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MATHEMATICS PANEL
QUARTERLY PROGRESS REPORT FOR THE PERIOD ENDING JULY 31, 1950

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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT
for the Period Ending July 31, 1950

A. S. Householder, Chief

DATE ISSUED: SEP 11 1950

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SUMMARY

T. W. Mullikin, of the University of Tennessee, is back with the Mathematics Panel for the summer as Research Participant. Also here as a Research Participant for the summer is Dr. C. L. Perry, of the University of Arkansas. Dr. L. J. Savage, of the University of Chicago, is now Consultant for the Mathematics Panel and the Biology Division, and in this capacity is spending the summer working on problems arising in biology.

Earl Burdett of the Engineering and Maintenance Division has taken up temporary residence in Chicago to assist in the construction of the Laboratory's electronic computer at Argonne. He is to be followed shortly by two Junior Engineers. Present estimates are that actual construction of the Oak Ridge machine will begin by the end of the calendar year and will be completed by about the end of 1951. While developmental work on the Williams storage tube still continues, it is of interest to note that two machines using this type of memory are now in actual operation. Both of these machines, the SEAC and the SWAC, were built by the National Bureau of Standards, the former at Washington, the latter at the Bureau's Institute for Numerical Analysis at Los Angeles.

Jack Moshman spent two weeks at the Argonne National Laboratory observing the methods used there in solving two-group pile problems by an iterative process on IBM machines for various geometries.

To facilitate training new people for computing on IBM equipment, F. C. Uffelman and Phyllis C. Johnson of Y-12 have planned the preparation of a group of lessons and notes to cover the machines available in the Y-12 computing laboratory. J. H. Fishel is writing the collator lessons, and Nancy M. Dismuke is writing the 604 lessons. Lessons for other machines are being written by members of the Y-12 group. Kathryn A. Pflueger is the latest member of the Panel to begin IBM training at Y-12.

Miss Johnson and Mrs. Dismuke spent two weeks at the Watson Laboratory and the IBM World Headquarters in New York gathering pointers on the use of IBM equipment and seeking assistance on an interpolation problem reported below.

Jack Moshman spoke to a group of biologists at an informal meeting in the Biology conference room on the use of IBM for recording and analyzing data.

[redacted]

Vivian Gordon, Ann Forbes, and Rooney Currey are on temporary loan to ANP at Y-12, doing 12-group pile calculations, according to a method developed at G.E., and other incidental computations. This work is not reported here. Since a series of the 12-group calculations are to be done over a period of time, these will be set up for the Card Programmed Calculator.

SPECIAL PROBLEMS AND PROJECTS

PROBLEM: Second Collison Density in Hydrogen

ORIGIN: E. P. Blizard, Physics Division

PARTICIPATING MEMBERS OF PANEL: G. E. Albert, T. W. Mullikin, M. L. Nelson

BACKGROUND AND STATUS: Neutrons of energy E impinge (perpendicular to the face) on a half space of hydrogen. The problem is to find the second collision density, supposing that

$$\frac{\lambda}{\lambda(\zeta)} = \left[\frac{E}{E'}\right]^{\nu/2} = \zeta^{-\nu} ,$$

where

ζ = cosine of scattering angle in laboratory,

E = energy before scattering,

E' = energy after scattering,

λ = mean free path at energy E,

$\lambda(\zeta)$ = mean free path after scattering through arc cosine ζ,

ν is a number between 1 and 2.

For unit source, the frequency of second collisions between t and $t + dt$ is $f(t)$ where t is measured in units of the mean free path at the initial energy E.

(1) $$e^t f(t) = \alpha \int_0^1 (1 + x)^{-\alpha} \left[1 - e^{-tx} \right] \frac{dx}{x}$$

(2) $$\alpha \equiv 2/(1 + \nu).$$

Since the frequency of first collisions is e^{-t}, equation (1) gives the ratio of the frequencies of second to first collisions. Two expansions were obtained for this function. One is

$$(3) \qquad \frac{1}{\alpha} e^{t} f(t) = \frac{t^{\alpha}}{\Gamma(\alpha)} \sum_{n=0}^{\infty} \frac{1}{(n+\alpha)^2} L_n^{\alpha}(t) ,$$

where L_n^{α} is the Laguerre Polynomial defined by

$$\left[\frac{d}{dx}\right]^n \{x^{n+\alpha} e^{-x}\} = \Gamma(n+1)\, x^{\alpha} e^{-x} L_n^{\alpha}(x) .$$

