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GALE YOUNG, Leader

PIECE-WISE GREULING SOLUTIONS FOR HYDROGEN

GALE YOUNG

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This report is one of a series on shielding and represents part of the work of a summer group working principally at Oak Ridge National Laboratory. Participants are drawn from Nuclear Development Associates (Navy contract), NEPA, Bureau of Ships and Bureau of Aeronautics (Navy Department), AEC, RAND, General Electric, Argonne and Oak Ridge National Laboratory.

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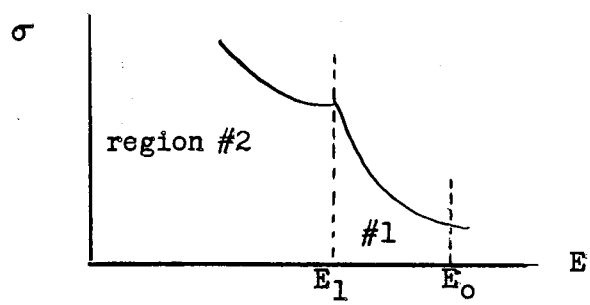
~ PIECE-WISE GREULING SOLUTIONS FOR HYDROGEN ~

Gale Young

At high energies the scattering cross-section σ of hydrogen varies as 1/E (1, 2), and advantage has been taken of this simplifying fact in certain idealized shielding calculation (2, 3, 4, 5). At lower energies the variation of σ with energy is less rapid (1, 6); around 1 Mev, for example, it goes as $1/v$. Any single-power law of variation can be dealt with in the "straight ahead" approximation by the method of Greuling (7, 8, 4); and some success has been had with the 1/E (3, 5) and $1/v$ (5) cases in treatments including simple angular scattering.

A single-power law will not fit hydrogen over a wide range. The purpose of the present note is to consider piecing together different Greuling solutions in different energy regions to get a better over-all approximation, as has been done elsewhere for γ ray problems (9).

Consider a case as sketched, with a 1/E upper region feeding into a $1/v$ lower region. The notation is as in (9) or (4), and the basic equation is



$$\frac{\delta F(x,E)}{\delta x} + \sigma F = \int_E^{E^*} F(x,t)\,\frac{\sigma(t)}{t}\,dt + S(x) \quad . \tag{1}$$

Upper Region

primary flux $$Q_p = e^{-\sigma_o x} \tag{2}$$

source term $$S_1 = \frac{\sigma_o}{E_o} e^{-\sigma_o x} \tag{3}$$

secondary spectrum $$F_1(x, E) = \frac{\sigma_o}{E_o} x e^{-\sigma_o x} \tag{4}$$

secondary integral flux $$Q_1 = \frac{E_o - E_1}{E_o} \sigma_o x e^{-\sigma_o x} \tag{5}$$

Bottom Region

source term $$S_2 = S_1 + \int_{E_1}^{E_o} F_1 (x, t) \frac{\sigma}{t} dt$$

$$= \frac{\sigma_o}{E_o} e^{-\sigma_o x} (1 + b x) \tag{6}$$

where $$b = \sigma_1 - \sigma_o .$$

In the bottom region σ varies as $E^{-\frac{1}{2}}$, and

$$\alpha = - \frac{\sigma(E)}{E \sigma'(E)}$$

has the value 2. Then from Greuling's work we know that the solution of (1) with $E^* = E_1$ and with a source term

$$e^{-\sigma_1 x} \tag{7}$$

is (4, page 17)

$$F_o (x, E) = x e^{-\sigma_1 x} \left(1 + \frac{\sigma - \sigma_1}{2} x\right) , \tag{8}$$

as can be verified directly by inserting in (1).

The source term (6) which it is desired to use is not of the form (7). However, it can be constructed from a space distribution of sources each of which decays in space like (7). Let P_o be the strength of such a source at $x = o$, and $P(x)$ the distributed source density. Then we require

that

$$P_o\, e^{-\sigma_1 y} + \int_o^y P(x)\, e^{-\sigma_1 (y-x)}\, dx = S(y)\,, \tag{9}$$

where $S(y)$ is the source to be constructed. From this it is seen that

$$P_o = S(0)\,, \tag{10}$$

and, upon multiplying through by $e^{\sigma_1 y}$ and differentiating,

$$P = S' + \sigma_1 S \tag{11}$$

For $S = S_2$ as in (6) we find

$$\left.\begin{aligned} P_o &= \frac{\sigma_o}{E_o} \\ P(x) &= \frac{\sigma_o}{E_o}\, b\, e^{-\sigma_o x}\,(2 + b x). \end{aligned}\right\} \tag{12}$$

