CENTRAL RESEARCH LIBRARY
DOCUMENT COLLECTION

[redacted]

ORNL-634
Physics-General

DECLASSIFIED

CLASSIFICATION CHANGED TO:
BY AUTHORITY OF: TID 111 3
BY: N Johnstone 10/23/56

INV. B2

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES
3 4456 0352630 7

LABORATORY RECORDS
1954

MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT
FOR THE PERIOD ENDING JANUARY 31, 1950

OAK RIDGE NATIONAL LABORATORY
CENTRAL RESEARCH LIBRARY
CIRCULATION SECTION
4500N ROOM 175
LIBRARY LOAN COPY
DO NOT TRANSFER TO ANOTHER PERSON
If you wish someone else to see this report, send in name with report and the library will arrange a loan.
UCN-7969 (3 9-77)

OAK RIDGE NATIONAL LABORATORY
OPERATED BY
CARBIDE AND CARBON CHEMICALS DIVISION
UNION CARBIDE AND CARBON CORPORATION
UCC
POST OFFICE BOX P
OAK RIDGE, TENNESSEE

[redacted]

[redacted]

ORNL 634
Physics-General

This document consists of 25 pages.
Copy 7 of 116. Series A.

Contract No. W-7405, eng 26

MATHEMATICS PANEL
Quarterly Progress Report for the Period Ending January 31, 1950

A. S. Householder, Chief

Date Issued MAR 10 1950

OAK RIDGE NATIONAL LABORATORY
operated by
CARBIDE AND CARBON CHEMICALS DIVISION
Union Carbide and Carbon Corporation
Post Office Box P
Oak Ridge, Tennessee

[redacted]

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES

3 4456 0352630 7

TABLE OF CONTENTS

SUMMARY 4

PUBLICATIONS 5

SPECIAL PROBLEMS 7

Tetrad Analysis (Yeast Genetics) 7

An Iterative Method for Locating Maxima and Minima 8

Statistical Consultation for Health Division 9

Diffusion of Isotopes in Multicompartment Biological Systems 9

Effects of Gaps in the Reactor Core, and of Ducts in a Shield 10

Fission Neutron Age in D_2O-H_2O Mixtures 10

Tabulation of the Fermi Functions 11

Sample Solutions of an Eigenvalue Problem 13

A Random Walk Whose Equation Is Hyperbolic 14

Reactivity Perturbations in a Temperature Modulated Pile 16

A Random Walk 19

Neutron Attenuation by Water 19

Approximate Calculation of the Neutron Flux in a Thick Shield Containing Some Heavy Elements 19

A Design Problem for the MTR 20

Criticality Computations 20

Homogeneous Pile 20

Statistical Interpretation of Data from Xenon Cross-Section Experiment 20

SUMMARY

Alston S. Householder, Chief

Jerome H. Fishel joined the Mathematics Panel on January 3. Mr. Fishel comes from Cleveland, Ohio, and attended Western Reserve University, where he received his B.S. with a major in Mathematics. Vivian Darnell, formerly of K-25, is temporarily a member of the Panel, for a period of 3 to 6 months beginning January 13, and will be assisting principally in MTR computations. Jack Moshman, now of the AEC, is expected to complete his present assignment in February and transfer to the Panel. Mr. Moshman's training has been chiefly in the field of statistics, in which subject he took his M.A. at Columbia University.

The fitting of the xenon curves is complete and a report is being issued. Ruth Arnette and Carl Perhacs have returned to Y-12 to resume work on the tissue problem for Health Physics.

The Panel is cooperating with the group which has responsibility for the homogeneous reactor and with Mr. J. Stein, of Westinghouse, who is assisting in that project. Mr. R. R. Coveyou is spending virtually his full time in this connection.

Some considerations (analytic in the sense that little or no numerical computations were made) appear among the problems listed below. Some numerical computations along lines already followed by the homogeneous reactor group are being made.

However, considerable thought has been given by Mr. Stein and Mr. Coveyou to a mathematical formulation of the "dynamics" of the homogeneous reactor. This formulation is being made so as to include all important causes (such as circulation of the working fluid) and to admit numerical computation.

Arrival of a Card Programmed Computer for installation at Y-12 is expected in February. This machine is to be used jointly by ORNL and NEPA. It represents a substantial improvement in both speed and versatility over present IBM equipment. Nancy Dismuke is preparing two problems for this machine, one being the preparation of tables of the Fermi functions and associated corrections, the other being a series of neutron age calculations for a number of mixtures of light and heavy water and certain light metals, in varying pro-

portions. Another problem for which the machine will be required is a computation of the spatial variation of neutron flux in the X-10 pile. Some discussion of these problems is given below.

By way of introducing the Card Programmed Computer, the IBM Corporation held two conferences on computing methods at the Homestead in Endicott, New York. A. S. Householder attended the first of these, in November, and presented a paper on the subject of fitting exponential decay curves. The second, held in December, was attended by Nancy Dismuke of the Mathematics Panel, by Jack Moshman, who is later to join the Panel, and by Phyllis Johnson of Y-12; Miss Johnson reported on the Monte Carlo age calculations being done for the Panel.

On December 2, A. S. Householder presented a paper by invitation at a meeting of the Texas Academy of Science on the Subject "The Relation of Mathematics to the Physical Sciences." This engagement was arranged through ORINS.