The other is

$$(4) \qquad \frac{1}{\alpha} e^{t} f(t) = \frac{-\Gamma'(\alpha)}{\Gamma(\alpha)} + \log t + -\sum_{k=1}^{n} (-1)^k \frac{\Gamma(\alpha+k)}{k\,\Gamma(\alpha)} t^{-k}$$

$$+ R_n(\alpha; t),$$

$$R_n(\alpha; t) \equiv (-1)^n \frac{\Gamma(\alpha+n+1)}{\Gamma(\alpha)} \int_t^{\infty} \frac{du}{u^{n+1}} \int_0^{\infty} \frac{e^{-ux}\, dx}{(1+x)^{n+1+\alpha}}$$

$$| R_n(\alpha; t) | \leq \frac{\Gamma(n+\alpha+1)}{\Gamma(\alpha)} t^{-n-1} .$$

The expression (3) is useful for small values of t while (4), an asymptotic expression, is useful for large values of t.

When $\alpha = 1$ ($\nu = 1$) we have the closed form

$$(5) \qquad \frac{1}{\alpha} e^t f(t) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)} + \log t + e^t E_1(t), \qquad \alpha = 1 ,$$

$$E_1(t) \equiv \int_t^\infty \frac{e^{-u}}{u} du .$$

The frequency function $f(t)$ is of considerable interest apart from this present context. For example it is noted that its moments are

$$\int_0^\infty t^n f(t)\, dt = \Gamma(n+1) \sum_{k=0}^{n} \frac{\alpha}{k+\alpha}$$

$$n = 0, 1, 2, \ldots .$$

For $\alpha = 1$, 0.9, and 0.8, that is $\nu = 1$, 1.22 ..., and 1.5, the corresponding functions $e^t f(t)$ have been plotted for t between 10 and 25.

In the interests of Mr. Blizard's application and for its pertinence to the general problem of neutron decay, a study of the distribution of the third collisions is contemplated.

PROBLEM: Evaluation of Monomeric and Dimeric Hydrolysis Constants of Thorium (IV)

ORIGIN: Dr. K. A. Kraus, Chemistry Division

PARTICIPATING MEMBERS OF PANEL: A. S. Householder, M. L. Nelson, T. W. Mullikin

STATUS: Completed

BACKGROUND: The chemistry of this problem will undoubtedly be discussed elsewhere by Dr. Kraus. Mathematically the problem here is to determine the constants A and B for which the observed values of N "best" fit the requirements

$$
\begin{array}{lllllll}
(1) & h^2 M & - A U & & & & = 0 \\
(2) & & - B U & + & h^2 D & & = 0 \\
(3) & M & + U & + & 2 D & & = t \\
(4) & 2 M & & + & 2 D & - t N & = 0
\end{array}
$$

where M, U, D and N are functions of h and t. For each of several values of t, a series of observations were made of N for various values of h.

Eliminating M, U, and D from equations (1), (2), (3), (4) gives a quadratic in N with coefficients which are functions of h and t. Direct application of least squares to this form is tedious. By regarding the (proper) solution of the quadratic for each observation as an independent variable, the problem is reduced to that of minimizing a function of many variables under a system of restraints. This problem was then solved by iterating a judiciously chosen linerarization of it.

PROBLEM: Monte Carlo Estimate of Ages and Collision Distribution in Tissue for 10 Mev Source Neutrons

ORIGIN: Dr. W. S. Snyder and Dr. J. Neufeld, Health Physics Division

PARTICIPATING MEMBERS OF PANEL: A. S. Householder, R. R. Coveyou, M. R. Arnette

BACKGROUND: See *Mathematics and Computing Panel Quarterly Progress Report, December, January, February, 1948-49*, A. S. Householder, ORNL 345, p. 11.

STATUS: The two thousand neutron histories have been completed on the IBM equipment at Y-12. A double entry table of all collisions occurring within definite ranges of E (u interval = .5, where $u = \ln E$) and r^2 (r interval = 1 cm) has been made, and other computations are being planned.

PROBLEM: Monte Carlo Estimates of Age in Water from 10 Mev

ORIGIN: E. P. Blizard, Physics Division

PARTICIPATING MEMBERS OF PANEL: M. R. Arnette, N. M. Dismuke

BACKGROUND AND STATUS: The Monte Carlo sample for the tissue problem (see above) can be used to provide these age estimates by sorting out all collisions with elements other than hydrogen and oxygen, and completing some histories to fill out an adequate sample.

PROBLEM: Basic Studies in the Monte Carlo Method

PARTICIPATING MEMBER OF PANEL G. E. Albert

REFERENCE: Same title, *Mathematics Panel Quarterly Progress Report for the Period Ending April 30, 1950*, ORNL 726, p. 8.

