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THE PRODUCTION OF TRANSURANIC ISOTOPES IN THE MTR

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CLASSIFICATION CHANGED TO:
BY AUTHORITY OF: TID-1149
BY: R. Morrison 7/10/57

THE PRODUCTION OF TRANSURANIC ISOTOPES IN THE MTR

H. E. Goeller

DATE ISSUED: MAR 15 1950

OAK RIDGE NATIONAL LABORATORY
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Post Office Box P
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The Production of Transuranic Isotopes in the MTR

0.0 Abstract

The production of transuranic isotopes from the irradiation of U^{235} fuel assemblies in the MTR has been recalculated on the basis of a 12 day irradiation at a power level of 30,000 KW for a loading of 3.2 Kg. of U^{235}.

1.0 Summary

This report is a re-evaluation of transuranic isotopes production in the 25 fuel assemblies irradiated for 12 days in the MTR, and supercedes several earlier references on this topic(1,2,3), which gave the production for 10, 15 and 20 days of irradiation. The production of U^{236} is omitted, since it is discussed fully in ORNL 611.

Calculations were made to evaluate the amounts of transuranic elements formed in the MTR. It was found that Np^{237} and Pu^{239} are formed in milligram quantities per assembly; Pu^{238}, Pu^{240}, and Pu^{241} in microgram quantities, and Am^{241} in sub-microgram quantities. Only Np^{237} and Pu^{239} are produced in quantities sufficient to consider recovery, and that separation of the small quantity of Pu^{239} is questionable when compared to Hanford production. The production of Pu^{238} directly from Np^{237} and of Am^{241} from Pu^{239} seems more advisable, since they can be produced in much higher concentrations by these methods, and thus are more easily separated and recovered. The isotopes considered in this report are listed below.

U^{236}	U^{239}
U^{237}	Np^{239}
Np^{237}	Pu^{239}
Np^{238}	Pu^{240}
Pu^{238}	Pu^{241}
	Am^{241}

[redacted]

2.0 Introduction

In the operation of the MTR, three additional isotopes of uranium, and eight transuranic isotopes will be produced in gram to microgram quantities. U^{240} and several other transuranic elements are produced in sub-microgram quantities, but have been neglected in this report since they will exist in too negligible concentrations to offer a health hazard or to be worth recovering. Production data for these thirteen isotopes have been calculated in order to evaluate the desirability of their separation and recovery, and to determine what activity hazards their production might present.

Although similiar calculations have already been made on this subject, recalculation was necessary to correct for the latest MTR operating specifications[8] which are as follows:

MTR Power Level	30,000 KW
Total 25 in MTR as Fuel Assemblies	3.2 Kg
Total Number of Fuel Assemblies (Slab loading)	23
Total 25 per Fuel Assembly	0.140 Kg
Composition of Initial Fuel Material	
U^{234}	1.1%
U^{235}	93.4%
U^{238}	5.5%
Total MTR Cycle Time	336 hrs. (14 days)
Irradiation Time per Cycle	288 hrs. (12 days)
Down Time between Cycles	48 hrs. (2 days)

Data were also expanded to permit the plotting of growth and decay curves for the various isotopes considered. The nuclear reactions involved are given in Figure 1, and their reaction constants in Table I. In addition, a summary of the general heavy isotope abbreviation system is also presented in Table I. In this scheme, any heavy element may be represented by a two digit number in which the first digit is the last number in the atomic number, and the second digit the last number in the atomic weight. Thus $_{92}U^{235}$ is written as 25, $_{94}Pu^{239}$ as 49; and $_{95}Am^{241}$ as 51.

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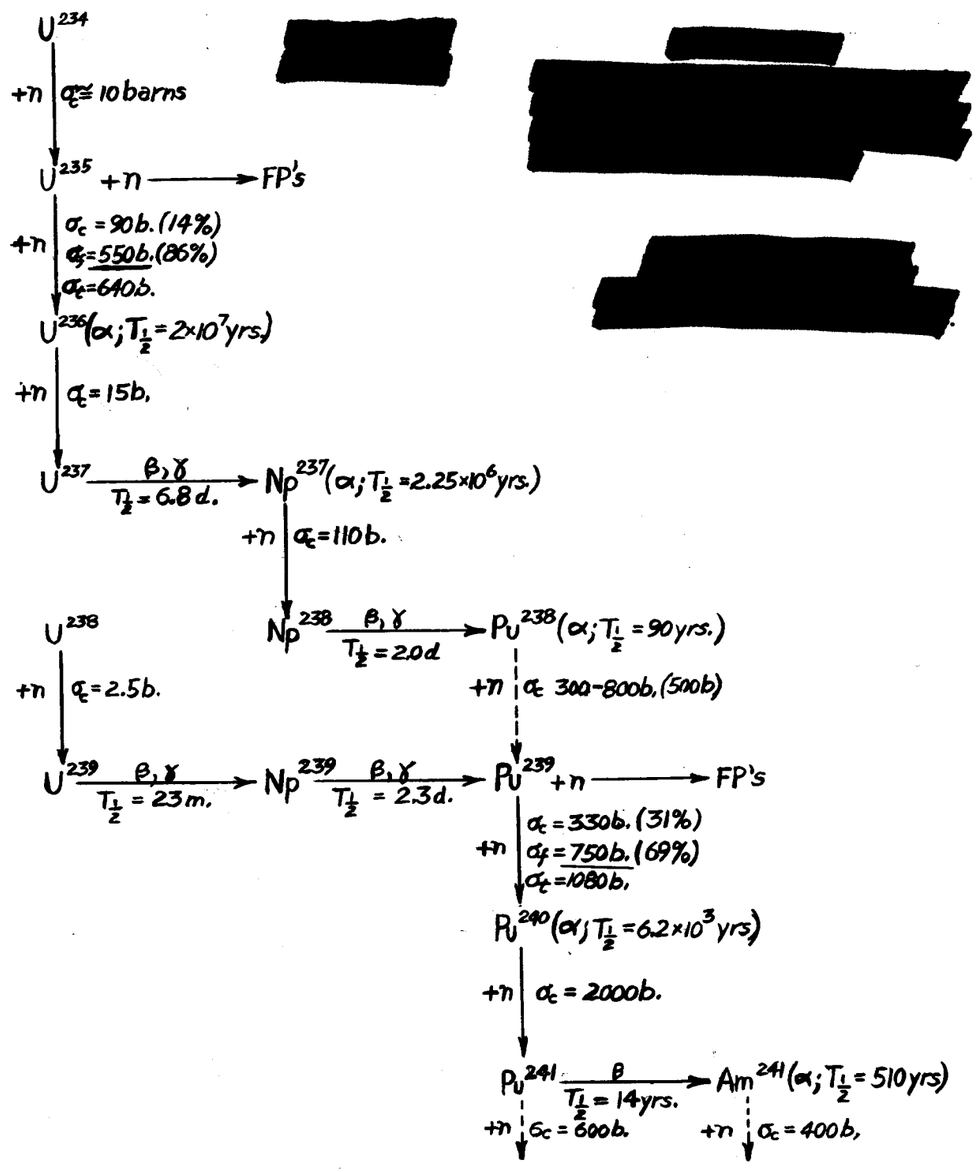

FIGURE 1

NUCLEAR REACTIONS OF U^{234}, U^{235}, AND U^{238}

[redacted]

In making these calculations the general growth and decay equation which appears below was used.

$$N_n = \frac{FN_1^{\circ}\lambda_1\lambda_2\cdots\lambda_{n-1}M_n}{M_1}\left[\frac{e^{-\lambda_1 t}}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)\cdots(\lambda_n-\lambda_1)} + \frac{e^{-\lambda_2 t}}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)\cdots(\lambda_n-\lambda_2)} + \cdots + \frac{e^{-\lambda_n t}}{(\lambda_1-\lambda_n)(\lambda_2-\lambda_n)\cdots(\lambda_{n-1}-\lambda_n)}\right]$$

N_1° = Mass of parent isotope initially present
N_1 = mass of parent isotope at any time, t.
N_n = mass of isotope n at any time, t.
M_1 = isotopic mass of parent isotope
M_n = isotopic mass of isotope n
λ = transformation constants of various isotopes in chain
$= \lambda_p + \lambda_d$
Where $\lambda_p = nv(\sigma_c + \sigma_f)$ for production of isotope involved
$\lambda_d = \ln 2/T_{1/2} = 0.693/T_{1/2}$ for decay of isotope involved
(usually λp and λd differ by several orders of magnitude so that one or the other may be neglected.)
λ_1 = transformation constant of parent isotope
λ_2 = transformation constant of 1st daughter isotope
λ_3 = transformation constant of 2nd daughter isotope
λ_n = transformation constant of (n-1)th dauther isotope
t = total reaction time
F = fraction of total reaction involved
(In the case of 26 production, $\lambda_1 = \lambda_{25} = nv(\sigma_c + \sigma_f)_{25}$.
Since $\sigma_c/\sigma_c + \sigma_f$ = 0.1405, the N26 produced is proportional to only 14% of the total 25 lost in the total reaction.)

The only restrictions on the use of this equation are:

1. Only the parent isotope is present at the start of the reaction.
2. The power level of the reactor remains nearly constant.
3. The values obtained are instantaneous values.

If any of the daughter isotopes are present at reactor startup, the production from these sources must be evaluated by the use of additional, but similiar, equations, and the total production then, is the sum of the solutions of the individual equations. This situation occurs where 27,37,38 and 48 are being produced from both 25 and 26 in class B through K assemblies. To determine the amount of any isotope present after reactor shutdown, new equations must be formed, in which only λ_d appear.

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Table I

Summary of Nuclear Constants

Isotope	Symbol	$\lambda_p = \eta v(\sigma_c + \sigma_f)$ (sec.$^{-1}$)	$\lambda_d = 0.693/T_{1/2}$ (sec.$^{-1}$)	λt*
U^{234}	24	2.00×10^{-9}	-	0.00207
U^{235}	25	1.28×10^{-7}	-	0.133
U^{236}	26	3.00×10^{-9}	-	0.00311
U^{237}	27	-	1.18×10^{-6}	1.22
Np^{237}	37	2.20×10^{-8}	-	0.0228
Np^{238}	38	-	4.00×10^{-6}	4.15
Pu^{238}	48	1.00×10^{-7}	-	0.104
U^{238}	28	5.00×10^{-10}	-	0.000518
U^{239}	29	-	5.03×10^{-4}	520.
Np^{239}	39	-	3.48×10^{-6}	3.51
Pu^{239}	49	2.16×10^{-7}	-	0.224
Pu^{240}	40	4.00×10^{-6}	-	4.15
Pu^{241}	41	1.20×10^{-7}	-	0.124
Am^{241}	51	8.00×10^{-8}	-	0.083

* $t = 12$ days $= 1.037 \times 10^6$ seconds

These general equations were used rather than condensed or simplified versions to avoid errors introduced by simplifying assumptions. The actual calculations were simplified by calculating the last member of the chain first, thereby evaluating all the constant and exponentials of the chain at one time. In this manner, the evaluation of the amount of all chain members present at any time was reduced to simple and rapid tabular calculations involving only multiplications. A sample calculation for Pu^{238} is given in the Appendix.

In the following section a short discussion is given on the production of 25 from 24. This is followed by two sections giving the production information on transuranic elements produced from 25, and from 28 respectively. Finally, production of 51 (Am^{241}) is discussed in some detail. Tabulated production data and growth and decay curves for the various heavy isotopes are collected at the end of this report in Tables III. to V. and Figures 2. to 14.

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3.0 Conversion of U^{234} to U^{235}

The 1.1% of 24 in the unirradiated Class A fuel gives rise to milligram quantities of 25 by neutron capture in the 24. For later class fuels, the amount of 24 increases to a final value of 2.7% in Class K material, and the amount of 25 produced increases proportionally. However, the amount produced is negligible so that its effect on the total 25 handled is insignigicant, since only 0.2% of the 24 present is converted per MTR pass to 25. The total 25 production from 24 the first year of MTR operation is 2.05 g, and the yearly steady state production only 3.23 g.

25 production by classes for one assembly and for steady state annual production is given in Table III. The total number of assemblies irradiated per year is the product of 23 assemblies per MTR loading and 26 MTR cycles per year, giving a total of 598 assemblies. Under steady state operations, the quantity of 25 produced is the same for all classes. As will be seen later, this is also true in the production of 39, 49, 40, 41, and 51 from 28.

