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FORCED CONVECTION HEAT TRANSFER IN THERMAL ENTRANCE REGIONS

PART I

H. F. POPPENDIEK

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FORCED CONVECTION HEAT TRANSFER IN THERMAL ENTRANCE REGIONS

PART I

H. F. Poppendiek

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GENERAL INTRODUCTION

In designing heat exchange equipment it is sometimes necessary to have information about forced convection heat transfer in the thermal entrance region of the heat exchanger duct system. A thermal entrance region results when a thermally established fluid flowing in a duct system suddenly flows over surfaces which possess some new temperature distribution. This new temperature distribution may be a uniform one, or may result from a uniform wall heat flux distribution, or may be characterized by some intermediate condition. The thermal entrance length can be defined as that distance from the entrance where the convective conductance* or thermal boundary layer thickness has attained a value which is one percent of the fully established value. Fundamental information about thermal entrance regions would make it possible 1) to establish the temperature distribution or heat flux in that portion of a duct where local high temperature areas (hot spots) exist, 2) to explore the thermal performance of short heat exchangers, and 3) to facilitate the design of research heat exchangers which are to be used to measure established heat transfer conductances far downstream.

Some of the pertinent theoretical entrance investigations that are found in the literature are briefly listed here. Classical conduction solutions equivalent to slug flow between parallel plates with uniform wall temperature and uniform wall heat flux entrance regions are given in references 1 and 2, respectively. Graetz derived a conduction solution for parabolic fluid flow in a pipe with a uniform wall temperature entrance region (reference 3).

*Also called heat transfer coefficient

Martinelli and Boelter have presented an analytical study of superposed free and forced laminar convection in a vertical pipe with a uniform wall temperature (reference 4). Latzko has developed turbulent forced convection solutions with both hydrodynamic and thermal entrance regions (with uniform wall temperature conditions); his solutions are valid for Prandtls' modulus equal to unity and for systems where heat is only transferred by eddy convection (reference 5). Sanders has obtained a turbulent flow convection solution for the uniform wall temperature entrance system by transforming the turbulent core to a laminar core of equivalent thermal resistance; this transformation is predicated on several postulates (reference 6). Seban and Shimazaki have obtained some numerical solutions for the entrance region by means of the finite difference method (reference 7).

Few experimental entrance investigations have been reported in the literature; some of those found are briefly noted here.

Boelter, Young, and Iversen have presented experimental local heat transfer data for air flow through a pipe with a uniform wall temperature entrance region and for a series of hydrodynamic entrance conditions (reference 8). The steam condensation method was used. The local conductance distributions of some of the different hydrodynamic entrance conditions studied are given in Figures 1 and 2. These data indicate that except for distribution (a), the entrance lengths for the specific conditions given are about $\frac{X}{D} = 17$ or greater. Although the Reynolds modulus range covered was only from 17,000 to 56,000, it appeared that the entrance length increased as Reynolds modulus increased.

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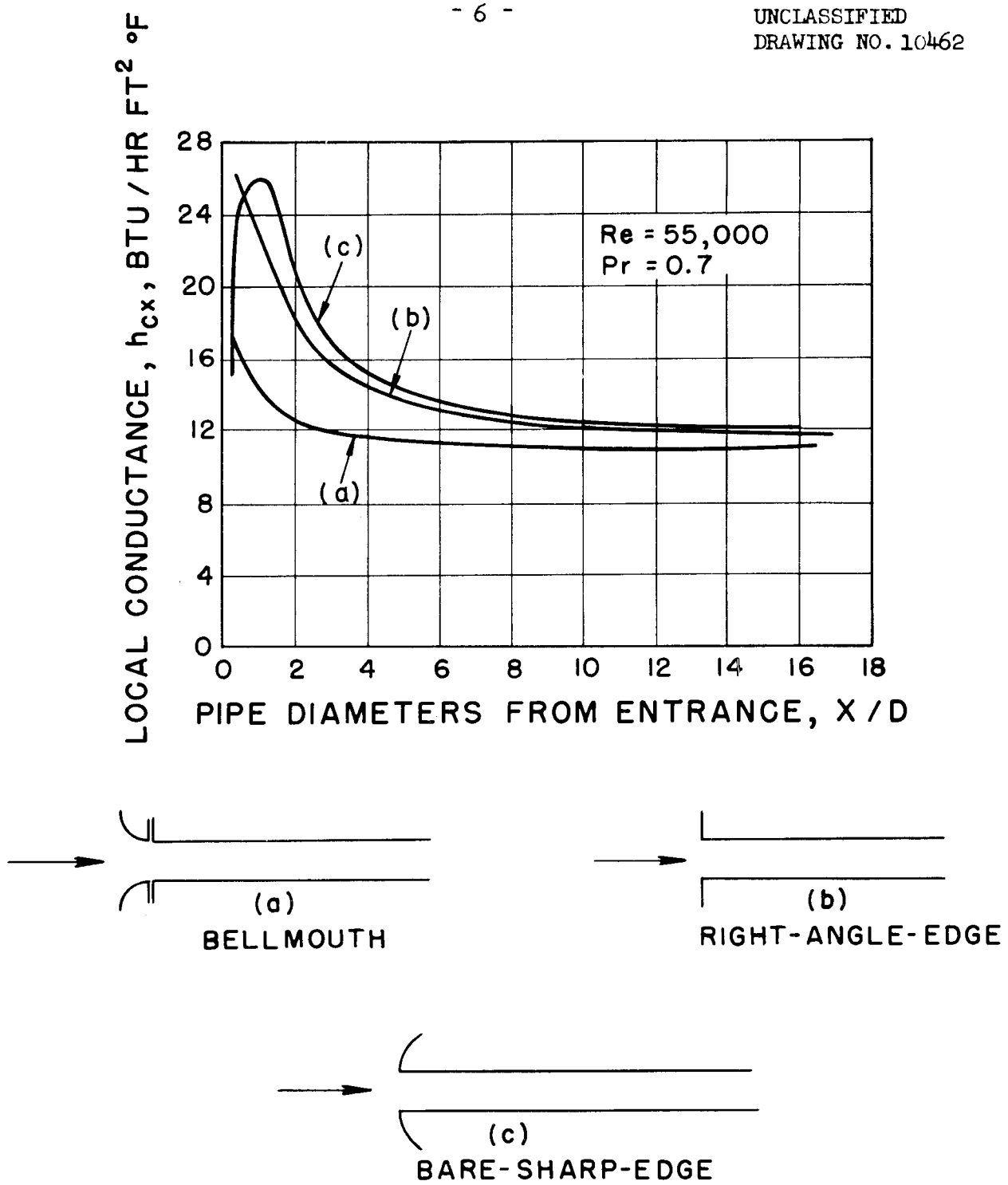

