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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT FOR THE PERIOD ENDING OCTOBER 31, 1950

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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT
for the Period Ending October 31, 1950

A. S. Householder, Chief

DATE ISSUED: DEC 29 1950

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SUMMARY

Three new members have joined the Mathematics Panel during the current quarter. Barbara McGill, of Knoxville, is a graduate of Florence State Teachers College. Dr. C. L. Perry, formerly of the University of Arkansas, did his graduate work at the University of Michigan in applied mathematics. Dr. Allyn W. Kimball comes from the Department of Biometrics, School of Aviation Medicine, Randolph Field, Texas. His graduate work was done in experimental statistics at the Institute of Statistics, Raleigh, North Carolina.

A formal contract with the Argonne National Laboratory has been signed and approved for the construction of the Laboratory's electronic computer. The contract calls for some research on design in order that experience gained in the construction of Argonne's own computer, as well as other developments, can be evaluated and utilized to the best advantage. However, a deadline date of July 1, 1951, has been set for freezing specifications, and it is hoped that construction can be completed in from six to nine months thereafter.

The National Bureau of Standards sponsored a meeting of the Association for Computing Machinery at the Shoreham Hotel in Washington, D. C., Sept. 7 to 9. Dr. Perry presented a paper describing the solving of a potential distribution problem on the Fairchild Linear Equation Solver (*Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1950,* ORNL-818, p. 14; this report, p. 8).

Vivian Gordon, Rooney Currey, and Ann Forbes remain on loan with the ANP Division. They were assisted temporarily by Carl Perhacs on a rush job. R. R. Coveyou is now on loan. Further details may be found in the ANP reports.

Plans for having the internal-conversion calculations for the L shell done on the Mark III were disrupted by the removal of the machine from Harvard University to the Naval Proving Ground at Dahlgren. Possible alternatives are the machine being made by Dr. Goertzel at New York University and the SWAC at Los Angeles. Tentative arrangements for use of the latter machine have been made with Albert Cahn of the Institute for Numerical Analysis.

The Panel seminar has been considering methods of interpolation and, more generally, of the approximate representation of functions. Beginning in December it will turn to methods of programing and coding for electronic

computing machines, starting with the Linear Equation Solver and progressing to the Princeton machine to be installed here. Dr. Perry will be chiefly responsible for these discussions.

It has been necessary to shelve temporarily several problems begun some time ago because of more urgent computational needs. Among these are the analysis of data on beta-ray spectroscopy for P. R. Bell and Dr. Rose's Ra-E problem. Work on the latter should be resumed as soon as the IBM computations have been completed for the internal-conversion problem. The former problem is at a stage which requires hand computation of elements of a matrix to be inverted at NEPA. It is hoped that at least one or two spectra can be analyzed during the coming quarter as a test of the proposed method.

SPECIAL PROBLEMS AND PROJECTS

Problem: **LIGHT ABSORPTION BY CELLS**

Origin: R. W. Koza, Biology Division

Participating Members of Panel: A. W. Kimball, C. L. Perry

Background: A circular cylindrical beam of light is passed through a spherical cell. The light absorbed by the cell is assumed to be proportional to the volume of the light beam intercepted by the cell. The beam is adjusted so that its axis is near the center of the cell. Mr. Koza wanted to know how the volume of intercepted light varied with the distance β from the center of the sphere to the axis of the cylinder. Easily calculated limits of this variation are given by the following inequalities:

$$V_0 \geqslant V_\beta \geqslant V_0 \sqrt{[R^2 - (\alpha + \beta)^2]/[R^2 - \alpha^2]} ,$$

where V_0 denotes the volume of the intercepted light when $\beta = 0$, V_β denotes the volume when $\beta \neq 0$, R is the radius of the sphere, and α is the radius of the cylinder.

Status: Completed.

Problem: **CHECKING THE MATHEMATICS IN KAPL REPORT NO. 362**

Origin: E. C. Miller, Metallurgy Division

Participating Member of Panel: C. L. Perry

Background: Mr. Miller asked the Mathematics Panel to proofread the mathematics in the above-mentioned report. Several misprints were found, and the solution to one of the differential equations was incorrect.

Status: Completed.

Problem: **DEFORMATION OF METALS**

Origin: Dr. A. G. H. Andersen, Metallurgy Division

Participating Member of Panel: C. L. Perry

Background: Dr. Andersen requested the Mathematics Panel to review the mathematics in the reports he was writing (*A Study of Stress-Strain-Time Functions of Metals*, ORNL 50-10-132 and ORNL 50-10-132A). In addition he wanted the solutions to the differential equation

$$\frac{d\delta}{dt} = \frac{\delta}{t} \frac{a}{b + c\delta^{\alpha} t^{\beta}} , \tag{1}$$

where a, b, c, α, and β are constants.

