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THE BARE INTERMEDIATE REACTOR: THE APPROACH TO CRITICAL

Nicholas M. Smith

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THE BARE INTERMEDIATE REACTOR: THE APPROACH TO CRITICAL

Nicholas M. Smith, Jr.

Introduction

The extension of bare reactor theory to the intermediate case is here undertaken. The usefulness of such a theory will lie in the study of the characteristics of the intermediate reactor, in devising experimental studies of nuclear constants, and in certain special engineering applications[1]

Model and Assumptions

One assumes[2] that the reactor is of sufficient size so that the more simple diffusion and slowing down theories may be applied; in particular it is assumed that all of the dimensions are somewhat greater than the transport mean-free-path, and that the extrapolation length is energy independent.

This latter assumption is one which makes necessary a large reactor. This is the usual assumption for thermal reactors but is no more restricting an assumption in this case than that for the theory of thermal reactors.

For further simplicity it is assumed that all neutrons are born of equal energy. Thus in the present form the theory is more applicable to "slightly" intermediate reactors. The generalization permitting consideration of the fission-spectrum is obvious and will be deferred until needed.

The notation will be written in terms of the Fermi-Age as the independent variable. This does not restrict the discussion to the Fermi-slowing down kernel; the theory is equally good for any slowing-down kernel.

The slowing-down kernel is defined to include the resonance escape probability.

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(1) See ORNL 672 ANP-NMS-2.

(2) The general model, assumptions, and method is similar to that employed by A. M. Weinberg in "Pile Neutron Physics II" lecture notes.

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Theory

Let $S(0,\underline{r})$ ≡ the source distribution resulting from fissions at all energies. (Everywhere in this paper the underscored variable $\underline{r}$ will refer to the vector; $d\underline{r}$ will refer to the volume element and is not a vector.) The zero refers to zero age.

Let $Q(\tau_0, \underline{r})$ ≡ the source distribution resulting from an external cause. These neutrons are taken to be born at age τ_0, which may be negative. The source may be a Ra-Be (α,n) source, for instance, or the delayed neutrons in a heat-exchanger having circulating fission fuel as on fluid.

$q(\tau,\underline{r})$ ≡ the slowing down density, *i.e.* the number of neutrons per second per cm^3 slowing down past age τ at a volume element at $\underline{r}$.

$P(\tau_0, \tau, \underline{r}, \underline{r}')$ ≡ the probability per cm^3 that a neutron born at age τ_0 at $\underline{r}'$ will reach age τ in a volume element at $\underline{r}$.

The differential equation describing the slowing-down process is assumed to be linear and homogenous, *i.e.* it is assumed that solutions are superposable. Thus, one has

$$q(\tau,\underline{r}) = \int_R S(0,\underline{r}')\, P(0, \tau, \underline{r}, \underline{r}')\, d\underline{r}' + \int_R Q(\tau_0,\underline{r}')\, P(\tau_0, \tau, \underline{r}, \underline{r}')\, d\underline{r}', \qquad (1)$$

where the symbol R indicates that the integration is over the volume of the (bare) reactor.

Defining $\phi(\underline{r})$ as the thermal flux, one has from elementary diffusion theory

$$\nabla^2 \phi(\underline{r}) - \kappa^2 \phi(\underline{r}) + [3\, q(\tau_s, \underline{r})/\lambda_s] = 0, \qquad (2)$$

where

$\kappa^2 = 3\, N\, \sigma_a/\lambda_s$,

N = no. atoms/cc,

σ_a = atomic absorption cross section at thermal energy,

λ_s = the thermal transport mean-free path, and the subscript s refers to thermal values.

Making use of the relations,

$$\phi(E,r)\ dE = \frac{q(E,\underline{r})}{\xi\ \Sigma_{sc}}\ \frac{dE}{E}\ ,$$

$$d\tau = \frac{\lambda}{3\ \xi\ \Sigma_{sc}}\ \frac{dE}{E}\ ,$$

where

$\phi(E,\underline{r})$ = the flux of energy E at $\underline{r}$,

ξ = the average loss of lethargy, u, per collision,

u = $\log\ (E_0/E\ )$,

Σ_{sc} = the macroscopic scattering cross section,

E = energy,

one may write

$$S(0,\underline{r}) = \int_0^{\tau_s} \frac{3\ \eta\ (\tau')\ \Sigma_f\ (\tau')}{\lambda\ (\tau')}\ q(\tau',\underline{r})\ d\tau'$$

$$+ \frac{k_s}{p\ (\tau_s)}\ \Sigma_s\ (\tau_s)\ \phi\ (\underline{r})\ , \qquad (3)$$

where

$\eta\ (\tau)\ d\tau$ = fission neutrons produced per absorption in uranium of neutron of age $d\tau$ about τ,

$\Sigma_f\ (\tau)$ = macroscopic fission cross section as a function of the age τ,

$\lambda(\tau)$ = transport mfp at age τ. (While it is assumed that $\lambda(\tau)$ is constant *outside* the reactor—in computing the extrapolation length—its variation inside is allowed.),

k_s = thermal multiplication constant of infinite medium,

$k_s = \eta(\tau_s)\, p(\tau_s)\, \Sigma_f(\tau_s)/\Sigma_a(\tau_s)$,

$\Sigma_a(\tau)$ = macroscopic total absorption cross section at age τ.

