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NOTE ON ZERO-ZERO TRANSITIONS

S. D. Drell

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NOTE ON ZERO-ZERO TRANSITIONS

S. D. Drell

I. INTRODUCTION

In the decay of a nucleus from an excited level of zero spin to a lower lying state of zero spin with no parity change, an orbital electron will be ejected from its path about the nucleus or, if the one excited level of the nucleus is higher than 2 mc^2 above the other level, an electron-positron pair may be created[1,2,3,4,5] as well. There will be a complete conversion of the energy released in the nuclear decay into freeing a bound electron and giving it kinetic momentum, or into creating the pair with a certain total momentum. This is because the selection rules strictly forbid single gamma emission, and the only other processes which could take up the energy made available in the nuclear transition are less probable (for example, double gamma emission[3]).

It is significant in these zero-zero, no parity change transitions that the perturbing fields which induce the internal conversion or pair formation exist only within the nucleus and vanish identically outside of the nuclear radius[1,2]. The entire interaction causing the electron to be ejected, from the

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1. R. H. Fowler, Proc. Roy. Soc. A129, 1; 1930.
2. H. Yukawa and S. Sakata, Proc. Phys.-Math. Soc., Japan, 17, 397; 1935.
3. J. R. Oppenheimer and J. Schwinger, Phys. Rev. 56, 1066; 1939.
4. R. Thomas, Phys. Rev. 58, 714; 1940.
5. The case for zero-zero transitions with a change in parity has been discussed by R. Sachs, Phys. Rev. 57, 194; 1940, and M. Goldberger, Phys. Rev. 73, 1119; 1948.

bound orbit in the conversion process and from a negative energy state in the internal pair formation, thus occurs within the charge-current distribution of the nucleus. This is to be contrasted with ordinary radiative internal conversion and pair formation.

Because of this unique property of zero-zero transitions it is of particular interest to consider the effect of finite nuclear radius on the rates of internal conversion and of pair formation for such a process. Presented here is a study of this effect. The ratio of K- to L-shell conversion is calculated using Rose's[6] wave functions for relativisitic Dirac electrons in the field of a finite charge distribution. This ratio is found to be independent of nuclear matrix elements and to agree, within 10%, with the ratio as calculated for electrons in a point Coulomb field.

We discuss briefly the three cases of zero-zero transitions with no parity change that have been proposed in the literature - internal conversion and pair formation from the 1.42 Mev transition[1,7] in RaC', pair formation from the 6.04 Mev transition[8] in O^{16}, and internal conversion from the 0.7 Mev transition[9] in Ge^{72}.

Agreement between theory and experiment in these instances may be interpreted as evidence in favor of the validity <u>within</u> the nucleus of Dirac electron theory and of our usual notions of electromagnetic forces.

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6. M. E. Rose; to be published.

7. C. D. Ellis and G. H. Aston, Proc. Roy. Soc. A129, 180; 1930. G. D. Latyshev, L. A. Sliv, I. F. Barchuk, and A. A. Bashilov, Izv. Akad. Nauk. SSSR, Ser. Fiz. 13, (No. 3) 340; 1949. (Cf. Physics Abstracts, Dec. 1949, Abstracts 7247-7253.)

8. W. A. Fowler and C. C. Lauritsen, Phys. Rev. 56, 840; 1939. V. K. Rasmussen, W. F. Hornyak, C. C. Lauritsen, and T. Lauritsen, Phys. Rev. 77, 617; 1950.

9. J. C. Bowe, M. Goldhaber, R. D. Hill, W. E. Meyerhof, and O. Sala, Phys. Rev. 73, 1219(A); 1948. A. C. G. Mitchell, B. D. Kern, and D. J. Zaffarano, Phys. Rev. 73, 1424; 1948, S. K. Haynes, Phys. Rev. 74, 423; 1948, J. Y. Mei, A. C. G. Mitchell, and C. M. Huddleston, Phys. Rev. 79, 19; 1950.

