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THERMAL NEUTRON FLUX DISTRIBUTIONS WHEN AIR VOIDS ARE PRESENT

DAVID W. WHITCOMBE

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THERMAL NEUTRON FLUX DISTRIBUTIONS WHEN AIR VOIDS ARE PRESENT

David W. Whitcombe*

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ABSTRACT

In this paper thermal neutron flux distributions have been found principally in two dimensions for a variety of shield materials which contain air voids. Problems are solved where the shield material contains one, two, or three air voids and for a straight through air duct. Most problems are solved for both finite and infinite geometries. A problem to indicate the effect of shifting a void is included as well as a three dimension problem with two air voids.

In all cases a plane source of neutrons (isotropic or collimated) is used with the boundary condition that the return neutron current shall vanish. The diffusion equation was reduced to linear difference equations and solved on the Fairchild-Linear Equation Solver. A more exact replacement for the Laplacian in the diffusion equation was also made. This permitted calculations of the streaming as well as an estimate of the error to be made.

THERMAL NEUTRON FLUX DISTRIBUTIONS WHEN AIR VOIDS ARE PRESENT

David W. Whitcombe*

INTRODUCTION

The solution for the neutron flux at all points of a finite or infinite medium with a void present has not been solved satisfactorily even for the simplest geometries. With the introduction of the Fairchild machine computer, these problems may now be approximately solved with few simplifying assumptions. The scheme is not limited to the consideration of only special geometries. For example, the solutions for square cylindrical ducts, cubical voids and even voids with angular bends are comparatively easy to obtain. Since in these computations the medium will be replaced by a grid (or lattice of finite area elements in the plane or a lattice of finite volume elements in space) it is desirable that the geometry of the problem correspond to a natural grid. That is, if the void is cubical and a plane source of neutrons is used, one would choose a grid of finite cubes the edge length of which was an integral divisor of the edge length of void.

A more interesting problem of this nature is one with two or more cubical voids. Then the scattering from one to the other may be found; little is known about this process.

ILLUSTRATIVE EXAMPLE

A very simple case may be solved by hand. This will show the procedure and the flexibility of the method. In Fig. 1

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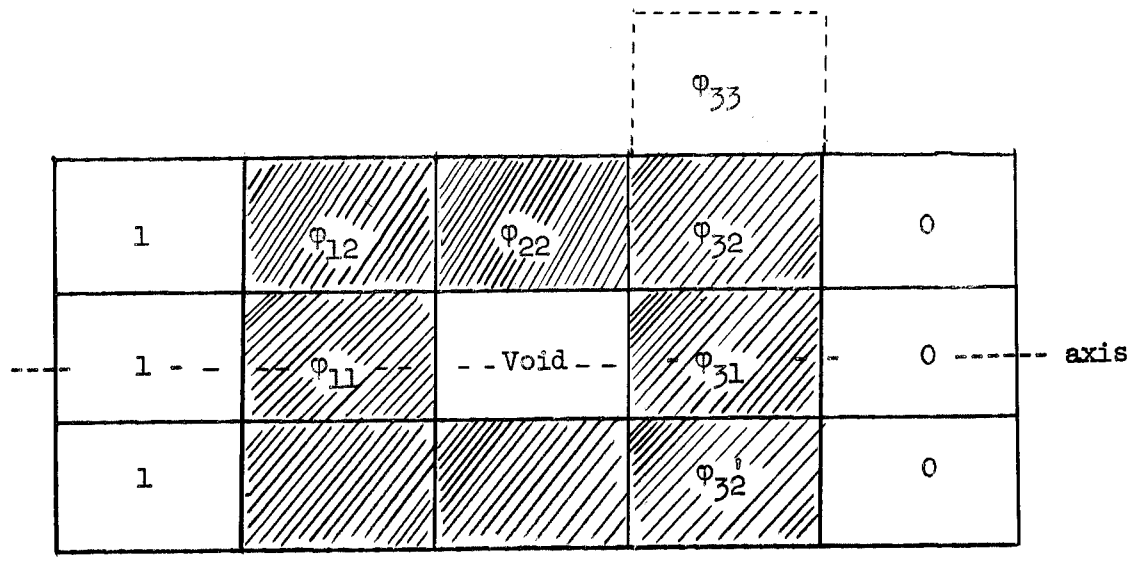

Fig. 1

the shaded material represents shield material with a void of square cross section in the middle. The void is infinite in length in the dimension perpendicular to the page. It is assumed that there is a unit isotropic source of thermal neutrons. A course grid is used. The cross sectional area of shield material is divided up into five representative squares each the same size as the void. The diffusion equation will give a relation among the φ's where the φ's give the flux at the center of the box. The boundary condition on the right edge of the material (that the return current is zero) is represented by the column of boxes which contain zeros. The concrete extends infinitely above and below the shaded portions in Fig. 1. This is handled by an image technique which here will be only approximate. That is, assume that the lattice is a "unit-cell" that repeats itself above and below. The box φ_{33} (above φ_{32}) has the same flux value as φ_{32}' and by the symmetry of the problem in the axis $\varphi_{32}' = \varphi_{32}$, hence $\varphi_{33} = \varphi_{32}$ under these assumptions.

The diffusion equation is written

$$\nabla^2 \varphi - k^2 \varphi = 0 . \tag{1}$$

For concrete* choose $k = \frac{1}{L} = \frac{1}{7.6 \text{ cm}}$. Also, take the edge of the void to be 4.3 cm, which is the lattice spacing used in a problem being prepared for the Fairchild machine. In two dimensions the $\nabla^2 \varphi$ is replaced by

$$\nabla^2 \varphi(x,y) = \frac{\varphi(x + h,y) + \varphi(x - h,y) + \varphi(x,y + h) + \varphi(x,y - h) - 4\varphi(x,y)}{h^2} . \tag{2}$$

Substitute (2) in (1) to obtain

$$\Sigma \varphi_n = \sigma \varphi(p) \tag{3}$$

where p refers to the point (x,y)

$$\sigma = k^2 h^2 + 4$$

$$= 4.32 \text{ for concrete}$$

and $\Sigma \varphi_n$ is the sum of the flux values about all the four neighbors, as indicated in Eq. (2) of the point p.

---

*Technical Report No. 24, Nuclear Shielding Studies, Victor Delano and Clark Goodman, p. 69.

It should be noted that equation (2) assumes that the neutrons in the concrete can go only in one of 4 equally probable directions. This approximation will be used in this example, but a more general replacement can be made.

Further it will be assumed that the neutron enjoys a free ride through the void.