Status: The reviewing of the reports has been completed. It was found that, when the variables δ and t in Eq. (1) are replaced by

$$\delta = x^{1/\alpha} \qquad \text{and} \qquad t = u^{1/\alpha} ,$$

the resulting equation can be reduced to the Bernoulli equation,

$$\frac{du}{dx} - \frac{bu}{\gamma x} - \frac{cu^2}{\gamma} = 0,$$

where $\gamma = a\alpha/\beta$. Using this result it was found that the solutions to Eq. (1) are, when $b \neq -a\alpha/\beta$,

$$t^{-\beta} = A\delta^{-b\beta/a} - \frac{c\beta}{a\alpha + b\beta} \delta^{\alpha} ,$$

and when $b = -a\alpha/\beta$,

$$t^{-\beta} = A\delta^{\alpha} - \frac{c\beta}{a} \delta^{\alpha} \log \delta .$$

Problem: **SEPARATION OF THE PILE EQUATION IN CYLINDRICAL COORDINATES**

Origin: Dr. D. K. Holmes, Reactor Technology Division

Participating Members of Panel: C. L. Perry, D. W. Whitcombe

Background and Status: Dr. Holmes asked the Mathematics Panel to evaluate the separation-of-variables technique for solving the boundary-value problem for the pile. The method turned out to be not feasible.

Problem: **(continuation)** **HEAT TRANSFER BY A FLUID FLOWING TURBULENTLY IN A CYLINDRICAL TUBE**

Origin: Dr. R. V. Bailey, Reactor Technology Division

Participating Members of Panel: A. S. Householder, C. L. Perry

Reference: *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1950,* ORNL-818, pp. 27-28.

Status: Dr. Bailey has returned to Mississippi State College. He has entrusted Dr. Poppendiek and Mr. Claiborne of the Reactor Technology Division with the determination of the coefficients of his differential equation. The Mathematics Panel is waiting for these coefficients before further investigation of Dr. Bailey's problem.

Problem: **HEAT TRANSFER BY A FLUID FLOWING WITH A SLUG FLOW IN A HOT PILE**

Origin: H. C. Claiborne, Reactor Technology Division

Participating Members of Panel: D. W. Whitcombe, C. L. Perry

Background and Status: This problem is equivalent to Mr. Claiborne's previous problem mentioned in *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1950.* Mr Claiborne has been helped in expanding his catalog of solutions to this problem. The catalog now includes cross sections that are ellipses and sectors of circles. Mr. Whitcombe has sent Mr. Claiborne a memorandum containing an approximate solution for the 60° sector.

Problem: **(continuation)** **DETERMINATION OF A POTENTIAL DISTRIBUTION**

Origin: Dr. W. M. Good and Dr. R. W. Lamphere, Physics Division

Participating Member of Panel: C. L. Perry

Reference: *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1950,* pp. 14-15.

[redacted]

Status: The Fairchild Linear Equation Solver Group at NEPA has completed the computations for the potential distribution. Their results have been used to plot the field in the symmetrical Van de Graaff generator. This plot agrees with the plot determined by the electrolytic tank analog of the Van de Graaff tube. The Mathematics Panel has also discussed the calculation techniques used for obtaining electron trajectories with Messrs. Case and Kington of the Van de Graaff staff. This phase of the problem is now complete. It has been suggested that the Fairchild Linear Equation Solver be used to investigate the influence of electrode roughness on the field distribution near the electrode.

Problem: **HEAT TRANSFER IN THE MOCK-UP**

Origin: S. E. Beall, Reactor Technology Division

Participating Member of Panel: C. L. Perry

Background: A cooling-system failure in the mock-up would produce a rise in temperature in the fuel elements. This rise depends on the heat generated by the fission products in the fuel elements and the heat transfer from the fuel elements to their surroundings. The quantity of fission products present at cooling failure depends on the power at which the mock-up has been operating. Mr. Beall is interested in obtaining an accurate estimate of the maximum temperature in the fuel element (on cooling-system failure) as a function of the power level of operation of the mock-up.

Status: The estimated time required for the calculations of this maximum temperature by the Mathematics Panel exceeded Mr. Beall's allowable time. Mr. Beall will use the estimate previously calculated by Mr. Baczewski.