FIGURE 1. LOCAL CONDUCTANCE DISTRIBUTIONS FOR SEVERAL HYDRODYNAMIC ENTRANCE CONDITIONS AT CONSTANT WALL TEMPERATURE. (REFERENCE 7)

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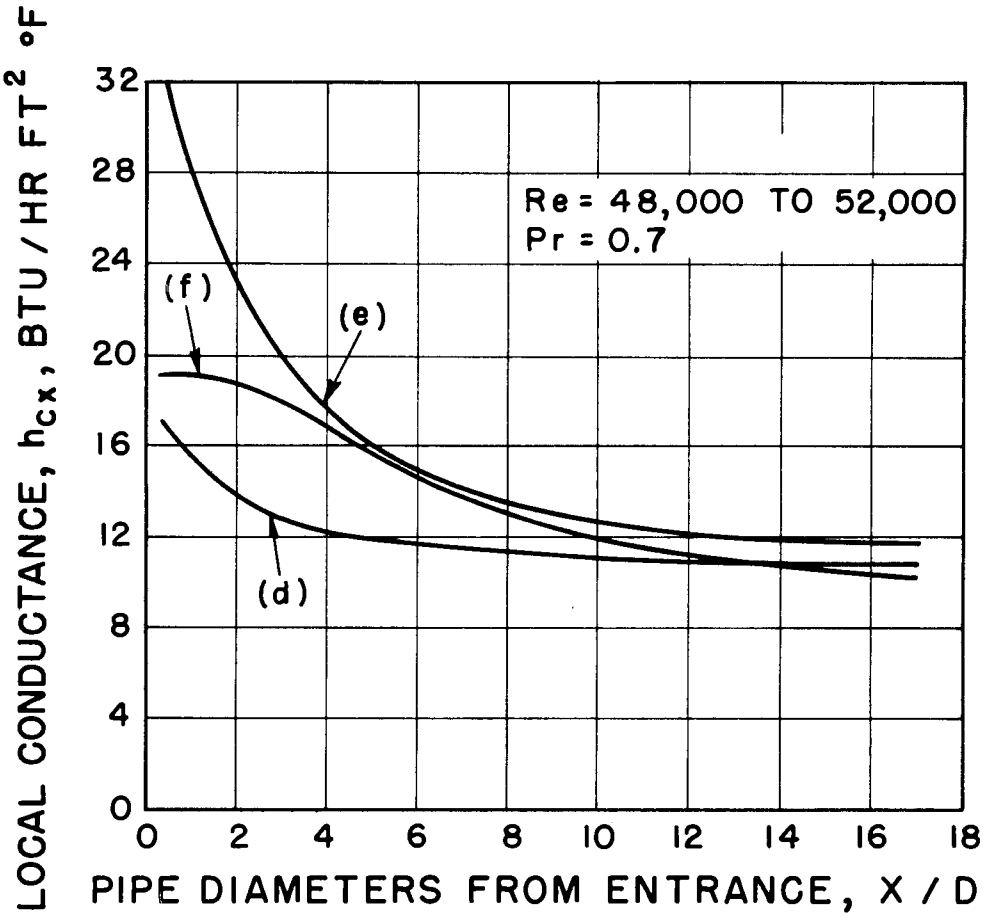

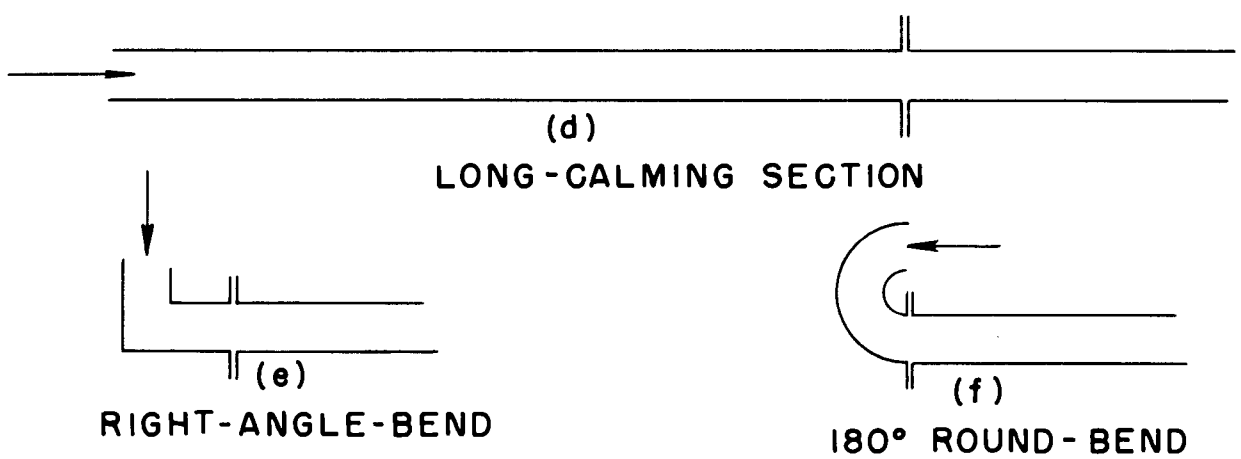

FIGURE 2. LOCAL CONDUCTANCE DISTRIBUTIONS FOR SEVERAL HYDRODYNAMIC ENTRANCE CONDITIONS AT CONSTANT WALL TEMPERATURE. (REFERENCE 7)

