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BINDING ENERGIES OF HEAVY ELEMENTS

K. WAY

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BINDING ENERGIES OF HEAVY ELEMENTS

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BINDING ENERGIES OF HEAVY ELEMENTS

K. Way

Evidence for Nuclear Structure

Evidence that certain nuclear species have particularly stable configurations and that additional particles are bound rather loosely to these special isotopes has recently been collected by Dr. Maria Mayer(1). Dr. Mayer has pointed out the high relative and absolute abundances of nuclei containing either 50 neutrons or protons, or 82 neutrons or protons, and the unusually large number of stable isotopes or isotones containing either one of these numbers of neutrons or protons. She has also recalled that two of the delayed neutron emitters are known to contain 51 and 83 neutrons respectively and that the fast neutron absorption cross sections of nuclei containing 50, 82, and 126 neutrons are remarkably low. These facts suggest weak bindings of the 51st, 83rd, and 127th neutrons.

Evidence of Abnormal Binding Energies in the Pb Region Found by Berthelot

The region of Pb is particularly interesting from the point of view of these 'magic' numbers since it contains isotopes with 82 protons and isotones with 126 neutrons and Pb^{208} actually contains a magic

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number of each kind of particle. A Berthelot [2] has pointed out that α decay energies indicate peculiar neutron and proton binding energies at the end of the naturally radio-active series. He has estimated some of these binding energies making use of known α and β decay energies and reasonable assumptions concerning the binding energies for nuclei somewhat heavier than Pb^{208} where abnormalities are not expected. His results show a difference in the case of Pb between the binding of the 126th and 127th neutrons of ~ 3 mev. The odd-even difference expected from the Bohr-Wheeler constants [5] is only ~ 1 mev. In three instances the difference between the binding of the 82nd and 83rd proton is found to be ~ 4 mev. where only ~ 1.5 mev. is estimated.

New Estimation of Binding Energies in Pb Region

In this report neutron and proton binding energies are given which have been calculated from α and β decay energies and experimental values of neutron and photo fission thresholds in a way explained in detail below. The results which are given in Tables 1 and 2 show discontinuities of the same kind as found by Berthelot and of about the same magnitude. * However, exceptionally low values for the binding of a 127th neutron to Bi_{83}^{209} and Po_{84}^{210} were not found. Rather than showing that a neutron 'shell' closes with 126 neutrons and a proton 'shell' with 82 protons, the results indicate that Pb^{208} acts as a very tight core and that all particles

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* note added in proof. The binding energy of a 126th neutron to Bi_{83}^{208} has recently been reported as 7.3 mev. by J. McElhinney, A. O. Hanson, and R. B. Duffield, B.A.P.S. 23, #4, 8 (1948).

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which are added to it have roughly an equivalent status. Thus, according to Table 1 the binding to this core of a neutron is about the same as that of a proton, and the binding of a neutron to (core + 2 protons) is very nearly the same as the binding of a proton to (core + 2 neutrons). Values given in the tables in brackets are binding energies calculated with Bohr-Wheeler constants.[5]

Method of Estimating Binding Energies

Binding energies of neutrons B_n and of protons B_p to a number of different members of the naturally radioactive families can be found from α and β decay energies if a single binding energy is known in each of the four families. For instance in α decay

$$B_n\,(A,\,Z) = M(A,\,Z) + n - M(A+1,\,Z)$$

$$B_n\,(A-4,\,Z-2) = M(A-4,\,Z-2) + n - M(A-3,\,Z-2)$$

$$B_n\,(A-4,\,Z-2) = B_n\,(A,\,Z) + E_{dis}\,(\alpha_2) - E_{dis}\,(\alpha_1)$$

$$\cong B_n\,(A,\,Z) + E\,(\alpha_2) - E\,(\alpha_1)$$

α_2 and α_1 are the particles emitted by the nuclei (A+1, Z) and (A, Z) respectively. E_{dis} is the total disintegration energy which includes the energy of the recoiling nucleus. Since differences are taken, a negligible error is made by using the α energies in place of the disintegration energies. For β decay the relations are entirely

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similar. In both cases, of course, it is necessary to use the energies of the particles with longest range in a particular disintegration and if these do not lead to the ground state of the daughter to add the energy of the γ ray which follows.