Preparations are being made for a meeting of the Southeast Section of the American Mathematical Society, to be held in Oak Ridge April 21 and 22. Professor Alfred Brauer, of the University of North Carolina, and Professor W. V. Parker of Alabama Polytechnic Institute, have been invited by the Society to give forty-minute papers. A session on non-statistical biometrics is planned, to include a paper by C. W. Sheppard, of the Biology Division, on the diffusion of isotopes in biological systems, a paper by Dr. Anatol Rapoport, of the University of Chicago, on a probabilistic theory of nerve nets, and a paper by Dr. John Z. Hearon on a topic yet to be selected.

Papers presented at last summer's Symposium on Monte Carlo at Los Angeles have been sent to the Bureau of Standards for publication.

Professor J. W. Tukey of Princeton University spent two days at the Laboratory January 25-27 as consultant on some statistical and Monte Carlo problems, and is expected back again in February.

PUBLICATIONS

Publications by members of the Panel for the quarter are:

Nelson, M. L., *A Monte Carlo Computation Being Made for Neutron Attenuation in Water* (declassified), ORNL 439.

Sheppard, C. W., and Householder, A. S., *The Interpretation of Experimental Investigations of Transfers Within a Two-Compartment System, Using Isotopic Tracers* (unclassified), ORNL 497.

SPECIAL PROBLEMS

PROBLEM (continuation): Tetrad Analysis (Yeast Genetics)

PARTICIPATING MEMBERS OF PANEL: A. S. Householder, E. A. Forbes, J. H. Fishel

BACKGROUND AND STATUS: A report is being prepared giving results to date and only a brief summary will be given here.

The case treated is that in which chromatid interference, as distinguished from chromosome interference, is neglected, and no exchange is supposed to occur between sister chromatids. Then if $p(r)$ is the probability of r exchanges between two given loci on the same chromosome, and we define

$$s = \sum_{0}^{\infty} (-2)^{-r} p(r),$$

the probabilities of the three types of ascus with respect to two characters is simply expressible in terms of $q = p(0)$ and s, and, in particular

$$P_1 - P_2 = q, \quad P_3 = 2(1 - s)/3.$$

With respect to the relation between a single gene and its centromere

$$x = (1 - s)/3$$

is the quantity taken as the map distance from centromere to gene locus. However, the quantities x are not additive, and, in fact, any special relation between the x-values associated with different segments of the same chromosome must necessarily presuppose some assumption as to the effect of interference. However, for genes on different chromosomes, if two subscripts refer to a pair of genes, and a single subscript to a segment bounded by a gene and the centromere, then always

$$s_{ij} = s_i s_j .$$

In the previous Quarterly Report[1] these quantities were designated w_{ij} and η_i, but arose only formally in the process of solving certain equations.

Data for three independent genes in pairs are required for estimates of the s_i, and hence the x_i. Data for four or more genes in pairs, at least three of which are independent, require a maximum likelihood or analogous procedure. It turns out that one of Neyman's BAN procedures is much simpler, and statistically equivalent. For this, one defines a function similar to chi-square, except that in the denominators are the observed, rather than the theoretical frequencies. This function is to be minimized with respect to the s_i. The equations are non-linear, but one can utilize an iterative procedure that turns out to converge fairly rapidly (see below), and this has been applied to some of Dr. Promper's data.

PROBLEM: An Iterative Method for Locating Maxima and Minima

PARTICIPATING MEMBERS OF PANEL: A. S. Householder, E. A. Forbes, J. H. Fishel

ORIGIN AND BACKGROUND: In the analysis of tetrad data discussed above there arises the problem of locating the minimum of a certain function which is algebraic but of rather high degree. Direct solution is virtually out of the question. The method of steepest descent could be applied but is rather laborious. The following iteration is found to be very effective for this case, and while more slowly convergent than the method of steepest descent, the greater simplicity of the individual steps is full compensation for the loss:

Let the function to be minimized be denoted $F(x_1, x_2, \ldots, x_n)$ and its derivatives F_i. Then the equations to be solved are

$$F_i = 0.$$

Suppose a preliminary approximation $x_1^{(0)}, \ldots, x_n^{(0)}$ is available. We seek to improve the approximation $x_i^{(0)}$. For this purpose we set

$$x_i = x_i^{(0)} + \delta_i, \quad x_j = x_j^{(0)} \qquad (j \neq i)$$

[1] Householder, A. S., *Mathematics Panel Quarterly Progress Report for the Period Ending October 31, 1949*, ORNL 516 (Dec. 12, 1949).

in the equation $F_i = 0$, expand in powers of δ_i to the linear term and solve for δ_i. One then treats $x_i^{(0)} + \delta_i$ as a new $x_i^{(0)}$ and passes to one of the other variables.

In minimizing Neyman's analogue of chi-square it turns out that the i-th derivative F_i is linear in F_i, so that the expansion is unnecessary and only the approximations $x_j^{(0)}$ $(j \neq i)$ need to be assumed. For the tetrad data about four complete cycles usually suffice to give adequate convergence.

A memorandum is being prepared giving the mathematical justification for the procedure. However, while the tetrad computations were under way the Quarterly Report for the National Applied Mathematics Laboratories appeared indicating that the process, along with certain others, is under consideration there.

PROBLEM: Statistical Consultation for Health Division

ORIGIN: Mary Owen, Health Division

PARTICIPATING MEMBER OF PANEL: A. S. Householder

BACKGROUND AND STATUS: The surveys of dispensary visits and of blood counts have been discussed in previous quarterly reports. The correlation matrix for the data on dispensary visits has been inverted and results received from K-25. Confidence limits for the coefficients are being determined and the final results will be summarized in a report by the Health Division. Analysis of variance for the blood counts is being carried out at K-25. Mrs. Owen of the Health Division is analysing responses to a questionnaire on vacation benefits and attitudes. Panel participation in all these problems consists in occasional consultation and advice on procedure.