Table II

Production of U^{235} by Neutron Capture in U^{234}

Class Mat'l Charged to MTR	g.24 per assembly	mg. 25 produced per assembly	No. Assemblies per year at steady state	g. 25 produced per year
A	1.65	3.18	101.2	.323
B	1.88	3.63	88.6	.323
C	2.16	4.16	77.6	.323
D	2.46	4.75	67.9	.323
E	2.82	5.44	59.5	.323
F	3.22	6.20	52.1	.323
G	3.68	7.10	45.6	.323
H	4.20	8.10	39.9	.323
J	4.80	9.25	35.0	.323
K	5.49	10.60	30.6	.323
Total annual steady state production				= 3.23

$$N_{25} = 1.93 \times 10^{-3}\ N_{24}^{o} \text{ for 12 day irradiation.}$$

4.0 The 25 Series-Production of U^{236}, U^{237}, Np^{237}, Np^{238} and Pu^{238}

The 25 series is that group of uranium isotopes and transuranic elements which result from neutron capture in 25 and 26. Since the production of 26 is fully discussed in ORNL-611, Section 4.3 (and 27 in Section 5.1.) Only the last four isotopes will be discussed here. All four isotopes are produced both directly from 25 and from 26 present in the fuel assemblies. Calculations were based on the amount of 25 and 26 present at the beginning of each irradiation. Since the amount of 25 entering the MTR is independent of the class, the amount of the above isotopes produced directly from 25 is the same for all classes. In the case of production directly from 26, none is produced in Class A material since no 26 is initially present; however, the production in later classes increases linearly with the amount of 26 present in each class. In all cases, the amount produced from 26 is much greater than that produced directly from 25.

Since 27 and 38 are short lived β emitters, and 37 and 48 are long lived α emitters, it is convenient to discuss 27 and 37 as one topic and 37 and 48 as another. 38

In the case of the 27-37 pair, both, after 12 days of irradiation, are present in milligram quantities. The 27 is a β and γ emitter with a half life of 6.8 days. The β particles emitted have maximum energies of 0.14 or 0.35 Mev and the γ quanta energies are 0.14, 0.23, and 0.53 Mev. The latter are emitted in the ratio of 15:6:1; thus the mean penetrating power of all γ radiation from 27 is that of a 0.27 Mev γ. Approximately one γ is emitted for each 35 disintergrations. All 27 decays eventually to 37 which is a 4.72 Mev α emitter with a half life of 2.25×10^6 years and a slow neutron capture cross section of 110 barns. Thus any 27 which is undecayed after 1200 Area and 1400 Area processing will enter the MTR in the next class assemblies as 37. Since 37 is present in milligram amounts, it appears feasible to recover it from 1200 Area chemical wastes. Its main use would be in the study of neptunium chemistry on a macroscopic scale since the other neptunium isotopes are short lived β emitters.

Production data for 27 and 37 are given in Table III. on the basis of one MTR assembly. Annual steady state and first year total production of 37 is given in Table V. (These Tables include all other heavy isotopes and will not be referred to further.) The total amount of 37 per assembly given in Table V. includes that formed during irradiation plus that formed by decay of 27 during the cooling period, and it is assumed that such decay is essentially complete.

In the case of the 38-48 pair, 38 is formed by slow neutron capture in 37. The 38 thus formed is a β and γ emitter which decays to 48 with a half life of 2.0 days. About 60% of the β particles have a maximum energy of 0.2 Mev, and the remaining 40%, 1.4 Mev. The γ quanta have energies of 1.2 and 0.075 Mev. The 48 is a 5.5 Mev α emitter with a half life 90 years, and because of this would be valuable as a tracer. However, since both 38 and 48 are present in only microgram quantities it appears more expedient to obtain 48 by the irradiation of 37 directly.

5.0 The 28 Series--Production of U^{239}, Np^{239}, Pu^{239}, Pu^{240}, and Pu^{241}

The 28 series includes all those isotopes produced directly from 28. As with 25 produced from 24, the products of the 28 series are directly proportional to the initial amount of 28 present at the start of an irradiation. Therefore, the amount of heavy isotopes of this series increases for later classes as the 28 concentration increases from 8.25 g. per assembly for Class A fuel to 27.4 g. for Class L. The total 28 lost per assembly is about 0.05% per pass through the MTR.

The 29 produced has little practical significance since it is a β and γ emitter with a half life of 23 minutes, and comes to its equilibrium value of 9.97×10^{-7} N_{28}^{8} grams in a matter of about two hours. Similiarly, it is essentially all gone a few hours after MTR shutdown. Of the β particles emitted, 10% have a maximum energy of 0.56 Mev, and 90%, 1.20 Mev. The γ quanta have energies of 0.076 and 0.3 Mev. The 29 decays to 39.

The 39 and 49 produced in the MTR are the same isotopes which are produced at Hanford on a much larger scale. The 39 is a β and γ emitter which decays to 49 with a half life of 2.3 days. The maximum β particle energies observed are 0.48 and 0.70 Mev; the γ quanta energies 0.5, 0.208, 0.226, and 0.276 Mev. The 49 is an α and γ emitter with a half life of 2.41×10^4 years, and a total slow neutron cross section of 1080 barns. The capture cross section is 330 barns and the fission cross section 750 barns for thermal neutrons. The α particle energy is 5.15 Mev and γ quanta energies 0.05 and 0.3 Mev.

The production of 49 in the MTR is in milligram quantities per assembly and is so small compared to the Hanford production capacity that it is considered undesirable to attempt 49 recovery within the MTR project. The 39 and 49 production per assembly (and also 40, 41, and 51) is given in Table IV. at the end of this report.

The 40 (Pu^{240}) produced in the MTR results from neutron capture in 49. Since about 70% of the 39 destroyed in the reactor is lost in fission, only 30% of the total 39 which disappears results in the formation of 40. The 40 is a 5.1 Mev α emitter with a half life of 6.2×10^3 years, a slow neutron capture cross section of about 2000 barns. It is of interest only as the precursor of

Am^{241}, which is produced by neutron capture in 40 to produce 41, which subsequently decays to 51 by β emission with a 14 year half life. The estimated capture cross section of 41 is 600 barns. The isotopes 40 and 41 are produced in microgram quantities. In previous calculations the capture cross section of Pu^{240} was assumed to be 20,000 barns; therefore the results of the present calculations indicate much lower yields of 41 and 51.

6.0 Production of Am^{241}

It has been suggested that the normal operations of the MTR might provide a good source of Am^{241}(51) which is a 5.5 Mev α emitter with a half life of 510 years. Since all other americium isotopes are short lived β emitters, it would be highly desirable to obtain an adequate quantity of 51 for the study of americium chemistry.

Calculations indicate that only sub-microgram quantities of 51 will be produced per assembly during irradiation, since all 51 is produced by β decay of 41 which has a relatively long half life of 14 years. After long cooling of 1200 Area chemical wastes, it is conceivable, though hardly feasible, that 51 could be milked from the wastes, since the low concentration would make this recovery most difficult.

A more productive method of obtaining 51 would be the irradiation of Pu^{239} slugs or assemblies in the MTR. In this manner the 51 concentration would be increased by a factor of 10^4 so that microgram quantities would be available at MTR shutdown, and milligram quantities after several years of cooling.

7.0 Heavy Isotope Growth and Decay Curves

Since a considerable amount of effort is necessary to obtain production figures for the formation of the various heavy isotopes formed in the MTR, and since this data should be extended to production values for other possible irradiation periods, heavy isotope growth and decay curves for various periods of irradiation and cooling were calculated and plotted. The thirteen plots are given in Figures 2 to 14 summarized below.

Figure 2 25 Consumption, and 26 and Total Fission Product Production.
Figure 3 Production of 27 and 37 from 25.
Figure 4 Decay of 27 and Growth of 37, Produced from 25.
Figure 5 Production of 27 and 37 from 26.
Figure 6 Decay of 27 and Growth of 37, Produced from 26.
Figure 7 Production of 38 and 48 from 25.
Figure 8 Decay of 38 and Growth of 48, Produced from 25.
Figure 9 Production of 38 and 48 from 26.
Figure 10 Decay of 38 and Growth of 48, Produced from 26.
Figure 11 Production of 39 and 49 from 28.
Figure 12 Decay of 39 and Growth of 49, Produced from 28.
Figure 13 Proeuction of 40 and 41 from 28.
Figure 14 Decay of 41 and Growth of 51, Produced from 28.

All values are in terms of initial amounts of 25,26,and 28. All decay curves are on the basis of a 12 day irradiation at 30,000 KW total power.

[redacted]

TABLE III

Production of 26, 27, 37, 38, & 48 From 25
for 12 day Irradiation in MTR
at 30,000 KW Total Power

$N_{26} = 1.754 \times 10^{-2} N^{o}_{25}$

$N_{27} = 1.918 \times 10^{-5} N^{o}_{25} + 1.793 \times 10^{-3} N^{o}_{26}$

$N_{37} = 8.58 \times 10^{-6} N^{o}_{25} + 1.303 \times 10^{-3} N^{o}_{26}$

$N_{38} = 2.66 \times 10^{-8} N^{o}_{25} + 4.76 \times 10^{-6} N^{o}_{26}$

$N_{48} = 2.50 \times 10^{-8} N^{o}_{25} + 5.94 \times 10^{-6} N^{o}_{26}$

Class Mat'l	To MTR			From MTR															
			26	27				37				38				48			
	N^{o}_{25}	N^{o}_{26}	Produced	Produced from		Total at MTR Shutdown	Total after Long Cooling	Produced from		Total at MTR Shutdown	Total after Long Cooling	Produced from		Total at MTR Shutdown	Total after Long Cooling	Produced from		Total At MTR Shutdown	Total after Long Cooling
				25	26			25	26			25	26			25	26		
	g.	g.	g.	mg.	mg.	mg.	mg.	mg.	mg.	mg.	mg.	μg.	μg.	μg.	μg.	μg.	μg.	μg.	μg.
A	140	0.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
B	140	2.81	2.46	2.69	0.00	2.69	0.00	1.20	0.00	1.20	3.89	3.72	0.00	3.72	0.00	3.50	0.00	3.50	7.2
C	140	6.03	2.46	2.69	5.04	7.70	0.00	1.20	3.66	4.86	12.56	3.72	13.3	17.0	0.00	3.50	16.7	20.2	37.2
D	140	9.70	2.46	2.69	10.81	13.5	0.00	1.20	7.86	9.06	22.6	3.72	28.7	32.4	0.00	3.50	35.8	39.3	71.7
E	140	13.90	2.46	2.69	17.39	20.1	0.00	1.20	12.64	13.84	33.9	3.72	46.2	49.9	0.00	3.50	57.6	61.1	111.0
F	140	18.7	2.46	2.69	24.92	27.6	0.00	1.20	18.11	19.31	46.9	3.72	66.2	69.9	0.00	3.50	82.6	86.1	156.0
G	140	24.2	2.46	2.69	33.53	36.2	0.00	1.20	24.37	25.57	61.8	3.72	89.0	92.7	0.00	3.50	111.1	114.6	207.3
H	140	30.5	2.46	2.69	43.39	46.1	0.00	1.20	31.53	32.73	78.8	3.72	115.2	118.9	0.00	3.50	143.7	147.2	266.1
J	140	37.6	2.46	2.69	54.69	57.4	0.00	1.20	39.74	40.94	98.3	3.72	145.2	148.9	0.00	3.50	181.2	184.7	333.6
K	140	45.7	2.46	2.69	67.42	70.1	0.00	1.20	48.99	50.19	120.3	3.72	179.0	182.7	0.00	3.50	223.3	226.8	409.5
L	-	-	2.46	2.69	81.94	84.6	0.00	1.20	59.55	60.75	145.4	3.72	217.5	221.2	0.00	3.50	271.5	275.0	496.2

[redacted]

[redacted]

Table IV

Production of 29, 39, 49, 40, 41, & 51 From 28
For 12 day Irradiation in MTR
At 30,000 KW Total Power

$N_{29} = 9.97 \times 10^{-7}\ N^{o}_{28}$
$N_{39} = 1.431 \times 10^{-4}\ N^{o}_{28}$
$N_{49} = 3.43 \times 10^{-4}\ N^{o}_{28}$
$N_{40} = 8.74 \times 10^{-6}\ N^{o}_{28}$
$N_{41} = 1.04 \times 10^{-6}\ N^{o}_{28}$
$N_{51} = 3.64 \times 10^{-10}\ N^{o}_{28}$

Class Mat'l	28 to MTR	28 from MTR	29		39		49		40		41		51	
			Total at MTR Shutdown	Total after Long Cooling	Total at MTR Shutdown	Total after Long Cooling	Total at MTR Shutdown	Total after Long Cooling	Total at MTR Shutdown	Total after Long Cooling	Total at MTR Shutdown	Total after Long Cooling	Total at MTR Shutdown	Total after Long Cooling*
	g.	g.	μg.	μg.	mg.	mg.	mg.	mg.	μg.	μg.	μg.	μg.	μg.	μg.
A	8.25	-	-	-	-	-	-	-	-	-	-	-	-	-
B	9.42	8.25	8.23	0.00	1.18	0.00	2.83	4.01	72.1	72.1	8.58	0.00	0.0030	8.58
C	10.78	9.42	9.39	0.00	1.35	0.00	3.23	4.58	82.3	82.3	9.80	0.00	0.0034	9.80
D	12.30	10.78	10.75	0.00	1.54	0.00	3.70	5.24	94.2	94.2	11.21	0.00	0.0039	11.21
E	14.08	12.30	12.26	0.00	1.76	0.00	4.22	5.98	107.5	107.5	12.79	0.00	0.0045	12.79
F	16.08	14.08	14.04	0.00	2.01	0.00	4.83	6.84	123.1	123.1	14.64	0.00	0.0051	14.65
G	18.38	16.08	16.03	0.00	2.30	0.00	5.52	7.82	140.5	140.5	16.72	0.00	0.0059	16.73
H	21.00	18.38	18.32	0.00	2.63	0.00	6.30	8.93	160.6	160.6	19.12	0.00	0.0067	19.13
J	24.00	21.00	20.94	0.00	3.01	0.00	7.20	10.21	183.5	183.5	21.84	0.00	0.0076	21.85
K	27.40	24.00	23.93	0.00	3.43	0.00	8.23	11.66	209.8	209.8	24.96	0.00	0.0087	24.97
L	-	27.40	27.32	0.00	3.92	0.00	9.40	13.32	239.5	239.5	28.50	0.00	0.0100	28.51

* Here, long cooling infers order of 100 years; elsewhere inference is 1 month to 1 year.

