Training School

[redacted]

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES
3 4456 0360548 1

ORNL-429
SERIES D
PHYSICS-GENERAL
75

MONTE CARLO METHODS FOR SHIELD COMPUTATION

OAK RIDGE NATIONAL LABORATORY
CENTRAL RESEARCH LIBRARY
CIRCULATION SECTION
4500N ROOM 175
LIBRARY LOAN COPY
DO NOT TRANSFER TO ANOTHER PERSON
If you wish someone else to see this report, send in name with report and the library will arrange a loan.
UCN-7969 (3 9-77)

OAK RIDGE NATIONAL LABORATORY
OPERATED BY
CARBIDE AND CARBON CHEMICALS CORPORATION
FOR THE
ATOMIC ENERGY COMMISSION

OAK RIDGE, TENNESSEE

[redacted]

[redacted]

ORNL 429
This document consists
of 12 pages. Series D.
Copy 7 of 124

Contract W-7405 eng 26*

ORNL SUMMER SHIELDING SESSION

Gale Young, Leader

DECLASSIFIED Per Letter Instructions Of
AEC-3-17-50
(with deletions)
JUN 28 1950
SUPERVISOR CENTRAL FILES

MONTE CARLO METHODS FOR SHIELD COMPUTATION

G. Goertzel and H. Kahn

DATE ISSUED: DEC 19 1949

OAK RIDGE NATIONAL LABORATORY
Operated by
Carbide and Carbon Chemicals Corporation
for the
Atomic Energy Commission
Post Office Box P
Oak Ridge, Tennessee

[redacted]

*Explanation on page 3.

[redacted]

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES
3 4456 0360548 1

[redacted]

ORNL-429, Series D
Physics-General

OAK RIDGE NATIONAL LABORATORY INTERNAL DISTRIBUTION

1. G. T. Felbeck (C&CCC)
2. 706-A Library
3. 706-A Library
4. 706-B Library
5. Biology Library
6. Health Physics Library
7. Training School Library
8. Training School Library
9. Central Files
10. Central Files
11. Central Files
12. Central Files
13. J. A. Swartout
14. J. H. Gillette
15. A. Hollaender
16. J. H. Frye, Jr.
17. E. J. Murphy
18. C. N. Rucker
19. W. D. Lavers
20. A. M. Weinberg
21. J. A. Lane
22. R. N. Lyon
23. M. M. Mann
24. E. P. Blizard
25. W. K. Ergen
26. A. H. Snell
27. Lewis Nelson
28. C. E. Larson (Y-12)
29. W. B. Hume (K-25)
30. Central Files (OP)
31. Central Files (OP)
32-41. Central Files (OP)

OAK RIDGE NATIONAL LABORATORY EXTERNAL DISTRIBUTION

42-53. Argonne National Laboratory
54. Armed Forces Special Weapons Project
55-56. Atomic Energy Commission, Washington
57. Battelle Memorial Institute
58-61. Brookhaven National Laboratory
62-63. Bureau of Ships
64-67. Carbide and Carbon Chemicals Corporation (K-25 Plant)
68-71. Carbide and Carbon Chemicals Corporation (Y-12 Plant)
72-75. General Electric Company, Richland
76. Hanford Operations Office
77-78. Idaho Operations Office
79-82. Knolls Atomic Power Laboratory
83-85. Los Alamos
86-88. Mound Laboratory
89. National Advisory Committee for Aeronautics
90. National Bureau of Standards
91. Naval Radiological Defense Laboratory
92. NEPA Project
93-97. New York Operations Office
98. North American Aviation, Inc.
99. Patent Branch, Washington
100. Sandia Laboratory
101-115. Technical Information Branch, ORE
116-120. University of California Radiation
121-124. Westinghouse Electric Corporation

[redacted]

This report is one of a series on shielding and represents part of the work of a summer group working principally at Oak Ridge National Laboratory. Participants are drawn from Nuclear Development Associates (Navy contract), NEPA, Bureau of Ships and Bureau of Aeronautics (Navy Department), AEC, RAND, General Electric, Argonne and Oak Ridge National Laboratory.

These reports are issued through ORNL on Contract No. W-7405, eng. 26.

MONTE CARLO METHODS FOR SHIELD COMPUTATION

G. Goertzel and H. Kahn

Introduction

There is at the present time no reliable method available to compute the attenuation of a shield. It is likely that suitable methods will be developed in the next few years.

The Monte Carlo method (1, 2)* for calculating attenuations is one of the most promising candidates for development. In this report we discuss the present state of the Monte Carlo method as applied to shielding computations and what we expect to be obtained with further development.