PROBLEM (continuation): Diffusion of Isotopes in Multicompartment Biological Systems

ORIGIN: Dr. C. W. Sheppard, Biology Division

PARTICIPATING MEMBER OF PANEL: A. S. Householder

BACKGROUND AND STATUS: Results on this problem are reported in ORNL 497.[2] However, this report is being revised somewhat for publication in the open literature, with some consideration of possible extensions.

[2] Sheppard, C. W. and Householder, A. S., *The Interpretation of Experimental Investigations of Transfers Within a Two-Compartment System, Using Isotopic Tracers, ORNL 497.* (Dec. 29, 1949).

Dr. Sheppard has designed and had constructed an electrical analogue machine for computing the change in radioactivity in the central compartment given a system of adjoining compartments of specified exchange characteristics.

PROBLEM: Effects of Gaps in the Reactor Core, and of Ducts in a Shield

ORIGIN: ANP

PARTICIPATING MEMBER OF PANEL: A. S. Householder

BACKGROUND AND STATUS: The problem is of some importance for reactor design, but difficult computationally. Various efforts have been made in the past to introduce approximations for computational simplicity, but the availability of improved computing machinery permits more complete treatment. With the help of Dave Whitcombe of NEPA, plans for a program of machine computation are being developed.

PROBLEM: Fission Neutron Age in D_2O-H_2O Mixtures

ORIGIN: Larry Noderer, Technical Division

PARTICIPATING MEMBERS OF PANEL: N. M. Dismuke, E. A. Forbes, V. M. Darnell

BACKGROUND AND STATUS: The availability of IBM's new Card-Programmed Calculator is expected to cut down on the work and time required to solve this type of problem by the Monte Carlo method [see ORNL 142[3] for details of method].

A rough draft of the scheme of computation of neutron histories is given here. It will be noted that the complete history of one neutron is determined before that of another is begun. This is in contrast to the logical procedure on standard IBM equipment which develops one stage of the histories of all neutrons before the next stage of any history is started. Completing one history at a time has the advantage, of course, that populations do not have to be predetermined.

Notation

$\rho_n = (x_n, y_n, z_n)$ = position vector of neutron at n-th collision.

$V_n = (u_n, v_n, w_n)$ = velocity vector of neutron after n-th collision.

[3] Householder, A. S., *Preliminary Stochastic Estimates of Slowing Down of Fast Neutrons in H_2O*, ORNL 142. (Sept. 17, 1948).

(l_n, m_n, n_n) = unit vector specifying direction of the neutron velocity after the n-th collision in the center of mass system.

T_n = mean time between the n-th and $(n+1)$st collisions.

r_1, r_2, r_3 = random numbers on the interval [0,1].

Table I: Spectrum Table

Unit interval has been subdivided so that length of k-th subdivision represents the probability that the source neutron has velocity in a specified range. An average velocity v_k is associated with this k-th subinterval.

Table II: Collision Type Table

The possible range of neutron velocities has been subdivided into intervals. Associated with each interval is a mean time between collisions, and a probability table (determined from cross sections) for specifying type of collision and mass, M, of colliding atom.

Calculation

Before the problem enters the CPC the following part of the calculation can be done:

(1) Unit vectors (l, m, n), specifying directions chosen at random from spherically symmetrical distributions, determined.

(2) $|V_0|$ determined from Table I and r_1.