Problem: **EFFECT OF OXYGEN TENSION ON SENSITIVITY OF BACTERIA TO X RAYS**

Origin: Dr. A. Hollaender, Biology Division

Participating Members of Panel: A. W. Kimball, B. S. McGill

Background and Status: Cultures of *E. coli* (B/r strain) were exposed to various doses of X radiation in suspensions through which either nitrogen or oxygen had been bubbled for 15 min. Four different conditions of growth were tried with each type of saturation: aerobic and anerobic each with and without glucose. For each dose the proportion of cells surviving was measured. Rough

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standard deviations computed for each point on the four response curves from each experiment indicated differences between the nutrient broths which were so obvious that no statistical tests of significance were required.

It was desired, however, to find some simple quantitative measurement that would differentiate between the various growth conditions. Plots of the proportion of cells killed against the dose revealed the sigmoid curve typical of these kinds of data. Accordingly, curves were fitted by the probit method, and the LD_{85} (dose killing 85 percent of the organisms) was chosen as the single measure for differentiating between the broths.

Problem: RADIOMANGANESE STUDIES IN THE DOG PANCREATIC FISTULA

Origin: Dr. W. T. Burnett, Jr., Biology Division

Participating Members of Panel: A. W. Kimball, C. Perhacs, B. S. McGill

Background: There is indirect evidence to indicate that manganese may be related to enzyme activity in the pancreas. In particular it is thought that manganese stimulates such activity and may perhaps form bonds with the enzymes. To investigate this relationship Mn^{54} was administered intravenously to a dog which had been prepared so that all juices secreted by the principal duct of the pancreas could be collected. After injection the fluid was collected at 2-hr intervals for a period of 12 days, except for the period from midnight to 8:00 A.M., during which time very little fluid was secreted. The Mn^{54} recovered in each sample was expressed as a percentage of the initial dose. Total nitrogen and protein nitrogen were also determined for each sample.

The investigator was interested in finding a mathematical expression for the rate of secretion of manganese by the pancreas following an initial dose. He was also interested in estimating the correlation between manganese recovered and enzyme activity.

Status: Metabolic activity varies considerably during a 24-hr period, from high levels, usually after meals, to low levels during periods of rest. Accordingly, in finding an expression relating the variation of percent Mn^{54} recovered, y, with time after injection, t, 24-hr totals were used. Since to some extent y varies with the total amount of fluid secreted, x, it was clear that the y's had to be adjusted for differences in the x's before any meaningful relationship between y and t could be established. The technique employed

to make these adjustments is similar to one used in the analysis of covariance. It was assumed that apart from variation with time, y is linearly related to x. After some preliminary calculations it was decided that a simple exponential would adequately describe the relationship between the adjusted y's and t. To achieve both objectives simultaneously the expression

$$y = a + bx + ce^{-t/10}$$

was fitted by least squares to the 12 points given by the 24-hr totals. The coefficient of the exponent of e in the above equation was determined a priori by trial and error. Conceivably a better fit could have been obtained if this quantity had also been estimated from the data, although it is not likely that the gain in precision would have been sufficient to warrant the extra computations. Tests of significance relating to the goodness of fit and to the significance of the individual regression coefficients accompanied the curve fitting.

For the estimation of correlations, the 2-hr sample values were used. The average level of manganese recovered decreased steadily during the 12-day period, whereas the nitrogen levels remained relatively constant. Nevertheless the diurnal patterns of variation were so similar and the variations in metabolic activity were so marked, it was not considered necessary to adjust the manganese measurements for variation with time. The manganese recovered was correlated with both protein nitrogen and nonprotein nitrogen. The difference between these two dependent correlation coefficients was tested statistically by the method described by Hotelling [*Ann. Math. Stat.* 11, 271 (1940)].

Detailed results of this experiment will be submitted to an appropriate journal for publication.

Problem: **RADIOACTIVITY OF ALIGNED AND UNALIGNED SAMPLES**

Origin: C. P. Stanford, Low Temperature Physics

Participating Members of Panel: A. S. Householder, G. E. Albert, A. W. Kimball, C. L. Perry

Background: Mr. Stanford wants to know whether or not there is a significant difference in the radioactivity of samples whose magnetic moments are

aligned and unaligned. He has set up an experiment in which a monitor and sample (aligned and unaligned samples) are placed in a neutron beam. The monitor and sample are removed from the beam, and radioactivity counts are made on them alternately. From these measurements the ratio of the activity of the sample to the activity of the monitor is determined. Let r and r^* represent the ratios for the aligned and unaligned samples, respectively. (Several r and r^* values are obtained from different runs.) The means of the r and r^* values and their distributions are compared to ascertain whether or not there is a significant difference between these ratios for the aligned and unaligned samples.