STATUS: A comparison was made between the use of importance function sampling, representative stratified sampling and systematic sampling plans for the evaluation of several simple definite integrals. No general results have been obtained.

The substance of a working paper "Quota Sampling and Importance Functions in Stochastic Solution of Particle Problems" by G. Goertzel, NDA, June 21, 1949, has been incorporated into a mathematically rigorous discussion of the same subject. This work is in a preliminary rough draft form.

PROBLEM: The Analysis of β-ray Spectroscopy

ORIGIN: P. R. Bell, Physics Division

PARTICIPATING MEMBERS OF PANEL: A. S. Householder, M. L. Nelson, G. E. Albert

BACKGROUND AND STATUS: In order to obtain a β spectrum, several stages of amplification are required, each stage introducing additional statistical variation. Thus the precise relation between the observed signal and the actual input can only be understood in terms of the theory of multiplicative processes. The theoretical question in these terms has to do with the statistical distribution of the responses produced by a monochromatic source of any given energy. This more basic problem is being attacked in connection with a general background study of multiplicative processes.

In the meantime, Mr. Bell has suggested that a Poisson distribution appears to be a reasonable approximation, though it is not theoretically correct. Consequently a numerical procedure is being devised for obtaining the source distribution on the assumption that the response does have a Poisson distribution.

Let $G(\xi)$ represent a measure of the total number of electrons in the source with energy not exceeding $\lambda\xi$, where λ is an unknown scale factor. Let

$$K(x, \xi) = \xi^x \exp(-\xi)/x!.$$

This is the Poisson distribution with mean and variance ξ, representing the probability that a source electron of energy $\lambda\xi$ will yield a response showing an energy λx. In some cases λ can be estimated independently. The intensity of the response at any given energy λx is given by

$$f(x) = \int_0^\infty K(x,\xi)\, dG(\xi),$$

where the integral is a Riemann-Stieltjes integral. Measurements are made for a limited number of energies represented by x_i. One might attempt to represent the functions $f(x)$ and $G(x)$ as linear combinations of some basic sets of functions, but from the nature of the graphs this approach looks unpromising. The alternative is to represent the integral as a finite sum and attempt to evaluate $G(\xi)$ approximately at a limited number of points ξ_a. This involves replacing the integral equation by the finite system

$$f_i = \sum K_{ia} \, [G_{a+1} - G_a]$$

where

$$G_a = G(\xi_a), \qquad f_i = f(x_i),$$

the x_i being points of observation while the ξ_a are any arbitrarily selected abscissae. Also

$$K_{ia} = K(x_i, \xi_a'),$$

where

$$\xi_a \leq \xi_a' \leq \xi_{a+1}.$$

By the mean value theorem, if the functions are sufficiently continuous, then for any set ξ_a there exists a set ξ_a' for which the finite system is exact. However, we do not know these and can only take each ξ_a' to be perhaps in the middle of the interval on which it is known to lie.

In principle one could take as many distinct ξ_a as there are x_i, but in view of the presence of errors of measurement, least-squares solution for a smaller number seems indicated. The ξ_a can be distributed according to convenience, but should be more dense where the variation in G is expected to be most rapid. Since $G_0 = 0$, we can rewrite the equations in the form

$$f_i = \sum_{1}^{\nu} (K_{i,\alpha-1} - K_{1,\alpha}) \; G_\alpha + K_\nu \; G_\nu \; ,$$

where ξ_ν is the largest ξ to be included.

Either the Fairchild machine or IBM may be used for the solution.

PROBLEM: Chromatography

ORIGIN: Dr. G. E. Boyd, Chemistry Division

PARTICIPATING MEMBERS OF PANEL: R. R. Coveyou, V. M. Gordon, M. L. Nelson

REFERENCE: "A Random Walk Whose Equation is Hyperbolic," *Mathematics Panel Quarterly Progress Report for Period Ending January 31, 1950,* ORNL-634, pp. 14-15.

STATUS: The tabulation referred to in the above reference has been temporarily laid aside, but will be completed and a final report prepared.

PROBLEM: Kinetics of Homogeneous Reactor

ORIGIN: C. B. Graham and J. M. Stein, Reactor Technology Division

PARTICIPATING MEMBERS OF PANEL: M. L. Nelson, R. R. Coveyou, E. A. Forbes, J. H. Fishel, A. R. Currey, V. M. Gordon

REFERENCE: *Mathematics Panel Quarterly Progress Report for the Period Ending April 30, 1950,* ORNL 726, pp. 16-17.