For simplicity, we write

$$\alpha(\tau) \equiv \frac{3\,\eta(\tau)\,\Sigma_f(\tau)}{\lambda(\tau)} \quad . \tag{4}$$

Combining equations (1), (3) and (4) one obtains

$$\begin{aligned} q(\tau,\underline{r}) = & \int_R \int_{\tau'=0}^{\tau_s} \alpha(\tau')\, q(\tau',\underline{r}')\, d\tau'\, P(0,\tau,\underline{r},\underline{r}')\, d\underline{r}' \\ & + \int_R \frac{k_s}{p(\tau_s)} \cdot \Sigma_a(\tau_s)\, \phi(\underline{r}')\, P(0,\tau,\underline{r},\underline{r}')\, dr' \\ & + \int_R Q(\tau_0,\underline{r}')\, P(\tau_0,\tau,\underline{r},\underline{r}')\, d\underline{r}' \quad . \end{aligned} \tag{5}$$

Characteristic Functions and Fundamental Reactor Theory

Following A. M. Weinberg we now consider the solutions, $Z(\underline{r})$, of the Helmholtz equation

$$\nabla^2 Z(\underline{r}) + B^2 Z(\underline{r}) = 0, \tag{6}$$

for which

$$Z(\underline{r}_b) = 0, \tag{7}$$

where $\underline{r}_b$ is the vector position of the external boundary, and B^2 a positive parameter called the *buckling*.

One gets a series of eigen values, B_n^2, and eigen functions $Z_n(\underline{r})$.

In general one has three components of B_n^2,

$$B_n^2 = B_{\alpha u}^2 + B_{\beta v}^2 + B_{\gamma w}^2 \ , \tag{8}$$

and $Z_n(\underline{r})$ factors into

$$Z_n\ (\underline{r}) = A_\alpha\ (u)\ B_\beta\ (v)\ \Gamma_\gamma\ (w)\ , \tag{9}$$

where α, β, γ are integers, u, v, w are any three mutually orthogonal coordinates, and $n = n\ (\alpha,\ \beta,\ \gamma)$.

It is demonstrable that the $Z_n(\underline{r})$ form an orthogonal and complete set. $Z_n(\underline{r})$ are defined herein as orthonormal, *i.e.* normalized such that

$$\int_R Z_i\ (\underline{r})\ Z_j\ (\underline{r})\ d\underline{r} = \delta_{ij}\ . \tag{10}$$

One defines also a vector $\underline{B}$,

$$\underline{B}_n = \hat{\underline{u}}\, B_{\alpha u} + \hat{\underline{v}}\, B_{\beta v} + \hat{\underline{w}}\, B_{\gamma w}\ , \tag{11}$$

where $\hat{\underline{u}}$, $\hat{\underline{v}}$, $\hat{\underline{w}}$ are unit vectors.

Consider further the Fourier Transform, $\overline{Z}_n\ (\omega)$ of $Z_n\ (r)$:

$$\overline{Z}_n\ (\omega) = \int_\infty e^{i\underline{\omega}\ \cdot\ \underline{r}}\ Z_n(\underline{r})\ d\underline{r}, \tag{12}$$

where the symbol, ∞, indicates that the integration is carried out over all space of the appropriate variable and by the Fourier Integral Theorem,

$$Z_n\ (\underline{r}) = \frac{1}{(2\pi)^3} \int_\infty e^{-i\ \underline{\omega}\ \cdot\ \underline{r}}\ \overline{Z}_n\ (\underline{\omega})\ d\underline{\omega}\ . \tag{13}$$

Taking the Fourier Transform of equation (6) one gets

$$(\omega^2 - B_n^{\,2})\ \overline{Z}_n\ (\underline{\omega}) = 0 \quad . \tag{14}$$

Thus $\overline{Z}_n(\underline{\omega})$ is identically zero unless $\omega^2 = B_n^{\,2}$, *i.e.*, $Z_n(\underline{\omega})$ exists only on the surface of a sphere of radius $\underline{B}_n$ in $\underline{\omega}$ - space.