With (8) the solution in the bottom region is then

$$\begin{aligned} F_2\,(x, E) &= \frac{\sigma_o}{E_o}\, x\, e^{-\sigma_1 x}\left(1 + \frac{\sigma - \sigma_1}{2}\, x\right) \\ &\quad + \int_o^x P(t)\, F_o\,(x-t;\, E)\, dt. \end{aligned} \tag{13}$$

Introducing $B(E) = \sigma - \sigma_1$, the integral in (13) may be expressed as

$$\frac{\sigma_o}{E_o}\,\frac{b}{2}\, e^{-\sigma_o x} \int_o^x e^{-bv}\left[(4 + 2bx)v + (2B - 2b + bBx)\, v^2 - bB\, v^3\right] dv$$

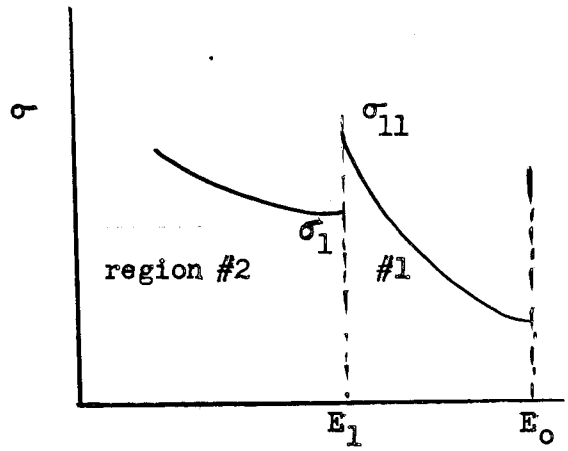
$$= \frac{\sigma_o}{E_o}\, e^{-\sigma_o x}\left[\frac{Bx}{b} - \frac{B}{b^2} + x - e^{-bx}\left(\frac{B\, x^2}{2} + x - \frac{B}{b^2}\right)\right], \tag{14}$$

Then (13) becomes

$$\begin{aligned} F_2\,(x, E) &= \frac{\sigma_o}{E_o}\left(1 + \frac{B}{b}\right) x\, e^{-\sigma_o x} \\ &\quad - \frac{\sigma_o}{E_o}\,\frac{B}{b^2}\left(e^{-\sigma_o x} - e^{-\sigma_1 x}\right); \end{aligned} \tag{15}$$

it can be directly verified that this satisfies (1) with $E^* = E_1$ and $S = S_2$ from (6). The first term of (15) is, as seen from (4), just $\left(1 + \frac{B}{b}\right)$ times what the solution would be if the $1/E$ upper region extended all the way down in energy; the last term is asymptotically unimportant.

The above has assumed that σ is continuous at E_1. If σ is discontinuous, as in the adjacent sketch,



let σ_{11} denote the right hand value and σ_1 the left hand value. Write $b = \sigma_{11} - \sigma_o$, $C = \sigma_1 - \sigma_o$, and $B = \sigma(E) - \sigma_1$ as before. Then instead of (15) we obtain

$$
\begin{aligned}
F_2(x, E) = {} & \frac{\sigma_o}{E_o}\left(\frac{b}{c}\right)^2\left(1 + \frac{B}{c}\right) x\, e^{-\sigma_o x} \\
& + \frac{\sigma_o}{E_o}\left[1 - \left(\frac{b}{c}\right)^2\left(2\frac{c}{b} - 1\right)\right]\left(1 + \frac{B}{2}x\right) x\, e^{-\sigma_1 x} \\
& - \frac{\sigma_o}{E_o}\left(\frac{b}{c}\right)^2\left[\frac{B}{c}\left(\frac{3}{c} - \frac{2}{b}\right) - 2\left(\frac{1}{b} - \frac{1}{c}\right)\right]\left(e^{-\sigma_o x} - e^{-\sigma_1 x}\right) \\
& - \frac{\sigma_o}{E_o}\left(\frac{b}{c}\right)^2 2\left(\frac{1}{b} - \frac{1}{c}\right) B\, x\, e^{-\sigma_1 x} .
\end{aligned}
\tag{16}
$$

The first term is $\left(\frac{b}{c}\right)^2 \left(1 + \frac{B}{c}\right)$ times what the solution would be if the upper 1/E region continued all the way down. The second term is similarly an energy factor times the solution (8) for a source at the top of the 1/v region.

Such solutions might be adaptable into calculations which allow for angular scattering (5).

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