The value of r (and r^*) was estimated in the following manner: The half-life of the sample is known (the monitor has the same half-life). The expected number of disintegrations from a given time $t = t_0$ to $t = \infty$ is $Ae^{-\lambda t_0}$, where λ is the half-life. Let us assume that the counter counts some constant multiple α of the total number of disintegrations. The counter also counts a background at the same time. Let us assume also that the background rate B is a constant. The expected number of counter counts δ_i in the time interval t_i to t_{i+1} can then be expressed in the form

$$\delta_i = \alpha A X_i + B Y_i ,$$

where

$$X_i = e^{-\lambda t_i} - e^{-\lambda t_{i+1}} ,$$

$$Y_i = t_{i+1} - t_i .$$

Let us now designate by Δ_i the actual number of counter counts. The values of αA and B were obtained by minimizing the sum

$$\sum_{i=0}^{n-1} \frac{(\Delta_i - \delta_i)^2}{\Delta_i} .$$

This is the minimization of the weighted sum of the squares of the deviations with the weight factor Δ_i^{-1} as estimate of the reciprocal variance of the

count. In the future the experiment will be planned so that the Δ_i's will be equal. The times devoted to the alternate counting of the sample and monitor will be in the ratio $\sqrt{\overline{\Delta}_i/\Delta_i}$, where Δ_i and $\overline{\Delta}_i$ are the numbers of counts from the sample and monitor, respectively.

Status: The method has been outlined, and some preliminary calculations have been made. A memorandum describing the calculations will be sent to the Low Temperature Physics Group, who will carry out the remaining calculations.

Problem: EFFECT OF IRRADIATION ON MITOTIC STAGES

Origin: Nyra Harrington, Biology Division

Participating Member of Panel: J. Moshman

Background: Previous investigations have shown an apparent decrease in the number of cells in grasshopper embryos in the metaphase and anaphase stages of mitosis. To examine this phenomenon more closely, late prophase stages were subjected to a similar scrutiny. Evidence indicates that this decrease results primarily from a delay of cells in the late prophase stage rather than directly from some effect in metaphase or anaphase.

Status: Completed.

Problem: A ROUGH TEST FOR REJECTION OF AN OUTLYING OBSERVATION

Origin: Dr. J. Furth, Biology Division

Participating Member of Panel: J. Moshman

Background: It has been customary to make a series of four hematocrit readings in routine hemotological assay and use the mean of the four as the "best" estimate. When one of the four observations was apparently anomalous, subjectively, it was discarded, and the resulting three were averaged. A more objective test, one that could be applied quickly, was desired.

Grubbs [*Ann. Math. Stat.* 21, 27-58 (1950)] found the distribution of the statistic

$$\frac{S_n^2}{S^2} = \frac{\sum_{i=1}^{n-1} (x_i - \overline{x}_n)^2}{\sum_{i=1}^{n} (x_i - \overline{x})^2} .$$

where

$$x_1 < x_2 < \ldots < x_n,$$

$$\bar{x}_n = \sum_{i=1}^{n-1} x_i/(n-1),$$

$$\bar{x} = \sum_{i=1}^{n} x_i/n,$$

in order to test whether the largest observation is too high. Similarly,

$$\frac{S_1^2}{S^2} = \frac{\sum_{i=2}^{n} (x_i - \bar{x}_1)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

may be used to test if the smallest observation is too low. In a sample where $n = 4$, the 5 percent significance point is 0.0494, small numbers being significant.

To avoid the task of squaring and adding many 3-digit numbers, the following procedure was proposed:

It is known that in a normal population there is a relationship between mean range and variance depending on sample size. If R_n is the mean range in a sample of size n, then

$$R_4 = 2.059\sigma$$

$$R_3 = 1.693\sigma$$

where σ is the square root of variance.

Now we can set $S^2 = (n - 1)\sigma^2$ and $S_n^2 = (n - 2)\sigma^2$, so that

$$\frac{S_n^2}{S^2} = \frac{(n-2)R_3^2/(1.693)^2}{(n-1)R_4^2/(2.059)^2} .$$

Therefore

$$S_n/S = 2.059\sqrt{2}\,R_3/(1.693\sqrt{3}\,R_4) = 2.912R_3/2.932R_4 \ .$$

On the 5 percent level of significance,

$$S_n/S \leqslant \sqrt{0.0494} \ ;$$

hence, the statistic

$$T = R_3/R_4 \leqslant 2.932\sqrt{0.0494}/2.912 = 0.224$$

Similarly, on the 10 percent level of significance,

$$S_n/S \leqslant \sqrt{0.0975}$$

and

$$T = R_3/R_4 \leqslant 0.314 \ .$$

It was decided to adopt the criterion of rejection when $T \leqslant 0.25$, which would be subject to rapid calculation and still give a probability of error of the first kind of about 6 percent; this was deemed satisfactory for these purposes.