To write the linear equation for φ_{11}, multiply φ_{11} by $\sigma = 4.32$ and subtract from it the φ-values of the 4 neighbors; since the void is a nearest neighbor one substitutes φ_{31}. Following this pattern there is an equation for each unknown, five in all or

$$(4a) \qquad 4.32\,\varphi_{11} - \varphi_{12} - \varphi_{31} - \varphi_{12} = 1$$

$$(4b) \qquad 4.32\,\varphi_{12} - \varphi_{12} - \varphi_{22} - \varphi_{11} = 1$$

$$(4c) \qquad 4.32\,\varphi_{22} - \varphi_{12} - \varphi_{32} - \varphi_{22} - \varphi_{22} = 0$$

$$(4d) \qquad 4.32\,\varphi_{32} - \varphi_{22} - \varphi_{31} - 0 - \varphi_{32} = 0$$

$$(4e) \qquad 4.32\,\varphi_{31} - \varphi_{11} - \varphi_{32} - 0 - \varphi_{32} = 0.$$

Like terms may be collected and then (4) may be solved by the method of substitution. That is solve (4e) for φ_{11} and substitute in the other four equations and repeat the elimination. The results may be expressed in the following convenient form in Fig. 2.

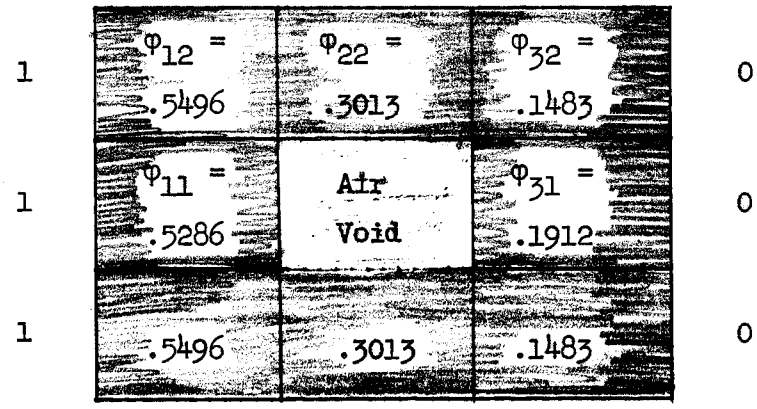

Fig. 2

where the shaded area is concrete, and an isotropic source of thermal neutrons is used. The values in the lower line were included from the symmetry of the problem. It should be noted that $\varphi_{11} < \varphi_{12}$ because the back scattering into φ_{11} is less than into φ_{12}. And $\varphi_{32} < \varphi_{31}$ because the neutrons in φ_{11} get a free ride through the void. Further the neutrons are attenuated roughly as an exponential because the geometrical progression is roughly satisfied. That is

$$\frac{1}{.5496} \sim \frac{.5496}{.3013} \sim \frac{.3013}{.1483}$$

or

$$1.82 \sim 1.82 \sim 2.03.$$

The neutrons would then be decayed by an exponential of the form

$$\frac{1}{1.82} \cong e^{-k\,h} .$$

Then $k\,h \approx .6$ and since $h = 4.3$ cm

$$k \approx \frac{.6}{4.3 \text{ cm}} \approx \frac{1}{7.2 \text{ cm}}$$

or L, the diffusion length is about 7.2 cm. This agrees very well with the value of L = 7.6 cm used in computations. It is not the same because so few points are used and because of the presence of the void.

REMARKS

The above sample problem illustrates all the technique necessary to solve for the neutron flux when a plane isotropic source of neutrons is used with boundary condition on the outside face that the return current shall vanish. Since there were only five unknowns these could be found by hand in less than a half hour. If there are as many as ten unknowns the time to obtain the solution much more than doubles itself. In the appendix of solved problems which follows, there are at least fifteen unknowns and up to 123 unknowns; in these cases it is much more expedient to obtain the solutions using the Fairchild Computing machine. This machine would permit dividing the region into 300 elements of area (or 600 if the medium possessed an axis or a plane of symmetry perpendicular to the source plane).

With the addition of this facility it would also be feasible to use a more precise replacement for ∇^2 than the one used above which considered only the nearest neighbors.

The procedure for making this more general replacement is explained in a paper by W. G. Bickley*. The diffusion equation $\nabla^2 \varphi - k^2 \varphi = 0$ then becomes

$$-20\, \varphi(p) + 4 \sum \varphi_n + \sum \varphi_c - 6\, k^2 h^2 \varphi(p) = 0$$

or

*W. G. Bickley, Finite Difference Formulae for the Square Lattice. Quart. I Mech. Appl. Math., Vol. 1, pp. 35-42 (1948).

$$\lambda\, \varphi(p) = 4 \sum \varphi_n + \sum \varphi_c$$

where the φ_n's are the nearest neighbors of $\varphi(p)$ and φ_c's are the nearest corners as shown below

$$\begin{array}{ccccc} \varphi_{c1} & - & \varphi_{n2} & - & \varphi_{c2} \\ \cdot & & \cdot & & \cdot \\ \varphi_{n1} & - & \varphi(p) & - & \varphi_{n3} \\ \cdot & & \cdot & & \cdot \\ \varphi_{c3} & - & \varphi_{n4} & - & \varphi_{c4} \end{array}$$

and $\lambda = 6\sigma - 4$.

This more exact replacement for ∇^2 includes a first order perturbation of the streaming. It also furnishes a means of estimating the error in the more elementary replacement for ∇^2 generally used in this paper. A calculation of the streaming and error will be made in the appendix.

It also seems that it would be possible to use the higher order terms in the Boltzman Taylor's Series Expansion. That is, replacements for ∇^4 and ∇^6 could be made. This would complicate the linear equations but this is no difficulty for the machine.

The advantage of the above method is its extreme flexibility, the ease with which the matrix elements are calculated, and the fact that the Fairchild machine gives quick accurate results. From the results of the above sample problem and the spot check of the theory with diffusion theory it seems that problems can be done in three dimensions with energy dependence, with only a 300 point representation. The Fairchild machine can solve linear equations in more than 300 unknowns but for many computations 300 will be sufficient.

APPENDIX OF SOLVED PROBLEMS

In all the problems that follow it is assumed that an isotropic source of thermal neutrons impinges on a slab of concrete, with voids indicated, and boundary condition on the outside face that the return neutron current shall vanish. To simplify the discussion the directions x, y, and z will be referred to as in Fig. 3.