STATUS: The computations reported in the preceding reference have been continued, and reports made to Mr. Stein. Need for further calculations may arise.

PROBLEM (continuation) Determination of a Potential Distribution

ORIGIN: Dr. W. M. Good and R. W. Lamphere, Physics Division

PARTICIPATING MEMBERS OF PANEL: A. S. Householder, C. L. Perry, C. W. Mullikin, C. Perhacs, A. R. Currey

BACKGROUND AND STATUS: Mr. Lamphere wants to know the electron trajectories in an acceleration tube. The electron trajectory depends on the field distribution in the tube and the electron energy. The field distribution in a symmetrical tube has been determined by C. Perhacs using the Liebman net method and a desk computer. The Fairchild Linear Equation Solver is Computing the same field distribution in which a Liebman net much finer than was practical by hand is being used. This field determination has also been made by J. Kington of the Physics Division using an electrolytic analog of the tube. The results from these three methods will soon be compared. Mr. Kington has also

determined the field in some asymmetric tubes. Electron trajectories have been determined by Messrs. Kington and Perhacs for several electron energies and tube fields.

PROBLEM: Atmospheric Differences of Mitotic Effects of Irradiation

ORIGIN: Dr. M. E. Gauldin, Biology Division

PARTICIPATING MEMBERS OF PANEL: J. Moshman, A. R. Currey

BACKGROUND: Various studies have been made on the mitotic effect of X-irradiation on grasshopper embryos under atmospheres containing various proportions of oxygen, ranging from 0% to 100%. Three possible metrics of radiation effect were considered in order to describe the atmospheric influence.

1) Duration of period of mitotic inhibition. There is a period of time in which the mitotic cycle is apparently suspended, but due to the fact that this period is known only in intervals of 22 minutes, the range of variation is too low to serve as an adequate metric.

2) Total cells observed in 374 minutes. The total number of cells undergoing a specific phase of mitosis is a better metric than the foregoing, but suffers from the defect that the confidence intervals of the radiation effect of varying proportions of oxygen, based upon the effect in a vacuum, are too wide to be very meaningful.

3) Time interval to peak of mitotic activity. Following the period of mitotic inhibition, there is an observable spurt of activity to some modal value, following which there is a leveling off to approximate the control activity. This measure appears to be the most desirable metric.

Fifty per cent confidence limits were computed giving the radiation effect in comparison with vacuum. It is known that if $\bar{x}$ and $\bar{y}$ are normally distributed with zero mean and variances σ_x^2 and σ_y^2 respectively, if we let

$$z = \frac{b + \bar{x}}{a + \bar{y}} ,$$

then

$$t = \frac{a\ z - b}{\sqrt{\sigma_y^2\ z^2 + \sigma_x^2}}$$

is normally distributed with zero mean and unit variance. This was the basis for the computation of confidence intervals. The derivation of this result is due to Geary (*Journal of the Royal Statistical Society* 93:442).

STATUS: The results revealed a steady rise in effect from vacuum to 21% oxygen (air) and a leveling off from air to 100% oxygen, except for an anomalous value for 10%. The latter value is being rechecked with an additional experiment before the final curve of radiation effect is fitted.

PROBLEM: **(continued)** Photoreactivation Effect on Paramecia

ORIGIN: Dr. R. F. Kimball, Biology Division

PARTICIPATING MEMBERS OF PANEL: J. Moshman, A. R. Currey

STATUS: Families of regression equations have been computed and significances assessed.

PROBLEM: Effect of Irradiation on the Transmission of Light Through Stained Cells

ORIGIN: N. Harrington, Biology Division

PARTICIPATING MEMBER OF PANEL: J. Moshman

BACKGROUND: It was conjectured that irradiation alters the amount of light passing through stained cells. An experiment was performed measuring the light transmission through irradiated and control cells under 10, 20, 30 and 40 minute hydrolysis periods. By means of a standard analysis of variance, it was found that there was no significant difference between irradiated and control cells, but a highly significant difference between hydrolysis periods. Furthermore there existed no significant interaction between hydrolysis and irradiation.

PROBLEM: Effect of Irradiation on Cell Diameters

ORIGIN: N. Harrington, R. W. Koza, Biology Division

PARTICIPATING MEMBER OF PANEL: J. Moshman

BACKGROUND: Mean diameters for control and irradiated cells under three hydrolysis periods were submitted. It was desired to ascertain the significance of observed differences.