Making use of the orthogonality and completeness of the $Z_n(\underline{r})$ one makes the expansions (15)

$$Q\ (\tau_0, \underline{r}) = \sum_n Q_n\ Z_n\ (\underline{r})\ , \tag{16}$$

$$q\ (\tau, \underline{r}) = \sum_n\ q(n,\tau)\ Z_n\ (\underline{r})\ , \tag{17}$$

$$\phi\ (\underline{r}) = \sum_n\ \phi_n\ Z_n\ (\underline{r})\ , \tag{18}$$

where Q_n, $q(n,\tau)$, ϕ_n are constants.

Combining equations (5), (13), (16), (17) and (18) one gets

$$\sum_n\ q(n,\tau)\ Z_n(\underline{r})$$

$$= \int_\infty \int_{\tau'=0}^{\tau_s} \alpha(\tau') \sum_n\ q(\tau', n)\ d\tau' \frac{1}{(2\pi)^3} \int_\infty e^{-i\underline{B}_n \cdot \underline{r}'} \overline{Z}_n\ (\underline{\omega})\ d\underline{\omega}\, P_\infty(0, \tau, |\underline{r}-\underline{r}'|)\ d\underline{r}'$$

$$+ \int_\infty \frac{k_s}{P_s} N\sigma_{as} \sum_n\ \phi_n \frac{1}{(2\pi)^3} \int_\infty e^{-i\underline{B}_n \cdot \underline{r}'} Z_n(\underline{\omega})\ d\underline{\omega}\, P_\infty(0, \tau, |\underline{r}-\underline{r}'|)\ d\underline{r}'$$

$$+ \int_\infty \sum_n\ Q_n \frac{1}{(2\pi)^3} \int e^{-i\underline{B}_n \cdot \underline{r}}\ \overline{Z}_n(\underline{\omega})\ d\underline{\omega}\, P_\infty(\tau_0, \tau, |\underline{r}-\underline{r}'|)\ dr'\ . \tag{19}$$

Here, since the $Z_n(\underline{r})$ exist throughout all space and since the finite and infinite slowing-down kernels satisfy the same linear and homogeneous differential equation, one may replace the finite by the infinite (displacement)

kernels, and the integral in $\underline{r}'$-space over the column of the reactor may be replaced by the integral over all space.*

On writing

$$-\underline{r}' = -\underline{r} + \underline{r} - \underline{r}'$$

and associating the exponential $e^{-\underline{B}_n \cdot \underline{r}}$ with $\overline{Z}_n(\underline{\omega})$ and $e^{-\underline{B}_n \cdot |\underline{r}-\underline{r}'|}$ with the slowing down kernel one gets (as in Weinberg's "Fundamental Pile Theorem"):

$$\sum_n q(n,\tau)\, Z_n(\underline{r})$$

$$= \sum_n Z_n(\underline{r}) \left\{ \overline{P}_\infty(B_n, 0, \tau) \left[\int_{\tau'=0}^{\tau_s} \alpha(\tau')\, q(n,\tau')\, d\tau' + \phi_n \frac{k_s}{p_s} N\sigma_{as} \right] + Q_n \overline{P}_\infty(B_n, \tau_0, \tau) \right\} . \tag{20}$$

By the orthorgonality of the $Z_n(\underline{r})$ one easily demonstrates that each term in the braces is identically equal to zero.

$$q(n,\tau) = \overline{P}_\infty(B_n, 0, \tau) \left[\int_0^{\tau_s} \alpha(\underline{r}')\, q(n,\tau')\, d\tau' + \phi_n \frac{k_s}{p_s} N\sigma_{as} \right]$$

$$+ Q_n \overline{P}_\infty (B_n, \tau_0, \tau) . \tag{21}$$

Now substitute equation (21) into equation (2), recalling that $\nabla^2 Z_n(\underline{r}) = -B_n^2 Z_n(\underline{r})$,

$$\sum_n Z_n(\underline{r}) \left\{ \phi_n(-B_n^2 - \kappa^2) + \frac{3}{\lambda} q(n,\tau_s) \right\} = 0 . \tag{22}$$

* The procedure followed by A. M. Weinberg "Pile Neutron Physics II" in developing the theory for thermal reactors is being followed.

Letting $L^2 = \kappa^{-2}$ (L is the thermal diffusion length.) and again employing the orthogonality of the $Z_n(\underline{r})$,

$$- \phi_n \left[(L^2 B_n^2 + 1) + \frac{k_s}{p_s} \overline{P}_\infty(B_n, 0, \tau_s) \right]$$

$$+ \frac{\overline{P}_\infty(B_n, 0, \tau_s)}{N\sigma_{as}} \int_0^{\tau_s} \alpha(\tau') \, q(n,\tau') \, d\tau'$$

$$+ \frac{\overline{P}_\infty (B_n, \tau_0, \tau_s)}{N\sigma_{as}} Q_n = 0. \tag{23}$$