Humble, Lowdermilk, and Grele (reference 9) have measured average heat transfer coefficients for the case of air flow through an electrically heated inconel tube with a bellmouth entrance. Because local outside tube wall temperatures were given in reference 9, it was possible for the writer of the present report to calculate local conductances for the inconel tube in the following manner. The tube was divided into twenty four small longitudinal sections. The heat generation and longitudinal heat conduction (to the bus bars) were determined for each section and the inside wall temperatures and convective heat fluxes calculated. The local mixed mean fluid temperatures were determined. The local heat transfer conductances which were then calculated are presented in Figure 3. Heat conduction to the bus bars was only significant near the ends of the tube; the conductances were not plotted in these regions. The uniform heat flux conductance data in Figure 3 indicates that for the specific conditions given the entrance length is about $\frac{X}{D} = 50$ and perhaps greater.

English and Barrett (reference 10) have measured local conductances for the case of mercury flow through electrically-heated nickel tubes. These tubes contained copper plated outer sheaths within which practically all of the total heat was generated. Figure 4 shows the local conductance distribution for mercury under approximately uniform heat flux conditions. The entrance length for the specific conditions given is about $\frac{X}{D} = 30$ or greater.

An attempt will be made to present a series of new forced convection heat transfer analyses for thermal entrance regions some of which are primarily applicable to liquid metal systems. These analyses are to be presented in several parts; the following report represents the first of these parts. The report numbers of the subsequent parts are to be consecutive.

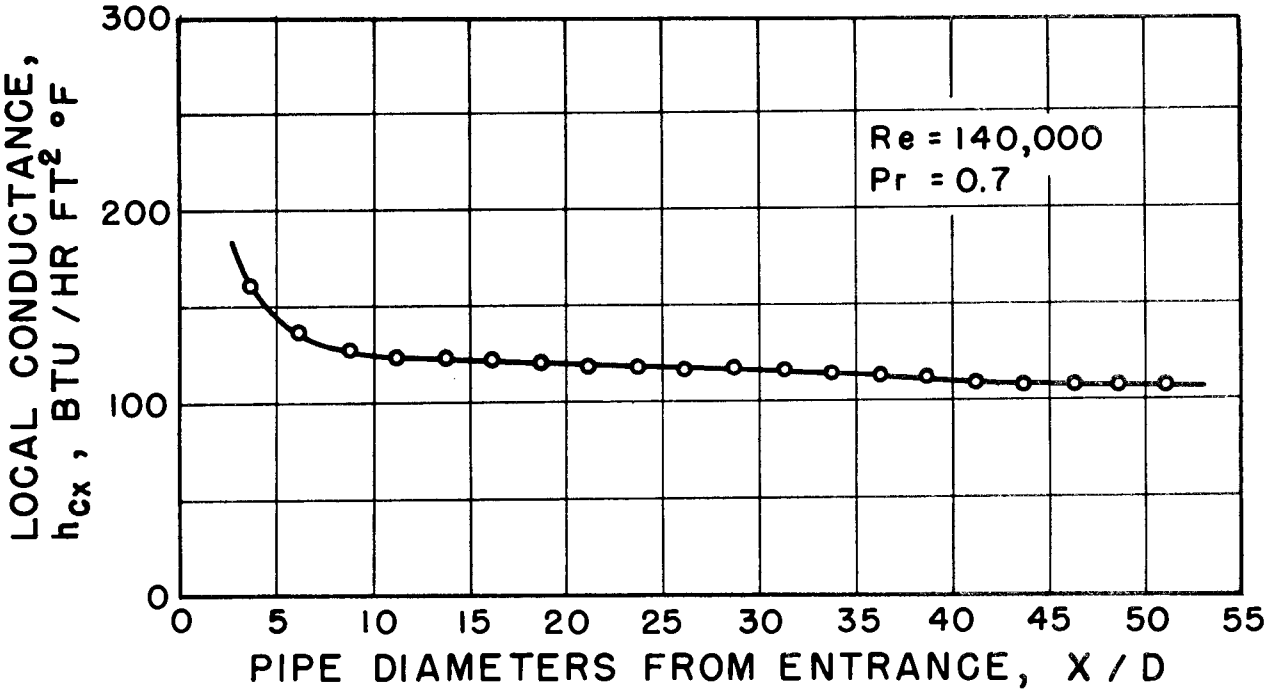

FIGURE 3. LOCAL CONDUCTANCES FOR AIR FLOW IN A PIPE UNDER UNIFORM HEAT FLUX CONDITIONS.