The Basic Method

The Monte Carlo method is a random sampling process. To apply it to a shielding computation one might propose to sample typical life histories of particles by playing the following series of games. One considers a particle traversing the shield. First he decides where the particle makes its collision. This is carried out by playing an appropriate game of chance.** He then decides, again with a game of chance, whether the particle is absorbed on the collision or follows some appropriate alternative. The decision is then made, with a game of chance, where the particle makes its next collision. This process is continued, collision after collision, until either the particle escapes the shield or is absorbed. As a result of this series of games of

---

* See list of References at end of report.

** Such as the spinning of a roulette wheel or, more conveniently, the selection of a set of digits from a table of random digits.

chance one has obtained a life history of a neutron incident on the shield, selected at random from the population of all possible life histories. This process is repeated until the number of particles which have penetrated the shield is large enough to give a good estimate of the probability of penetration.

The relative probable error of an estimate obtained in this manner is given by

$$0.67 \left((1 - p)/(Np) \right)^{\frac{1}{2}} ,$$

where p is the probability of penetration and N is the number of particle histories, so that Np is the number of particles which have penetrated the shield. In a shield p is much less than one, so that for a relative probable error of 10% in an estimate of the penetration, one must trace enough histories that almost one hundred particles penetrate the shield. Thus one needs 100/p particles, which for a p of 10^{-6} means handling a hundred million particles. This is prohibitively large.

The necessity for the introduction of the probable error as in the last paragraph arises from the statistical nature of the Monte Carlo method. The significance of the probable error and thus the meaning of the type of information obtained by Monte Carlo is briefly discussed in the appendix.

Modifications

Various modifications (3, 4, 5, 6, 7, 8, 9) of the above described primitive Monte Carlo technique have been proposed in order to reduce to manageable proportions the work necessary in carrying out a penetration calculation. Three of these, each of which make it possible to carry out a computation are:

1 Splitting,

2 Statistical estimation, and

3 Importance sampling. *

It is our belief that the third method is of broadest applicability and is to be preferred to the others. However, various combinations of the three methods may be used and in many problems may be advantageous.

Statistical estimation is a combination of the Monte Carlo method with analytical procedures for handling certain parts of the problem. For example, the first and last collisions may be partly treated analytically. (5)

Both the splitting and importance sampling methods are based on the concept of the importance of a region in phase space. (3, 5, 6, 7, 8) A particle is characterized, say just before it makes a collision, by its position in the shield and by the direction and magnitude of its velocity. The probability that the particle will eventually penetrate the shield depends on its velocity and position just before a collision. This is the ideal importance of the particle at the particular time in its history considered and is thus the ideal importance of the corresponding point in phase space. Of course, the ideal importance as so defined is not known, or there would be no problem left to solve. However, an estimate of the importance of the various parts of phase space can be made. This estimated importance function is used in both the splitting and importance sampling methods.

---

* Importance sampling is the term used herein for what has in several reports been called "quota sampling." (3, 5, 6, 7) We believe "importance sampling" to be more descriptive.

In the splitting technique, whenever a particle passes from a region of lower importance to one of twice the importance, it is split in two and thenceforth treated as two particles. If it passes back into a region with half the importance, it is forced to play a game of chance in which half the time it will be killed. In this way, more of the computation effort goes into studying the more important regions.

In the importance sampling method, the importance function is used to modify each game of chance to be played in such a manner that the particles are encouraged to travel into more important regions and to avoid the less important regions. Thus, each particle eventually penetrates the shield.

In both the importance sampling and the splitting methods the games of chance have been modified from the form suggested by the physical situation. This is compensated by giving a weight to each particle in accordance with the modification of the game of chance. Thus, each particle which penetrates the shield represents not one particle but that small fraction of a particle given by its weight.

The modifications of the games of chance as used in both the splitting and importance sampling methods are completely compensated by appropriate use of weights, so that the expected answer is not at all changed. Thus, these modifications are not approximations that affect the expected value of the final result. Rather, good choice of importance function and good technique have the effect of decreasing the number of particles which must be studied in order to obtain a given probable error. It is here, into the importance function, that one's prior physical knowledge is inserted. The better this knowledge, the better the importance function and thus the more efficient the computation.

Conclusions

Shielding computation embodying the above principles have been carried out by T. E. Harris and H. Kahn at Rand, S. W. W. Shor at Massachusetts Institute of Technology, G. Goertzel at Nuclear Development Associates*, and H. Kahn and L. Nelson at Oak Ridge National Laboratory.* In light of the results of and the experience gained in these calculations we feel that the following conclusions may be tentatively accepted:

1 An attenuation computation for a specific shield of homogeneous material and plane slab geometry can probably be carried out at present on hand computing machines in from two to six man weeks.** Some fifty particles are necessary and with sagacious choice of importance function the estimated probable error should lie in the range from 5 to 30 percent. Of course, the ability to properly choose the importance function will depend on past experience. However, a pilot computation will in general aid in choosing the importance function if the prior knowledge and experience of the computer is not sufficient without this assistance.