Proton binding energies follow from neutron binding energies in the following way:

$$B_p(A, Z) = M(Z, A) + p - M(A+1, Z+1)$$

$$B_n(A, Z) = M(Z, A) + n - M(A+1, Z)$$

$$B_p(A, Z) = B_n(A, Z) + p - n + \beta_2 + E(\beta_2)$$

$$= B_n(A, Z) + H - n + E(\beta_2)$$

Where β_2 is the β particle emitted in the β decay of (A+1, Z) to (A+1, Z+1) and H is the mass of the hydrogen atom.

The α and β energies used are listed in Table 3. In several instances the β decay schemes are not known. In such cases decisions as to the most probable total ($\beta + \gamma$) disintegration energy were made after consulting the literature and taking stability and cycle predictions into account. By cycle prediction is meant the value for any disintegration found by requiring disintegration energies which lead from the same initial to the same final nucleus by different paths to be equal.

The four fundamental neutron energies from which all others were derived were found by making use of the following two assumptions:

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Assumption I

The single value of ~ 5.3 mev. which was found for the photo fission threshold of five nuclei from Th_{90}^{232} to Pu_{94}^{239} by Koch, McElhinney, and Gasteiger[3] is actually the energy which must be given to any compound (target plus captured neutron) nucleus in this mass range to produce fission.

Assumption II

If a nucleus in this mass range has a threshold for neutron fission, the binding energy of the neutron to the target nucleus is equal to 5.3 mev. minus the neutron threshold energy. The neutron fission thresholds of Th^{232}, Pa^{231}, U^{238}, and Np^{237} as given in the wall paper book[4] were used to find the binding energies of neutrons to these four nuclei.

Check on Binding Energies

From the four fundamental neutron binding energies the binding energies to four consecutive nuclei such as Th^{230}, Th^{231}, Th^{232}, Th^{233} can be found from α and β decay energies. The sum of the binding energies of neutrons to each of these four nuclei should be equal to the mass difference $[M(Th^{230}) + 4n's - M(Th^{234})]$. The difference in mass between Th^{234} and $(Th^{230} - He^{4})$ is given directly by the decay energy of the two β disintegrations and one α disintegration by means of which Th^{234} (UX_1) transforms into Th^{230} (Io). The sum first found from the

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four primary neutron binding energies was 21.58 mev., that from the direct decay 22.28 mev., a difference of 3%. In the actual calculations the photo fission threshold was taken to be 5.4 mev. instead of 5.3 mev. and two β disintegration energies which are not very well known were adjusted in a favorable direction so that this difference vanished. Thus, the actual primary neutron binding energies used were:

Nucleus	Neutron Fission Threshold	Assumed Neutron Binding Energy
Th^{232}	1.1	4.3
Pa^{231}	0.35	5.05
U^{238}	0.9	4.5
Np^{237}	0.15	5.25

Predictions About Fissionability

The values in brackets in Table 2 which were calculated with Bohr-Wheeler[5] constants show that the neutron binding energies in the region of Th and U are very similar to those found by the liquid drop model. New predictions as to thermal fissionability are not, therefore, to be expected for any given assumption such as I. In Table 2 the superscripts +, -, ? indicate respectively that the target nucleus is known to be thermally fissionable, is known to be not thermally fissionable, or that an upper limit as to its fissionability

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has been given. If assumption I is valid, thermal fissionability is to be expected for Th^{233} and U^{237}. If its range of validity is extended slightly, Th^{231} should also be thermally fissionable and Pu^{240} should not.

Sources of Inaccuracy

It is difficult to give an estimate of the reliability of the results. Assumptions I and II on which everything is based may be completely wrong. Even if correct there are inaccuracies in the photo and neutron fission thresholds of possibly 5%. Many α decay energies are in doubt since it is not known which γ's are in cascade with which β's. The establishment of reliable decay schemes for β decaying nuclei of the naturally radioactive families would lead to better estimates of most of the binding energies. In particular a precise knowledge of the disintegration energies of Pa^{232}, Pa^{233}, and Np^{238} would make possible a rather rigid test of Assumptions I and II.

References:

(1) M. G. Mayer, Phys. Rev. 74, 235, 1948

(2) A. Berthelot, J. Phys. Rad. VIII, 3, 17, 1942

(3) H. W. Koch, J. McElhinney, and F. L. Gasteiger, CP-3758, 1947.