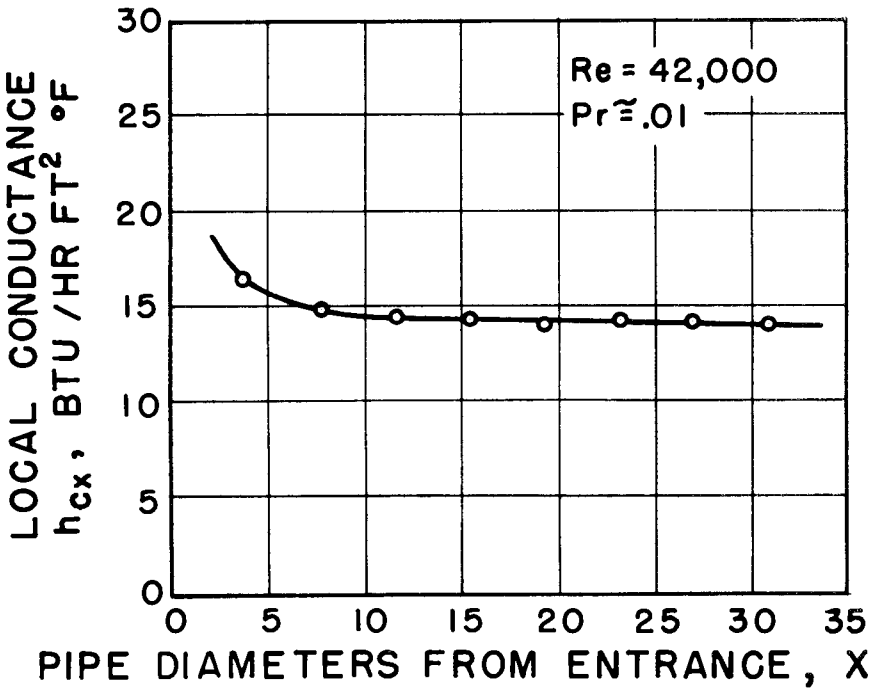

FIGURE 4. LOCAL CONDUCTANCES FOR MERCURY FLOW IN A PIPE UNDER UNIFORM HEAT FLUX CONDITIONS. (REFERENCE 10)

FORCED CONVECTION HEAT TRANSFER IN THERMAL ENTRANCE REGIONS

PART I

Heat Transfer In the Thermal Entrance Region of Parallel Plate and Pipe Duct Systems, with Uniform Wall Temperatures, Containing Liquid Metals which Have Turbulent Velocity Profiles and Low Prandtl's Moduli.

SUMMARY

Part I concerns itself with an analytical study of forced convection heat transfer in the thermal entrance region of parallel plate and pipe duct systems, with uniform wall temperatures, containing liquid metals that are characterized by low Prandtl's moduli and turbulent velocity profiles. Local heat transfer and convective conductance expressions in terms of Reynolds' and Prandtl's moduli are developed.

NOMENCLATURE

English Letters

a,	thermal molecular diffusivity of the fluid, ft^2/hr
A_c,	fluid flow cross sectional area, ft^2
b,	one half the distance between the parallel plates, ft
B,	parameter in the turbulent velocity expression, ft/hr
b_n,	Fourier-Bessel series coefficients, ^oF
C, C_1, C_2, C_3,	constants in equations 13, 14, and 16
c_n,	series coefficients in equation 19, ^oF
c_p,	heat capacity of the fluid, Btu/lb ^oF
h_{CX},	local unit thermal convective conductance, Btu/hr ft^2 ^oF
J,	Bessel function of the first kind
k,	thermal conductivity of the fluid, Btu/hr ft^2 (^oF/ft)
m,	exponent in the turbulent velocity expression
$\left(\frac{q}{A}\right)_X$,	local convective heat flow, Btu/hr ft^2
t,	fluid temperature, ^oF
t_{mX},	local mixed mean fluid temperature, ^oF
t_o,	initial fluid and wall temperature, ^oF
t_w,	duct wall temperature, ^oF
U,	mean fluid velocity, ft/hr
u,	fluid velocity at distance, y, from the duct wall, ft/hr
x, y,	Cartesian coordinates in Figure 1, ft
Y,	Bessel function of the second kind

Greek Letters

β_n,	eigenvalues of equation 18
γ,	fluid density, lbs/ft^3
$\Phi_1(X)$,	a function of X only
$\Phi_2(Y)$,	a function of Y only
ε_H,	thermal eddy diffusivity, $\frac{ft^2}{hr}$
ν,	fluid kinematic viscosity, ft^2/hr
ρ,	fluid mass density, lbs hr^2/ft^4
ζ,	friction factor defined in equation 27
$\frac{\Delta p}{\Delta x}$,	axial pressure drop, lbs/ft^3

Terms

$$T = t - t_w$$

$$T_o = t_o - t_w$$

$$a = \frac{k}{\gamma c_p}$$

$$Z = \left(\frac{2}{m+2}\right) \frac{(Y)}{F}^{\frac{m+2}{2}}$$

$$Z_1 = \left(\frac{2}{m+2}\right) \frac{1}{F}$$

$$F^2 = \frac{a}{b\,B}$$

$$f(Z) = T_o / \left(\left(\frac{m+2}{2}\right) ZF\right)^{\frac{1}{m+2}}$$

Dimensionless Moduli

$$X = \frac{x}{b} \qquad Pr = \frac{\gamma c_p \nu}{k}$$

$$Y = \frac{y}{b} \qquad Re = \frac{2bU}{\nu}$$

INTRODUCTION

A pertinent thermal entrance system encountered when dealing with liquid metals is one in which 1) the thermal eddy diffusivity is small compared to the thermal molecular diffusivity (that is, when the reciprocal of Prandtl's modulus is large compared to the eddy diffusivity - kinematic viscosity ratio) and 2) the established velocity profile is a turbulent one.

The first condition arises when considering fluids that have high thermal conductivities, low heat capacities, and low densities; a large fraction of the liquid metals fall into this category. Table I reveals a comparison of Pr^{-1} and the ratio $\frac{\varepsilon_H}{\nu}$ for a series of Prandtl's and Reynolds' moduli for a pipe system. The term $\frac{\varepsilon_H}{\nu}$ is an arithmetic mean value for a fluid annulus, half the radius in thickness, and contiguous to the pipe wall; this is the important layer as far as heat transfer is concerned.