Status: This rule is only a first approximation. It is planned to run a series of empirical tests to examine the closeness of agreement between Grubbs' statistic and the proposed T statistic, and possibly to investigate the power of the test.

Problem: **AN INVESTIGATION OF BLOOD VOLUMES IN TUMOR-BEARING MICE**

Origin: Dr. J. Furth, Biology Division

Participating Members of Panel: J. Moshman, B. S. McGill

Background: It has been noted that granulosa-cell tumors are associated with hypervolemia in addition to an unusual type of congestive change in the viscera of the animal.

Detailed statistical analyses were made of differences observed between various strains of tumor-bearing mice and control animals with respect to cell, plasma, and total blood volumes as well as the peripheral hematocrit. It was found that in every case the hematocrit of the tumor-bearing mice was significantly ($P < 0.01$) lower than in the respective control mice. Plasma and total blood volumes also showed significant differences except in the case of the females of one strain with ovarian carcinoma. Sex differences on the whole were not significant.

An attempt was made to test the dependence of the various blood indices upon tumor size in the host animals. When the tumor-bearing mice were placed in dichotomous classes according to tumor size, it was found that in each class the distribution of the index was nonnormal in character, the frequency curves being very platykurtic, approaching a rectangular distribution in shape. Accordingly differences were tested by a nonparametric method of E. J. G. Pitman [*J. Roy. Stat. Soc., Supplement*, 4, 117-130 (1937)], and only in the strain V granulosa tumor were hematocrit differences significant on the 5 percent level. It is noteworthy, however, that all hematocrits were smaller in the animals with larger tumors.

Status: A report on the results of this study is being prepared and will shortly be offered to *Cancer Research* for publication under the joint authorship of J. Furth and J. Moshman.

Problem: **THE RELATION OF PERIPHERAL VENOUS HEMATOCRIT TO AVERAGE BODY HEMATOCRIT**

Origin: Dr. J. Furth, Biology Division

Participating Member of Panel: J. Moshman

Background: In the determination of cell (C), plasma (P), and total blood (T) volumes, it is customary to make a direct determination of the average body hematocrit (H) and either cell or plasma volume. Then, if f is some constant factor,

[redacted]

$$C/fH = T \tag{1}$$

or

$$P/(1 - fH) = T \tag{2}$$

under the restriction $C + P = T$. The factor f relates the average body hematocrit to the venous hematocrit.

Wish *et al.* [*Proc. Soc. Exptl. Biol. Med.* 74, 644-648 (1950)] in a study based on 20 normal and tumor-bearing mice concluded the mean value of f to be 0.88. The range of mean values for their mice, after classification into strain, was 0.80 to 0.94.

In an independent check of this result in 27 cases where both cell and plasma volumes were determined directly, Eqs. (1) and (2) were solved for f:

$$C/fH = P/(1 - fH),$$

$$f = C/H(P + C).$$

It was found that the mean f was 0.89 with a range among the 27 from 0.68 to 1.16. When classified into the various normal and tumor-bearing strains, the range of mean values was 0.83 to 1.05. Values for the four tumor-bearing strains were higher than for the two normal strains. Subjected to a one-way classification analysis of variance, it was found that the probability of homogeneity of the means was about 0.025.

Status: Further investigations are planned to relate f to various physiological and pathological states of the animal.

Problem: **(continuation) DETERMINATION OF NEUTRON FLUX IN THE ORNL PILE**

Origin: Dr. D. K. Holmes, Physics Division

Participating Member of Panel: J. Moshman

Background: In the discussion of the adaptation of von Neumann's weighting scheme in the previous quarterly report (ORNL-818, *p.* 17) there was an error due to an ambiguous use of the letter n.

Consider t strata based upon t regions in which a neutron may have a collision outside the core. Assume that we start with n/k neutrons and split

[redacted]

each neutron into k_i neutrons if it falls into the ith stratum. In this case the variance of the mean flux is

$$s'^2 = \frac{k}{n} \frac{\Sigma p_i s_i^2}{k_i}, \tag{1}$$

where p_i is the probability of a neutron's falling into the ith stratum and s^2 the variance of that stratum.

