the variation of $(B^2\phi)^2$ is zero),

$$= - \frac{\int_\tau \phi \nabla^2 \phi \, d\tau}{\int_\tau \phi^2 \, d\tau}$$

a form familiar in Quantum Mechanics.

virgin flux

The flux of fission neutrons which have made no collisions.

wave equation (space form of) (Helmholtz equation)

The differential equation

$$\nabla^2 \phi + B^2 \phi = 0$$

where ∇^2 is the divergence of the gradient (the Laplacian operator, ϕ is the neutron flux in the pile equation, and B^2 a constant (real) called in reactor physics the "geometric buckling." It's negative value is also sometimes referred to as the "Laplacian" with symbol Δ. Since this latter term causes confusion with the Laplacian operator, its use is not popular.

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<u>wave equation (space form of) (Helmholtz equation)</u> (continued)

The name "wave equation" is short for what is correctly described as "the space form of the wave equation," or as the scalar "Helmholtz equation." Mathematically the equation

$$\nu^2 \nabla^2 U - \frac{\partial^2 U}{\partial t^2} = 0,$$

where U is any physical quantity, t the time, and ν a constant called the phase velocity, is known as the wave equation. When the space variables are separated from the time variables by assuming that

$$U = ST$$

where S is a function of space coordinates and T a function of time only there results

$$T + \omega^2 T = 0, \text{ and}$$

$$\nabla^2 S \quad \frac{\omega^2}{\nu^2} S = 0.$$

In cartesian coordinates, (x, y, z), the Laplacian of ϕ becomes

$$\nabla^2 \phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi$$

(continued)

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wave equation (space form of) (Helmholtz equation) (continued)

and the solution of the Helmholtz equation is

$$\phi = C_{B_1 B_2 B_3} e^{\pm i(B_1 x + B_2 y + B_3 z)}$$

where $B_1^2 + B_2^2 + B_3^2 = B^2$;

Thus if B^2 is positive, the solution leads to sines and cosines; if B^2 is negative, the solution leads to hyperbolic sines and cosines. The imposition of boundary conditions puts restrictions on the possible values of the B_i's and thus leads to eigenvalues. The solutions for ϕ then are called eigenfunctions. This last statement supplies in general to any geometry.

The additional restriction that the flux be everywhere finite, positive or zero, leads in the bare pile to the acceptance of the lowest ordered eigenvalue or fundamental mode. For a bare rectangle of sides a, b, and c, the eigenvalue (of the fundamental mode) is

$$B^2 = \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} + \frac{\pi^2}{c^2} \text{ , and}$$

$$\phi = \phi_o \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \cos \frac{\pi z}{c}$$

(origin in the center).

(continued)

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<u>wave equation (space form of) (Helmholtz equation)</u> (continued)

In cylinder coordinates (r, φ, z) the Laplacian of ϕ becomes:

$$\nabla^2\phi \quad \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\phi^2} + \frac{\partial^2\phi}{\partial z^2}$$

and the solution of the Helmholtz equation can be written

$$\phi = C\, e^{\pm i(B_z Z \pm \ell\phi)} \quad K_\ell\,(B_r\, r)$$

where $B^2 = B_r^2 + B_z^2$

$$\ell = 0, 1, 2, \text{etc.}$$

and $Z_\ell(x)$ is the <u>Bessel Function</u> (cylindrical harmonics) of order ℓ. If B^2 is positive, $Z_\ell(B_r\, r)$ becomes $J_\ell(B_r\, r)$ or $N_\ell(B_r\, r)$, the Bessel functions of first and second kind respectively. If B^2 is negative, then $Z_\ell(B_r\, r)$ becomes the Hankel functions, as the argument is imaginary (or equivalently, the $I_\ell(B_r\, r)$ and $K_\ell(B_r\, r)$ functions of first and second kind respectively with imaginary arguments).

For a bare cylindrical reactor of radius R and height, there results since $\ell = 0$, the eigenvalue

(continued)

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<u>wave equation (space form of) (Helmholtz equation)</u> (continued)

$$B^2 = \frac{\pi^2}{h^2} + \frac{(.7655\,\pi)^2}{R^2}$$

and $\quad \phi = \phi_o \cos \frac{\pi z}{h} J_o (B_r r)$

$$B_r = \frac{.7655\,h}{R}$$

In Polar (spherical) coordinates (r, θ, ϕ) the Laplacian becomes

$$\nabla^2 \phi = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \phi}{\partial \theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 \phi}{\partial \phi^2}\right]$$

and the solution of the Helmholtz equation can be written:

$$\phi = \sum_{\ell=0}^{\infty} C_{k\ell} Y(\theta, \phi)\, r^{-\frac{1}{2}}\, Z_{\ell+\frac{1}{2}}(Br)$$

where

$$Y_\ell(\theta, \phi) = \sum_{m=-\ell}^{\ell} C_m P_\ell^m (\cos\theta)\, e^{-im\varphi}$$

and is known as <u>surface spherical harmonic</u>

$C_{k\ell}$ are constants

C_m are constants

(continued)

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wave equation (space form of) (Helmholtz equation) (continued)

$$P_\ell^m(x) = (1 - x^2)^{m/2} \frac{d^m P_\ell(x)}{d x^m}$$

and are called Associated Legendre functions, or associated spherical harmonics, the functions

$$\frac{d^m P_\ell(x)}{d x^m}$$ are called Helmholtz functions;

where

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \frac{d^\ell \left[(x^2 - 1)^\ell \right]}{d x^\ell}$$, are called

surface zonal harmonics, or Legendre polynomials, or Lengendre coefficients; and

where

$Z_{\ell + \frac{1}{2}}(Br)$ is the Bessel function of order $\ell + \frac{1}{2}$.

For a bare spherical reactor of radius R, we get $m = \ell = 0$, and the imposition of the boundary conditions results in the eigenvalue

$$B = \frac{\pi}{R}$$, and the eigenfunction

$$\phi = \phi_0 \frac{\sin Br}{Br}$$.

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wave equation (space form of) (Helmholtz equation) (continued

If the surface zonal and spherical harmonics are multiplied by r^m or r^{-m+1} they are called solid zonal and solid spherical harmonics respectively.

The zonal, cylindrical, and spherical harmonics are orthogonal and complete. See Morgenau & Murphy, "The Mathematics of Physics and Chemistry" D. Van Nostrand Co., New York City.

See also pile equation and characteristic equation.

weighting factor

The ratio of the fractional change in the effective multiplication constant, $\delta k_{eff}/k_{eff}$, to the fractional change in a particular property, i.e.

$$\zeta = \frac{\frac{\delta k_{eff}}{k_{eff}}}{\frac{\delta \zeta}{\zeta}} .$$

Zeta (ζ) may be the macroscopic scattering or absorption cross-sections, diffusion constants, or any other physical property in the pile. The weighting factors are functions of position, and are computed by means of perturbation theory.

See: perturbation theory.

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zonal harmonics

a. surface

b. solid

See discussion under wave equation.