[redacted]

[redacted]

Table V

Total Heavy Isotope Production In MTR (After Long Cooling)—12 day Irradiation at 30,000 KW Total Power

23 Assemblies loaded/ Cycle x 26 Cycle/yr. = 598 Assemblies/yr.

STEADY STATE PRODUCTION					25 SERIES						28 SERIES							
Class Mat'l	% To MTR	% From MTR	No. Assbys to MTR	No. Assbys from MTR	26/Assby g.	Total Kg.	37/Assby mg.	Total g.	48/Assby μg.	Total mg.	49/Assby mg.	Total g.	40/Assby μg.	Total mg.	41/Assby μg.	Total mg.	51/Assby μg.	Total μg.
A	16.92	-	101.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
B	14.81	16.92	88.6	101.2	2.46	0.248	3.89	0.394	7.2	0.729	4.01	0.407	72.1	7.30	8.58	0.87	0.0030	0.30
C	12.97	14.81	77.6	88.6	2.46	0.218	12.56	1.113	37.2	3.296	4.58	0.407	82.3	7.29	9.80	0.87	0.0034	0.30
D	11.36	12.97	67.9	77.6	2.46	0.191	22.6	1.745	71.7	5.564	5.24	0.407	94.2	7.31	11.21	0.87	0.0039	0.30
E	9.95	11.36	59.5	67.9	2.46	0.167	33.9	2.302	111.0	7.537	5.98	0.407	107.5	7.30	12.79	0.87	0.0045	0.30
F	8.71	9.95	52.1	59.5	2.46	0.146	46.9	2.791	156.0	9.282	6.84	0.407	123.1	7.30	14.64	0.87	0.0051	0.30
G	7.63	8.71	45.6	52.1	2.46	0.128	61.8	3.220	207.3	10.800	7.82	0.407	140.5	7.32	16.72	0.87	0.0059	0.30
H	6.68	7.63	39.9	45.6	2.46	0.112	78.8	3.593	266.1	12.134	8.93	0.407	160.6	7.32	19.12	0.87	0.0067	0.30
J	5.85	6.68	35.0	39.9	2.46	0.098	98.3	3.922	333.6	13.311	10.21	0.407	183.5	7.32	21.84	0.87	0.0076	0.30
K	5.12	5.85	30.6	35.0	2.46	0.086	120.3	4.211	409.5	14.333	11.66	0.407	209.8	7.34	24.96	0.87	0.0087	0.30
L	-	5.12	-	30.6	2.46	0.075	145.4	4.449	496.2	15.184	13.32	0.407	239.5	7.33	28.50	0.87	0.0100	0.30
Total annual production						1.469		27.749		92.170		4.070		73.13		8.70		3.00
1st Yr Production																		
A	--	--	311	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
B	--	--	241	311	2.46	0.764	3.89	1.210	7.2	2.239	4.01	1.247	72.1	22.42	8.58	2.67	0.0030	0.933
C	--	--	46	241	2.46	0.593	12.56	3.027	37.2	8.965	4.58	1.104	82.3	19.83	9.80	2.36	0.0034	0.819
D	--	--	--	46	2.46	0.112	22.6	1.040	71.7	3.298	5.24	0.241	94.2	4.33	11.21	0.52	0.0039	0.179
Total 1st yrs. Production						1.469		5.277		14.502		2.592		46.58		5.55		1.931

[redacted]

Bibliography

1. Ott, H. C., and S. G. English, "Expected Formation of Heavy Isotopes in Proposed Clinton Heterogeneous Pile", Central Files No. 47-3-8, March 19, 1947.

2. Feder, H. M., "Heavy Isotope Formation in the Proposed Research Pile", July 18, 1947.

3. Goeller, H. E., "Heavy Isotope Formation in Proposed Research Pile as a Function of Percent 25 Enrichment", Central Files No. 47-8-428, August 26, 1947.

4. "Nuclei Formed in Fission: Decay Characteristics, Fission Yields, and Chain Relationships", Journal of American Chemical Society, 68: 2411-2442, 1946.

5. Sullivan, W. H., "Trilinear Chart of Nuclear Species", John Wiley & Sons, 1949.

6. Haines, G., and K. Way, "Tables of Neutron Cross Sections for Elements Po-Cm", ORNL-86, September 9, 1948.

7. "Tables of the Exponential Function", WPA Project No. 765-97-3-10, 1939.

8. Mallon, D. J., "Materials Testing Reactor Design Data Sheets", Central Files No. 49-10-232, October 18, 1949.

9. Goeller, H. E., "The Cooling of Irradiated 25 MTR Fuel Assemblies and Its Effect on 25 Inventory in the MTR Project", ORNL-611, February 15, 1950.

10. Goeller, H. E., "The Production of Fission Products in the MTR and Their Decontamination in the 1200 Area", ORNL-613.

Drawing #8363

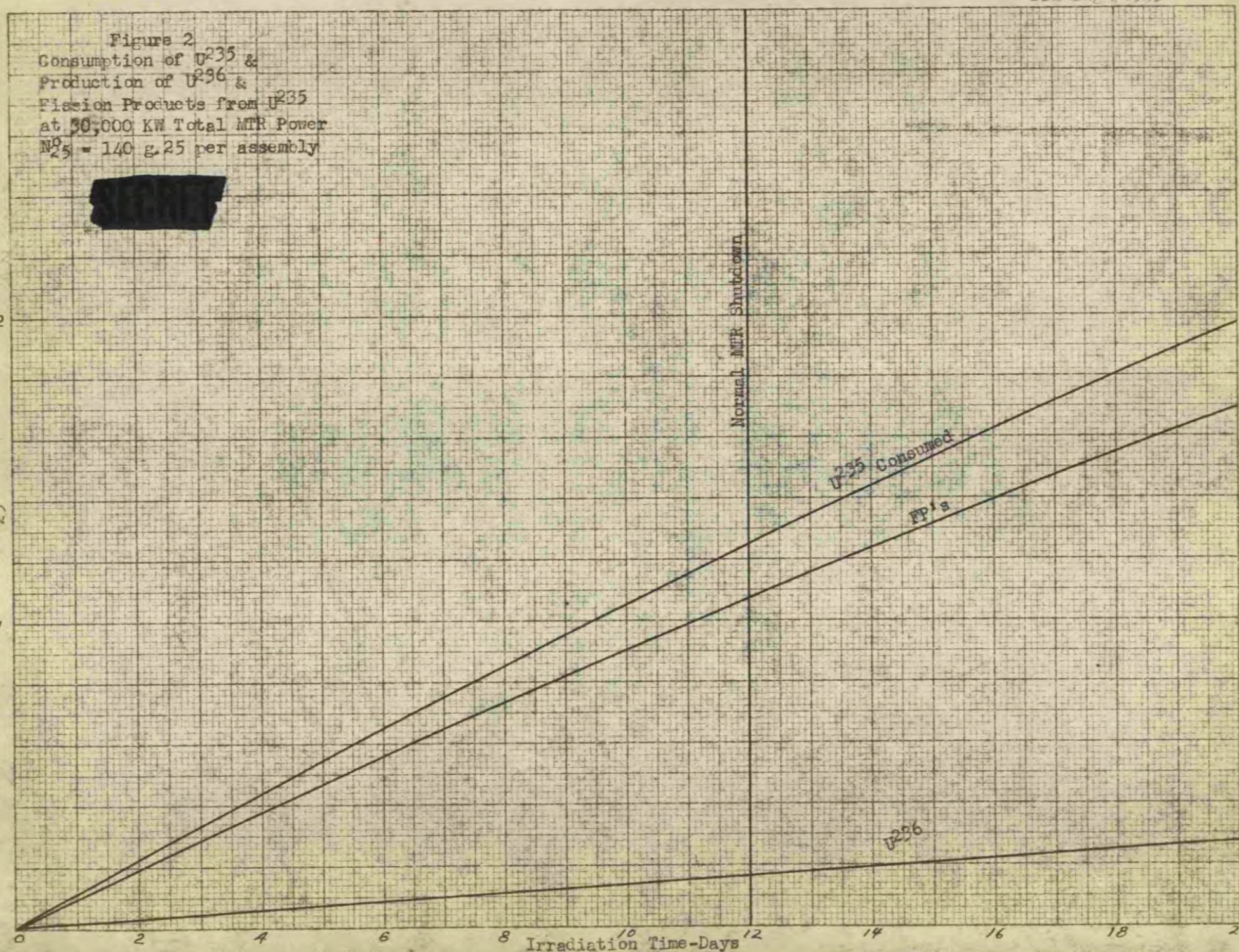

Figure 2
Consumption of U^{235} &
Production of U^{236} &
Fission Products from U^{235}
at 30,000 KW Total MTR Power
N^0_{25} = 140 g.25 per assembly

[redacted]

DWG. 8363

[redacted]

[redacted]

DWG. 8364

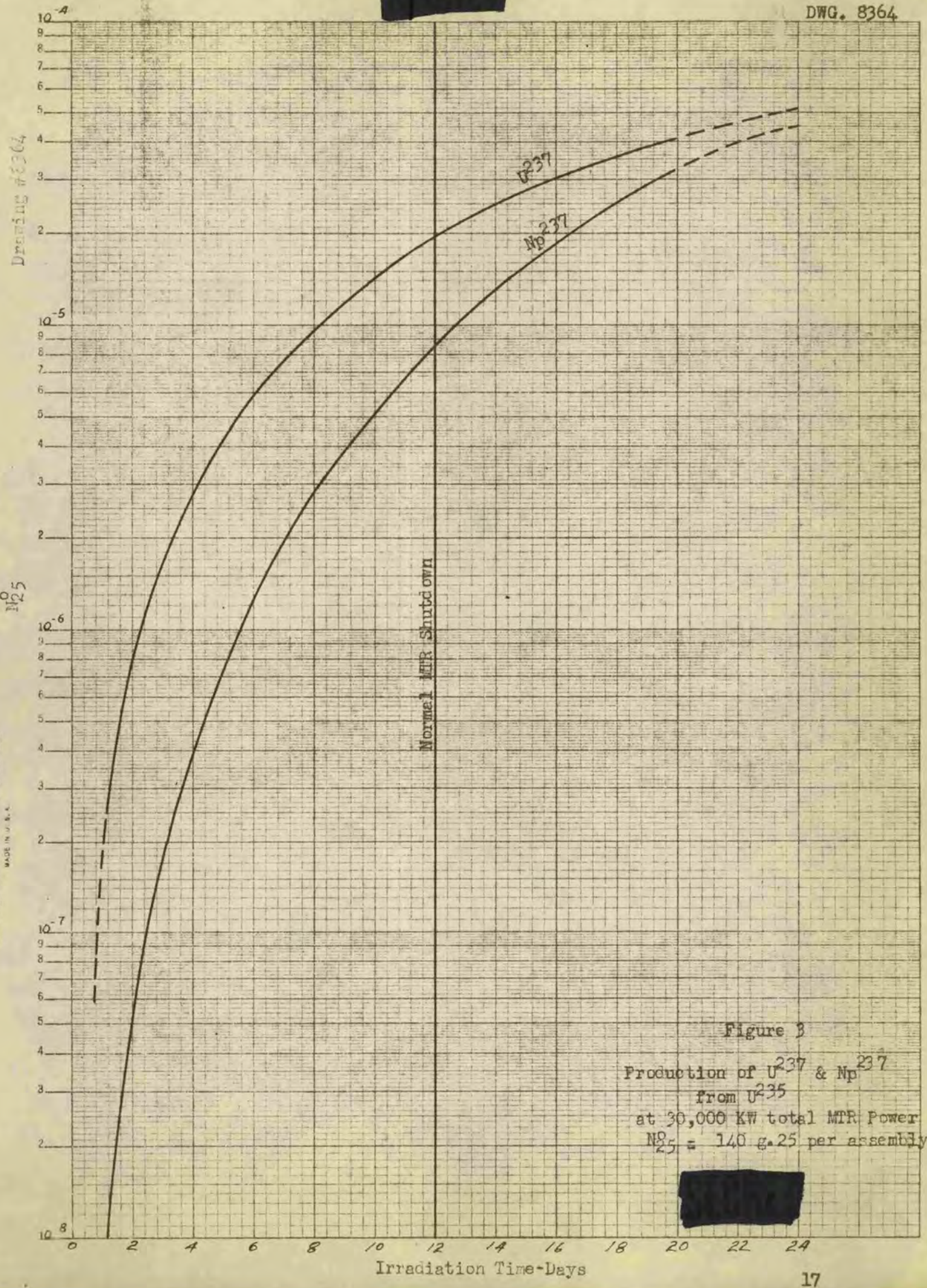

Figure 3

Production of U^{237} & Np^{237}
from U^{235}
at 30,000 KW total MTR Power
N^0_{25} = 140 g.25 per assembly