II. CALCULATION

A. Matrix Element

The number of electronic transitions per second from state 0 to state f, resulting from interaction with potentials $\vec{A}$, ϕ that are created during a nuclear transition, is given by

$$N_e d\Omega = \frac{2\pi e^2}{\hbar} \sum_{\text{spin}} \left| \int d\vec{r}\, \psi_f^* (\alpha \cdot A + \phi)\, \psi_o \right|^2 d\Omega . \qquad (1)$$

ψ_f is normalized to unit energy, the $(\alpha_1, \alpha_2, \alpha_3)$ are the four by four Dirac spin matrices, and the spin sum extends over final and initial spin states. For ψ_o a normalized bound state wave function, (1) gives the internal conversion rate (for two K-electrons if state 0 is the K-shell). For ψ_o a negative energy continuum state, normalized to unit energy, (1) gives the rate of pair formation per unit energy interval. The total pair production rate is then given by the integral of (1) over available energy states from which the negative energy electron may be ejected; that is by

$$\int_{mc^2}^{\Gamma - mc^2} N_e (E)\, dE, \qquad (2)$$

where Γ is the energy difference between the initial and final nuclear states and E is the total positron energy.

In the usual way we write for the radiating potentials which are created during the nuclear transition between initial and final nuclear states φ_e and φ_g, separated in energy by Γ,

$$\vec{A}(\vec{r}) = \frac{-ie\hbar}{2Mc} \sum_P \int \frac{e^{i\frac{\Gamma}{\hbar c}|\vec{r}-\vec{r}_P|}}{|\vec{r}-\vec{r}_P|} \left\{ \varphi_g^*(\vec{r}_P)\overrightarrow{\mathrm{grad}}_P\, \varphi_e(\vec{r}_P) - \varphi_e(\vec{r}_P)\mathrm{grad}_P\, \varphi_g^*(\vec{r}_P) \right\} d\vec{r}_P \,, \tag{3}$$

$$\phi(\vec{r}) = e \sum_P \int \frac{e^{i\frac{\Gamma}{\hbar c}|\overrightarrow{r-r_P}|}}{|\overrightarrow{r-r_P}|} \varphi_g^*(\vec{r}_P)\, \varphi_e(\vec{r}_P)\, d\vec{r}_P . \tag{4}$$

The sum extends over individual proton coordinates $\vec{r}_P$.[10]

Consider the case in which both the initial and final nuclear states have spin zero and the same parity. By the following argument, developed even more generally by Yukawa and Sakata[2], we shall show that the fields vanish outside of the nucleus. We see directly from (3) that the vector potential is radial for spherically symmetric (spin zero) nuclear states. Consequently the magnetic field

$$\vec{H} = \overrightarrow{\mathrm{curl}}\, A$$

vanishes. We have then by Maxwell's equation

$$\frac{\partial \vec{E}}{\partial t} = -4\pi \vec{j} = -4\pi \frac{e\hbar}{2\,iM} \left\{ \varphi_g^* \,\mathrm{grad}\, \varphi_e - \varphi_e \,\mathrm{grad}\, \varphi_g^* \right\} .$$

Thus the electric field differs from zero only inside of the nucleus where the mixed current density exists. By an appropriate choice of gauge, (3) can be made to vanish for $|\vec{r}| > R$, the nuclear radius. We see from the orthogonality of the nuclear states that the scalar potential (4) vanishes to the dipole approximation[11] for $|\vec{r}| > R$. To within a few percent it is the matrix element of the scalar potential ϕ between initial and final states that accounts for the entire contribution to (1) in this particular case of a zero-zero transition. This can be shown by two simple

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10. In place of this sum, Yukawa and Sakata introduce an effective charge Qe. (cf. ref. (2)).