Subject to the restriction $\Sigma\, p_i k_i = k$, we wish to determine k_i so that s'^2 is minimized. Let

$$G(k_1, k_2, \ldots, k_t, \lambda) = \frac{k}{n} \frac{\Sigma p_i s_i^2}{k_i} + \lambda\, \Sigma\, p_i k_i - \lambda k$$

where λ is a Lagrange multiplier. Then

$$\frac{\partial G}{\partial k_i} = \frac{k}{n} \frac{p_i s_i^2}{k_i^2} + \lambda\, p_i \qquad (i = 1, 2, \ldots, t)$$

$$\partial G/\partial \lambda = \Sigma\, p_i k_i - k \tag{2}$$

If we set the partial derivatives equal to zero, we find that

$$k_i^2/s_i^2 = \lambda n/k = m^2,$$

say, where m is a constant. Hence for any i and j,

$$k_i/s_i = k_j/s_j$$

or each k_i is proportional to the s_i of that stratum.

Let

$$k_j = s_j k_1/s_1 \qquad (j = 2, 3, \ldots, t) \tag{3}$$

[redacted]

Substituting in Eq. (1),

$$s'^2 = \frac{k}{n} \frac{\sum p_i s_i^2 s_1}{k_1 s_i} = \frac{k}{n} \frac{\sum p_i s_1 s_i}{k_1} \tag{4}$$

$$s'^2 = \frac{k}{n} \cdot \frac{s_1}{k_1} \sum p_i s_i .$$

But after substituting Eq. (3) in Eq. (2),

$$\sum p_i s_i k_1 / s_1 = k$$

or

$$\frac{k_1}{s_1} = \frac{k}{\sum p_i s_i} \tag{5}$$

Finally, substituting Eq. (5) in Eq. (4), we have

$$s'^2 = \frac{(\sum p_i s_i)^2}{n},$$

which, in general, will be less than the usual estimate

$$s'^2 = \sum p_i s_i^2 / n.$$

Status: All preliminary work has been completed. Punch cards have been prepared with random numbers, random direction cosines, energy-spectrum values proportional to the curve $f(E) = \sqrt{2/\pi e}\ e^{-E} \sinh \sqrt{2E}$, radial origin in the rod proportional to $I_0(\kappa r)$, and other items needed for the final calculations. Programing is now in progress.

[redacted]

[redacted]

Problem: **SOLUTION OF EQUATION OF MOTION FOR CONTROL RODS ON HOMOGENEOUS REACTOR EXPERIMENT**

Origin: A. G. Grindell, Engineering Division

Participating Member of Panel: D. W. Whitcombe

Background: The control rods for the Homogeneous Reactor Experiment at Y-12 are arranged so that they are held by the tension of a spring. Mr. Grindell was interested in the variation of distance with time when the control rods are lowered into water. To express the motion he set up the following differential equation:

$$\ddot{S} + k_4 S(\dot{S})^2 + k_5 \dot{S} = k_6,$$

where the nonlinear term expresses the skin friction and the linear term in S expresses the spring tension and the buoyancy force. In this particular problem $k_6 \approx 30$, $k_4 \approx 10^{-2}$, and $k_5 \approx 10^{-2}$; hence it is clear that the trajectory will be a small perturbation from a parabola.

A power-series solution should then converge rapidly; the expansion is made about $t = 0$ since the initial conditions are given as $S(0) = 0$ and $\dot{S}(0) = 8.7$. Then we write

$$S = \Sigma S_n t^n.$$

The derivatives and powers of S are easily found and substituted into the differential equation. If powers of t are then equated, a recurrence relation is obtained. From this any number of S_n's may be computed. The solution has now been used twice at Y-12, and it was found that six terms were sufficient.

Problem: **DUCTING DIFFERENCE EQUATIONS**

Origin: Dr. R. L. Echols, NEPA

Participating Member of Panel: D. W. Whitcombe

Background: The solution for the neutron flux at all points of a finite or infinite medium which contains a duct cannot be found analytically even for the simplest geometries. However, numerical solutions can be found by replacing the differential equations by the corresponding difference equations. In these computations the medium will be replaced by a grid of area (or volume) elements, and it is desirable that the geometry of the problem correspond to a natural grid.