(3) $l\ n\ r_3$

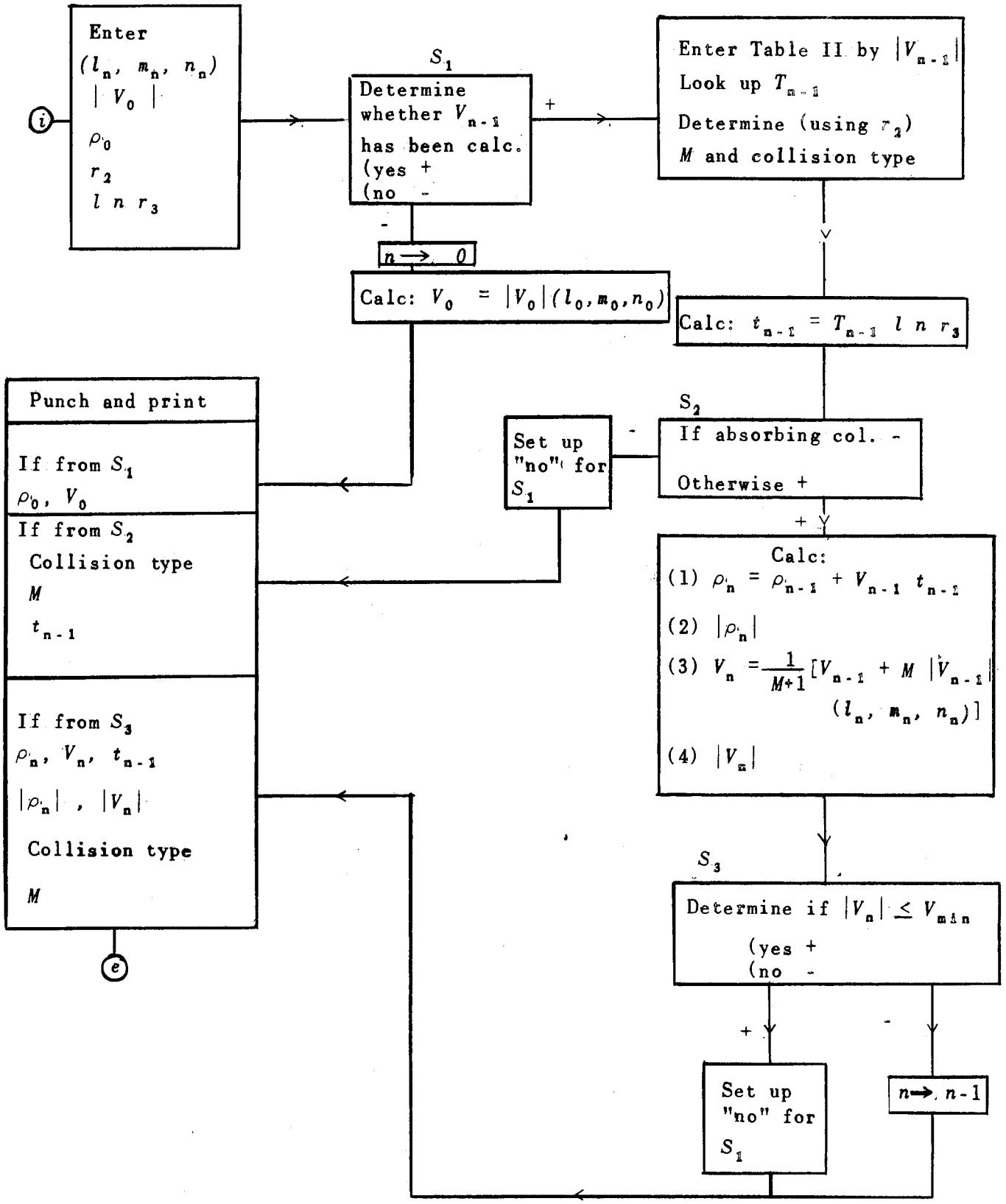
After the CPC is entered, one set of instruction cards, which does not change throughout the calculation, carries the problem through a cycle from i to e (see diagram), regardless of which loop is taken. Data cards carrying (l_n, m_n, n_n), $|V_0|$, ρ_0, r_2, $l\ n\ r_3$, which must be replaced after each cycle, are merged with the instruction cards.

PROBLEM: Tabulation of the Fermi Functions

ORIGIN: Mr. P. R. Bell and Dr. M. E. Rose, Physics Division

PARTICIPATING MEMBERS OF PANEL: N. M. Dismuke, E. A. Forbes

BACKGROUND AND STATUS: In experimentally determining the end point of a β spectrum, use is generally made of a Kurie plot, which requires the calcula-


Enter
(l_n, m_n, n_n)
$|V_0|$
ρ_0
r_2
$l\ n\ r_3$
S_1
Determine whether V_{n-1} has been calc.
(yes +
(no -
+
-
$n \rightarrow 0$
Calc: $V_0 = |V_0|(l_0, m_0, n_0)$
Enter Table II by $|V_{n-1}|$
Look up T_{n-1}
Determine (using r_2) M and collision type
Calc: $t_{n-1} = T_{n-1}\ l\ n\ r_3$
S_2
If absorbing col. -
Otherwise +
-
Set up "no" for S_1
+
Calc:
(1) $\rho_n = \rho_{n-1} + V_{n-1}\ t_{n-1}$
(2) $|\rho_n|$
(3) $V_n = \frac{1}{M+1}[V_{n-1} + M\ |V_{n-1}|\ (l_n, m_n, n_n)]$
(4) $|V_n|$
S_3
Determine if $|V_n| \leq V_{min}$
(yes +
(no -
+
-
Set up "no" for S_1
$n \rightarrow n-1$
Punch and print
If from S_1
ρ_0, V_0
If from S_2
Collision type
M
t_{n-1}
If from S_3
ρ_n, V_n, t_{n-1}
$|\rho_n|$, $|V_n|$
Collision type
M
i
e

tion of certain functions or coefficients. Tabulation of these functions or coefficients will be helpful to experimenters.

The following range and spacing for the independent variables ν, p, and Z has been specified for tabulation, where ν is the degree of forbiddenness, Z is the nuclear charge number, and p the electron (or positron) momentum in m_0c^2 units.

ν =	0, 1, 2, 3	
Z =	± 1 (1) ± 96	
p =	0 (.1) 1	$\lvert z \rvert \leq 96$
	1 (.2) 3	..
	3 (.5) 5	..
	5 (1) 9	..
	9 (2) 15	..
	15 (5) 25	$\lvert z \rvert \leq 25.$

From the point of view of computation the problem is primarily that of determining $\lvert \Gamma (s_\nu + i\ y) \rvert$. ($e^{\pi y}$ and p^{2s_ν} appear also, as multiplicative factors.) s_ν lies in the range .6 - 1.0, 1.8 - 2, 2.9 - 3, and 3.9 - 4 and $0 \leq \lvert y \rvert < 7.5$. Although polynomials are rather easily computed by IBM machines, polynomial fittings are not very practical over such large ranges of argument since many fittings would be required. A better procedure, in the problem at hand, appears to be to work in the logarithm and to make use of the existing table of $ln\ \Gamma\ (x + i\ y)$ for $x = 9(.1)\ 10$ and $y = 0(.1)\ 10$ which has been computed by the Bureau of Standards. This table can be extended to the desired range of s_ν by the recursion formula $ln\ Z + ln\ \Gamma\ (Z) = ln\ \Gamma\ (Z + 1)$, and then differenced for interpolation purposes. Working in the logarithm will also have the advantage that multiplications by $e^{\pi y}$ and $p^{2\nu}$ are greatly simplified for machine handling.