11. The validity of neglecting retardation within the nucleus is discussed below following equation (5).

arguments. The matrix elements of the α_i between ψ_o and ψ_j^* are reduced relative to those of the unit matrix by terms of the order of[12]

$$\sim \frac{Z\alpha}{2}\left(1 + \frac{1}{3}\,\frac{A^{1/3}}{Z}\,\frac{W_f}{mc^2}\right)\frac{r}{R}\ ,$$

where $\alpha = e^2/\hbar c \cong \frac{1}{137}$ is the fine structure constant, W_f is the total energy of the electron (or positron) in the final continuum state, and Z and A are the atomic and mass numbers of the decaying atom. Also we expect the amplitude of the vector potential to be reduced relative to the scalar potential in the ratio of the mean nucleon velocity to the velocity of light. Consequently, evaluation of (1) reduces to a calculation of

$$N_e d\Omega = \frac{2\pi e^4}{\hbar}\sum_{\text{spin}}\left|\sum_P \iint d\vec{r}\, d\vec{r}_P\ \psi_f^*(r)\ \varphi_g^*(r_P)\ \frac{1}{r_>}\psi_o(\vec{r})\ \varphi_e(r_P)\right|^2 d\Omega. \qquad (5)$$

In (5) we have neglected retardation within the nucleus. The error introduced by this approximation is entirely negligible in the cases which we consider here, for which

$$\frac{\Gamma R}{\hbar c} = \frac{\Gamma}{mc^2}\,\frac{1}{2}\,\alpha A^{1/3} << 1.$$

B. Electronic Wave Functions in the Field of a Finite Nucleus

Rose[6] has developed a method of obtaining solutions by quadrature to the Dirac equation for an electron in the field of a finite nucleus. We write the electron's wave function in the form[13]

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12. Cf. equation (8) below.

13. G. Goertzel and M. E. Rose, MDDC-1514 (USAEC Oak Ridge report).

$$\psi = \begin{pmatrix} i\, F_{\ell j}\, \sigma_r\, \Omega^m_{\ell j} \\ G_{\ell j}\, \Omega^m_{\ell j} \end{pmatrix} , \tag{6}$$

where $F_{\ell j}$ and $G_{\ell j}$ are real radial functions, the operator σ_r is given by

$$\sigma_r = \begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta \end{pmatrix} ,$$

and the two-component spinors Ω^m_j are given for type (a) and type (b) states by

$$\Omega^m_{j-\frac{1}{2},j} = \begin{pmatrix} \sqrt{\frac{j+m}{2j}}\; Y^{m-\frac{1}{2}}_{j-\frac{1}{2}} \\ -\sqrt{\frac{j-m}{2j}}\; Y^{m+\frac{1}{2}}_{j-\frac{1}{2}} \end{pmatrix} ,$$

$$\Omega^m_{j+\frac{1}{2},j} = \begin{pmatrix} \sqrt{\frac{j-m+1}{2j+2}}\; Y^{m-\frac{1}{2}}_{j+\frac{1}{2}} \\ \sqrt{\frac{j+m+1}{2j+2}}\; Y^{m+\frac{1}{2}}_{j+\frac{1}{2}} \end{pmatrix} . \tag{7}$$

The Y^m_ℓ are normalized spherical harmonics. For type (a) solutions ($j = \ell + \frac{1}{2}$), the large components $G_{j-\frac{1}{2},j}$ vary as r^ℓ for small r, and the small components go as $r^{\ell+1}$. The situation is reversed for type (b) functions, the index being $\ell+1$ for $G_{j+\frac{1}{2},j}$ and ℓ for $F_{j+\frac{1}{2},j}$. Since the electrons experience the perturbation only during that fraction of the time which they spend within the nuclear radius, we may restrict our considerations to states with zero "orbital angular momentum". Contributions from $\ell = 1$ states will be reduced in the order of

$$(R/\hbar/mc)^2 \sim (\tfrac{1}{2}\alpha A^{1/3})^2 .$$

The solutions obtained by Rose's[6] method for the radial functions of a Dirac electron in the field of a nucleus of finite radius R, within which the charge is taken to be uniformly distributed[14], can be written:

Type a; j = 1/2; total energy W

$$G = C_G\left(1 - \frac{1}{2}\frac{r^2}{R^2}\left\{\frac{3}{4}Z^2\alpha^2 + \frac{1}{2}Z\alpha^2 A^{1/3}\frac{W}{mc^2} - \frac{1}{12}\alpha^2 A^{2/3}\left[1 - \frac{W^2}{m^2c^4}\right]\right\} + O\left(\frac{Z\alpha r}{R}\right)^4\right)$$

$$F = C_G\left(\frac{-W}{mc^2} + 1 - \frac{3Z}{A^{1/3}}\right)\frac{1}{6}\alpha A^{1/3}\frac{r}{R}\left\{1 - \frac{1}{5}\frac{r^2}{R^2}\frac{3Z/A^{1/3}}{\left(\frac{3Z}{A^{1/3}} + \frac{W}{mc^2} - 1\right)} + \cdots\right\} . \tag{8}$$

For type (b) solutions it is necessary only to interchange F and G in (8) and to change the sign of W and of Z.[6]

C. Rate of Transition

We obtain, upon inserting (6), (7), and (8) into (5),

$$N_e\, d\Omega = \frac{4\pi^2}{9} e^4/\hbar \left|C_{G_f}\right|^2 \left|C_{G_i}\right|^2 \left\langle \sum_P r_P^2 \right\rangle^2 . \tag{9}$$

There are correction terms of order

$$\frac{1}{10}\alpha^2 Z^2 \left(3 + 1\,\frac{A^{1/3}}{Z}\frac{W_f}{mc^2}\right)\frac{1}{R^2}\frac{\left\langle \sum_P r_P^4 \right\rangle}{\left\langle \sum_P r_P^2 \right\rangle}$$

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14. Studies for various charge distributions within the nucleus (ref. 6) indicate that the wave functions are insensitive to the particular model assumed.

and smaller which may be neglected within the uncertainty in the nuclear matrix element of $\sum_P r_P^2$ between initial and final nuclear states. If we neglect these higher terms and insert for the normalization constants those values which apply in the case of a point nucleus, we obtain for the rates of conversion and of pair formation results identical with those given by Yukawa and Sakata[2] and by Thomas[4].

D. K- to L-Shell Conversion Ratio

The transition rate (9) depends critically on the value one assigns to the nuclear matrix element. However, the K-to L-shell conversion ratio is an experimentally measurable quantity for which theory can predict a number independent of $\left\langle \sum_P r_P^2 \right\rangle$.

Corrections to the leading order term, which is exhibited in equation (9), cancel out to within a few percent in the conversion ratio. We have thus, accurate to order $Z A^{1/3} \alpha^2 \dfrac{\left\langle \sum_P r_P^4 \right\rangle}{5 R^2 \left\langle \sum_P r_P^2 \right\rangle}$,

$$N_K : N_{L_I} : N_{L_{II}} = \left| C^K_{G_f} C_{G_K} \right|^2 : \left| C^{L_I}_{G_f} C_{G_{L_I}} \right|^2 : \left| C^{L_{II}}_{F_f} C_{F_{L_{II}}} \right|^2 \tag{10}$$

for the ratio of K- to L_I- to L_{II}-shell conversion. C_{G_K} denotes the normalization constant for a type (a), K-shell function; $C^K_{G_f}$, for the final continuum function of type (a) of the electron ejected from the K-shell. Actually the conversion from the L_{II}-subshell will be greatly reduced in comparison with L_I conversion[15]. This is quite analogous to the case for a point nucleus, where $C_{F_{L_{II}}} / C_{G_{L_I}} \approx Z \alpha$.

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15. M. E. Rose and D. K. Holmes; to be published.

We can neglect conversion from the L_{III} subshell by the arguments of the preceding sections, since L_{III} electrons have one unit of orbital angular momentum.

III. DISCUSSION

Rose and Holmes[15] have found that taking into account the finite size of the nucleus does not appreciably alter the G function (equation 8) for type (a) wave functions. The F function is seriously modified, but is of less concern to us here.[16] The error one makes in calculating the ratio of K- to L-shell conversion, using Coulomb wave functions for a point nuclear source, is estimated to be less than 10%.[17]

There are in the literature three proposed cases of zero-zero transitions with no change of parity:[5,18] a 1.42 Mev transition[7] in RaC', a 6.04 Mev transition[8] in O^{16}, and a 0.7 Mev transition[9] in Ge^{72}.