[redacted]

To illustrate the procedure consider Fig. 1, where the shaded material represents concrete, the column of 1's refers to a unit collimated-beam source

1	ϕ_2	ϕ_3	ϕ_4	0
1	ϕ_1	Void	ϕ_5	0
1				0

Fig. 1

of thermal neutrons, and the column of 0's indicates that the return current on the outside face of the concrete is zero. The diffusion equation

$$\nabla^2\phi - k^2\phi = 0$$

was replaced by

$$\sigma\phi_p - \sum_n \phi_n = 0 \text{ and } \sigma = 4 + k^2h^2,$$

where h is the lattice spacing, p refers to an arbitrary cell, and $\sum_n$ is the sum of the four neighbors of ϕ_p (*i.e.*, above, below, and on the two sides). To apply these difference equations it is considered that Fig. 1 is a unit cell which is repeated above and below. The results are shown in Fig. 2.

1	0.5496	0.3013	0.1483	0
1	0.5286	Air Void	0.1912	0
1	0.5496	0.3013	0.1483	0

Fig. 2

Because of the symmetry there are only five equations and five unknowns. When the lattice spacing is taken smaller there will be many more unknowns, and then the Fairchild machine will be used to solve the linear equations. A report will soon be issued which will contain an appendix of problems similar to the above but where the solutions were obtained on the Fairchild machine.

Problem: STEADY-STATE HEAT TRANSFER DUE TO SLUG FLOW IN A 60° SECTOR

Origin: H. C. Claiborne, Reactor Technology Division

Participating Member of Panel: D. W. Whitcombe

Background: When a hot material is passed around pipes, the temperatures reached may be excessive and the problem of cooling arises. The natural method of cooling is to pass a coolant through the pipe. The process is known as "slug flow" when the coolant flows with constant velocity.

The design group at Y-12 is interested in pipes of various cross sections, but present concern is with a pipe whose cross section is a 60° sector of a circle.

The differential equation to be solved is

$$\phi_{rr} + r^{-1}\phi_r + r^{-2}\phi_{\theta\theta} = c'$$

where $\phi = \phi(r, \theta)$ gives the temperature distribution within and on the boundary of the sector, (r, θ) being the usual polar coordinates and chosen so that $\phi(r, \theta) = \phi(r, -\theta)$. The boundary conditions require that the normal derivative of ϕ on the boundary shall be a constant c. Then the solution was expressed as

$$\phi = \phi_0 + \frac{c^1 r^2}{4} - 2cr \cos\theta + \frac{2ca}{\pi} \sum_{n=1}^{\infty} (-)^{n+1} \left[\frac{r}{a}\right]^{6n} \frac{\cos 6n\theta}{n[(6n)^2-1]}$$

where a is the radius of the sector and ϕ_0 is the temperature at the apex of the sector. The series was tabulated for the two lines $\theta = 0$ and $\theta = \pi/6$, and the temperatures were calculated along these lines, so that the variation could be studied.

Problem: **(continuation)** **FERMI FUNCTIONS**

Origin: Dr. M. E. Rose and P. R. Bell, Physics Division

Participating Members of Panel: J. H. Fishel, N. M. Dismuke

Participating Y-12 Personnel: P. Johnson, E. Mitchell

Background: See previous quarterly reports

Status: The F_0's, Fermi functions for allowed transitions, have been completed. Work on coefficients for forbidden transitions is approximately half completed.

Problem: **(continuation)** **INTERPOLATING POLYNOMIAL FOR INTERNAL CONVERSION COEFFICIENTS**

Origin: Dr. M. E. Rose, Physics Division; Mathematics Panel

Participating Members of Panel: N. M. Dismuke, J. H. Fishel

Participating Y-12 Personnel: P. Johnson, W. G. Driggers

Background: See previous quarterly reports

Status: Matrix elements for the least-squares fitting were calculated on IMB equipment. The matrix inversion is being done on the NEPA machine. When inversion is complete (1) one iteration (possibly two or three) will be carried out to improve the accuracy, and (2) residuals will be calculated, using the Y-12 machines. Thereafter the complete tabular entries will be computed from the least-squares polynomials with corrections interpolated from the residuals.

Problem: **MONTE CARLO ESTIMATE OF AGES AND COLLISION DISTRIBUTIONS IN TISSUE FOR 10-MEV SOURCE NEUTRONS**

Origin: Dr. W. S. Snyder and Dr. J. Neufeld, Health Physics Division

Participating Members of Panel: A. S. Householder, R. R. Coveyou, M. R. Arnette, N. M. Dismuke

Background: See *Mathematics Panel Quarterly Progress Report for December, January, February, 1948-1949.*

Status: Energy-loss computations for the 30-cm slab have been completed on the IBM machines at Y-12 and have been given to Dr. Snyder. Other computations are to be started within a week or two.