PROBLEM: Sample Solutions of an Eigenvalue Problem

ORIGIN: Dr. M. E. Rose, Physics Division

PARTICIPATING MEMBERS OF PANEL: N. M. Dismuke

BACKGROUND AND STATUS: Application of Rose's "three shot method" in solving an eigenvalue problem has been completed. Integration of the differ-

ential equations using the extrapolated value of the parameter W indicated that this value was approaching the required eigenvalue.

PROBLEM: A Random Walk Whose Equation Is Hyperbolic

ORIGIN: Dr. Henri Levy, Chemistry Division

PARTICIPATING MEMBER OF PANEL: Lewis Nelson

BACKGROUND: Dr. Levy has suggested the random walk discussed below as a simple case of neutron diffraction by crystals. The model used probably has little intrinsic interest to the chemist and physicist. However, it is a random walk (Monte Carlo) problem whose differential equation is hyperbolic and consequently of interest to those who are investigating Monte Carlo procedures for solving partial differential equations.

Suppose that a system of "point" particles move in a plane. Suppose further that each motion is either in the positive x-direction or in the positive y-direction. At each geometric point P a particle either continues its direction of motion, or it is deflected through a right angle as follows:

(i) those moving in the positive x-direction can be deflected to their left only,

(ii) those moving in the positive y-direction can be deflected to their right only.

More precisely, let:

$X(P)$ be the number of particles per unit length which are moving in the positive x-direction at the point P,

$Y(P)$ be the same measure of those moving in the positive y-direction at the point P,

$\alpha(P)$ be probability per unit length of a particle being "lost" from X at the point P,

$\gamma(P)$ be probability per unit length of particles being "gained" by X from Y at the point P,

$\beta(P)$ similarly measure Y's gain from X,

$\sigma(P)$ similarly measure Y's loss.

The densities X and Y then satisfy

(1) $X_x = -\alpha X + \gamma Y,$

(2) $Y_y = \beta X - \sigma Y.$

In case there is no regeneration of particles

$$\beta \leq \alpha \qquad \gamma \leq \sigma.$$

Eliminating Y and its derivatives yields the hyperbolic equation in X,

$$(3) \qquad -\beta\gamma^2 X + (\gamma\sigma - \gamma_y)\ (\alpha X + X_x + \gamma\ (\alpha X + X_x)_y = 0.$$

An obvious permutation produces a similar hyperbolic equation in Y, namely

$$(4) \qquad -\gamma\beta^2 Y + (\alpha\beta - \beta_x)\ (\sigma Y + Y_y) + \beta\ (\sigma Y + Y_y)_x = 0.$$

Note: In the case $\alpha = \beta = \gamma = \sigma$ and are constant, say α, the transformation

$$(5) \qquad F(x, y) = \exp\ (\alpha x + \alpha y)\ X(x, y)$$

$$(6) \qquad G(x, y) = \exp\ (\alpha x + \alpha y)\ Y(x, y)$$

applied to (1) and (2) gives

$$(7) \qquad F_x = \alpha G \quad \text{or} \quad F_{xy} = \alpha^2 F$$

$$(8) \qquad G_y = \alpha F \quad \text{or} \quad G_{xy} = \alpha^2 G$$

Under the initial conditions

$$X(0, y) = 1 \qquad 0 < y$$

$$Y(x, 0) = 0 \qquad 0 < x$$

Equations (7) and (8) are easily solved by applying a Laplace Transform and inverting the solution of the resulting algebraic equation. One finds

$$X(x,y) = \exp(-\alpha x - \alpha y) \sum_0^\infty I_n\ (2\ \alpha\ \sqrt{xy})\ (y/x)^{n/2}$$

$$Y(x, y) = X(x, y) - \exp(-\alpha x - \alpha y)\ I_0\ (2\ \alpha\ \sqrt{xy}).$$

PROBLEM: **Reactivity Perturbations in a Temperature Modulated Pile**

ORIGIN: **C. B. Graham, C. E. Winters, *et al.*, Technical Division**

PARTICIPATING MEMBERS OF PANEL: **Lewis Nelson, R. R. Coveyou, Ann Forbes**

BACKGROUND AND STATUS: Attempts are being made to answer by analytic and/or computational techniques such questions as: What results from a sudden change in reactivity in the homogeneous pile (as now being designed by this Laboratory) if it has been operating in a steady-state prior to the change? The following outlines some preliminary analytic considerations.

We consider a homogeneous pile and assume that its temperature is uniform. The pile is spherical and immersed in a constant temperature bath. We further assume that the heat transfer coefficient from pile to bath is independent of temperature, and that the reactivity of the pile is a linear function of the temperature with negative coefficient for the temperature-dependent term.

We ignore, in this first approximation, the effect of delayed neutrons. We assume that the pile has been at equilibrium for some time, and that at time $t = 0$ we introduce suddenly a perturbation consisting of a finite increment Δ_0 to the reactivity of the pile. Let $\Delta(t)$ be the excess reactivity. Choose the zero of temperature as the operating temperature of the pile, the unit of temperature as that rise in temperature which would reduce the reactivity of the pile to zero, starting from the original equilibrium conditions. Then we have

$$(1) \qquad \Delta(t) = \Delta_0 - T(t),$$

where $T(t)$ is the pile temperature as function of time.