The data were obviously non-normal. The analysis consisted of an application of Mosteller's k-sample slippage test (*Annals of Mathematical Statistics*, 19:58). All three groups showed a significant slippage to the right.

Non-parametric confidence intervals with confidence coefficients of 95%, 98% and 99% were fitted to each of the three sets of data by the method of Pitman (*Journal of the Royal Statistical Society Supplement*, 4:119).

PROBLEM: **(continued)** Determination of the Neutron Flux in the ORNL Pile

ORIGIN: Dr. D. K. Holmes, Physics Division

PARTICIPATING MEMBER OF PANEL: J. Moshman

BACKGROUND AND STATUS: The programming of the Monte Carlo analysis is under way. The first attempt will be to examine the practicality of using the position of the first collisions outside the core as a basis of stratification.

If t strata exist, with the probability of a neutron falling in the i-th stratum being p_i and an estimate of the variance of i-th stratum being s_i, then the variance of the mean flux will be

$$s^2 = \frac{1}{n} \sum_{i=1}^{t} p_i \, s_i^2 \tag{1}$$

where n is the total sample size.

One method of weighting would be to use an adaptation of von Neumann's and call each neutron k_i neutrons if it falls in the i-th stratum. Then

the variance of the mean is

$$s'^2 = \sum_{i=1}^{t} \frac{p_i \, s_i^2}{k_i} .$$

Letting

$$\sum_{i=1}^{t} p_i \, k_i = n ,$$

and to determine the k_i, subject to the above restriction, such that s^2 is minimized, let

$$G(k_1, k_2, \ldots k_t, \lambda) = \sum \frac{p_i \, s_i^2}{k_i} + \lambda \sum p_i \, k_i - \lambda n ,$$

where λ is a Lagrange multiplier. Then

$$\frac{\partial G}{k_i} = \frac{-p_i \, s_i^2}{k_i^2} + \lambda, \qquad i = 1, 2, \ldots, t,$$

$$\frac{\partial G}{\partial \lambda} = \sum p_i \, k_i - n .$$

Setting the partial derivatives equal to zero we see that

$$\frac{s_i}{s_j} = \frac{k_i}{k_j} ,$$

namely, that each k_i is proportional to the s_i of that stratum and finally

(2) $$s'^2 = \frac{1}{n} \left[\sum p_i \, s_i \right]^2 .$$

It is apparent that (2) will give, generally, a lower value of s^2.

A third possibility exists in an adaptation of the usual procedure of stratification. If the population of neutrons consist of N fission neutrons emitted per unit time, of which N_i are in the i-th stratum, then the variance of the mean flux would be

$$s''^2 = \frac{N - n}{N\,n} \sum p_i\, s_i^2 + \sum n_i \left[\frac{N_i\, s_i}{n_i} - \frac{\sum (N_j\, s_j)}{n} \right]^2$$

$$(3) \qquad - \frac{1}{Nn} \sum N_i\, (s_i - \sum p_j\, s_j)^2.$$

If the n_i, ($\Sigma\, n_i = n$) the sample size in the i-th stratum, is chosen proportional to the $N_i\, s_i$, or equivalently the $p_i\, s_i$, the middle term in (3) vanishes. If N is large, then essentially

$$(4) \qquad s''^2 = s^2 - \frac{1}{N\,n} \sum N_i\, (s_i - \sum p_j\, s_j)^2 \quad ,$$

which is also an improvement over (1). The relative merits of (2) and (3) depend upon the size of the last term in (4). If the strata are chosen to be widely different, then that term will be large and s''^2 small. A trial sample will be run to determine the feasibility of using (3) to estimate the variance. If this method of stratification is chosen, it will be impossible to allocate the sample optimally at first, but will be necessary to follow each neutron through its first collision and then throw away each excess neutron after the stratum quota is filled. This question will have to be decided in light of the results of the trial run.

PROBLEM: Internal Conversion in K-shell—Calculation of Interpolating Polynomial

ORIGIN: Dr. M. E. Rose, Physics Division

PARTICIPATING MEMBERS OF PANEL: M. L. Nelson, R. R. Coveyou, N. M. Dismuke, E. A. Forbes

STATUS: The original data were the internal coefficients (electric and magnetic) as functions of γ-energy k, atomic number z, and multipole order l:

$$E_l\ (k,\ z) \text{ and } M_l\ (k,\ z)$$

For each type and multipole order we have the values of the functions at points of a certain grid in the k-z plane. For purposes of regularization, the values of $\log E_l$ ($\log M_l$) were calculated as functions of $x = \log z$, $y = \log k$. Each of the ten functions is being fitted least squares by a polynomial of 21 terms and maximum degree 5 in x and y combined. It is anticipated that the computations of these polynomials will be finished shortly and checking of the results against the data will then show the reliability of approximation.