TABLE I

Comparison of Pr^{-1} and $\frac{\varepsilon_H}{\nu}$ for Various Re and Pr Moduli

Pr	Re = 5,000		Re = 10,000		Re = 50,000	
	Pr^{-1}	(ε_H/ν)mean	Pr^{-1}	(ε_H/ν)mean	Pr^{-1}	(ε_H/ν)mean
.01	100	7	100	13		
.005	200	7	200	13	200	56
.001	1000	7	1000	13	1000	56

Table I illustrates generally the region in which the thermal eddy diffusivity is small compared to the thermal molecular diffusivity.

Turbulent velocity profiles for parallel plate and pipe duct systems can be satisfactorily represented by power expressions (references 11 and 12). For example, Dönch expresses his experimental parallel plate velocity data by an eighth power law (Figure 5). Pipe velocity data are usually represented by a seventh power law.

The following ideal system was postulated to approximate the forced convection heat transfer system under consideration (see Figure 6) and was utilized in the analytical study:

1. The established turbulent velocity profile is represented by the power law, $u = B\left(\frac{y}{b}\right)^m$.
2. The initial fluid and wall temperatures are t_o.
3. The uniform wall temperature for $x > 0$ is t_w.
4. The thermal eddy diffusivity is small compared to the thermal molecular diffusivity and is neglected.
5. Longitudinal heat conduction is small compared to transverse heat conduction and is neglected; this postulate, which is frequently made, has been verified experimentally.
6. Fluid properties are invariant with temperature.

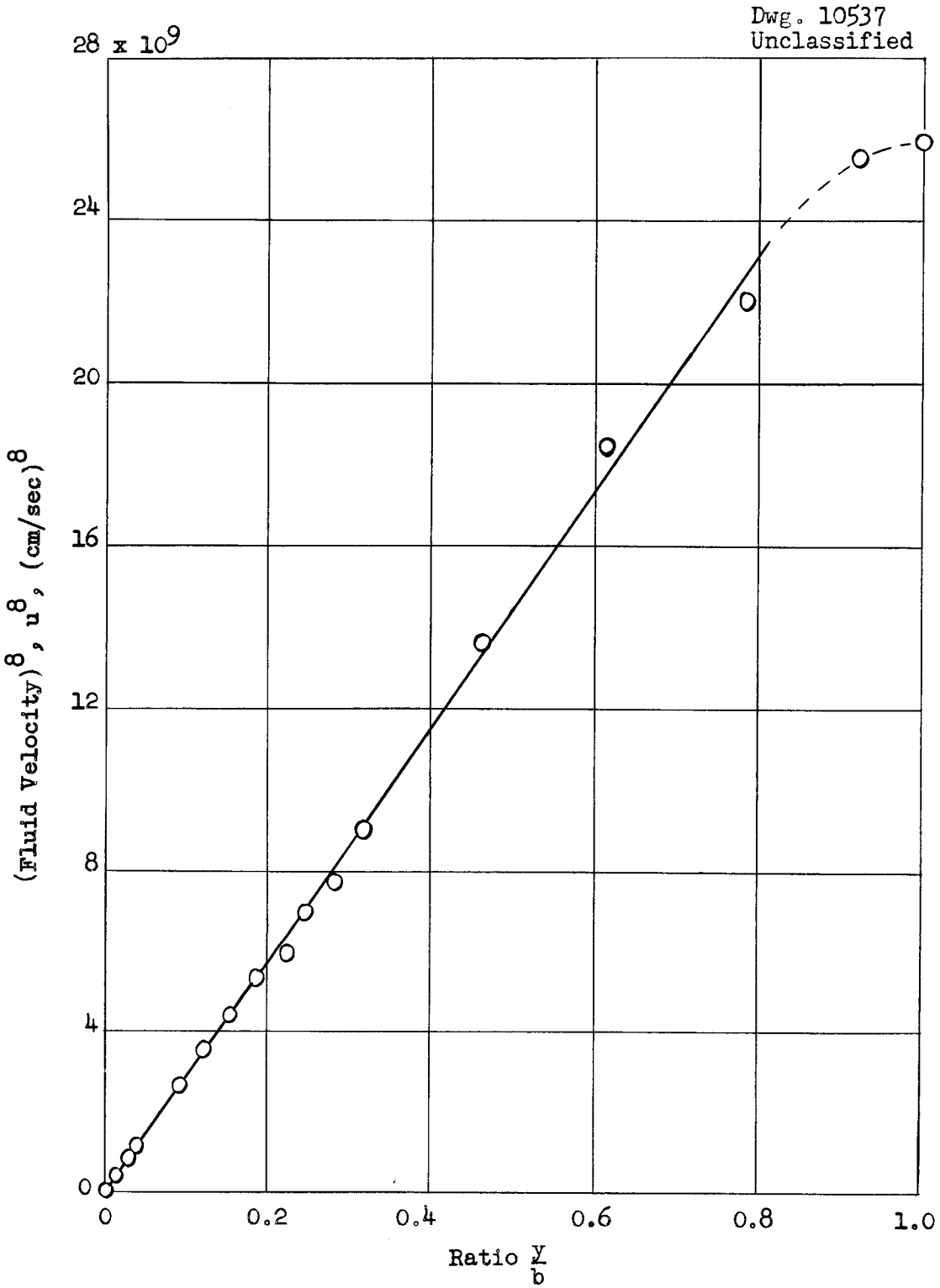

Figure 5. The eighth power velocity distribution for turbulent flow between two parallel plates (measured by Dönch, reference 11)