If we choose the unit of power as the original power of the pile, the unit of time as the neutron life-time of the pile, we have

$$(2) \qquad \dot{P}(t) = \Delta(t)\, P(t)$$

with $P(t)$ the power as function of time.

The temperature and power are related by equation (4) which uses the definitions (3)

(3) c_0 = "Heat capacity" of the pile; *i.e.* the amount of energy necessary to raise the pile temperature 1 unit, where the unit of energy is unit power for unit time,

c_1 = "Heat transfer coefficient" of the pile; *i.e.*, the temperature drop in the resting pile per unit time per unit temperature difference between pile and bath,

$-T_0$ = temperature of the bath.

$$(4)\quad \dot{T}(t) = c_0\, P(t) - c_1 \{ T(t) + T_0 \} .$$

We generalize equation (4) slightly to read:

$$(4')\quad \dot{T}(t) = c_0\, P(t) - c_1 \{ \alpha\, T(t) + T_0 \} ,$$

where α is to be specified.

The case $\alpha = 1$ is Equation (4); the case $\alpha = 0$ gives the case of constant heat dissipation from the pile. Each of these cases is of some importance; the first corresponds to our present model. If we assume a reactor with "slug flow" of working fluid through a heat exchanger, then, until one cooling cycle is complete, the heat dissipation follows roughly the second of these cases.

We have now the system consisting of (1), (2) and (4').

At the original equilibrium, *i.e.* when $t < 0$, equation (4') prevails and one concludes that $c_0 = c_1\, T_0$. Using this and eliminating T, P and their derivatives from (1) (2) and (4') gives

$$(5)\quad \ddot{\Delta} = \Delta\,\dot{\Delta} + \alpha\, c_1\, \Delta^2 - c_1\, (\alpha\, \Delta_0 + T_0)\, \Delta - \alpha\, c_1\, \dot{\Delta} .$$

The boundary conditions are

$$(6)\quad \Delta(0) = \Delta_0$$

$$\dot{\Delta}(0) = -\dot{T}(0) = 0.$$

In the case of constant heat dissipation, ($\alpha = 0$), Equation (5) specializes to

$$(7)\quad \ddot{\Delta}(t) = \Delta(t)\, \dot{\Delta}(t) - c_1\, T_0\, \Delta(t).$$

The problem is now the examination of the behavior of the solution of (5) with respect to stability and type of oscillation.

If $\Delta(t)$ and $\dot{\Delta}(t)$ are initially small enough, we can replace (5) by the approximate equation

$$(8) \qquad \ddot{\Delta}(t) + \alpha\, c_1\, \dot{\Delta}(t) + c_1\, (\alpha\, \Delta_0 + T_0)\, \Delta(t) = 0.$$

A rigorous analysis, not reproduced here, shows that, under restrictions which are satisfied here, the behavior of a solution of (5) is asymptotically similar to that of a solution of (8), for small initial values of $\Delta(t)$ and $\dot{\Delta}(t)$.

The roots of the characteristic equation of (8) are $(-\gamma \pm i\, K)$ where we have abbreviated

$$2\gamma \equiv \alpha\, c_1$$

$$K^2 \equiv c_1\, (\alpha\, \Delta_0 + T_0) - \gamma^2 .$$

In our case, K is real and the general solution of (8) is

$$\Delta(t) = e^{-\gamma t}\, (A \cos Kt + B \sin Kt).$$

If we were sure that (8) is an adequate approximation to (5) for all t, the solution satisfying the original boundary conditions would be

$$\Delta(t) = \Delta_0\, e^{-\gamma t} \cos (Kt).$$

However, the analysis does not guarantee this. We can say, however, that

$$(9) \qquad \Delta(t) = A\, e^{-\gamma t} \cos \{ K(t + \delta) \}$$

is a good approximation to the solution for large t and appropriately chosen amplitude A and phase shift δ.

In the case of constant heat loss, we get, similarly,

$$(10) \qquad \Delta(t) = A \cos \{ K(t + \delta) \}$$

as an asymptotic solution for large t, where $K^2 = c_1\, T_0$.

PROBLEM: **A Random Walk**

ORIGIN: **Shielding**

PARTICIPATING MEMBER OF PANEL: **Lewis Nelson**

BACKGROUND AND STATUS: The Institute for Numerical Analysis, Bureau of Standards, Los Angeles, has started a study of numerical inversion of large order matrices. In this connection the Panel was asked for information and suggestions. It is known that if inversion of large order matrices were quick and practical, many shielding problems could be quickly and practically solved. In order to provide an example of a large scale matrix with *known* inverse the following random walk was considered.

Particles are constrained to motion on a line. A particle moving to the right is transmitted by an "obstacle" with probability t, reflected with probability r and absorbed with probability a. A particle moving to left has the corresponding probabilities τ, ρ, and α. There are a finite number of obstacles and the probabilities vary from one obstacle to another. Given a source of particles, what is their steady-state distribution? The associated matrix was inverted in closed form, but it is not practical to reproduce the analysis and result here because of space limitations.

PROBLEM: **Neutron Attenuation by Water**

ORIGIN: **Shielding Session**

PARTICIPATING MEMBER OF PANEL: **Lewis Nelson**

STATUS: The life histories of 1200 neutrons have been computed by a Monte Carlo method. The computation was carried out by the Central Statistics Laboratory of K-25. These life histories are to be analysed.