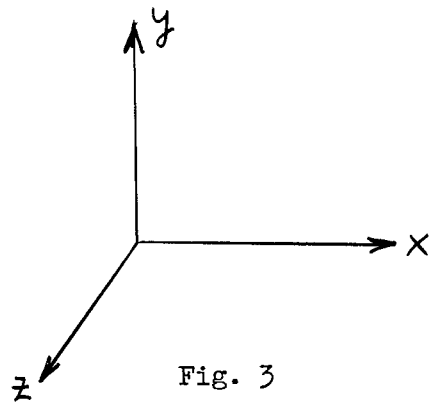

Fig. 3

If one wishes a solution to a ducting problem with thermal neutrons where a plane collimated source is used then it is only necessary to multiply the given results by four since the probability that a collimated beam neutron will go in the x direction is one and the probability that an isotropic source neutron will go in the x direction is one-quarter. This applies in all cases except the one where the more exact replacement for $\nabla^2\varphi$ was used.

Most of the following problems are solved for $\sigma = 4.32$ which is the case when the lattice spacing is 4.3 cm and the diffusion length L = 7.6 cm. The numbers which represent the solution however may be interpreted in a more general manner. Since σ is defined by

$$\sigma = 4 + \left(\frac{h}{L}\right)^2$$

all solutions are obtained for

$$h = \sqrt{\sigma - 4}\, L .$$

So when $\sigma = 4.32$ one obtains

$$h = .565\ L.$$

With the aid of Table I,

TABLE I

THERMAL NEUTRON DATA

MATERIAL	L(cm)	$k^2(cm^{-2})$	λ_s
Concrete	7.6	.01728	1.5
Lead	12.5	.0072	2.75
Carbon	50	.000426	2.49
Water	2	.25	< 2

it is clear that the calculations apply to carbon, for example, where $h = (.565)50 = 28.3$ cm. However one cannot expect the results to be accurate when the lattice spacing is this large. The lattice spacing should not be greater than the scattering mean free paths. If $h > \lambda_s$ the results will less accurately represent the physical situation.

The calculations with $\sigma = 4.32$ apply also to water. Then $h = .565(2) = 1.130$ cm and if there are only 5 boxes then the overall thickness of water is 5.65 cm. This does not seem like a sufficient amount of water for a ducting experiment but the results are still of interest in ducting design. To find results of interest in the case of water, problems will have to be solved with more lattice elements in the x direction than the 5 used above. Some calculations are included for $\sigma = 4.133$ and $\sigma = 4.043$. These calculations will enable one to increase or decrease a geometrical configuration uniformly in size or to compare some geometrical configuration as the material is varied.

I- REFERENCE PROBLEM

The problem with no voids and an isotropic source of thermal neutrons is one-dimensional. That is the shield material ($\sigma = 4.32$) is infinite in the $\pm$ y direction. Fig. 4 represents a section in the x-y plane. Then the difference equations may be set up and solved easily by hand to obtain

1	.5706	.3239	.1809	.0958	.0413	0

Fig. 4

One may form the ratios of the flux in one element to that in the next to obtain

$$\frac{1}{.5706}, \quad \frac{.5706}{.3239}, \quad \frac{.3239}{.1809}, \quad \frac{.1809}{.09577}, \quad \frac{.0958}{.0413}$$

or

$$1.752, \quad 1.762, \quad 1.798, \quad 1.888, \quad 2.32$$

and it is clear that the attenuation is nearly exponential. It is clear that the attenuation is nearly exponential. It is not exactly exponential because there is no back scattering beyond the 5 elements, i.e. it would only be exponential if the shield were also infinite in the x direction.

II - TWO AIR VOIDS

a) The shield ($\sigma = 4.32$) is infinite in the $\pm$ y direction, finite in the x direction. Figure 5 represents a section in the x-y plane. The slant lines above and below the figure indicate that solid material is above and below. The voids are placed as follows and one obtains:

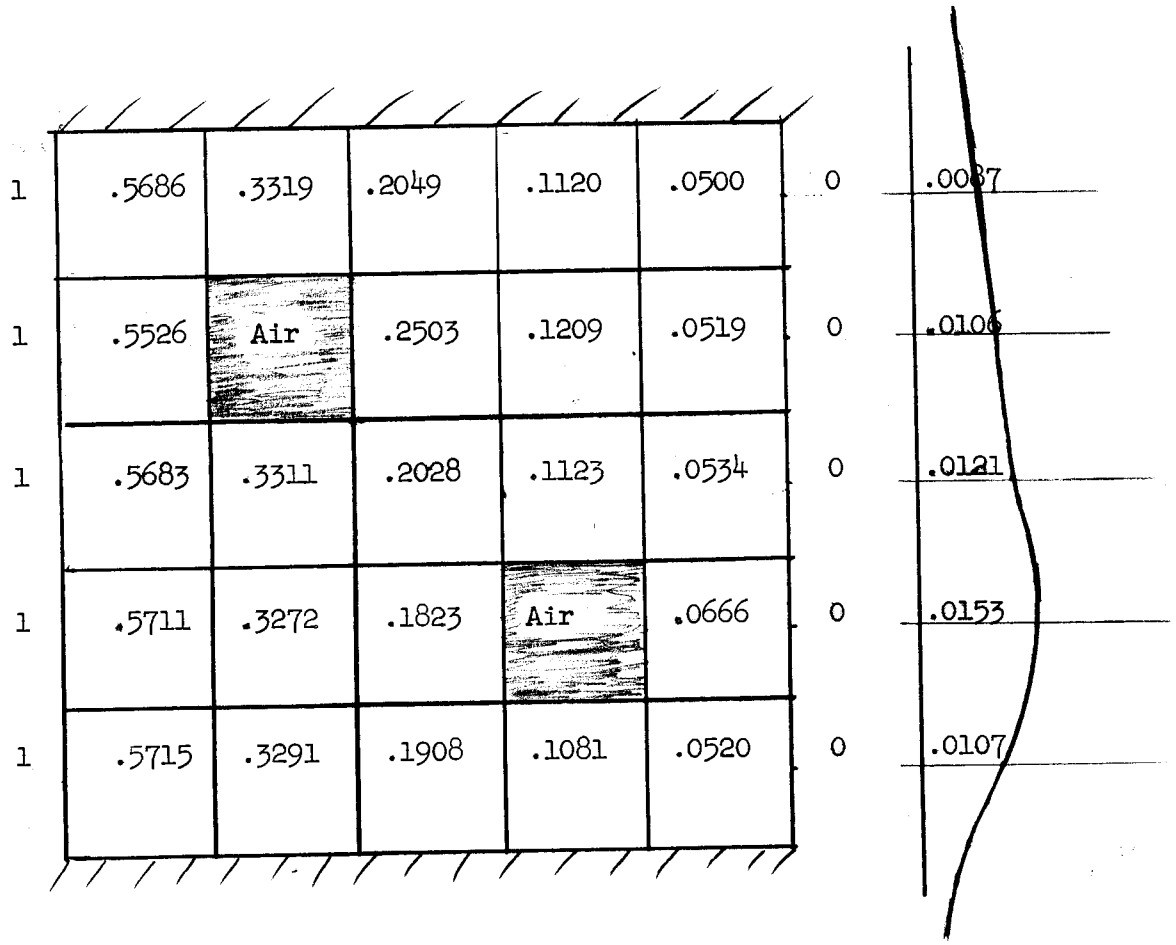

Fig. 5

The plot at the right indicates the surplus exit flux over the exit flux of the reference problem.

b) Shield material (σ = 4.32, 4.133, 4.043) is finite in the $\pm$ y and the x directions. Fig. 6 represents a section in the x-y plane.