Ellis and Aston[7] first observed the conversion line in RaC' and Fowler[1] explained the absence of photons by describing it as a zero-zero transition. Further work has recently been reported by Latyshev[7] and his co-workers in Russia for which is claimed excellent agreement with theory in so far as concerns the relative rates for internal conversion and pair production, and for the ratio of K- to L-conversion.

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16. Rose and Holmes (cf. ref. 15) have shown it to have a significant effect on the shapes of forbidden β-spectra.

17. On the other hand, taking the finite size of the nucleus into consideration will effect the calculated ratio of L_I to L_{II} conversion. No experimental data appears to be available on this point.

18. M. Goldhaber, C. O. Muehlhause, and S. H. Turkel, Phys. Rev. 71, 372; 1947.

The 6.04 Mev level in O^{16} decays predominantly by pair formation. The probability for conversion of an orbital electron is reduced relative to the pair formation probability in this case because there are very many energy states available for the electron-positron pair at 6 Mev. The momentum distribution for the positrons[8] and the angular correlation between the electron and positron of the pair[19] agree satisfactorily with theoretical predictions for zero-zero pair production within the nucleus.

Bowe et al[9] first observed a strongly converted 700 Kev line in Ge^{72}. No gamma rays of this energy have been seen. The lifetime of the state is $1/3 \times 10^{-6}$ sec.[9,20] To achieve a consistent interpretation of these data one must assume, as proposed by the above investigators, that this a zero-zero transition.[21] A very high multipole order ($\Delta \ell > 5$) would be required to give a large enough conversion coefficient. It would, however, be incompatible with the lifetime which indicates $\Delta \ell = 3$ for a radiative transition. There are no data available at this time on the K- to L-shell conversion ratio which, on the basis of equation (10) is theoretically predicted to be about eight to one.[22]

In conclusion one can make estimates on the basis of formula (5) for the lifetimes of levels decaying via zero-zero transitions with no change of parity. (For pair formation we integrate (5) over available energy states as indicated

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19. S. Devons and G. R. Lindsey, Nature 164, 539; 1949. J. R. Oppenheimer, Phys. Rev. 60, 964; 1941.

20. F. K. McGowan, S. DeBenedetti, J. E. Francis, Jr., Phys. Rev. 75, 1761; 1949.

21. Experiments in progress at ORNL by F. K. McGowan seem to indicate that the zero-zero transition in Ge^{72} does not take place to the ground state. The presence of delayed gamma rays appears to argue against having the transition occur to ground. (Cf. also ref. 7). However, if the levels occur high up in the energy level scheme, one would expect them both to have odd parity, or a line directly to the presumably even ground state of Ge^{72} would be observed. All other levels between them and ground would be expected to have a spin ≥ 3, or the 0.7 Mev line would be washed out by cascade to ground.

22. This ratio would also be consistent with ordinary high order multipole conversion.

by (2).) Fowler[1] has estimated a lifetime of roughly 2.5×10^{-11} sec. for the RaC' level on the basis of competition with α-particle emission. Devons, Hereward, and Lindsay[23] have measured a mean life of 7×10^{-11} sec. for the O^{16} level. The lifetime of the Ge^{72} 0.7 Mev level has been determined[9,20] to be $1/3 \times 10^{-6}$ sec. In order to calculate a theoretical lifetime we replace the matrix element $\left\langle \sum_P r_P^2 \right\rangle$ by the square of the nuclear radius times an effective overlap factor for the initial and final states. Thus

$$\left\langle \sum_P r_P^2 \right\rangle = \left(\frac{1}{2} \alpha A^{1/3}\right)^2 \lambda .$$

For these three cases the number λ turns out to be of the order of 1/4 to 1/8. The consistency in the magnitude of λ in these three examples indicates an element of similarity in the different excited states.

It is a pleasure to acknowledge my appreciation to Dr. M. E. Rose for his suggestion of this investigation and for his helpful interest during its progress.

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23. Devons, Hereward, and Lindsay, Nature 164, 586; 1949.