[redacted]

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[redacted]

DWG. 8365

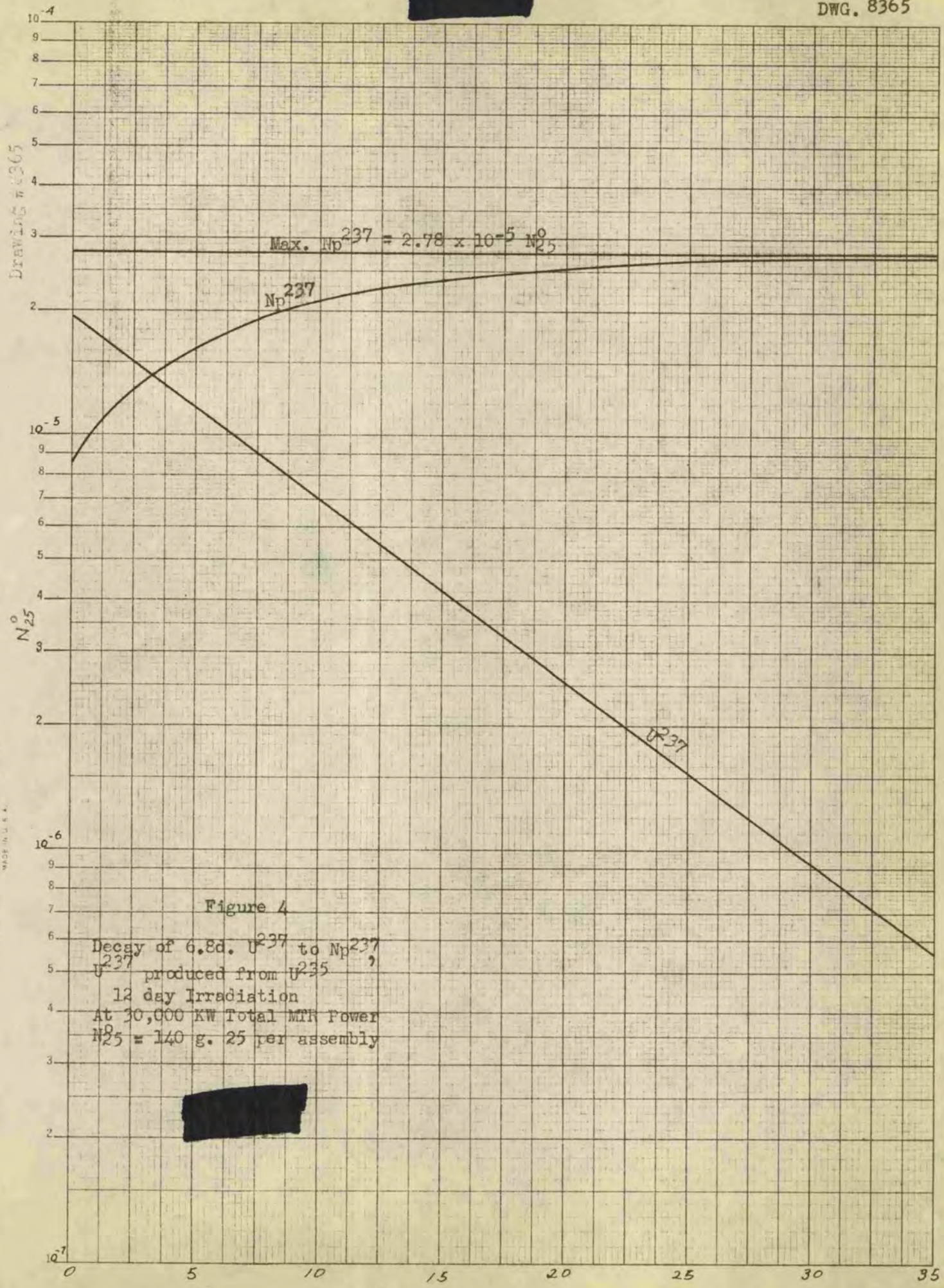

Figure 4

Decay of 6.8d. U^{237} to Np^{237},
U^{237} produced from U^{235}
12 day Irradiation
At 30,000 KW Total MTR Power
N^{o}_{25} = 140 g. 25 per assembly

[redacted]

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DWG. 8366

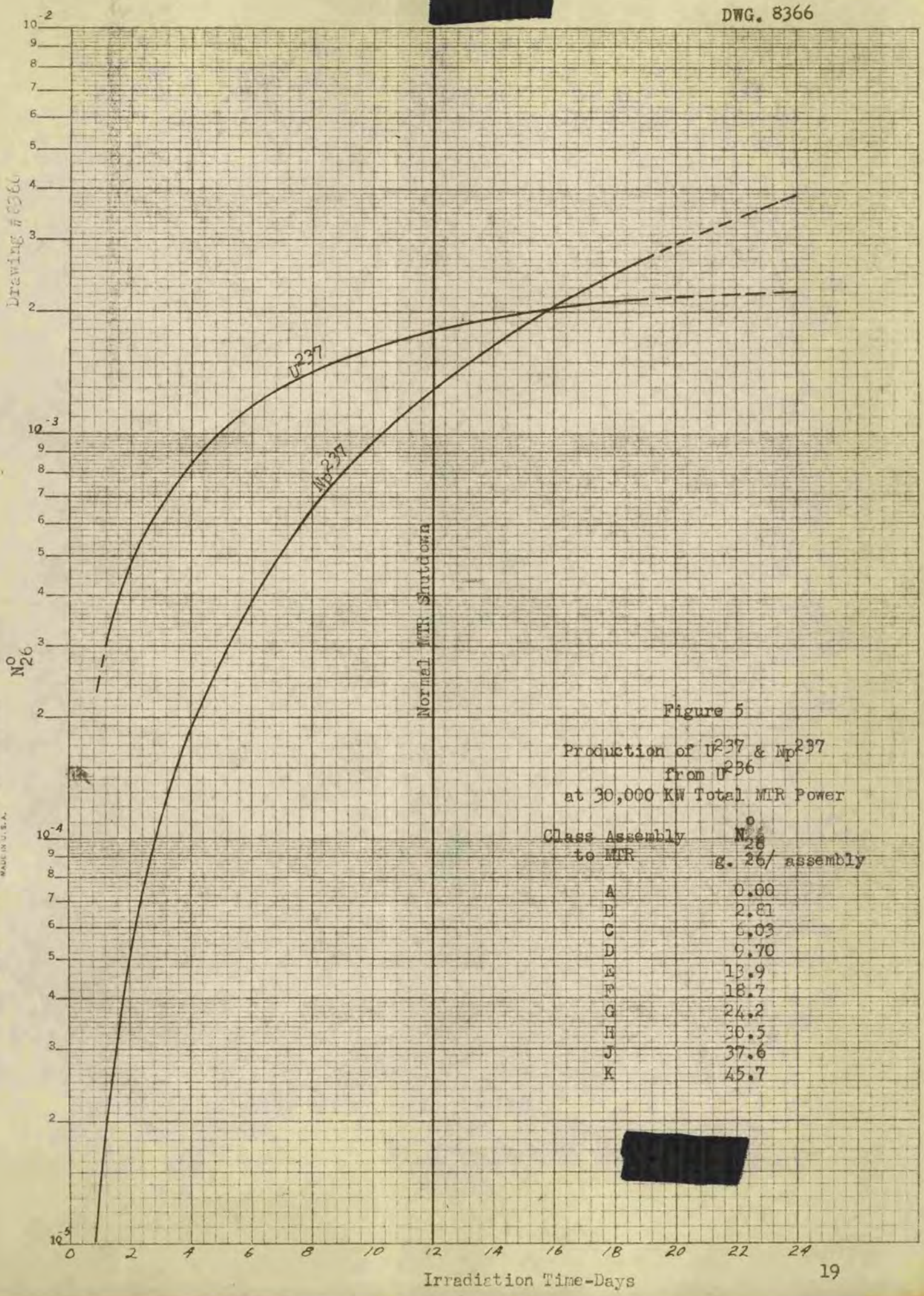

Figure 5

Production of U^{237} & Np^{237}
from U^{236}
at 30,000 KW Total MTR Power

Class Assembly to MTR	N^{o}_{26} g. 26/ assembly
A	0.00
B	2.81
C	6.03
D	9.70
E	13.9
F	18.7
G	24.2
H	30.5
J	37.6
K	45.7

[redacted]

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Semi-Logarithmic, 3 Cycles x 10 to the Inch, 5th Item mounted
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DWG. 8368

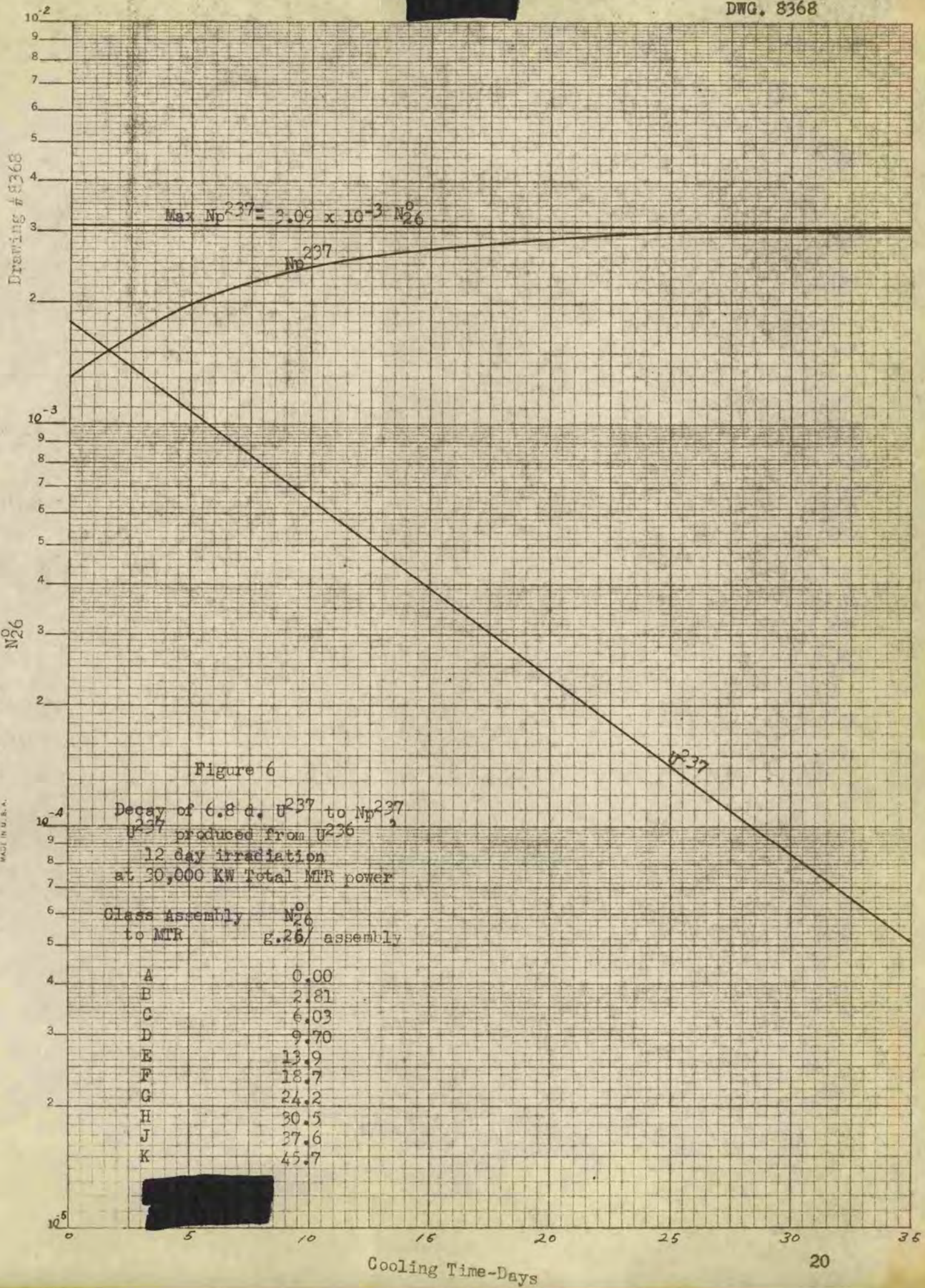

Figure 6

Decay of 6.8 d. U^{237} to Np^{237},
U^{237} produced from U^{236}
12 day irradiation
at 30,000 KW Total MTR power