	0	0	0	0	0	
1	.3809 .4229 .4478	.1651 .2062 .2326	.0895 .1206 .1416	.0411 .0604 .0742	.0158 .0249 .0317	0
1	.4802 .5414 .5782	Air Void	.1806 .2317 .2657	.0723 .1041 .1268	.0272 .0425 .0540	0
1	.5129 .5825 .6245	.2428 .3085 .3512	.1383 .1913 .2278	.0636 .0957 .1190	.0294 .0468 .0598	0
1	.4925 .5571 .5959	.2325 .2948 .3352	.1105 .1547 .1853	Air Void	.0362 .0551 .0689	0
1	.3822 .4247 .4499	.1586 .1979 .2232	.0704 .0982 .1175	.0348 .0533 .0668	.0164 .0262 .0336	0
	0	0	0	0	0	

Fig. 6

There is no surplus flux plot at the right because the reference problem does not correspond to finite geometry. In Fig. 6 the values on top are for $\sigma = 4.32$; middle, $\sigma = 4.133$; bottom, $\sigma = 4.043$.

III - TWO RIGHT ANGLE BENDS

a) The shield material ($\sigma = 4.32$) is infinite in the $\pm$ y direction, finite in the x direction. Fig. 7 represents a section in the x-y plane.

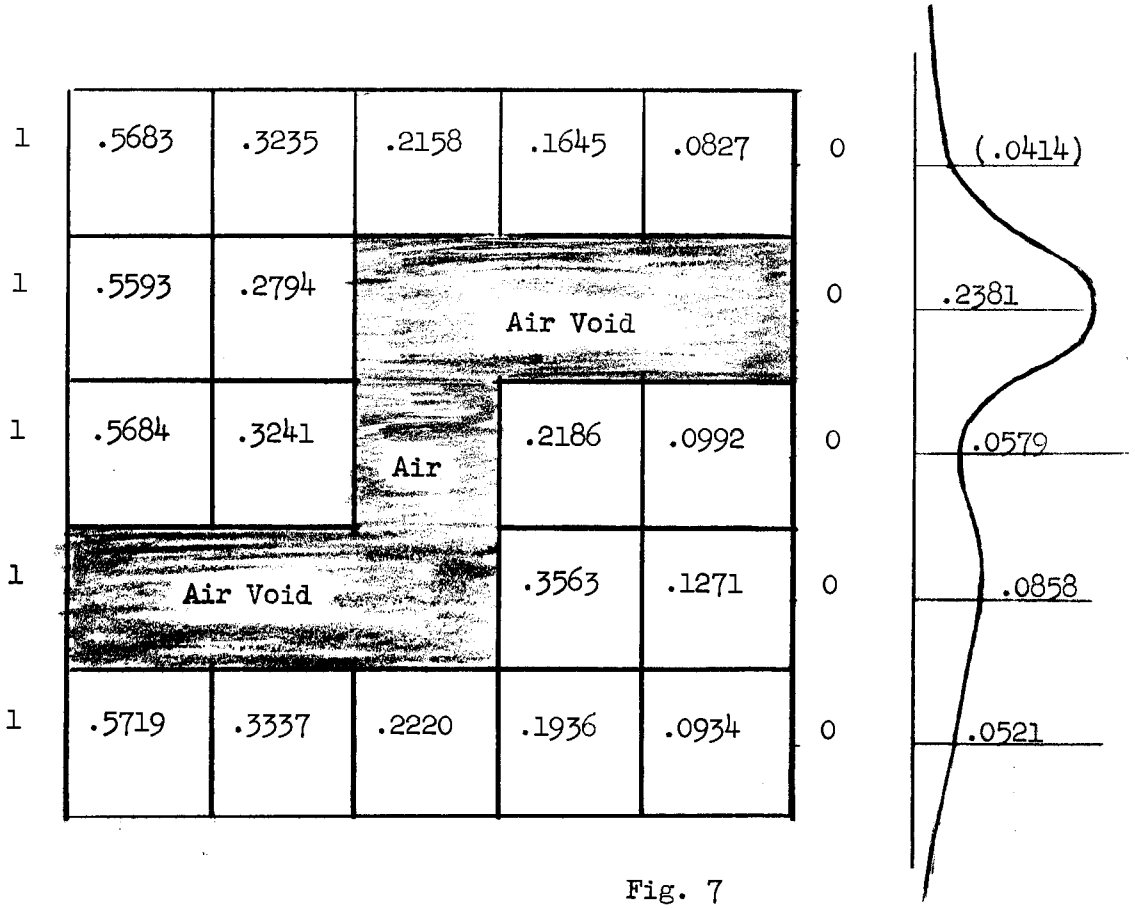

Fig. 7

The change of flux in each element due to the presence of the void is shown in Fig. 8.

-.0023	-.0004	+.0349	+.0687	+.0414
o.0113	-.0445			
-.0022	+.0002		+.1228	+.0579
			+.2605	+.0858
+.0013	+.0098	+.0411	+.0978	+.0521

Fig. 8

It is interesting to note that five of the boxes have fewer neutrons than before the insertion of the duct.

b) The shield material (σ = 4.32, 4.133, 4.043) is finite in the $\pm$ y and the x directions. Fig. 9 represents a section in the x-y plane.