(4) H. H. Goldsmith, H. C. Ibser, MUC-HHG-7, 1945

(5) N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426, 1939.

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Table 3

α and β Disintegration Energies Used in Calculation of Binding Energies of Heavy Elements

Nucleus	Radiation	Decay Energy	Reference	Nucleus	Radiation	Decay Energy	Reference
Am-242	β	0.6	c	U-232	α	5.31	c
241	α	5.5	c	231	K	~0.2	stability
				230	α	5.9	c
Pu-241	β	0.02	c				
240	α	5.1	c	Pa-234	β, γ	2.32	Bradt
239	α	5.15	c	233	β, γ	0.4	c
238	α	5.5	c	232	β⁻ n	1.45 1.15	c cycle
Np-239	β, γ	1.2	p	231	α	5.012	c
238	β⁻γ (K⁻)	1.55 0.35	c cycle	230	β⁻ (β⁺)	1.1 2.2	c cycle
237	α	4.75	c				
				Th-234	β	0.2	c
U-239	β	1.20	c	233	β	1.6	c
238	α	4.18	c	232	α	4.00	c
237	β, γ	0.5	c	231	α	0.2	c
236	α	4.5	c	230	α	4.66	c
235	α	4.59	CC-3715	229	α	5.05	c
234	α	4.76	c	228	α	5.38	c
233	α	4.82	c	227	α	6.05	c
				226	α	6.3	c

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α and β Disintegration Energies Used in Calculation of Binding Energies of Heavy Elements (continued)

Ac-228	β	1.55	c
	α	4.5	c
Ac-227	α, γ	0.05	c
226	α	5.0	c
225	α	5.80	c
Ra-224	α	5.66	c
223	α	5.72	c
222	α	6.5	c
Ra 226 - Th 226		0.18	cycle
Fr-221	α	6.30	c
Em-220	α	6.28	c
219	α	6.82	c
218	α	7.1	c
At-218	α	6.63	c
217	α	7.00	c
Po-218	α	6.00	c
	β	0.02	cycle
216	α	6.77	c
215	α	7.37	c
	β	0.4	
214	α	7.68	c
213	α	8.32	c

Po-212	α	8.78	c
211	α	7.43	c
210	α	5.30	c
Bi-214	α	5.50	c
	β	3.15	c
213	α	5.86	c
	β	1.3	c
212	α	6.05	c
	β	2.20	c
211	α	6.62	c
	β	0.66	cycle
210	α	4.87	c
	β	1.17	c
Pb-214	β	0.65	c
212	β	0.88	cycle
211	β	1.40	c
210	β	0.07	c
209	β	0.70	c
Tl-210	β	5.33	cycle
209	β	3.76	"
208	β	4.93	c
207	β	1.47	c
206	β	1.6	c

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c - Value taken from chart of Known Isotopes of Heavy Elements CLM-KW-3, December 1946

cycle - Value arrived at only from cycle differences

Bradt - Value from private communication from H. L. Bradt reported in Nucleonics, Part 2, May 1948, page 181

P - K. Philipp, J. Riedhammer, Zeits, f. Naturforsch, 1, 372 (1946) report coincidences between 0.48 β and 0.50 γ

s - G. T. Seaborg and I. Perlman, BC-59 (1947)

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TABLE I

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NEUTRON AND PROTON BINDING ENERGIES