PROBLEM: **Approximate Calculation of the Neutron Flux in a Thick Shield Containing Some Heavy Elements**

ORIGIN: **Dr. Forrest H. Murray, Technical Division**

PARTICIPATING MEMBERS OF PANEL: **Carl Perhacs and Jerry Fishel**

BACKGROUND AND STATUS: These are computations for exploring the practicality of the method which Dr. Murray proposes for estimating the shielding properties of various mixtures of elements. Some computations have been made

and others are to be made.

PROBLEM: **A Design Problem for the MTR**

ORIGIN: **N. F. Lansing, Technical Division**

PARTICIPATING MEMBERS OF PANEL: **Ann Forbes, Vivian Darnell**

BACKGROUND AND STATUS: A numerical computation was required for designing the geometric location of the detectors for the flux-sensitive control system. The work continues.

PROBLEM: **Criticality Computations**

ORIGIN: **M. C. Edlund, Technical Division**

PARTICIPATING MEMBERS OF PANEL: **Ann Forbes, Jerry Fishel, Vivian Darnell**

BACKGROUND AND STATUS: Two group, two region criticality computations were performed.

PROBLEM: **Homogeneous Pile**

ORIGIN: **C. B. Graham and I. Spiewak, Technical Division**

PARTICIPATING MEMBERS OF PANEL: **R. R. Coveyou, Ann Forbes, Vivian Darnell**

BACKGROUND AND STATUS: A numerical integration of a set of equations which describe the homogeneous pile in a limited sense was started. These equations allow for several effects, *e. g.* delayed neutrons, not included in the analytic treatment described in "Reactivity Perturbations in a Temperature Modulated Pile" of this report. They are not regarded as a sufficient description of the pile and solution of a more adequate system is expected to be undertaken. The present computations are 50% complete.

PROBLEM: **Statistical Interpretation of Data from Xenon Cross-Section Experiment**

ORIGIN: **Dr. Seymour Bernstein, *et al.***

PARTICIPATING MEMBERS OF PANEL: **All**

STATUS: **Completed**

BACKGROUND: The data presented to the Mathematics Panel for analysis were estimates of the transmission factors of the different samples, as func-

tions of neutron energy, sample number, and time, together with statistical weights assigned to these transmission factors.

We have

(1) $T(n,E,t) = T_0 \ (n,E) \exp \{ -\sigma(E) \ X(n,t) \}$,

(2) $X(n,t) = \{ \lambda \ I(n)/(\lambda-\mu) \} \{ \exp [-\mu(t-t_0)] - \exp [-\lambda(t-t_0)] \}$.

n = sample number,

E = neutron energy,

t = time of measurement,

t_0 = time of precipitation,

λ = I^{135} decay constant.

μ = Xe^{135} decay constant,

$I(n)$ = number of I^{135} atoms per unit cross-sectional area of neutron beam present in n-th sample at precipitation time t_0,

$X(n,t)$ = number of Xe^{135} atoms per unit cross-sectional area of neutron beam present in n-th sample at time t,

$T_0(n,E)$ = transmission factor of n-th sample without Xe at energy E,

$T(n,E,t)$ = transmission factor of n-th sample including Xe^{135} for energy E at time t.

Setting

(3)
$$S(n,E,t) = - \log T(n,E,t)$$
$$S_0(n,E) = - \log T_0(n,E) ,$$

and combining (1) and (2) above, we have

(4) $$S(n,E,t) = S_0(n,E) + \{\lambda \ I(n)\sigma(E)/(\lambda-\mu)\} \{\exp [-\mu(t-t_0)] - \exp [- \lambda(t-t_0)] \} .$$

The measured values of $S(n,E,t) = -\log T(n,E,t)$ were treated as observations after recalculating the statistical weights by the formulae

(5) $W(x) = 1/V(x)$

(6) $V\{-\log x\} = [\partial(-\log x)/\partial x]^2 \; V(x) = V(x)/x^2$,

(6a) $W(-\log x) = x^2 W(x)$

$W(x)$ = Statistical weight assigned to an estimate of the value of x.

$V(x)$ = the "variance" of x, *i.e.* $V(x)$ is the square of the "standard deviation" of x.

We have several difficulties to contend with in the estimation of $\sigma(E)$ from the data and the above formula:

(a) Since the $I(n)$ are not directly known, it is apparent that separate estimates of $I(n)$ and $\sigma(E)$ are *not* possible; one can only estimate the product of $I(n)\,\sigma(E)$ for each sample and the energies concerned. Because of this complication, each run had been measured at $E = .03$ ev, and, if we write

(7) $$\sigma_R(E) = \sigma(E)/\sigma(.03) = I(n)\,\sigma(E)/I(n)\,\sigma(.03) ,$$

we see that the relative cross sections $\sigma_R(E)$ can be estimated from the data.

(b) t_0, the time of precipitation is not precisely defined, since the precipitation process is not instantaneous, and not all Xe^{135} is removed at the time the sample is sealed. However, one can show that, once a sample has been sealed, it behaves as if there had been a precisely defined precipitation time. Hence, it was decided to make t_0 one of the parameters to be estimated. (As things turned out, it was found that this difficulty had been over-estimated; it was found possible to fit the data to curves of the type of equation (4), using the precipitation time estimated by the chemists. One sample (12) did require a considerable shift in t_0 to produce even a reasonable fit, but this sample and the measurements obtained from it were later discarded, on this and other evidence of irregularity.)