ANALYTICAL STUDY

A. Derivation of the Temperature Solution

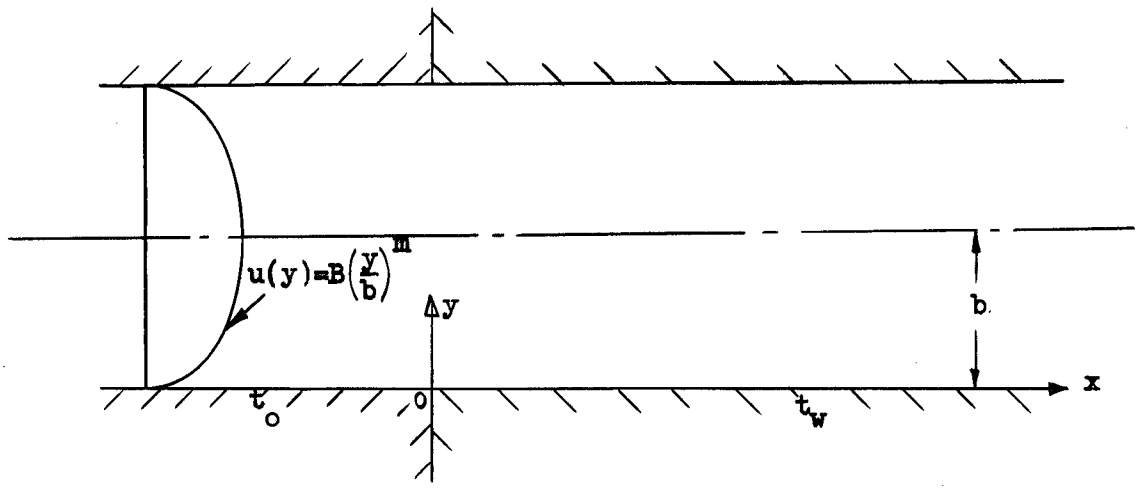

Figure 6. Parallel plate heat transfer - fluid flow system

The heat transfer differential and boundary equations for the system under consideration are

$$B\left(\frac{y}{b}\right)^m \frac{\partial t}{\partial x} = a \frac{\partial^2 t}{\partial y^2} \tag{1}$$

$$t(x,o) = t_w \tag{2}$$

$$\frac{\partial t}{\partial y}(x,b) = o \tag{3}$$

$$t(o,y) = t_o \tag{4}$$

where $u(y) = B\left(\frac{y}{b}\right)^m$, turbulent velocity profile

x,y, cartesian coordinates in Figure 1

t, fluid temperature

a, thermal diffusivity of the fluid

b, one half the distance between the parallel plates

B, parameter in the turbulent velocity expression (a function of Reynolds' modulus)

m, exponent in the velocity expression

Equations 1 to 4 can be expressed in the following more general form:

$$\frac{\partial T}{\partial X} = \frac{F^2}{Y^m} \frac{\partial^2 T}{\partial Y^2} \tag{5}$$

$$T(X,0) = 0 \tag{6}$$

$$\frac{\partial T}{\partial Y}(X,1) = 0 \tag{7}$$

$$T(0,Y) = T_o \tag{8}$$

where,

$$T = t - t_w$$

$$T_o = t_o - t_w$$

$$F^2 = \frac{a}{bB}$$

$$X = \frac{x}{b}$$

$$Y = \frac{y}{b}$$

This boundary value problem can be solved by the separation of variables technique.

$$\text{Let} \quad T = \Phi_1(X).\Phi_2(Y) \tag{9}$$

where $\Phi_1(X)$ and $\Phi_2(Y)$ are functions of X and Y, respectively.
Upon the substitution of these functions into the differential equation there results,

$$\frac{1}{\Phi_1(X)} \frac{d\Phi_1(X)}{dX} = \frac{F^2}{Y^m \Phi_2(Y)} \quad \frac{d^2\Phi_2(Y)}{dY^2} = -\beta^2 \tag{10}$$

where β^2 is a constant.

$$\text{Thus} \quad \frac{d\Phi_1(X)}{dX} + \beta^2 \Phi_1(X) = 0 \tag{11}$$

$$\text{and} \quad \frac{d^2\Phi_2(Y)}{dY^2} + \frac{\beta^2}{F^2} Y^m \Phi_2(Y) = 0 \tag{12}$$

The solutions of equations (11) and (12) are

$$\Phi_1(X) = C_1 e^{-\beta^2 X} \tag{13}$$

$$\Phi_2(Y) = C_2 Y^{\frac{1}{2}} J_{\frac{1}{m+2}}\left[\left(\frac{2}{m+2}\right) \frac{\beta}{F} Y^{\frac{m+2}{2}}\right] + C_3 Y^{\frac{1}{2}} Y_{\frac{1}{m+2}}\left[\left(\frac{2}{m+2}\right) \frac{\beta}{F} Y^{\frac{m+2}{2}}\right] \tag{14}$$

where J and Y are $\frac{1}{m+2}$ order Bessel functions of the first and second kind, respectively, and where C_1, C_2, and C_3 are constants.

Thus the general solution is

$$T = C_1 e^{-\beta^2 X} Y^{\frac{1}{2}} \left[C_2 J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{\beta}{F} Y^{\frac{m+2}{2}} \right) + C_3 Y_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{\beta}{F} Y^{\frac{m+2}{2}} \right) \right] \tag{15}$$

The $\lim_{Y \to 0} Y^{\frac{1}{2}} Y_{\frac{1}{m+2}}$ is finite and not equal to zero, but $T(X,0) = 0$; thus C_3 must be equal to zero.

$$\text{Thus } T = C e^{-\beta^2 X} Y^{\frac{1}{2}} J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{}{F} Y^{\frac{m+2}{2}} \right) \tag{16}$$

where C is a constant.

$$\text{Also } \frac{\partial T}{\partial Y} = C e^{-\beta^2 X} \left[\frac{1}{2Y^{\frac{1}{2}}} J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{\beta}{F} Y^{\frac{m+2}{2}} \right) + \frac{\beta}{F} Y^{\frac{m+1}{2}} J'_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{\beta}{F} Y^{\frac{m+2}{2}} \right) \right] \tag{17}$$

$$\text{where } J'_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{\beta}{F} Y^{\frac{m+2}{2}} \right) = \frac{\partial J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{\beta}{F} Y^{\frac{m+2}{2}} \right)}{\partial \left(\left(\frac{2}{m+2}\right) \frac{\beta}{F} Y^{\frac{m+2}{2}} \right)}$$