Employing a modification of the usual definition,

$$k_{eff}(\tau_s, n) \equiv \frac{k_s}{p_s} \frac{\overline{P}_\infty(B_n, 0, \tau_s)}{1 + L^2 B_n^2} , \tag{24}$$

and calling it the thermal effective multiplication constant corresponding to the nth mode, and

$$k_{ex}(\tau_s, n) = k_{eff}(\tau_s, n) - 1 , \tag{25}$$

one gets

$$\phi_n = - \frac{1}{(1 + L^2 B^2) \, k_{ex}(\tau_s, n) \, N\sigma_{as}} \left\{ \overline{P}_\infty(B_n, 0, \tau_s) \int_0^{\tau_s} \alpha(\tau') \, q(n,\tau') \, d\tau' \right.$$

$$\left. + Q_n \overline{P}_\infty(B_n, \tau_0, \tau_s) \right\} . \tag{26}$$

Now, subtituting equation (26) into the expression for $q(n,\tau)$, equation (21), one gets

$$q(n,\tau) = \overline{P}_\infty(B_n,\ 0,\ \tau) \left\{ \int_0^{\tau_s} \alpha(\tau')\ q(n,\tau')\ d\tau' \left[1 - \frac{k_{eff}(\tau_s,n)}{k_{ex}(\tau_s,n)} \right] \right.$$

$$\left. - \frac{k_{eff}(\tau_s,n)}{k_{ex}(\tau_s,n)}\ Q_n\ \overline{P}_\infty(B_n,\ \tau_0,\tau_s) \right\} + Q_n\ \overline{P}_\infty(B_n,\tau_0,\tau)\ . \tag{27}$$

Now define

$$\psi_s = \frac{\overline{P}_\infty(B_n,\ \tau_0,\tau_s)}{\overline{P}_\infty(B_n,\ 0,\tau_s)}\ , \tag{28}$$

and

$$\psi(\tau) = \frac{\overline{P}_\infty(B_n,\ \tau_0,\ \tau)}{\overline{P}_\infty(B_n,\ 0,\ \tau)}\ ; \tag{29}$$

we note that

$$\psi_s \doteq \psi(\tau) \equiv \psi\ . \tag{30}$$

Thus

$$\frac{q(n,\tau)}{\overline{P}_\infty(B_n,\ 0,\ \tau)} = \frac{1}{1 - k_{eff}(\tau_s,n)} \left[\int_0^{\tau_s} \alpha(\tau')\ q(n,\tau')\ d\tau' + \psi\ Q_n \right]\ . \tag{31}$$

The right hand side of equation (31) is independent of τ and is therefore a function of n alone, $f(n)$:

$$q(n,\tau) = f(n)\ \overline{P}_\infty(B_n,\ 0,\ \tau)\ . \tag{32}$$

Substituting (32) into the right hand side of (31) there results

$$f(n) = \frac{\psi\, Q_n}{1 - \dfrac{1}{1 - k_{eff}(\tau_s, n)} \displaystyle\int_0^{\tau_s} \alpha(\tau')\, \overline{P}(B_n, 0, \tau')\, d\tau'} \quad . \tag{33}$$

The denominator of equation (33) is the effective multiplication constant for the entire reactor. The critical reactor will have the ~~effective~~ excess multiplication constant of its lowest ordered eigenvalue equal to zero, yielding the characteristic equation determining criticality of the intermediate reactor,

$$1 - k_{eff}(\tau_s, 0) = \int_0^{\tau_s} \alpha(\tau')\, \overline{P}(B_0,\ 0,\ \tau')\, d\tau' \quad . \tag{34}$$

The thermal flux is found by substitution of equations (32) and (33) into equation (26).

$$\frac{\phi_n}{Q_n}\left\{\frac{N\sigma_{as}}{1 + L^2\, B^2}\right\} = \overline{P}_\infty(B_n,\ 0, \tau_s)\ \frac{\displaystyle\int_0^{\tau_s} \alpha(\tau')\, \overline{P}_\infty(B_n,\ 0,\ \tau')\, d = 1}{1 - k_{eff}(\tau_s, n) + \displaystyle\int_0^{\tau_s} \alpha(\tau')\, \overline{P}_\infty(B_n,\ 0,\ \tau')\, d\tau'}$$

$$+ \overline{P}_\infty(B_n,\ \tau_0,\ \tau_s) \quad . \tag{35}$$

To complete the solution recall that

$$q(\tau,\ \underline{r}) = \sum_n q(n,\ \tau)\ Z_n(\underline{r}), \tag{17}$$

and

$$\phi(r) = \sum_n \phi_n Z_n (\underline{r}) \quad , \tag{18}$$

and that

$$Q_n = \int_R Q(\tau_0, \underline{r}) Z_n(\underline{r}) \, d\underline{r} \quad . \tag{36}$$