	0	0	0	0	0	
1	.3840 .4275 .4534	.1616 .2027 .2293	.0766 .1079 .1296	.0434 .0655 .0815	.0201 .0316 .0400	0
1	.4972 .5639 .6041	.2375 .3022 .3442	.1258 .1775 .2134	.0909 .1312 .1598	.0435 .0650 .0804	0
1	.5262 .6003 .6452	.2415 .3047 .3452	Air Void			
1	.5345 .6121 .6595	.2796 .3566 .4066	Air	.1799 .2343 .2710	.0769 .1056 .1255	0
1	Air Void			.3299 .3749 .4038	.1088 .1371 .1560	0
1	.5031 .5726 .6149	.2520 .3224 .3685	.1385 .1923 .2294	.1364 .1777 .2059	.0632 .0860 .1017	0
1	.3867 .4316 .4584	.1673 .2108 .2391	.0841 .1172 .1400	.0575 .0811 .0977	.0279 .0404 .0493	0
	0	0	0	0	0	

Fig. 9

IIII - SHIFTING A VOID

Five problems are solved where Figures 10a - 10e represent x-y plane sections, and the concrete is infinite in the ± y direction, finite in the x-direction.

a)

1	.5702	.3228	.1789	.0934	.0400	0	(-.0013)
1	.5701	.3224	.1779	.0911	.0395	0	(-.0018)
1	.5700	.3220	.1761	.0829	Air Void	0	(+.0416)
1	.5701	.3224	.1779	.0911	.0395	0	(-.0018)
1	.5702	.3228	.1789	.0934	.0400	0	(-.0013)

Fig. 10a

b)

1	.5702	.3229	.1798	.0961	.0429	0	.0016
1	.5699	.3221	.1780	.0965	.0464	0	.0051
1	.5696	.3205	.1707	Air Void	.0610	0	.0197
1	.5699	.3221	.1780	.0965	.0464	0	.0051
1	.5702	.3229	.1798	.0961	.0429	0	.0016

Fig. 10b

c)

1	.5703	.3241	.1840	.1017	.0450	0	.0037
1	.5693	.3217	.1850	.1086	.0479	0	.0066
1	.5672	.3114	Air Void	.1347	.0534	0	.0121
1	.5693	.3217	.1850	.1086	.0479	0	.0066
1	.5703	.3241	.1840	.1017	.0450	0	.0037

Fig. 10c

d)

1	.5724	.3311	.1937	.1059	.0464	0	.0051
1	.5692	.3332	.2060	.1115	.0482	0	.0069
1	.5533	Air Void	.2516	.1215	.0504	0	.0091
1	.5692	.3332	.2060	.1115	.0482	0	.0069
1	.5724	.3311	.1937	.1059	.0464	0	.0051

10d

e)

1	.5838	.3488	.2020	.1092	.0475	0
1	.5892	.3723	.2127	.1129	.0486	0
1	Air Void	.4575	.2316	.1174	.0497	0
1	.5892	.3723	.2127	.1129	.0486	0
1	.5888	.3488	.2020	.1092	.0475	0

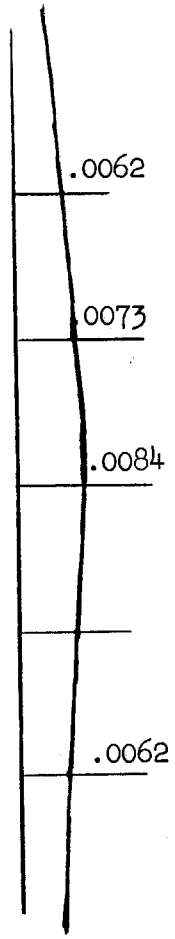

Fig. 10e

If one should wish to shield for the increase in radiation due to the presence of the void it is clear from Figures 10a - 10e that the patch is flatter when the void is closer to the source. The amount of patch material can be calculated in all cases.

V - THREE AIR VOIDS

The shield material (σ = 4.32, 4.133, 4.043) is infinite in the $\pm$ y direction, finite in the x direction. Fig. 11 represents a section in the x-y plane.

1	.5684 .6814 .7644	.3312 .4694 .5812	.2054 .3274 .4333	.1135 .2027 .2848	.0507 .0976 .1424	0
1	.5528 .6609 .7410	Air Void	.2525 .3707 .4709	.1253 .2145 .2956	.0532 .1001 .1445	0
1	.5670 .6789 .7609	.3289 .4652 .5752	.2071 .3290 .4344	.1222 .2129 .2953	.0540 .1015 .1464	0
1	.5677 .6793 .7608	.3153 .4449 .5496	Air Void	.1413 .2346 .3177	.0580 .1066 .1520	0
1	.5701 .6832 .7659	.3243 .4595 .5688	.1910 .3108 .4152	.1150 .2052 .2879	.0554 .1045 .1507	0
1	.5708 .6844 .7676	.3245 .4599 .5692	.1787 .2906 .3880	Air Void	.0663 .1200 .1694	0
1	.5712 .6852 .7689	.3280 .4658 .5773	.1901 .3102 .4152	.1090 .1981 .2805	.0523 .1006 .1465	0

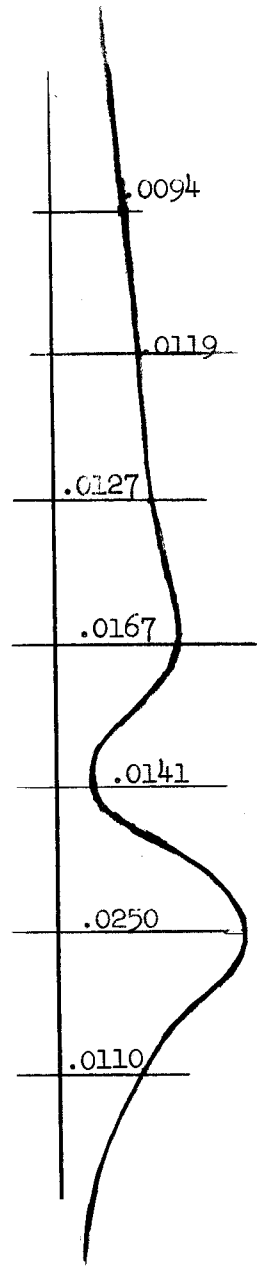

Fig. 11

The plot at the right indicates the surplus flux over the flux given in the reference problem for the case $\sigma = 4.32$ which is the top number in each box of three numbers.

VI - TWO AIR VOIDS IN THREE DIMENSIONS

The shield material ($\sigma = 4.32$) is finite in the x, and infinite in the $\pm$ y, $\pm$ z directions. A unit isotropic of slow neutrons impinging on the front face is assumed. The boundary condition on the back face is that the return current vanishes. See Fig. 12.