Hand calculations indicated the need for carrying a fairly large number of significant figures. The machine calculation will carry 15 significant figures. The C.P.C. will be used for performing 15 × 15 digit multiplications, additions and sums of products. Since available storage capacity is soon exhausted by such large factors, many intermediate steps will be punched on data cards for later use, making the calculation semi-automatic.

PROBLEM: Age in D_2O, H_2O

ORIGIN: L. C. Noderer, Reactor Technology Division

PARTICIPATING MEMBER OF PANEL: N. M. Dismuke

STATUS: In progress.

PROBLEM: Fermi Functions

ORIGIN: Dr. M. E. Rose and P. R. Bell Physics Division

PARTICIPATING MEMBER OF PANEL: No Panel member participating, Phyllis C. Johnson is directing the calculations.

STATUS: In progress.

PROBLEM: **(continuation)** K-capture and β-decay, Correction for Finite Size of Nucleus

ORIGIN: Dr. D. K. Holmes and Dr. M. E. Rose, Physics Division

PARTICIPATING MEMBER OF PANEL: E. A. Forbes

BACKGROUND AND STATUS: This will involve some IBM computations to be done and reported later. Preliminary hand computations have been completed.

PROBLEM: Solution of a 7th Degree Algebraic Equation

ORIGIN: S. H. Hanauer, Physics Division

PARTICIPATING MEMBER OF PANEL: T. W. Mullikin

STATUS: The equation had the form

$$a + s = \sum_{1}^{6} a_i/(a_i + s) = 0$$

with given values of the a's and a's. Solutions were obtained by Newton's method.

PROBLEM: Aircraft Shield Evaluation

ORIGIN: Dr. C. Goodman and the ANP

PARTICIPATING MEMBER OF PANEL: D. W. Whitcombe

BACKGROUND: The summer visitors at Y-12 with the ANP are assembling a report to revise the thinking on the feasibility of nuclear energy for the propulsion of aircraft since the time of the Lexington report. The shield revision was under the direction of Dr. Clark Goodman. It was his hope to decide the relative merits and faults of about 20 shields, since the Lexington shield. It became apparent to Dr. Goodman that finding the pertinent parameters and analysing the existing shields was a big problem and would require outside assistance. During three weeks spent with Dr. Goodman, I read reports on all of the most promising shields, including those of Dr. Echols' Theoretical Physics Group at NEPA, the Thompson-Stern Shield at NEPA, and the Bethe shield. From the reports and from discussions it was possible to analyse the shields according to the following subheads:

(1) Title of Report and Cycle

(2) Reactor Crew Separation (100 ft in most cases)

(3) Reactor Size (2, 3, 4 ft in most cases)

(4) Reactor Power (400,000 kw in most cases)

(5) Shield Weight and Structure

(6) Basic Assumption

(7) Remarks and Reliability.

In columns adjacent to the column listing of the subheads (1) through (7), the various shields were summarized in the form of a large table. The dimensions of the table were about 5 × 9 ft which will be reduced to 8½ × 11 in. when it is included in the ANP report.

PROBLEM: Temperature Distribution in a Uranium Slug

ORIGIN: G. H. Boss, Metallurgy Division

PARTICIPATING MEMBER OF PANEL: D. W. Whitcombe

BACKGROUND: Several months ago a rather elaborate experiment was set up in the Metallurgy building to check some of the physical properties of uranium. Currents of the order of 15,000 amperes were run through the slugs and from these data it was hoped that the radial distribution of temperature from the center of the slug to the radial face could be found for the steady state case. The equation to be solved here is written

$$T_{rr} + \frac{1}{r} T_r + M E^2 (1 - \alpha T + \beta T^2) = 0.$$

where $T(r)$ is the radial distribution of temperature, M, α, β are the constants of the uranium, and E is the voltage drop across the rod. The subscripts r refer to derivatives with respect to r. Although the problem is non-linear it is found that a power series in even powers of r converges rapidly.

A satisfactory solution was found by approximating the power series by a Bessel function. This solution fits the observed central temperatures within 5% for most of the cases. An improvement is improbable since the experiments were not repeatable to any greater accuracy. This is because the bars were exposed to severe stresses, causing warping. In a few special cases the bars even developed cracks. A memo on the above problem will be issued in the near future.