The constant β can be evaluated from the boundary condition given by equation (7),

$$\frac{\partial T}{\partial Y}(X,1) = 0 = C e^{-\beta^2 X} \left[\frac{1}{2} J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{\beta}{F} \right) + \frac{\beta}{F} J'_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{\beta}{F} \right) \right]$$

Thus the bracketed term (the eigenfunction) is equal to zero.

$$\frac{F}{2} J_{\frac{1}{m+2}}(Z_1\beta_n) + \beta_n J'_{\frac{1}{m+2}}(Z_1\beta_n) = 0 \tag{18}$$

where, β_n, are the eigenvalues (n = 1,2,...)

$$Z_1 = \left(\frac{2}{m+2}\right)\frac{1}{F}$$

$$Z = \left(\frac{2}{m+2}\right)\frac{(Y)}{F}^{\frac{m+2}{2}}$$

Thus $$T = Y^{\frac{1}{2}} \sum_{n=1}^{\infty} c_n e^{-\beta_n^2 X} J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right)\frac{\beta_n}{F} Y^{\frac{m+2}{2}}\right) \tag{19}$$

where c_n are constants to be evaluated from the last boundary condition given by equation (8):

$$T(0,Y) = T_o = Y^{\frac{1}{2}} \sum_{n=1}^{\infty} c_n J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right)\frac{\beta_n}{F} Y^{\frac{m+2}{2}}\right) \tag{20}$$

or $$\frac{T_o}{\left(\left(\frac{m+2}{2}\right)ZF\right)^{\frac{1}{m+2}}} = \sum_{n=1}^{\infty} c_n J_{\frac{1}{m+2}}(\beta_n Z) = f(Z) \tag{21}$$

Because $\int_0^{Z_1} Z^{\frac{1}{2}} f(Z)dZ$ exists and is uniformly convergent, it is possible to expand f(Z) in a Dini series (reference 13).

$$f(Z) = \sum_{n=1}^{\infty} b_n J_{\frac{1}{m+2}}(\beta_n Z) \tag{22}$$

where

$$b_n = c_n = \frac{\int_0^{Z_1} f(Z) Z J_{\frac{1}{m+2}}(\beta_n Z) dZ}{\int_0^{Z_1} Z \left[J_{\frac{1}{m+2}}(\beta_n Z) \right]^2 dZ} = \frac{\int_0^{Z_1} f(Z) Z J_{\frac{1}{m+2}}(\beta_n Z)\, dZ}{\frac{1}{2\beta_n^2}\left[Z_1^2 \frac{F^2}{4} + Z_1^2 \beta_n^2 - \left(\frac{1}{m+2}\right)^2 \right] \left[J_{\frac{1}{m+2}}(\beta_n Z_1) \right]^2} \tag{23}$$

and β_n are the roots of

$$\frac{F}{2} J_{\frac{1}{m+2}}(Z_1 \beta_n) + \beta_n J'_{\frac{1}{m+2}}(Z_1 \beta_n) = 0 \tag{18}$$

Thus the temperature solution is

$$T = Y^{\frac{1}{2}} \sum_{n=1}^{\infty} c_n e^{-\beta_n^2 X} J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right) \frac{\beta_n}{F} Y^{\frac{m+2}{2}} \right)$$

Where β_n are the roots of

$$\frac{F}{2} J_{\frac{1}{m+2}}(Z_1 \beta_n) + \beta_n J'_{\frac{1}{m+2}}(Z_1 \beta_n) = 0$$

$$c_n = \frac{\int_0^{Z_1} f(Z) Z J_{\frac{1}{m+2}}(\beta_n Z)\, dZ}{\frac{1}{2\beta_n^2}\left[Z_1^2 \frac{F^2}{4} + Z_1^2 \beta_n^2 - \left(\frac{1}{m+2}\right)^2 \right] \left[J_{\frac{1}{m+2}}(\beta_n Z_1) \right]^2} \tag{24}$$

$$Z = \left(\frac{2}{m+2}\right)\left(\frac{Y}{F}\right)^{\frac{m+2}{2}} \qquad T = t - t_w$$

$$F^2 = \frac{a}{bB} \qquad T_o = t_o - t_w$$

$$a = \frac{k}{\gamma c_p} \qquad X = \frac{x}{b}$$

$$Y = \frac{y}{b} \qquad (24) \text{ (Con't.)}$$

B. Determination of the Hydrodynamic Parameter, 'B'

The parameter 'B' can be determined in terms of the mean velocity.

$$U = \frac{\int_{A_c} u \, dA_c}{A_c} = \frac{2\int_o^b u(y)\, l \, dy}{2bl}$$

$$= \frac{\int_o^b By^m dy}{b^{m+1}} = \frac{B}{b^{m+1}} \left(\frac{1}{m+1}\right) b^{m+1}$$

$$= \frac{B}{(m+1)} \qquad (25)$$

where,

U, mean fluid velocity

A_c, fluid flow cross sectional area

Thus $B = (m + 1)U$ (26)

Note that the mean fluid velocity in the parallel plate system is related to the pressure drop and the friction factor as in the case of flow in a pipe (reference 9).