Class Assembly to MTR	N_{26}^{o} g.26/ assembly
A	0.00
B	2.81
C	6.03
D	9.70
E	13.9
F	18.7
G	24.2
H	30.5
J	37.6
K	45.7

[redacted]

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DWG. 8369

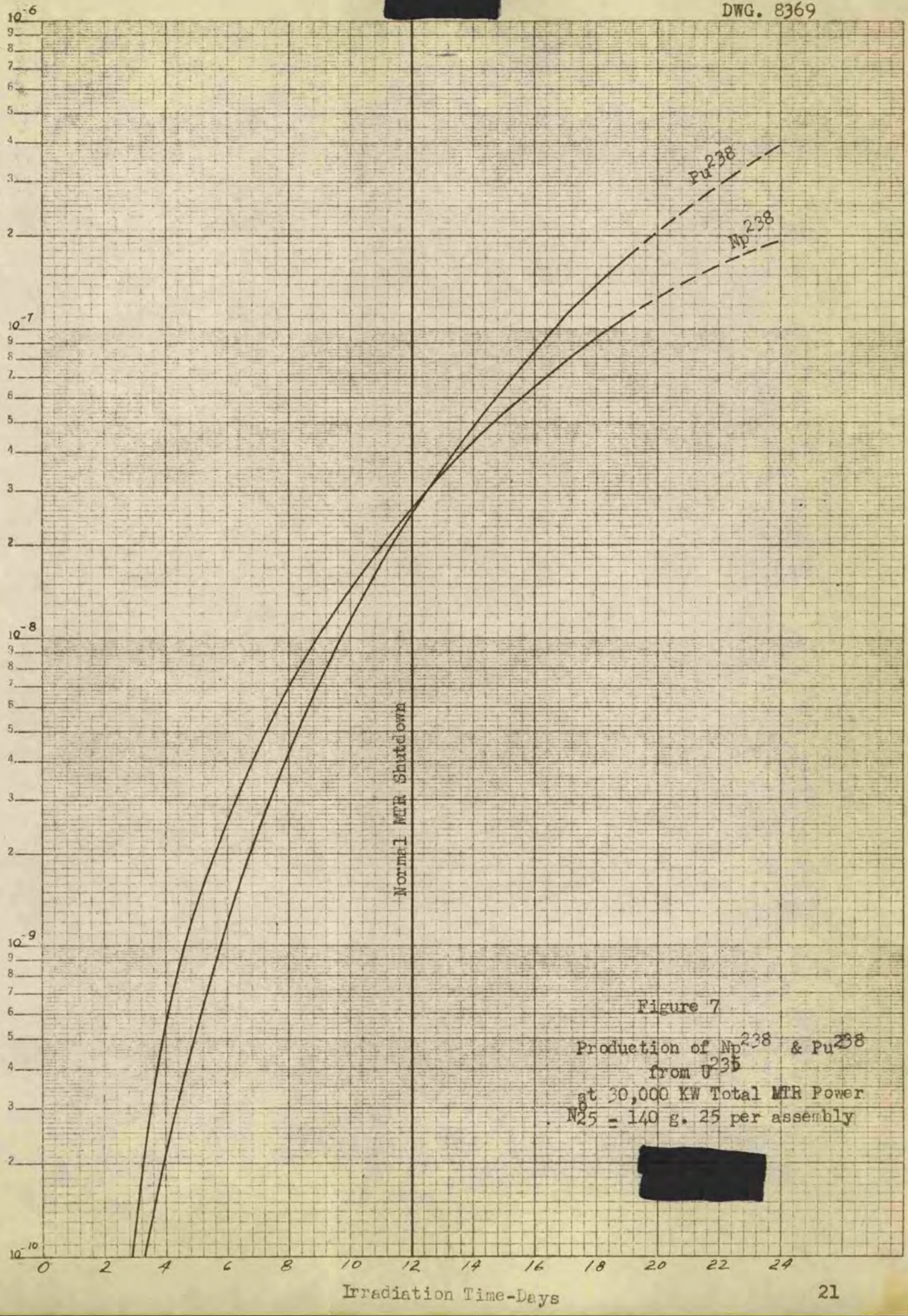

Figure 7

Production of Np^{238} & Pu^{238}
from U^{235}
at 30,000 KW Total MTR Power
N_{25}^{0} = 140 g. 25 per assembly

[redacted]

[redacted]

DWG. 8370

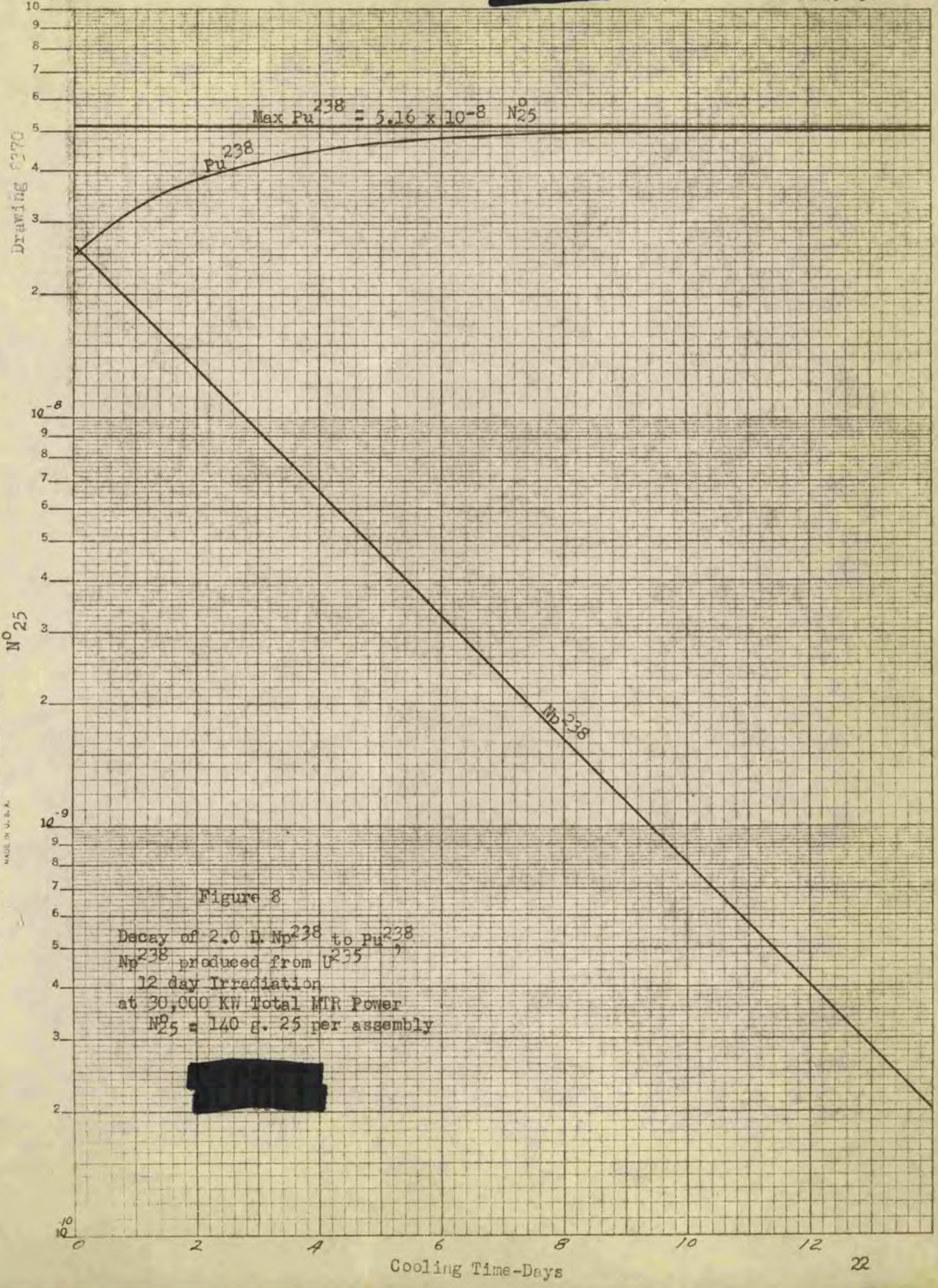

Figure 8

Decay of 2.0 D. Np^{238} to Pu^{238}
Np^{238} produced from U^{235}
12 day Irradiation
at 30,000 KW Total MTR Power
N^o_{25} = 140 g. 25 per assembly

[redacted]

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[redacted]

DWG. 8371

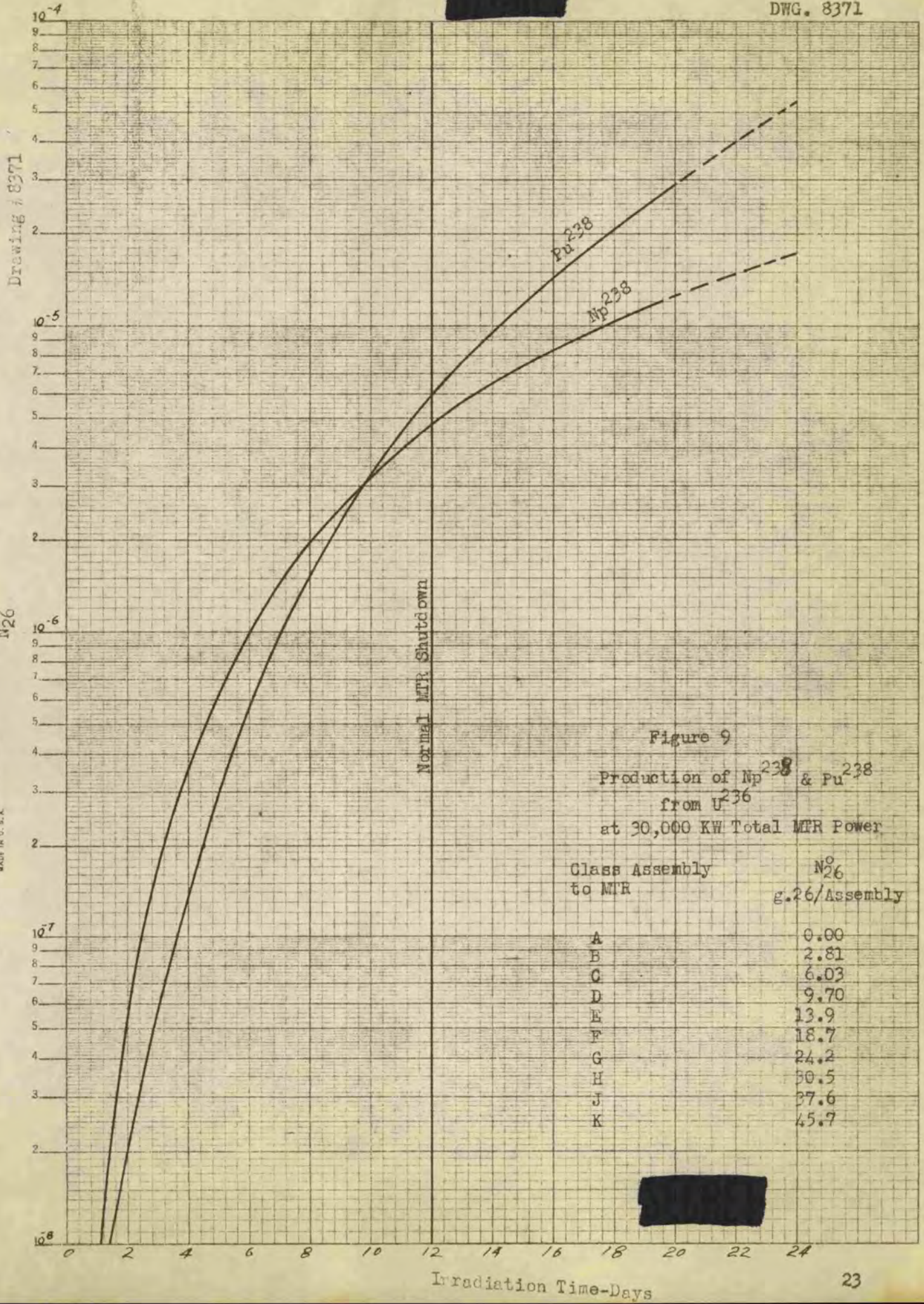

Figure 9

Production of Np^{238} & Pu^{238}
from U^{236}
at 30,000 KW Total MTR Power

Class Assembly to MTR	N^{o}_{26} g.26/Assembly
A	0.00
B	2.81
C	6.03
D	9.70
E	13.9
F	18.7
G	24.2
H	30.5
J	37.6
K	45.7

[redacted]

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Semi-Logarithmic, 4 Cycles × 10 to the Inch.
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DWG. 8372