PROBLEM: A Diffusion Solution for a Black Thermal Neutron Pipe Duct Containing a Pure Scattering Material

ORIGIN: Dr. N. M. Smith and ANP Shielding Group

PARTICIPATING MEMBER OF PANEL: D. W. Whitcombe

BACKGROUND: The above report was issued as ORNL-751 in July. The problem is solved in terms of r_0 and z_0, the extrapolated boundaries of the cylinder, and these are related to the actual boundaries. The exit current has been found at $r = 0$, $z = z_1$. In a sample problem for the case of bismuth it is found that the attenuation is 10^{-8} for a duct of 24 cm radius, 5 ft length, and a 4 cm transport-mean-free-path.

PROBLEM: A Machine Solution for the Fundamental Transport Equation in One Dimension

ORIGIN: Dr. A. S. Householder, Dr. R. L. Echols and E. S. Bettis

PARTICIPATING MEMBER OF PANEL: D. W. Whitcombe

BACKGROUND: The equation referred to as the fundamental transport equation is written

$$(1) \qquad \varphi(x) = \lambda \int_0^a \varphi(y)\, E_1(|y - x|)\, dy + S(x).$$

The solution to the above equation, with the E_1 kernel was solved for two different source terms: (1) $S(x) = e^{-x}$, *i.e.*, a collimated beam source, (2) $S(x) = E_1(x)$, an isotropic source. The extent of the upper limit a was 10 mean free path units, which corresponds in this case to about 3 cm of body tissue. The above problem was solved in the case of the collimated beam and for infinite geometry by Dr. Snyder at X-10, using a rather neat consideration of upper and lower bounds. A table is included here containing Dr. Snyder's solution compared with the one obtained on the Fairchild machine:

$\varphi(x)$	DR. SNYDER'S COLLIMATED BEAM	FAIRCHILD MACHINE SOLUTION	
		COLLIMATED BEAM	ISOTROPIC SOURCE
$\varphi(x_1)$	3.1	1.7	.88
$\varphi(x_2)$	3.4	1.6	.73
$\varphi(x_3)$	2.8	1.3	.58
$\varphi(x_4)$	2.6	1.1	.47
$\varphi(x_5)$	2.2	.90	.37
$\varphi(x_6)$	1.8	.69	.29
$\varphi(x_7)$	1.6	.52	.22
$\varphi(x_8)$	1.4	.38	.16
$\varphi(x_9)$	1.2	.26	.11
$\varphi(x_{10})$	1.0	.15	.08

It should be noticed that the values obtained by Dr. Snyder are larger. This is because all the back scattering for $x > 10$ was absent in the Fairchild machine solution. Further one should note $\varphi(x_2) > \varphi(x_1)$ in Dr. Snyder's solution but not in the Fairchild solution. Whether this anomaly is due to the coarseness of the grid, or actually exists, is not known.

The above problem was preliminary to a larger one, corresponding to about 35 cm of body tissue (essentially infinite). The larger problem will also be solved for both of the above mentioned sources. All the matrix elements have been computed for this larger problem, including a change in the exponential transformation suggested by the solution to the smaller problem. This problem will be put on the machine soon. It will be interesting to see if $\varphi(x_2) > \varphi(x_1)$.

PROBLEM: Heat Transfer by a Fluid Flowing with a Parabolic Flow Distribution in a Hot Pipe

ORIGIN: C. Claiborne, Reactor Technology Division

PARTICIPATING MEMBERS OF PANEL: C. L. Perry, R. R. Coveyou, D. W. Whitcombe

BACKGROUND AND STATUS: Mr. Claiborne wanted to determine the isotherms in a fluid flowing in a hot circular pipe with a parabolic velocity distribution. He was particularly interested in the isotherms near the entrance of the tube. The following is the boundary value problem used to describe the temperature distribution:

$$k\left[1-\left(\frac{r}{r_0}\right)^2\right]\frac{\partial T}{\partial z}=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2},\quad |r|<r_0,\quad z>0;$$

$$\frac{\partial T}{\partial r}(z,\ 0)=0,\quad T(z,\ r_0)=T_0,\quad z>0;$$

$$T(0,\ r)=0,\quad \left\{\lim_{z\to\infty} T(z,\ r)=0 \text{ or } \frac{\partial T}{\partial z}(0,\ r)=0\right\},\quad |r|<r_0;$$

where k, T_0 and r_0 are constants. Mr. Claiborne stated that his present interest in the problem did not warrant the expenditure of a great deal of work on the problem. It has been found that Graetz (1885), Nusselt (1910), Groeber (1921), Drew (1931), Jakob and Eck (1933) and recently Boelter and Martinelli have been interested in this problem. The assumption that $\partial^2 T/\partial z^2$ is negligible changes the above problem to a simpler problem that can be solved by the separation of variables method. Mr. Claiborne has stated that this assumption will give him results that are as accurate as he requires. He has been referred to the results published by the above authors. The Mathematics Panel has found the solution (in terms of the inverse Laplace transform of a confluent hypergeometric function) to the original problem.