(c) There existed strong evidence that some of the samples leaked. Here a quite arbitrary assumption was made, justified by success in achieving a

reasonable fit to formula (4) for each sample, namely, that escape of Xe^{135} from a leaky sample was proportional to the partial pressure in the sample due to Xe^{135}, *i.e.*, to the amount of Xe^{135} present. This assumption again leads to an equation of type (4), but with μ now representing an "effective" decay constant for Xe^{135}, slightly higher than the true nuclear decay constant. Hence, in fitting equation (4) to the data, μ was also taken as an adjustable parameter.

In fitting equation (4) to the data from a given sample, one must estimate $S_0(n,E)$ and $I(n)\ \sigma(E)$ for each energy at which measurements were taken on the sample, and must also estimate μ and t_0. Since each sample was measured at 3-5 energies, this demands the evaluation of 8-12 parameters for each sample. A direct attack on this problem by a method of "linearized least squares" was attempted, but was found to be impractical with the machine used.

Since each sample was measured at .03 ev several times, the following procedure was adopted. First, estimates of transmission factors measured at the same energy at times which were very close together were "grouped", *i.e.*, a mean, weighted with the original statistical weights, was calculated, and a statistical weight equal to the sum of the weights of the components was assigned. (This procedure is valid, in general, if the curvature of the fitting curve is negligible in the interval over which each group of data is averaged.) The parameter μ was then estimated by a step-by-step procedure, using only the data at .03 ev. After this was done, it was discovered that good fits could be obtained without tampering with t_0, the precipitation time as estimated by the chemists. Using this value of μ, $S_0(n,E)$ and $I_n\ \sigma(E)$ were estimated for each energy separately, a rather easy procedure. Equation (4) is linear in these two parameters for fixed n and E. Finally, it was determined that the fits obtained to equation (4) for other energies, using μ as estimated from the data at $E = .03$ were as good as the best fit we could get at $E = .03$.

From the estimated values of $I(n)\ \sigma(E)$ obtained by this process, estimates of $\sigma_R(E)$ were calculated, together with statistical weights assigned to them.

The next step is the fitting of the estimated relative cross sections $\sigma_R(E)$ to the Breit-Wigner single level nuclear resonance formula in the form

$$\sqrt{E}\ \sigma(E) = A\ \Gamma/[\ 4(E-E_0)^2 + \Gamma^2\ ]\ , \tag{8}$$

$$\Gamma = \Gamma_\gamma + (E/E_0)^{1/2}\ \Gamma_n\ , \tag{9}$$

where A, E_0, Γ_γ , Γ_n are parameters to be estimated.

The substitutions

$$(10) \qquad \lambda = 1/\sqrt{E\,\sigma}$$

$$Z = \sqrt{E_0}$$

$$Z = \sqrt{E}$$

$$\Gamma_n/\Gamma_\gamma = \omega$$

reduce equation (8) to the form

$$(11) \qquad \lambda = [\,C(Z^2 - Z_0^2)\,]/[\,Z_0 + \omega Z] + D(Z_0 + \omega Z),$$

with

$$(12) \qquad C = 4\sqrt{E_0}/A\,\Gamma_\gamma$$

$$D = \Gamma_\gamma/A\sqrt{E_0}\,.$$

This transformation has the advantage that equation (11) is linear in *two* parameters (C,D), while (8) is linear only in A. Considerable difficulty was experienced in arriving at the correct statistical weights to be assigned to the estimates of λ, as described in the preceding Quarterly Report.[4]

Again, these points were grouped, in accordance with the considerations described on page 22. It was found impractical to estimate the four parameters ω, Z_0, C, D, simultaneously. Indeed, considerable evidence developed that ω could not rationally be estimated at all by direct least squares procedures. This conclusion being supported by physical reasoning suggested by Dr. E. Feenberg, it was decided to find a least squares fit for the parameters Z_0, C, D for several assumed values of ω. Even with this simplification, it was again found necessary to resort to step-by-step procedures to get reliable estimates for these parameters. The decision to assign ω arbitrarily was justified after the event by the results; even though each assigned value of ω led to widely different estimates of Z_0, C, D, the resulting cross-section estimates differed very little. It was found possible, finally, to select with reasonable certainty, a "most likely" value of ω, but the likelihood that this is the "true" value determined by the experiment is only infinitesimally

[4] Householder, A. S., *Mathematics Panel Quarterly Progress Report for the Period Ending October 31, 1949*, ORNL 516 (Dec. 12, 1949).

larger than the likelihood that any other value of ω of a quite wide range is the true one.

Finally, estimates of A, E_0, Γ_n, Γ_γ were calculated, and used as initial estimates of these quantities in a linearized least squares procedure applied directly to equation (8). It resulted that A_0, E_0 had reasonably small variances of estimate, Γ_γ was determined with somewhat less certainty, while Γ_n was rather badly determined.

The final conclusion reached is that the experiment served to determine the relative cross sections as a function of energy within a per cent or two over a large energy range ($E = \sim .02$ to $\sim .20$). Further, the estimates finally derived for the Breit-Wigner parameters A, E_0 are reasonably good, but Γ_γ and Γ_n are not well determined.

The difficulties of technique encountered in this problem were such that it has been decided to investigate the problem, considered purely mathematically, in some detail, for use in possible future applications. Numerical results will be reported by S. Bernstein, *et al.*, in their final report on the experiment.