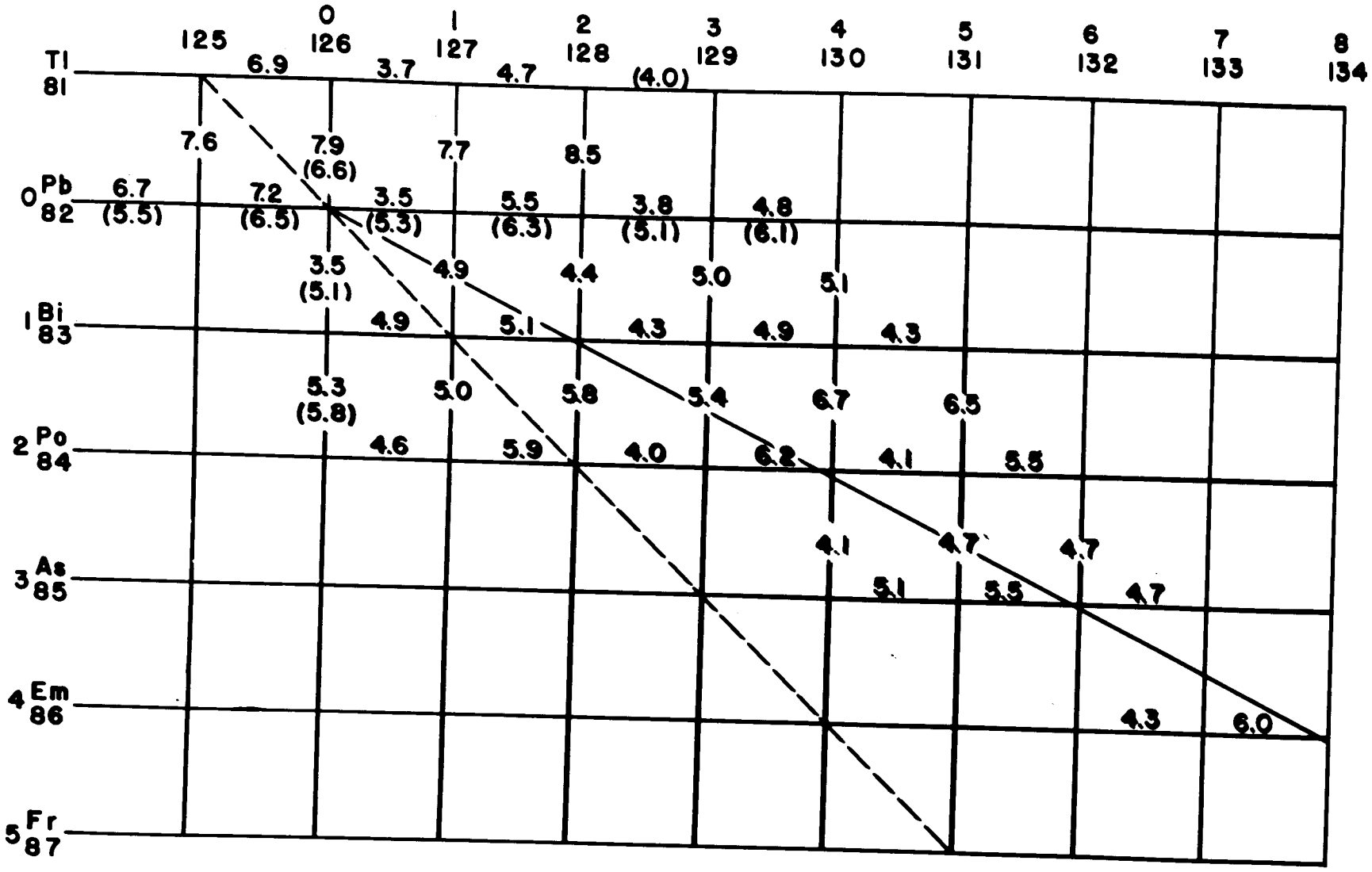

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TABLE 2

NEUTRON AND PROTON BINDING ENERGIES

		9 135	10 136	11 137	12 138	13 139	14 140	15 141	16 142	17 143	18 144	19 145	20 146	21 147
6	Ra 88	6.1?	5.2	6.7										
7	Ac 89				5.2?									
8	Th 90		5.4	6.7+ (6.7)	5.5? (5.6)	7.1+ (6.5)	5.0− (5.3)	6.4? (6.2)	$\boxed{4.3}$− (5.1)	6.6 (6.0)				
9	Pa 91				4.9	4.1 7.4+	4.4 (5.0) $\boxed{5.1}$−	4.5 (5.2) 7.1+	5.1	6.1 5.3?				
10	U 92				5.9+ (5.9)	6.4 6.7+ (6.8)	5.7 (5.6) 6.0+ (5.7)	6.7 (5.8) 7.2+ (6.6)	6.8 5.2− (5.4)	6.7 6.5+ (6.3)	5.2 (5.2)	6.1 (6.1)		$\boxed{4.5}$− (4.9)
11	Np 93										4.9 $\boxed{5.3}$−	5.0 6.1+	4.7	
12	Pu 94										6.0 5.7+	6.5 6.5+	6.9	5.2?
13	Am 95												4.5	5.6+
													5.4	