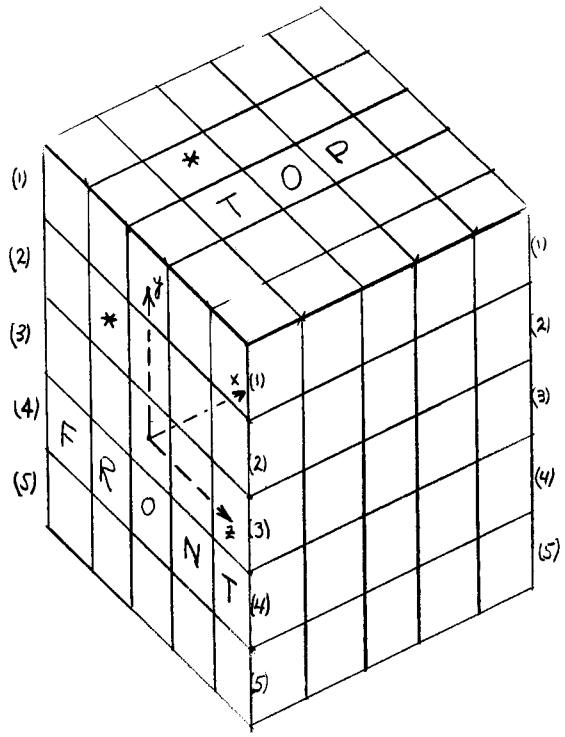

Fig. 12

One of the two voids is located by the intersection of a line in the -y direction, through the * in the top face and a line in the x direction through the * in the front face. The other air void is located in the diagonally opposite position and symmetrical with the center of the cubical array. The location of the air voids will become clear from a consideration of Figures 13a - 13e which represnt the x-z sections (1) through (5) and which also contain the solution as found on the Fairchild machine. The cubical array contains 125 boxes, two of which are air voids. The distribution of flux in these 125 volume elements will now be given, 25 at a time.

(1) - (1) Section

1	.5666	.3193	.1811	.0964	.0418	0
1	.5628	.3108	.1848	.0978	.0420	0
1	.5665	.3192	.1810	.0964	.0417	0
1	.5681	.3210	.1796	.0956	.0416	0
1	.5682	.3211	.1798	.0957	.0418	0

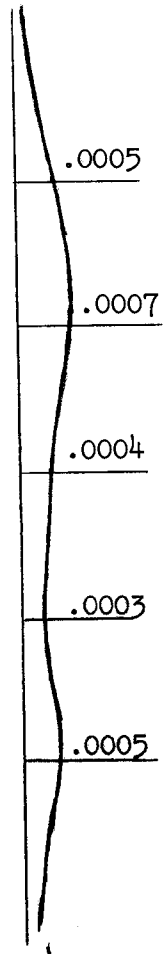

Fig. 13a

(2) - (2) Section

1	.5639	.3193	.1858	.0978	.0420	0
1	.5478	Air Void	.2186	.1029	.0428	0
1	.5638	.3192	.1857	.0978	.0421	0
1	.5674	.3206	.1803	.0960	.0419	0
1	.5675	.3207	.1804	.0960	.0417	0

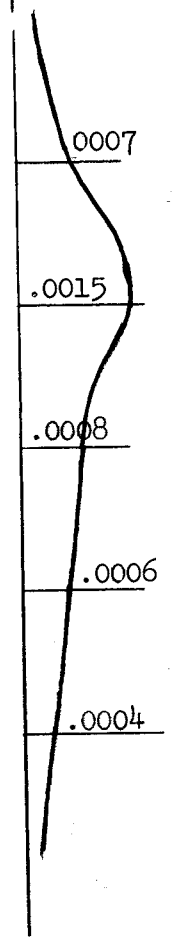

Fig. 13b

(3) - (3) Section

1	.5631	.3094	.1772	.0952	.0415	0
1	.5535	.2619	.1749	.0956	.0417	0
1	.5631	.3093	.1769	.0953	.0421	0
1	.5668	.3180	.1771	.0953	.0436	0
1	.5669	.3183	.1780	.0952	.0420	0

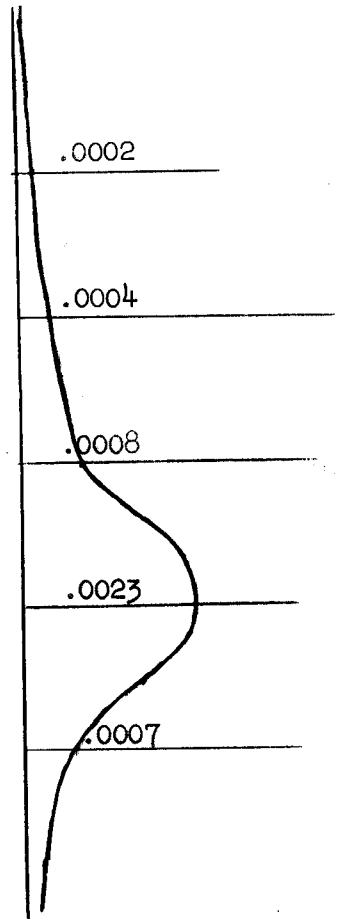

Fig. 13c

(4) - (4) Section

1	.5658	.3158	.1770	.0947	.0416	0
1	.5631	.3083	.1755	.0946	.0417	0
1	.5658	.3159	.1760	.0949	.0435	0
1	.5674	.3183	.1710	Air Void	.0546	0
1	.5675	.3192	.1770	.0950	.0435	0