STATUS: Further investigation of the problem, or Math Panel solution, is now only of academic interest.

PROBLEM: Steady State, Turbulent, Fluid Flow in a Rectangular Pipe

ORIGIN: Dr. H. F. Poppendiek, Reactor Technology Division

PARTICIPATING MEMBERS OF PANEL: A. S. Householder, C. L. Perry

BACKGROUND: Dr. Poppendiek has found that the following boundary value problem describes the velocity distribution, u, in a turbulent fluid flowing in a rectangular pipe. The differential equation and boundary conditions he obtained are:

$$(u + c)\left[\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + \left[\frac{\partial u}{\partial y}\right] + \left[\frac{\partial u}{\partial z}\right]^2 = c', \quad |y| < l_1, \quad |z| < l_2$$

$$u = 0, \qquad |y| = l_1 \quad \text{or} \quad |z| = l_2;$$

where c and c' are constants. The following change of dependent functions

$$\psi = u^2 + 2\,c\,u - c'\,(y^2 - l_1^2)$$

reduces the above problem to the harmonic problem:

$$\frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = 0, \quad |y| < l_1, \quad |z| < l_2, \quad \psi(l_1, z) = 0, \quad \psi(y, l_2) = c' \ (l_1^2 - y^2).$$

STATUS: A rapidly converging series solution to the problem has been given for the general case. The Fairchild Linear Equation Solver can be used to obtain the numerical solution for particular flows. The analysis is complete except for the evaluation of the flow velocities in the cases in which Dr. Poppendiek is interested.

PROBLEM: Fluid Flow in Cylindrical Pipes

ORIGIN: C. Claiborne, Reactor Technology Division

PARTICIPATING MEMBER OF PANEL: C. L. Perry

BACKGROUND AND STATUS: The flow distribution was desired for a fluid flowing uniformly in cylinders whose cross sections were squares, isosceles right triangles, and equilateral triangles. These problems were shown to be equivalent to the corresponding torsion problems of elasticity. Mr. Claiborne was referred to the published solutions of the torsion problem for the above geometries.

PROBLEM: Heat Transfer by a Fluid Flowing Turbulently in a Cylindrical Tube

ORIGIN: Dr. R. V. Bailey, Reactor Technology Division

PARTICIPATING MEMBERS OF PANEL: A. S. Householder, C. L. Perry

BACKGROUND: Dr. Bailey is interested in the numerical determination of the temperature distribution in a fluid flowing turbulently in a circular cylindrical hot tube. The thermal and hydrodynamic boundaries are assumed to be established simultaneously. The parabolic partial differential equation describing the distribution is

$$\frac{\partial}{\partial s}\left\{p\ (s, z)\ \frac{\partial T}{\partial s}\right\} = q\ (s,\ z)\frac{\partial T}{\partial z} + r\ (s,\ z)\ \frac{\partial T}{\partial z},$$

where T is the temperature, s is a radial coordinate, z is the distance along the axis of the tube and p, q and r are given smooth functions. J. von Neumann has discussed the numerical solution of the above type of equation (Los Alamos report number 657).

STATUS: The Mathematics Panel is investigating the possibility of using the IBM equipment to find the numerical solution to the above problem. Dr. Bailey is determining the functions p, q, r and the number of difference equations necessary to obtain the accuracy he desires near the mouth of the tube.

PROBLEM: A Determination of the Proportion of Mice Carrying a Lethal Gene

ORIGIN: Dr. W. L. Russell, Biology Division

PARTICIPATING MEMBERS OF PANEL: L. J. Savage, A. S. Householder, J. Moshman

BACKGROUND: This is a continuation of work reported under the same title in *Mathematics Panel Quarterly Progress Report for the Period Ending April 30, 1950,* ORNL 726, p. 13.

Past work on the problem has been concerned primarily with testing sequentially whether individual females in a population of mice carry sex-linked lethal genes, and secondarily with estimating the frequency of lethal bearing mice in the population. If estimation is regarded as primary and testing as secondary different sequential sampling plans promise to be much more efficient.

The work is still incomplete, but some progress toward finding sampling schemes efficient for estimation has been made by examining the information (in Fisher's sense) and the average sample number of sampling schemes in general.