$$\frac{\Delta p}{\Delta x} = \zeta \frac{\rho}{2b} \frac{U^2}{2} \tag{27}$$

$$\zeta = \Phi(Re) \tag{28}$$

where, $\frac{\Delta p}{\Delta x}$, axial fluid pressure drop

ζ, friction factor

ρ, fluid mass density

$Re = \frac{2bU}{\nu}$, Reynolds' modulus

ν, kinematic fluid viscosity

C. Heat Flow Solution

The local convective heat flow expression is

$$\left(\frac{q}{A}\right)_X = -k \left(\frac{\partial t}{\partial y}\right)_{y=0} = -\frac{k}{b}\left(\frac{\partial T}{\partial Y}\right)_{Y=0}$$

$$= -\frac{k}{b}\left[\frac{1}{2Y^{\frac{1}{2}}} \sum_{n=1}^{\infty} c_n e^{-\beta_n^2 X} J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right)\frac{\beta_n}{F} Y^{\frac{m+2}{2}}\right)\right.$$

$$\left. + Y^{\frac{1}{2}} \sum_{n=1}^{\infty} c_n e^{-\beta_n^2 X} \frac{\beta_n}{F} Y^{\frac{m}{2}} J'_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right)\frac{\beta_n}{F} Y^{\frac{m+2}{2}}\right)\right]_{Y=0} \tag{29}$$

where,

$$\lim_{Y\to 0} \frac{J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right)\frac{\beta_n}{F} Y^{\frac{m+2}{2}}\right)}{Y^{\frac{1}{2}}} = \frac{\left[\left(\frac{1}{m+2}\right)\frac{\beta_n}{F}\right]^{\frac{1}{m+2}}}{\Gamma\left(\frac{1}{m+2}+1\right)}$$

$$\lim_{Y\to 0} Y^{\frac{m+1}{2}} J'_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right)\frac{\beta_n}{F} Y^{\frac{m+2}{2}}\right) = 0$$

Thus $$\left(\frac{q}{A}\right)_X = -\frac{k}{b}\,\frac{\left[\left(\frac{1}{m+1}\right)\frac{1}{F}\right]^{\frac{1}{m+2}}}{\Gamma\left(\frac{1}{m+2}+1\right)}\sum_{n=1}^{\infty} c_n e^{-\beta_n^2 X}\,\beta_n^{\frac{1}{m+2}} \tag{30}$$

D. Convective Conductance Solution

The local unit thermal convective conductance is defined as

$$h_{cX} = \frac{\left(\frac{q}{A}\right)_X}{t_w - t_{mX}} \tag{31}$$

where, t_w, duct wall temperature

t_{mX}, the local mixed mean fluid temperature

The local mixed mean fluid temperature is defined as

$$t_{mX} = \frac{2\int_0^b ut\,1dy}{2U\int_0^b 1dy}$$

$$= t_w + \frac{B}{Ub}\int_0^b \left(\frac{y}{b}\right)^m \left(\frac{y}{b}\right)^{\frac{1}{2}} \sum_{n=1} c_n e^{-\beta_n^2 \frac{x}{b}}\, J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right)\frac{\beta_n}{F}\left(\frac{y}{b}\right)^{\frac{m+2}{2}}\right)dy \tag{32}$$

The substitution of equations 30 and 32 into equation 31 yields

$$h_{cX} = \frac{\dfrac{\left[\left(\dfrac{1}{m+2}\right)\dfrac{1}{F}\right]^{\frac{1}{m+2}}}{\Gamma\left(\dfrac{1}{m+2}+1\right)} \displaystyle\sum_{n=1}^{\infty} c_n e^{-\beta_n^2 \frac{x}{b}} \beta_n^{\frac{1}{m+2}}}{\dfrac{B}{Uk}\displaystyle\int_0^b \left(\frac{y}{b}\right)^m \left(\frac{y}{b}\right)^{\frac{1}{2}} \sum_{n=1}^{\infty} c_n e^{-\beta_n^2 \frac{x}{b}} J_{\frac{1}{m+2}}\left(\left(\frac{2}{m+2}\right)\frac{\beta_n}{F}\left(\frac{y}{b}\right)^{\frac{m+2}{2}}\right) dy} \tag{33}$$

E. Generalization

The solutions which have been derived may be expressed in more general forms involving Reynolds' and Prandtl's moduli. For example, it can be shown that

$$\left[\left(\frac{2}{m+2}\right)\frac{\beta_n}{F}(Y)^{\frac{m+2}{2}}\right] = \left[\left(\frac{2}{m+2}\right)\left(\frac{m+1}{2}\right)^{\frac{1}{2}}\beta_n\right]\left[\mathrm{Re}\ \mathrm{Pr}\ Y^{m+2}\right]^{\frac{1}{2}} \tag{34}$$

$$F^2 = \left(\frac{2}{m+1}\right)\left(\frac{1}{\mathrm{Re}\ \mathrm{Pr}}\right) \tag{35}$$

$$Z_1 = \left(\frac{2(m+1)}{(m+2)^2}\cdot \mathrm{Re}\ \mathrm{Pr}\right)^{\frac{1}{2}} \tag{36}$$

and c_n and β_n are functions of Re, Pr, and m. Re and Pr are Reynolds' and Prandtl's moduli, respectively.

DISCUSSION

Martinelli and Boelter (reference 4) have shown that heat transfer in a pipe system can be satisfactorily approximated by heat transfer in a parallel plate system with the pipe velocity profile. This approximation is valid because the flow annuli in the vicinity of the pipe wall (rather than the flow annuli near the pipe center) are the important heat transfer layers, and they may be treated as flow layers in a parallel plate system. Thus, heat flows and convective conductances may be estimated for the pipe system by utilizing the parallel plate solutions containing new hydrodynamic parameters 'B' and 'm'.

The determination of the eigenvalues, β_n, and the series coefficients, c_n, from equations 18 and 23, respectively, must be accomplished before the heat transfer solutions given by equation 24, 30 and 33 can be evaluated. Although this typical kind of operation is tedious, it only involves the plotting of known functions and the performing of simple graphical integrations. It is planned to undertake this task in the future.

H.F. Poppendiek

H. F. Poppendiek

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