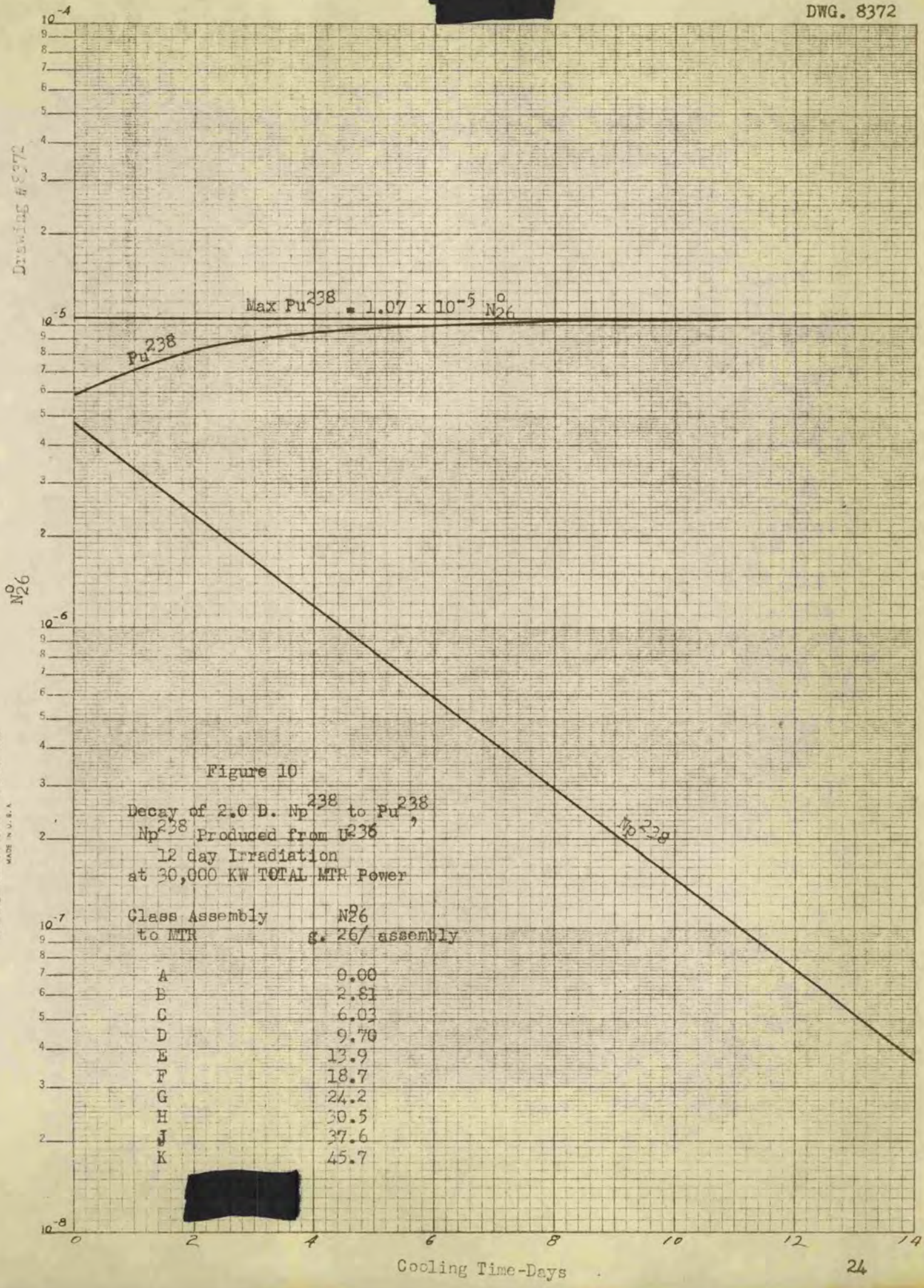

Figure 10

Decay of 2.0 D. Np^{238} to Pu^{238},
Np^{238} Produced from U^{236}
12 day Irradiation
at 30,000 KW TOTAL MTR Power

Class Assembly to MTR	N^0_{26} g. 26/ assembly
A	0.00
B	2.81
C	6.03
D	9.70
E	13.9
F	18.7
G	24.2
H	30.5
J	37.6
K	45.7

[redacted]

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DWG. 8373

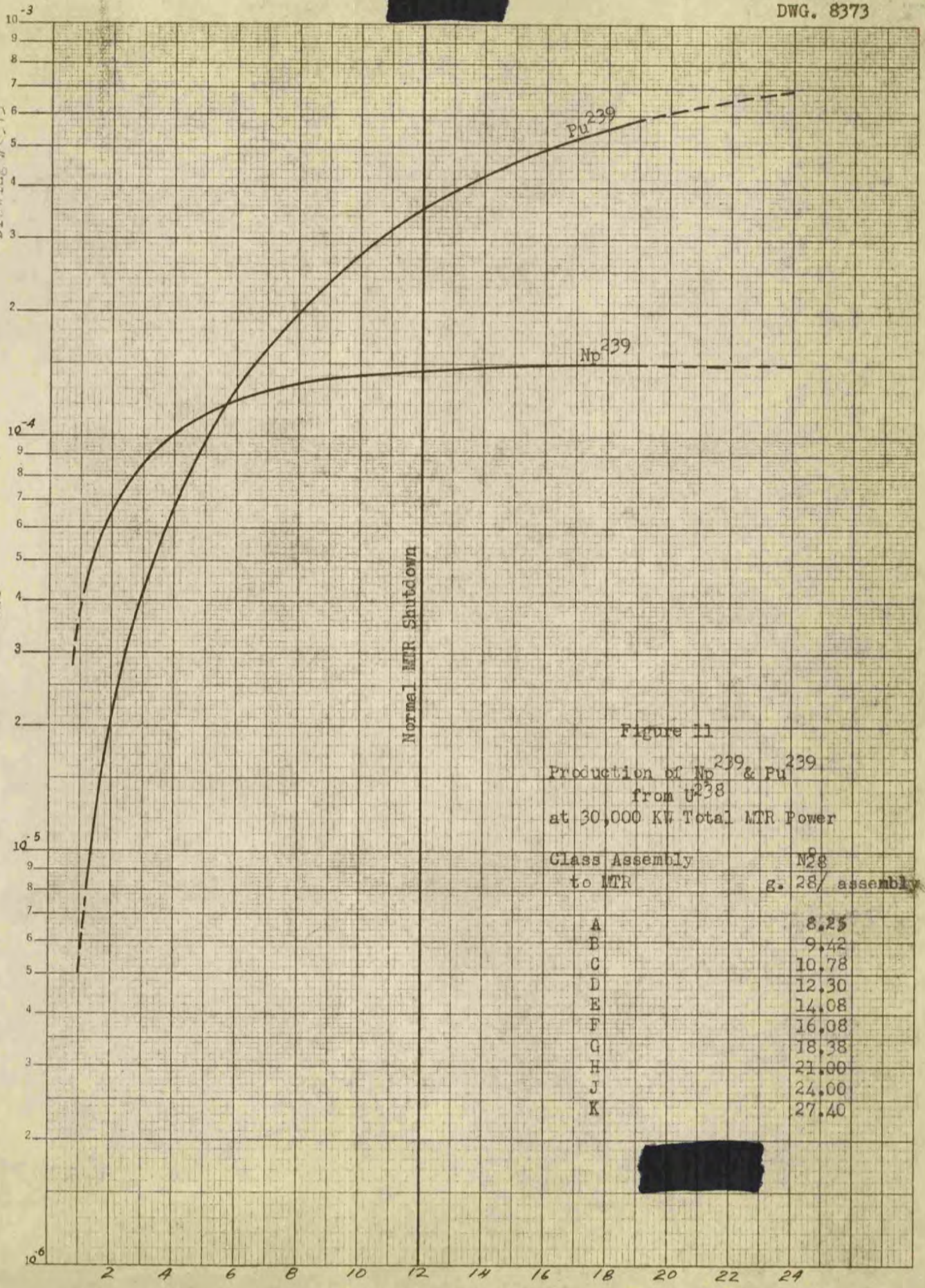

Figure 11

Production of Np^{239} & Pu^{239}
from U^{238}
at 30,000 KW Total MTR Power

Class Assembly to MTR	N_{28}^{0} g. 28/ assembly
A	8.25
B	9.42
C	10.78
D	12.30
E	14.08
F	16.08
G	18.38
H	21.00
J	24.00
K	27.40

[redacted]

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Semi-Logarithmic, 3 Cycles × 20 to the inch
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DWG. 8374

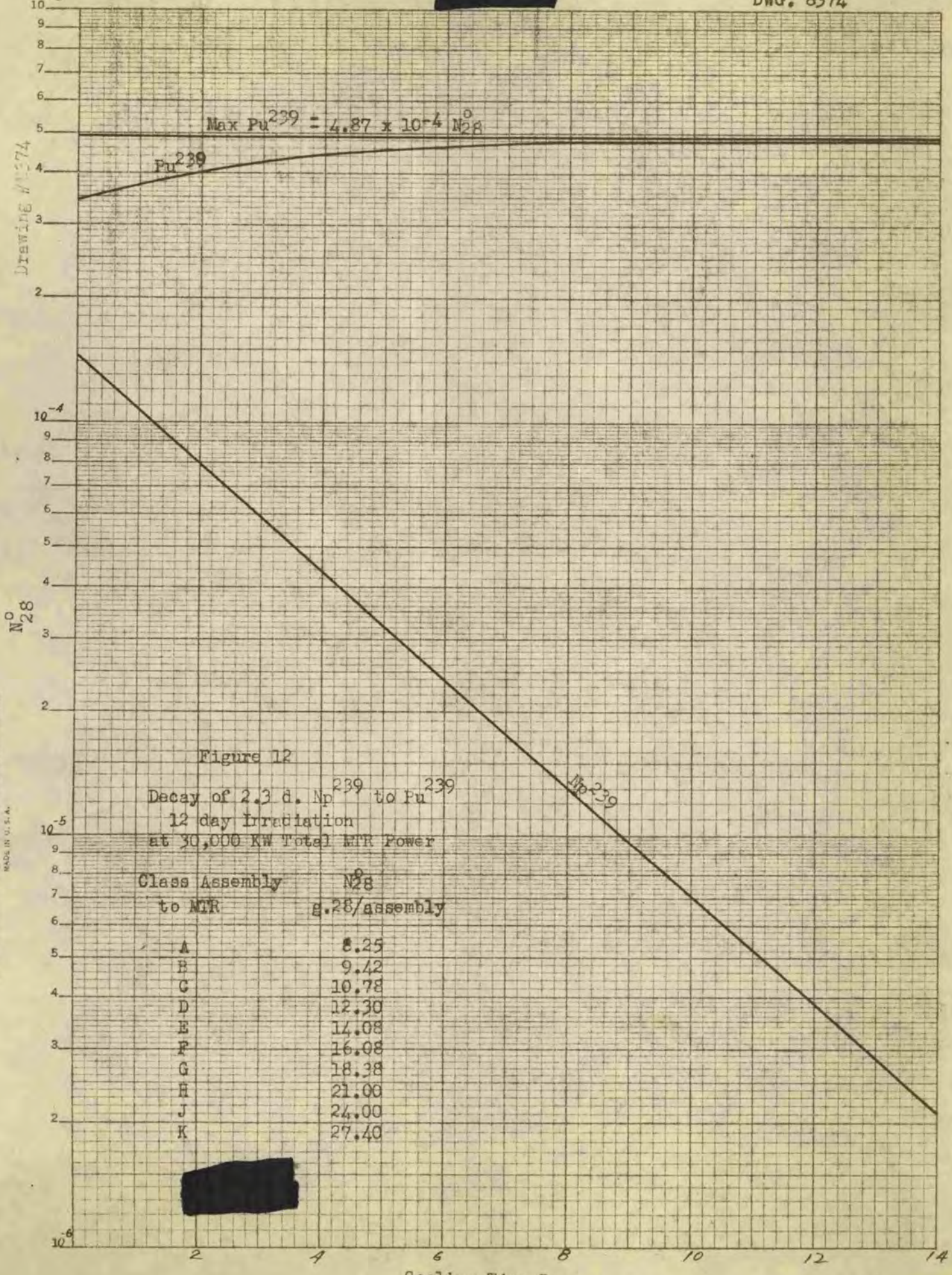

Figure 12

Decay of 2.3 d. Np^{239} to Pu^{239}
12 day Irradiation
at 30,000 KW Total MTR Power

Class Assembly to MTR	N^{o}_{28} g.28/assembly
A	8.25
B	9.42
C	10.78
D	12.30
E	14.08
F	16.08
G	18.38
H	21.00
J	24.00
K	27.40

[redacted]