Problem: **CALCULATION OF STOPPING POWERS OF GOLD, ALUMINUM, COPPER, AND LEAD FOR PROTONS OF VARIOUS ENERGIES**

Origin: Dr. J. Neufeld, Health Physics Division

Participating Member of Panel: C. Perhacs

Background: See Health Physics Division progress reports (ORNL-786, p. 14; ORNL-877, p. 9)

Status: Completed.

Problem: **COMPUTING AND PLOTTING OF A SERIES OF GRAPHS**

Origin: Dr. K. Z. Morgan, Health Physics Division

Participating Member of Panel: C. Perhacs

Background: The curves are to be used as lecture and textbook material for the Health Physics Division.

Status: In progress.

Problem: **PARTICLE DISPLACEMENT IN A CYCLOTRON**

Origin: Dr. B. Cohen, Electromagnetic Research

Participating Member of Panel: C. L. Perry

Background: Dr. Cohen has determined by analysis the particle displacement in a cyclotron with a uniform field. The analysis consisted in solving a pair of simultaneous second-order ordinary differential equations. The actual field in a cyclotron, however, is not uniform. The solution to the differential equations that describe the motion in a nonuniform field has been found by the IBM Group by numerical integration of the differential equations. The solutions obtained for the nonuniform field differed considerably from the solution for the uniform field. The numerical integration was performed for angular steps of 1°. F. C. Uffelman of the IBM Group wanted to know whether or not it was worth while to integrate the differential equations again, using a smaller increment. The additional time required and an estimate of the results were computed. The numerical integration was performed for intervals of 2° and for intervals of 0.25°. The additional IBM time and change in the accumulated error were close to the estimates.

Status: Completed.

[redacted]

Problem: **MULTIGROUP PROBLEM**

Origin: Dr. D. K. Holmes, Dr. N. M. Smith, and Col. M. J. Neilsen, ANP

Participating Members of Panel: J. H. Fishel, K. A. Pflueger, N. M. Dismuke

Participating Y-12 Personnel: P. Johnson, E. Mitchell, E. McDonough, W. G. Driggers

Status: The problem is divided into two parts: (1) cross-section averaging, and (2) integration to obtain the flux from which various pile constants are calculated. For details of formulas see report Y-F10-5. A few calculations were made for part (1), but the number of groups was then increased so that the original plan had to be revised. Calculation of part (2) requires the use of the combination 604, 402, 521, and 941 and of the 604 alone. Detailed planning and board wiring for part (2) is in progress.

Problem: **MONTE CARLO ESTIMATES OF AGE IN WATER FOR 10-MEV SOURCE NEUTRONS**

Origin: E. P. Blizard, Physics Division

Participating Members of Panel: A. S. Householder, M. R. Arnette, N. M. Dismuke, R. R. Coveyou

Background and Status: The Monte Carlo sample for the tissue problem is being used for this age estimate. Instead of rejecting all collisions with elements other than hydrogen and oxygen and then completing enough histories to fill out an adequate sample as was first planned, various hand calculations have been made and it has been decided that a better approximation to the average energy loss in water is obtained by leaving in the collisions with nitrogen and carbon and considering them as oxygen collisions.

Computation of the statistics of this problem are in progress; the first six factorial moments of the r^2 distribution are now being computed for the following energies: 0.1, 0.3, 1.0, and 3.0 Mev.

Problem: **PUTTING NUCLEAR DATA ON CARDS**

Origin: Dr. Katherine Way, National Bureau of Standards

Purpose: To facilitate looking at common characteristics of available data directly, without picking out such information from data listed by nucleus.

[redacted]

Participating Member of Panel: N. M. Dismuke

Participating AEC Personnel: I. Wachtel, T.I.S.

Status: In order to make a proposal for putting nuclear data on punched cards, the group of three named above spent two weeks working together on the problem at the Bureau of Standards in Washington. The resulting proposal will be circulated in the near future as ORNL-883.

Two general possibilities were considered: (1) use of punched cards for hand sorting and (2) use of punched cards on tapes for machine sorting. Since the second possibility requires assistance from a service group as well as use of more or less expensive machine equipment, it does not appear to be the most satisfactory at the moment.

The detailed scheme makes use of 6½- by 7½-in. McBee Keysort cards for sorting with needles. In planning the card layout the prime objectives were to include the material which at present is difficult to collect quickly from existing data sources and to make the coding scheme simple enough to be very nearly self-explanatory.