2 It is quite likely that, with increasing experience, with development of the methods and of special techniques, inhomogeneous shields with arbitrary geometry may be studied in times of the same order of magnitude as in 1.

3 It will often be possible to carry out parametric studies, such as determining the effect of small changes in shield composition, without a great increase in the amount of work required.

Because of these conclusions, we feel that there are two fundamental reasons for continuation of the development of the Monte Carlo method:

1 It is useful for shield design purposes and may turn out to be the most useful method available.

2 It is needed at present as a check and counterfoil for the various analytic techniques which have been and are now being developed. The Monte Carlo methods are more useful in this respect than experiments, since there exists the certainty that the comparison of Monte Carlo and analytic results are based on the same physical data and assumptions.

---

* The NDA and ORNL computations were carried out in connection with the Oak Ridge National Laboratory Shielding Work Session, Summer 1949.

** This estimate does not include a duplicate computation for checking.

Recommendations

Three lines of approach to further development of the Monte Carlo methods are recommended:

1 The actual carrying out of useful computations with present day techniques. Many such results are desirable for comparison with experiment, for design studies, and for comparison with analytic techniques. The experience so obtained will be invaluable in striving for improvement of the calculation process.

2 A formal study of the statistical situation involved in the modified calculations. This is desirable so that one may determine with more certainty the reliability of his results and thus avoid considering more particles than necessary with the concomitant increase in work.

3 A study of how to use the results of a calculation to determine an improved importance function. It is conceivable that if this can be done well, two separate small calculations may be so performed that the final result (that of the second calculation with an improved importance function) is more reliable than would be the result of a single large computation. It is clear that improving the probable error of a computation by considering more particles is a relatively inefficient way of improving reliability.

APPENDIX

As was mentioned in the text, the fact that a Monte Carlo computation is a random sampling procedure means that one cannot say that the probability of penetration is p, but must rather state that an estimate of the probability of penetration, p, is given by m, where m is the result of the computation. The latter statement is incomplete unless one has a measure of how well m serves as an estimate of p.

This situation is clearly not unknown to the experimental physicist, as the results of measurements are in this form. On the other hand, it is not a usual situation for theoretical work, but arises in the present case because one is not carrying out a mathematical computation in the usual (analytic) sense, but is carrying out a mathematical experiment with the aid of tables of random digits.

The result of the Monte Carlo computation is a set of weights for particles, these weights corresponding to a sample of particle histories selected at random from the population of all possible particle histories. The arithmetic mean of the weights of all histories in the population is the desired penetration probability, p. An estimate of this is m, the arithmetic mean of the sample weights. The reliability of this estimate depends on the distribution function of sample arithmetic means. This is not known, but it is known that for sufficiently large samples the distribution of sample means is normal. If the assumption of normality is made, one estimates the parameters of the normal distribution from the sample by the usual statistical procedures. Thus, one uses the sample mean, m, and the probable error, d, estimated from the sample to approximately determine the normal distribution from which the sample was selected. (Actually, Student's distribution, not the normal, should be used here. These are identical for samples greater than 30 particles.)

On the basis of the above, one may make the following statement about the population mean, p:

> "The probability that p lies between m - bd and m + bd is given by k, where the relation between k and b is given in the following table."

k	b
0.50	1.000
0.70	1.537
0.90	2.439
0.95	2.758
0.99	3.819

The major difficulty with the above discussion is the fact that it is not known whether one can safely use the assumption that the sample mean is normally distributed in any given calculation. This point is under investigation.

References

1 S. Ulam and N. Metropolis, "The Monte Carlo Method, " to be published.

2 H. Kahn, "Stochastic (Monte Carlo) Attenuation Analysis," to appear in Nucleonics. Rand Report P-88.

3 T. E. Harris and H. Kahn, Rand Report R-148, "Estimation of Particle Transmission by Random Sampling."

4 W. DeMarcus and L. Nelson, "Methods of Probabilities in Chains Applied to Particle Transmission through Matter," NEPA-1135-STR-13.

5 H. Kahn, Rand Research Memorandum 248, "Particle Histories in Plane Slabs." T. E. Harris and H. Kahn, Research Memorandum 249, "Estimation of Particle Transmission." This report is an extension of portions of RM-248 and has been replaced by Reference 3.

6 G. Goertzel, "Quota Sampling and Importance Functions in Stochastic Solution of Particle Problems," ORNL-434.

7 G. Goertzel, "A Proposed Particle Attenuation Problem," ORNL-423

8 S. W. W. Shor, Phys. Rev. 76, 474A (1949); MIT thesis.

9 Much work on Monte Carlo, especially with regard to the splitting method, has been carried out by Hammer, Mayer, Metropolis, Richtmeyer, Ulam, von Neumann, and others.