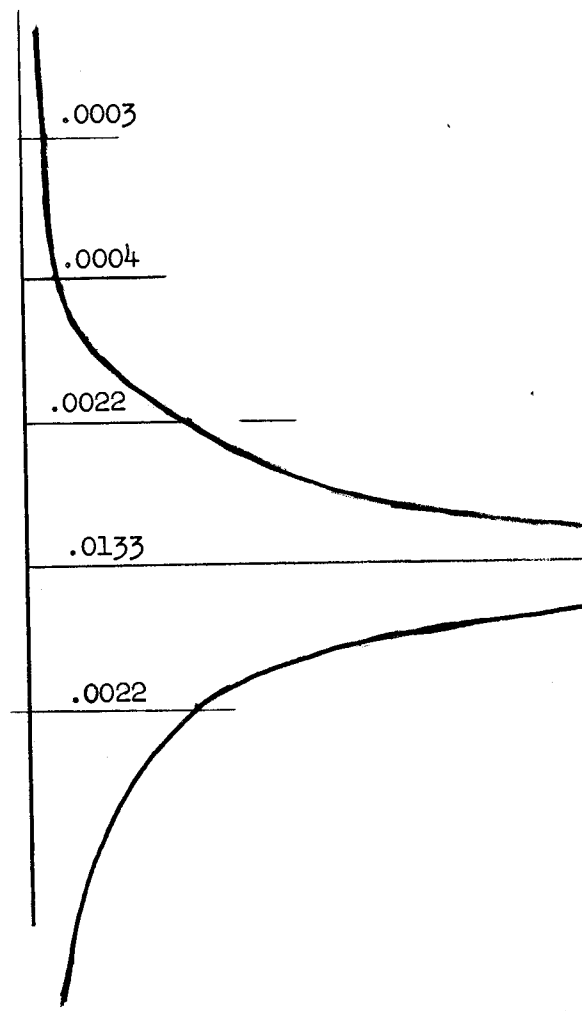

Fig. 13d

(5) - (5) Section

1	.5673	.3193	.1788	.0952	.0415	0	.0002
1	.5659	.3167	.1787	.0954	.0416	0	.0003
1	.5672	.3192	.1784	.0953	.0420	0	.0007
1	.5681	.3203	.1777	.0953	.0436	0	.0023
1	.5682	.3206	.1786	.0952	.0420	0	.007

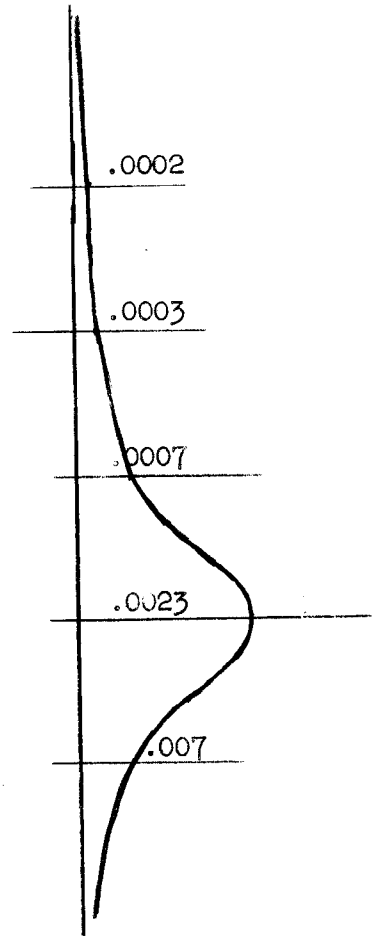

Fig. 13e

The plots at the right indicate the transmission in the usual way. The place that would require the most shielding is easily predicted as the point opposite the void in the (4) - (4) section. It is noted that the effect of each void is about the same in the transmission plot for section (1) - (1), since the void nearer the source is also closer to the top.

VII - A "STRAIGHT-THROUGH" AIR DUCT

For a unit isotropic source, the figure represents an x-y section. The top set of numbers is for $\sigma = 4.32$, the middle set for $\sigma = 4.133$, the lower set for $\sigma = 4.043$.

a) When the shield material is finite in the x, $\pm$ y directions one obtains the results given in Fig. 14.

	0	0	0	0	0	
1	.3852 .4307 .4587	.1627 .2067 .2366	.0724 .1044 .1282	.0320 .0516 .0671	.0120 .0208 .0281	0
1	.5014 .5731 .6183	.2451 .3189 .3699	.1179 .1733 .2148	.0540 .0880 .1150	.0197 .0344 .0466	0
1	.5356 .6186 .6717	.2767 .3647 .4261	.1380 .2050 .2554	.0636 .1042 .1366	.0193 .0335 .0453	0
1	Air Duct					
1	.5365 .6186 .6717	.2767 .3647 .4261	.1380 .2050 .2554	.0636 .1042 .1366	.0193 .0335 .0453	0
1	.5014 .5731 .6183	.2451 .3189 .3699	.1179 .1733 .2148	.0540 .0880 .1150	.0197 .0344 .0466	0
1	.3852 .4307 .4587	.1627 .2067 .2366	.0724 .1044 .1282	.0320 .0516 .0671	.0120 .0208 .0281	0
	0	0	0	0	0	

Fig. 14

b) When the shield material is finite only in the x direction, one obtains the results in Fig. 15.

1	.5706 .6866 .7740	.3239 .4640 .5819	.1809 .3029 .4154	.0958 .1820 .2671	.0413 .0853 .1306	0
1	.5706 .6866 .7740	.3239 .4640 .5819	.1809 .3029 .4154	.0958 .1820 .2671	.0413 .0853 .1306	0
1	.5706 .6866 .7740	.3239 .4640 .5819	.1809 .3029 .4154	.0958 .1820 .2671	.0413 .0853 .1306	0
1	Air Duct					
1	.5706 .6866 .7740	.3239 .4640 .5819	.1809 .3029 .4154	.0958 .1820 .2671	.0413 .0853 .1306	0
1	.5706 .6866 .7740	.3239 .4640 .5819	.1809 .3029 .4154	.0958 .1820 .2671	.0413 .0853 .1306	0
1	.5706 .6866 .7740	.3239 .4640 .5819	.1809 .3029 .4154	.0958 .1820 .2671	.0413 .0853 .1306	0

.9587

Fig. 15

The plot on the right represents the surplus exit flux due to the presence of the air duct for the case $\sigma = 4.32$.

It will be noted that above and below the air duct the flux has exactly the value one would obtain if no duct were present. Along the duct the source flux is transmitted without attenuation.

The theory used here (four point replacement for ∇^2) ignores the streaming effects. These will be calculated as well as an estimate of the error in the next paragraph.

2) Streaming Effects and Calculation of Error

If the more general replacement for ∇^2 referred to in the text is now used, one will obtain a better estimate for the flux distribution. The solution will be found for a unit isotropic source of thermal neutrons. Fig. 16 represents an x-y section and the numbers are calculated for a shield material for which $\sigma = 4.32$, finite only in the x-direction.

1	.5751	.3318	.1898	.1031	.0454	0	.0041
1	.5764	.3369	.1951	.1070	.0477	0	.0064
1	.5735	.3553	.2086	.1159	.0541	0	.0128
1	Air Duct						.9587
1	.5735	.3553	.2086	.1159	.0541	0	.0128
1	.5764	.3369	.1951	.1070	.0477	0	.0064
1	.5751	.3318	.1898	.1031	.0454	0	.0041

Fig. 16

The plot at the right shows the surplus exit flux over the reference problem. This may be compared with the plot in the problem that immediately precedes. It will be noticed that the effect of a more general replacement for ∇^2 causes a variance in the flux values in the shield material in the y-direction that results from the streaming* of neutrons. Also the streaming causes an obvious smoothing in the transmission plots.