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DWG. 8375

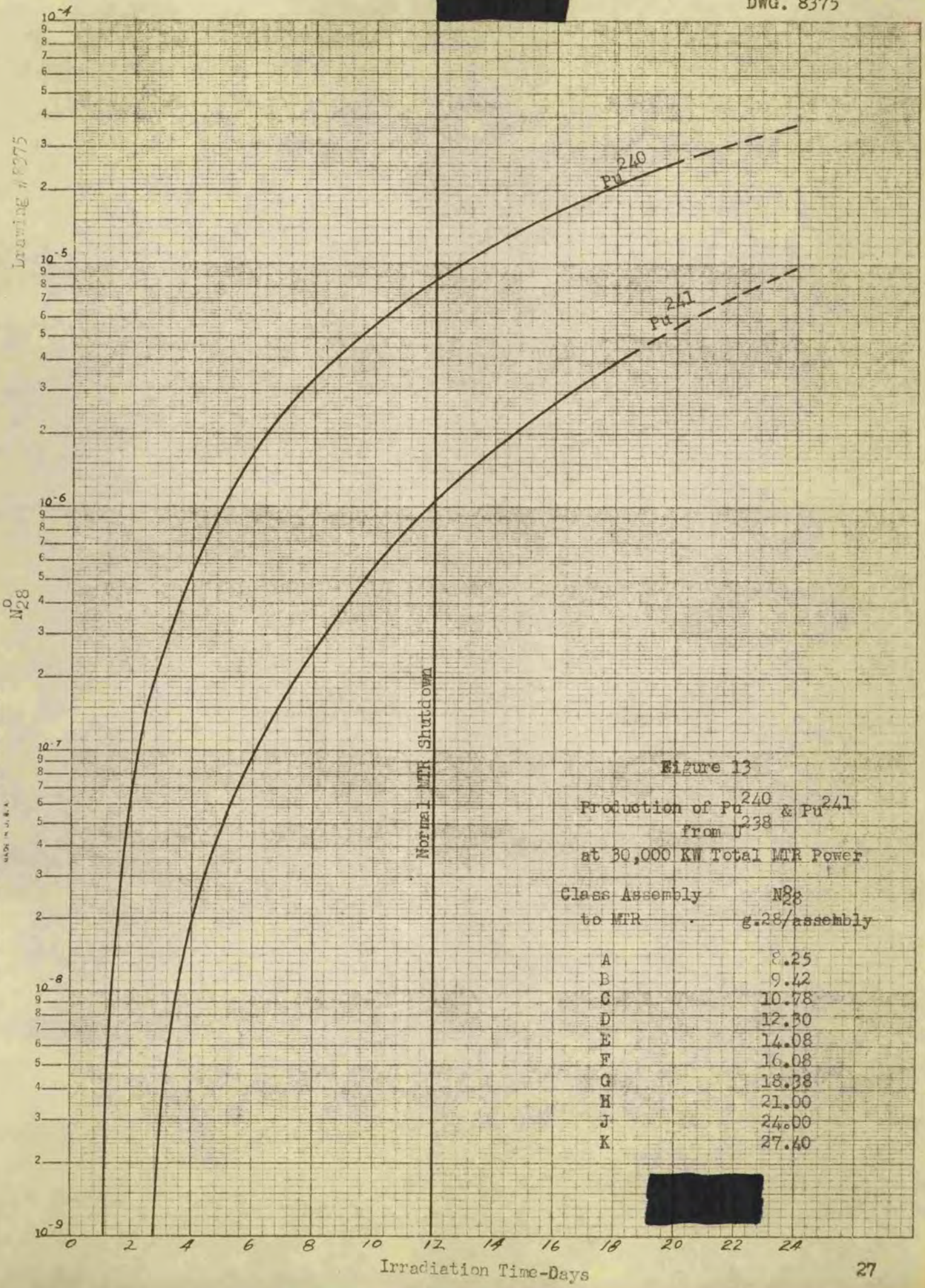

Figure 13

Production of Pu^{240} & Pu^{241} from U^{238} at 30,000 KW Total MTR Power

Class Assembly to MTR	N_{28}^{0} g.28/assembly
A	8.25
B	9.42
C	10.78
D	12.30
E	14.08
F	16.08
G	18.38
H	21.00
J	24.00
K	27.40

[redacted]

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DWG. 8376

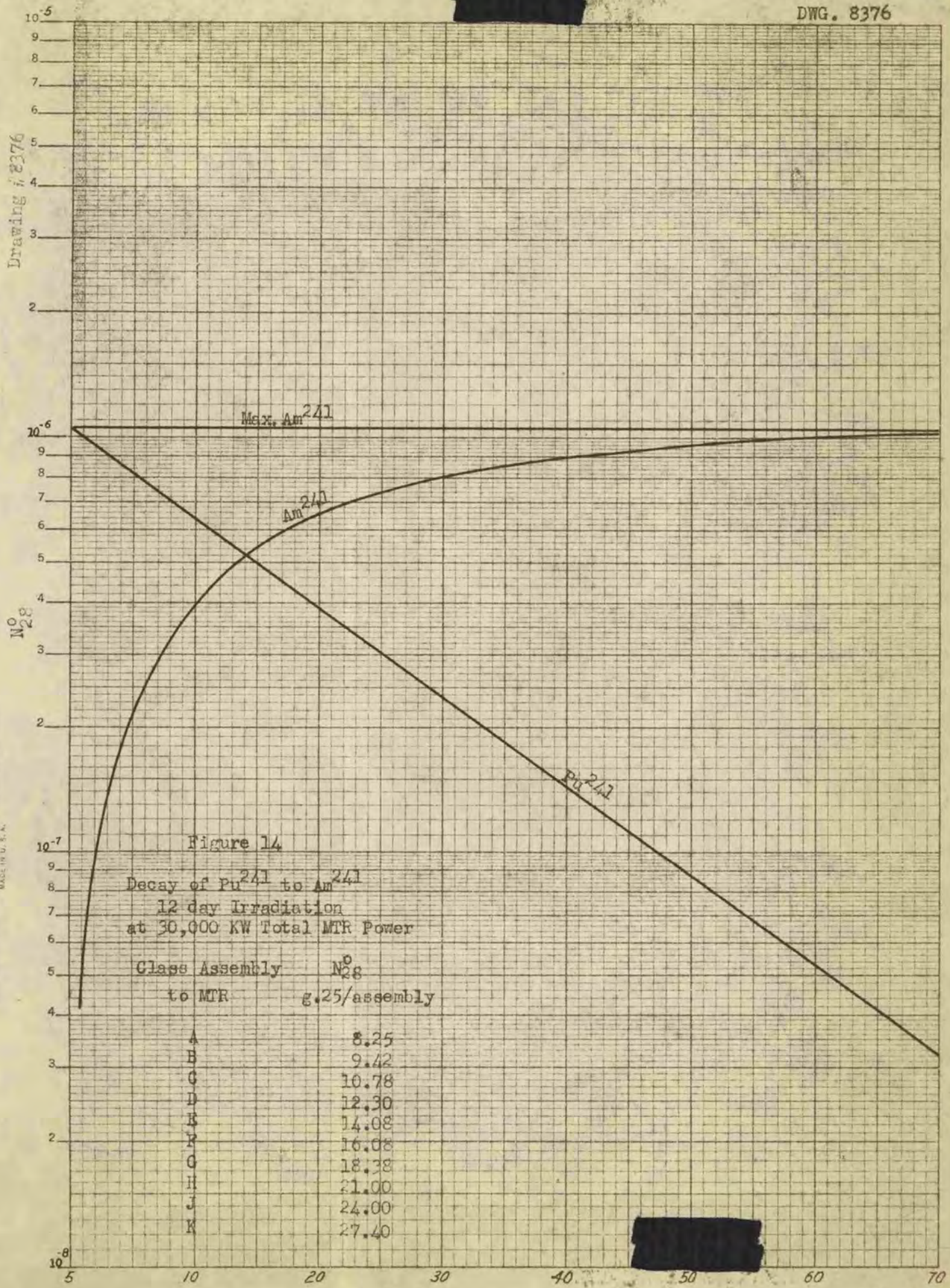

Figure 14

Decay of Pu^{241} to Am^{241}
12 day Irradiation
at 30,000 KW Total MTR Power

Class Assembly to MTR	N^o_{28} g.25/assembly
A	8.25
B	9.42
C	10.78
D	12.30
E	14.08
F	16.08
G	18.38
H	21.00
J	24.00
K	27.40

[redacted]

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APPENDIX

HEAY ISOTOPE CALCULATION

SAMPLE CALCULATION - PRODUCTION OF Pu^{238}

PRODUCTION EQUATION

$$N_{48} = F N_{25}^{0} \lambda_{25}\lambda_{26}\lambda_{27}\lambda_{37}\lambda_{38} \frac{M_{48}}{M_{25}} \left[\frac{e^{-\lambda_{25}t}}{(\lambda_{26}-\lambda_{25})(\lambda_{27}-\lambda_{25})(\lambda_{37}-\lambda_{25})(\lambda_{38}-\lambda_{25})(\lambda_{48}-\lambda_{25})} \right.$$

$$+ \frac{e^{-\lambda_{26}t}}{(\lambda_{25}-\lambda_{26})(\lambda_{27}-\lambda_{26})(\lambda_{37}-\lambda_{26})(\lambda_{38}-\lambda_{26})(\lambda_{48}-\lambda_{26})} + \frac{e^{-\lambda_{27}t}}{(\lambda_{25}-\lambda_{27})(\lambda_{26}-\lambda_{27})(\lambda_{37}-\lambda_{27})(\lambda_{38}-\lambda_{27})(\lambda_{48}-\lambda_{27})}$$

$$+ \frac{e^{-\lambda_{37}t}}{(\lambda_{25}-\lambda_{37})(\lambda_{26}-\lambda_{37})(\lambda_{27}-\lambda_{37})(\lambda_{38}-\lambda_{37})(\lambda_{48}-\lambda_{37})} + \frac{e^{-\lambda_{38}t}}{(\lambda_{25}-\lambda_{38})(\lambda_{26}-\lambda_{38})(\lambda_{27}-\lambda_{38})(\lambda_{37}-\lambda_{38})(\lambda_{48}-\lambda_{38})}$$

$$\left. + \frac{e^{-\lambda_{48}t}}{(\lambda_{25}-\lambda_{48})(\lambda_{26}-\lambda_{48})(\lambda_{27}-\lambda_{48})(\lambda_{37}-\lambda_{48})(\lambda_{38}-\lambda_{48})} \right]$$

EVALUATION OF TERMS FROM FIGURE

$F = 0.1405$	$\lambda_{25} = 1.28 \times 10^{-7}\ sec^{-1}$	$\lambda_{25}t = 0.133$
$N_{25}^{0} = 140$ g.	$\lambda_{26} = 3.00 \times 10^{-9}$	$\lambda_{26}t = 0.00311$
$M_{25} = 235$ g./g. atom	$\lambda_{27} = 1.18 \times 10^{-6}$	$\lambda_{27}t = 1.22$
$M_{48} = 238$ g./g. atom	$\lambda_{37} = 2.20 \times 10^{-8}$	$\lambda_{37}t = 0.0228$
$t = 1.037 \times 10^{6}$ sec.	$\lambda_{38} = 4.00 \times 10^{-6}$	$\lambda_{38}t = 4.15$
	$\lambda_{48} = 1.00 \times 10^{-7}$	$\lambda_{48}t = 0.104$

Substitute λt for all λ terms since they are closer to integers and since the t's cancel

Evaluate $\lambda_{25}t\lambda_{26}t$; $\lambda_{26}t\lambda_{26}t\lambda_{27}t$; etc

$\lambda_{25}t\lambda_{26}t$	$= 0.133 \times 3.11 \times 10^{-3}$	$= 4.14 \times 10^{-4}$
$\lambda_{25}t\lambda_{26}t\lambda_{27}t$	$= 4.14 \times 10^{-4} \times 1.22$	$= 5.05 \times 10^{-4}$
$\lambda_{25}t\lambda_{26}t\lambda_{27}t\lambda_{37}t$	$= 5.05 \times 10^{-4} \times 2.28 \times 10^{-2}$	$= 1.151 \times 10^{-5}$
$\lambda_{25}t\lambda_{26}t\lambda_{27}t\lambda_{37}t\lambda_{38}t$	$= 1.151 \times 10^{-5} \times 4.15$	$= 4.78 \times 10^{-5}$

EVALUATE:

$$F N_{25}^{0} \lambda_{25} \lambda_{26} \lambda_{27} \lambda_{37} \lambda_{38} \lambda_{48} \frac{M_{48}}{M_{25}} = 0.1405 \times N_{25}^{0} \times 4.78 \times 10^{-5} \times \frac{238}{235}$$

$$= 6.71 \times 10^{-6} N_{25}^{0}$$

THEN

$$N_{48} = 6.71 \times 10^{-6} N_{25}^{0} \Sigma$$

where Σ = term in brackets on pg. i

EVALUATION OF Σ

(FROM HERE ON $\lambda = \lambda t$)

1. Set up λ terms by descending value and determine value of $(\lambda - \lambda)$ terms.

$\lambda_{38} = 4.15$	$(\lambda_{38}-\lambda_{27}) = 2.93$	$(\lambda_{27}-\lambda_{25}) = 1.087$	$(\lambda_{25}-\lambda_{48}) = 0.029$	$(\lambda_{48}-\lambda_{37}) = 0.0812$
$\lambda_{27} = 1.22$	$(\lambda_{38}-\lambda_{25}) = 4.017$	$(\lambda_{27}-\lambda_{48}) = 1.116$	$(\lambda_{25}-\lambda_{37}) = 0.1102$	$(\lambda_{48}-\lambda_{26}) = 0.10089$
$\lambda_{25} = 0.133$	$(\lambda_{38}-\lambda_{48}) = 4.046$	$(\lambda_{27}-\lambda_{37}) = 1.1972$	$(\lambda_{25}-\lambda_{26}) = 0.12989$	
$\lambda_{48} = 0.104$	$(\lambda_{38}-\lambda_{37}) = 4.1272$	$(\lambda_{27}-\lambda_{26}) = 1.21689$		$(\lambda_{37}-\lambda_{26}) = 0.01969$
$\lambda_{37} = 0.0228$	$(\lambda_{38}-\lambda_{26}) = 4.14689$			
$\lambda_{26} = 0.00311$				

2. Evaluate products of $(\lambda - \lambda)$ terms. (See pg. iii)

3. Evaluate exponentials

$e^{-\lambda_{25}t} = e^{-0.133} = 0.8754650921$

$e^{-\lambda_{26}t} = e^{-0.00311} = 0.9968948311$

$e^{-\lambda_{27}t} = e^{-1.22} = 0.2952301669$

$e^{-\lambda_{37}t} = e^{-0.0228} = 0.9774579558$

$e^{-\lambda_{38}t} = e^{-4.15} = 0.0157644164$

$e^{-\lambda_{48}t} = e^{-0.104} = 0.9014957055$

a. Exponentials for numbers to four deimal places are given to 20 decimal places in WPA Tables

b. $e^{-\lambda_{26}t}$ was calculated from the relationship

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} - + - \cdots \quad \text{where } x = \lambda t$$

$$\begin{aligned} 1 &= 1.0000000000 \\ \lambda t &= -0.0031100000 \\ &\quad 0.9968900000 \\ (\lambda t)^2/2 &= +0.0000048361 \\ &\quad 0.9968948361 \\ (\lambda t)^3/3! &= -0.0000000050 \\ e^{-\lambda_{26}t} &= 0.9968948311 \end{aligned}$$

$(\lambda t)^2 = (3.11 \times 10^{-3})^2 = 9.6721 \times 10^{-6}/2 = 4.8361 \times 10^{-6}$

$(\lambda t)^3 = (3.11 \times 10^{-3})^3 = 30.08 \times 10^{-9}/6 = 5.0 \times 10^{-9}$

Calculation of $(\lambda-\lambda)$ products

26	27	37	38	48
$(\lambda_{26}-\lambda_{25})(\lambda_{27}-\lambda_{25})$: $(-.12989)(+1.087) = -0.14119043$		$(\lambda_{37}-\lambda_{25})$ $\times(-.1102) = 0.0155918539$	$(\lambda_{38}-\lambda_{25})$ $\times(+4.017) = +0.06250124771$	$(\lambda_{48}-\lambda_{25})$ $\times(-0.029) = -0.001812536184$
$(\lambda_{25}-\lambda_{26})(\lambda_{27}-\lambda_{26})$ $(+.12989)(+1.21689) = +0.1580618421$		$(\lambda_{37}-\lambda_{26})$ $(+0.01969) = +0.003112237671$	$(\lambda_{38}-\lambda_{26})$ $\times(+4.14689) = +0.01290610728$	$(\lambda_{48}-\lambda_{26})$ $\times(+0.10089) = +0.001302097163$
$(\lambda_{25}-\lambda_{27})(\lambda_{26}-\lambda_{27})$ $(-1.087)(-1.21689) = +1.32275943$		$(\lambda_{37}-\lambda_{27})$ $\times(-1.1972) = -1.583607590$	$(\lambda_{38}-\lambda_{27})$ $\times(+2.93) = -4.639970239$	$(\lambda_{48}-\lambda_{27})$ $\times(-1.116) = +5.178206787$
$(\lambda_{25}-\lambda_{37})(\lambda_{26}-\lambda_{37})$ $(+0.1102)(-0.01969) = -0.002169838$		$(\lambda_{27}-\lambda_{37})$ $\times(+1.1972) = -0.002587730034$	$(\lambda_{38}-\lambda_{37})$ $\times(+4.1272) = -0.01072135148$	$(\lambda_{48}-\lambda_{37})$ $\times(+0.0812) = -0.000870573740 2$
$(\lambda_{25}-\lambda_{38})(\lambda_{26}-\lambda_{38})$ $(-4.017)(-4.14689) = +16.65805713$		$(\lambda_{27}-\lambda_{38})$ $\times(-2.93) = -48.80810739$	$(\lambda_{37}-\lambda_{38})$ $\times(-4.1272) = +201.4408208$	$(\lambda_{48}-\lambda_{38})$ $\times(-4.046) = -815.0295610$
$(\lambda_{25}-\lambda_{48})(\lambda_{26}-\lambda_{48})$ $(+0.029)(-0.10089) = -0.00292581$		$(\lambda_{27}-\lambda_{48})$ $\times(+1.116) = -0.00326520396$	$(\lambda_{37}-\lambda_{48})$ $\times(-0.0812) = +0.000265134561 6$	$(\lambda_{38}-\lambda_{48})$ $\times(+4.046) = +0.001072734436$

Note:
1. $\lambda_{38} >$ Other λ's, therefore all $\lambda-\lambda_{38}$ terms are negative, and products are alternately $-,+,-,+$. Also, since $\lambda-\lambda_{38}$ terms are largest, products are greatest, and of least significance.
2. $\lambda_{26} <$ Other λ's therefore all $\lambda-\lambda_{26}$ terms and their products are positive
3. In general, the smaller the value of λ, the greater its final significance
4. For checking purposes; note the numerical and sign symmetry about the dotted axis
5. The products in any column above the double line are the $\lambda-\lambda$ products applicable to the calculations for the isotope at the head of the column
6. If, for example, the amount of 48 produced from 26 originally present is required, omit the top row and left hand column of $\lambda-\lambda$ products, form the new products, and divide the appropriate exponentials by these products.

EVALUATION OF Σ (CONTINUED)

4. Finally divide each exponential by its appropriate λ-λ product and add the resulting terms algebraically.

25	Term	0.8754650921 / − 0.001812536184	= − 483.005581
26	"	0.9968948311 / + 0.001302097163	= + 765.607099
27	"	0.2952301669 / + 5.178206787	= + .057014
37	"	0.9774579558 / − 0.0008705737402	= − 1122.774453
38	"	0.0157644164 / − 815.0295610	= − .000019
48	"	0.9014957055 / + 0.001072734436	= + 840.119667
			0.003727

$$\Sigma = 3.73 \times 10^{-3}$$

$$N_{48} = 6.71 \times 10^{-6} \Sigma N_{25}^{0}$$
$$= 6.71 \times 10^{-6} \times 3.73 \times 10^{-3} N_{25}^{0} = 2.501 \times 10^{-8} N_{25}^{0}$$
$$= 2.501 \times 10^{-8} \times 1.40 \times 10^{2} = 3.50 \times 10^{-6} \text{ g / assembly, or}$$
$$= \underline{3.50 \ \mu g. \text{ / assembly}}$$

THE ABOVE DATA CAN BE EXTENDED TO CALCULATE N_{48} FOR OTHER PERIODS OF IRRADIATION AS FOLLOWS:

CALCULATE N_{48} PRESENT AT 2.4, 7.2 & 19.2 DAYS IRRADIATION

1. Obtain exponentials.

Isotope	For 2.4 d. λt	$e^{-\lambda t}$	For 7.2 d. λt	$e^{-\lambda t}$	For 19.2 d. λt	$e^{-\lambda t}$
25	0.0266	0.9737506639	0.0798	0.9233009881	0.2128	0.8083177846
26	0.000622	0.9993781934	0.001866	0.9981357399	0.004976	0.9950363599
27	0.244	0.7834876343	0.732	0.4809461353	1.952	0.1419898678
37	0.00456	0.9954503810	0.01368	0.9864131461	0.03648	0.9641773773
38	0.830	0.4360492913	2.490	0.0829099666	6.640	0.0013070272
48	0.0208	0.9794148279	0.0624	0.9395070088	0.1664	0.8467074835

2. Obtain the reciprocals of the λ-λ products, since repeated multiplications by the same number (the λ-λ product) are simpler than repeated divisions.

25	1/−0.001812536184 = −551.713123	37	1/−0.0008705737402 = 1148.667770
26	1/+0.001302097163 = +767.991843	38	1/−815.0295610 = 0.001227
27	1/+5.178206787 = +0.193117	48	1/+0.001072734436 = 932.197164

3. Multiply each $\frac{1}{(\lambda-\lambda)}$ product by the 3 exponentials and add the terms obtained algebraically.

Isotope Term	For 2.4 d.	For 7.2 d.	For 19.2 d.
25	− 573.231020	− 509.397272	− 445.959529
26	+ 767.514301	+ 766.560106	+ 764.179808
27	+ 0.151305	+ 0.092879	+ 0.027421
37	− 1143.441769	− 1133.060989	− 1107.519478
38	− 0.000535	− 0.000102	− 0.000002
48	+ 913.007725	+ 875.805769	+ 789.298315
	0.000007	0.000391	0.026535
Σ	7×10^{-6}	3.91×10^{-4}	2.65×10^{-2}

4 Use Σ's obtained above in gen'l equation.

$$N_{48} = 6.71\times10^{-6}\,\Sigma N^{0}_{25}$$

$= 6.71\times10^{-6}\times 7\times10^{-6}\,N^{0}_{25} = 4.65\times10^{-11}N^{0}_{25}$ Produced in 2.4 days

$= 6.71\times10^{-6}\times 3.91\times10^{-4}\,N^{0}_{25} = 2.62\times10^{-9}\,N^{0}_{25}$ " " 7.2 "

$= 6.71\times10^{-6}\times 2.65\times10^{-2}\,N^{0}_{25} = 1.78\times10^{-7}\,N^{0}_{25}$ " " 19.2 "

THE GENERAL EQUATION IS APPLICABLE TO FISSION PRODUCT PRODUCTION IF THE FISSION YIELD, Y, IS INSERTED AS A FRACTION

$$N_{FP} = F\,Y\,N_0\,\lambda_1\lambda_2\cdots\lambda_{n-1}\times\left[\Sigma\right]$$

NOTE ON THE HANDLING OF VERY LARGE λ'S

If the λ for any member > 10 times the next largest λ it may safely be omitted from the equation. Thus in the reaction

$$U^{238} + n \rightarrow U^{239} \xrightarrow[23m]{\beta} Np^{239} \xrightarrow[2.3d]{\beta} Pu^{239} + n \rightarrow$$

$\lambda_{29}t = 520$. This is so much larger than $\lambda_{39} = 3.51$, the next largest λ, that the reaction may be considered to be

$$U^{239} + n \rightarrow Np^{239} \xrightarrow[2.3d]{\beta} Pu^{239} + n \rightarrow$$

and the general equation written in terms of this reaction. This assumption is true for all irradiation times > 3-4 hours.

The truth of this assumption can be proven as follows.

1. Since $\lambda_{29} t = 520$ $\quad e^{-\lambda_{29} t} \ll 1 \quad (\sim 10^{-200})$
2. $\lambda_{29} - \lambda_x \cong \lambda_{29}$ and is the same in all λ-λ terms, thus its omission is the same as the omission of a constant factor, 1, since $\frac{\lambda_{29}}{\lambda_{29} - \lambda_x} \cong 1$
3. The denominator of the $e^{-\lambda_{29} t}$ term is $(\lambda_{29})^n$

Thus $e^{-\lambda_{29} t}/(\lambda_{29})^n \lll 1$ and negligible compared to other Σ_i terms.

In the reaction

$$N^0_{25} + n \rightarrow Y\left[Xe^{140} \xrightarrow[16s.]{\beta} Cs^{140} \xrightarrow[\substack{\text{Short} \\ (\sim 1s.)}]{\beta} Ba^{140} \xrightarrow[12.8d.]{\beta} La^{140} \xrightarrow[40\,hr.]{\beta} Ce^{140}_{(S)}\right]$$

$\lambda_{Xe^{140}}$ & $\lambda_{Cs^{140}}$ are both so large compared to λ_{25}, $\lambda_{Ba^{140}}$ & $\lambda_{La^{140}}$ that the general equation can be set up for the reaction

$$N^0_{25} + n \rightarrow Y\left[Ba^{140} \xrightarrow[12.8d.]{\beta} La^{140} \xrightarrow[40\,hr.]{\beta} Ce^{140}_{(S)}\right]$$

except for irradiation time of less than 2-3 minutes.

NOTES ON EVALUATING EXPONENTIALS

Since nuclear reaction cross sections and half lives are never accurate to more than 1 or 2%, the values of λ's should not be carried to more than two, or at the most three significant figures. Similiarly λt's should not carried to more than three digits. This permits the exact evaluation of $e^{-\lambda t}$ to as many as 20 decimal places. Similiarly in finding a number of values of N for plotting growth and decay curves, the λt's must be exact fractions or multiples of each other if exponential tables are used.