The error in the ∇^2 replacement may be calculated from the flux values given in problem VII-2 and VII-1-a, top values (for $\sigma = 4.32$) and varies from a maximum in the first column of 10 per cent to a maximum of 23 per cent in the last column. For calculations of this nature it then seems that the usual replacement for ∇^2 (4 point formula) is tenable unless one is particularly interested in streaming effects.

3) Solution for a Collimated beam source of thermal neutrons, i.e. it is assumed that the unit source of neutrons have a probability of one of going in the x-direction into the neighboring box. The shield material ($\sigma = 4.32$) is finite in the x-direction. The figures represent x-y sections.

a) With the more general ∇^2 replacement one obtains the results shown in Fig. 17.

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*For a discussion of streaming, see ORNL-403, Solution of the Diffusion Equation with Streaming for three Basic Geometries, D. W. Whitcombe and N. M. Smith, Jr.

1	1.8872	1.0592	.5892	.3140	.1372	0
1	1.8712	1.1344	.5836	.3176	.1412	0
1	1.8324	.9292	.5860	.3316	.1564	0
1	Air Duct					
1	1.8324	.9292	.5860	.3316	.1564	0
1	1.8712	1.1344	.5836	.3176	.1412	0
1	1.8872	1.0592	.5892	.3140	.1372	0

Fig. 17

3b) Solution of the above problem (VII-2-a) using the usual replacement for ∇^2 gives the results in Fig. 18.

1	2.2824	1.2956	.7236	.3832	.2052	0
1	2.2824	1.2956	.7236	.3832	.2052	0
1	2.2824	1.2956	.7236	.3832	.2052	0
1	Air Duct					
1	2.2824	1.2956	.7236	.3832	.2052	0
1	2.2824	1.2956	.7236	.3832	.2052	0
1	2.2824	1.2956	.7236	.3832	.2052	0

Fig. 18

This solution was calculated using the solution in VII-1-b and multiplying the values by 4 since in the isotropic case the probability that a source neutron will go in the x-direction is one-quarter and in the collimated beam case this probability is one. No such simple law holds when the more exact replacement for ∇^2 is used; then each problem must be run separately. It is seen that

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the error for the collimated beam case is about the same (20 per cent to 25 per cent) but that now the usual ∇^2 replacement appears to represent an upper bound while it represented a lower bound when the source was isotropic.

CONCLUSIONS AND ACKNOWLEDGMENTS

It is regrettable that the "appendix of solved problems" does not contain solutions where the void material is other than air, where there is a dependence on energy, and where many more elements of area were used. However, it is felt that the time required to complete such an appendix where these parameters also varied would be prohibitive and in many cases beyond the range of the Fairchild machine. These problems can be examined one-by-one, as they arise.

The author would like to express his gratitude to Herman Kahn of the Rand Corporation for pointing out the value in these calculations; Robert Coveyou and Dr. C. L. Perry at the Oak Ridge National Laboratory for guiding him in the use of the difference equations; and to J. J. Stone at the Fairchild Engine and Airplane Corporation, NEPA Project, for obtaining the solutions on the Fairchild machine.

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	.5836	.3419	.2028	.1255	.0881	.0728	.0662	.0521	.0342	.0166	
1	.6430	.4149	.2707	.1834	.1383	.1164	.1036	.0817	.0546	.0270	0
	.6817	.4659	.3213	.2292	.1799	.1533	.1353	.1069	.0721	.0359	
	.7077	.5015	.3582	.2637	.2122	.1824	.1603	.1267	.0859	.0429	
	.5828	.3398	.1982	.1178	.0882	.0767	.0711	.0546	.0353	.0170	
1	.6416	.4112	.2629	.1706	.1375	.1206	.1096	.0850	.0561	.0275	0
	.6797	.4607	.3107	.2120	.1782	.1575	.1417	.1106	.0738	.0365	
	.7053	.4953	.3453	.2431	.2097	.1865	.1668	.1306	.0877	.0436	
	.5823	.3379	.1918	.0947							
1	.6405	.4077	.2517	.1328							0
	.6781	.4556	.2952	.1618		AIR DUCT					
	.7033	.4890	.3265	.1831							
	.5827	.3389	.1938	.0977							
1	.6409	.4087	.2537	.1356							0
	.6785	.4566	.2970	.1641							
	.7037	.4899	.3280	.1849							
	.5839	.3427	.2050	.1317			.1081	.0764	.0461	.0212	
1	.6427	.4142	.2696	.1831	A		.1510	.1097	.0686	.0325	0
	.6808	.4635	.3166	.2222	I		.1848	.1366	.0872	.0420	
	.7064	.4979	.3504	.2510	R		.2103	.1571	.1015	.0493	
	.5848	.3456	.2130	.1554			.1857	.1198	.0653	.0282	
1	.6443	.4188	.2811	.2128	D		.2317	.1562	.0900	.0405	0
	.6830	.4695	.3310	.2565	U		.2664	.1841	.1094	.0504	
	.7090	.5050	.3671	.2887	C		.2918	.2049	.1241	.0580	
					T		.4149	.1876	.0868	.0347	
1							.4533	.2246	.1127	.0477	0
		AIR DUCT					.4800	.2517	.1324	.0578	
							.4987	.2713	.1470	.0655	
							.4105	.1851	.0857	.0343	
1							.4477	.2213	.1112	.0471	0
							.4737	.2479	.1306	.0571	
							.4921	.2672	.1451	.0647	
	.5848	.3452	.2099	.1379	.0968	.0977	.1649	.1119	.0623	.0272	
1	.6447	.4192	.2795	.1979	.1481	.1430	.2059	.1459	.0859	.0391	0
	.6836	.4708	.3312	.2448	.1902	.1805	.2377	.1725	.1047	.0487	
	.7099	.5070	.3687	.2800	.2228	.2096	.2618	.1926	.1190	.0561	
	.5843	.3437	.2063	.1307	.0924	.0818	.0888	.0688	.0431	.0202	
1	.6441	.4176	.2758	.1906	.1436	.1263	.1275	.0999	.0646	.0311	0
	.6831	.4693	.3277	.2380	.1859	.1637	.1596	.1257	.0826	.0403	
	.7094	.5056	.3655	.2737	.2187	.1930	.1845	.1457	.0965	.0474	

The figure represents a 2 dimensional shield material with thermal diffusion length L and the air duct as shown. Each box has edge length h. The top number in each box gives the flux distribution when h = .546 L, the 2nd set for h = .446 L, the 3rd set for h = .387 L, and the bottom set for h = .346 L. For a complete discussion of such solutions see ORNL-899 - David W. Whitcombe.

The solutions were obtained on the Fairchild machine under the direction of J. Stone.