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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

FOR THE PERIOD ENDING APRIL 30, 1951

OAK RIDGE NATIONAL LABORATORY
OPERATED BY
CARBIDE AND CARBON CHEMICALS COMPANY
A DIVISION OF UNION CARBIDE AND CARBON CORPORATION
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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

for the Period Ending April 30, 1951

A. S. Householder, Chief

DATE ISSUED: JUL 2 1951

OAK RIDGE NATIONAL LABORATORY
operated by
CARBIDE AND CARBON CHEMICALS COMPANY
A Division of Union Carbide and Carbon Corporation
Post Office Box P
Oak Ridge, Tennessee

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Mathematics Panel quarterly progress reports previously issued in this series are as follows:

ORNL-345	December, January, February, 1948-1949
ORNL-408	Period Ending July 31, 1949
ORNL-516	Period Ending October 31, 1949
ORNL-634	Period Ending January 31, 1950
ORNL-726	Period Ending April 30, 1950
ORNL-818	Period Ending July 31, 1950
ORNL-888	Period Ending October 31, 1950
ORNL-979	Period Ending January 31, 1951

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SUMMARY

Dr. Nat Edmonson, Miss Marina Tsagaris, and Miss Phyllis Brown have transferred from NEPA to the Mathematics Panel, and will be working with the ANP group. Another addition to the same group is Mrs. Mildred Guernsey, of Knoxville. Dr. Ward C. Sangren, formerly of Miami University, Oxford, Ohio, is another new member of the Panel. Dr. Sangren received his doctor's degree at the University of Michigan in applied mathematics in 1950.

Mr. B. W. Drucker, ORINS Fellow from the University of North Carolina, is planning to write his thesis at the Laboratory on some problems relating to the numerical solution of differential equations on automatic computing machinery. Dr. J. W. Givens, of the University of Tennessee, is a new consultant, and will be working chiefly on algebraic and logical problems associated with the design and use of automatic computers. New consultants in biometrics are Dr. J. Z. Hearon, of the University of Chicago, and Dr. H. L. Lucas, of North Carolina State College.

There were one hundred and four in attendance at the joint meeting of the Institute of Mathematical Statistics and the Biometric Society in March. Social events included a smoker, sponsored by Carbide and Carbon, and a banquet with Dr. Weinberg as speaker. Among the speakers at the scientific sessions were Dr. G. E. Albert and Dr. J. W. Tukey, consultants, and Dr. A. W. Kimball.

Two major computational projects were completed this quarter, the RaE and the interpolation for the K-shell internal conversion coefficients. These have been described in previous quarterly reports (see p. 3 for ORNL numbers of reports). The calculation of the Fourier coefficients for B. S. Borie, Jr., is to be done on the differential analyzer at Wayne University.

The SEAC is about to begin actual calculation of the L-shell internal conversion coefficients. While the proposed moving of the machine to new quarters was canceled, the coding of the problem has taken longer than was anticipated because storage requirements for the problem are excessive. Ruth Arnette, who assisted in the coding during the early stages, is returning to Washington to assist in final checks in the coding and in checking the results as they come out.

Dr. J. W. Tukey visited the Laboratory on March 13 and 14. In addition to his consultation on biometric problems, he proposed a plan for constructing a possible analog computer for P. R. Bell's problem in beta-ray spectroscopy, and much improved circuitry for Dr. W. A. Arnold's instruments for measuring light production in plants. Dr. Arnold was able to introduce the innovation immediately; the feasibility in practice of the analog computer is being studied.

Developmental work at Argonne on the Oak Ridge digital computer has continued on the memory systems and on a new type of toggle. The work is reported in detail in the last quarterly progress report of the Argonne National Laboratory (ANL-4587). At the Institute for Advanced Study the control system has not yet been designed, whereas at Los Alamos this part of the machine has been built and tested. It is planned that the control for the Oak Ridge machine will be designed this summer after a study has been made of the control for the Los Alamos machine, and of whatever information can be obtained from IAS. Meanwhile, members of the Panel are studying coding techniques and are programming a number of standard routines such as the computation of elementary functions, solving linear systems and algebraic equations, and others that can be expected to recur frequently in any extensive calculation. The purpose is both to gain experience and to accumulate a library of routines for future use. This work will not be further discussed in this report.

In the ORINS Traveling Lecture Program, members of the Panel made the following appearances:

Dr. Lewis Nelson

Mathematics for Engineers, Young and Old, Georgia Institute of Technology, February 27.

An Abel Integral Equation, Alabama Polytechnic Institute, March 1.

Mathematics for Engineers, Young and Old, Alabama Polytechnic Institute, March 1.

An Abel Integral Equation, Duke University, April 25.

Dr. C. L. Perry

Programming for an Automatic Computing Machine, University of Oklahoma, April 25.

Dr. A. W. Kimball

Some Elementary Notions of Sequential Analysis, University of Georgia, February 22.

Factorial Designs, University of Georgia, February 23.

Dependent Tests of Significance in the Analysis of Variance, University of Georgia, February 24.

Dr. A. S. Householder

Nervous Systems and Computing Machines, Tulane University, February 13.

Iterative Methods for Solving Equations, Tulane University, February 14.

Mathematics in Oak Ridge, Mississippi State College (Mississippi-Louisiana Section, Mathematical Association of America), February 16.

Nervous Systems and Computing Machines, Mississippi State College (Mississippi-Louisiana Section, Mathematical Association of America), February 17.

Iterative Methods for Solving Equations and *Nervous Systems and Computing Machines*, Vanderbilt University, February 19.

A paper by Dr. C. L. Perry entitled "The Bending of Thin Elliptic Plates" appeared in the American Mathematical Society's Vol. III of *Symposia on Applied Mathematics*.

SPECIAL PROBLEMS AND PROJECTS

Problem: **EFFECT OF ROUND-OFF ON OPERATIONS WITH MATRICES**

Participating Member of Panel: A. S. Householder

Origin and Background: The build-up of error due to round-off can at times completely invalidate the results of any computation, and in automatic computations, which are voluminous and not accessible to direct observation as the work progresses, careful preliminary analysis is a prerequisite. Unfortunately, not many guiding rules are to be found in the literature. Some results relating to iterative operations with matrices, which seem to be new, are at hand; one of these will be quoted.

Given the system $Ax = y$, if x_0 is an approximate solution, obtained by any method, and if $r_1 = y - Ax_0$, then the same sequence of operations which, when applied to y_1 yield x_0, will, in general, when applied to r_1 yield a correction x_1 to x_0 such that $x_0 + x_1$ is a better approximation than is x_1. By defining $r_2 = r_1 - Ax_1$ and proceeding sequentially one obtains in principle an infinite series which converges to the true solution under very general hypotheses. In practice, however, each step may introduce round-off errors so that real improvement is to be expected in at most a finite number of the steps. It is desired to estimate this number of steps.

If $A = (\alpha_{ij})$, let

$$b(A) = \max_{i,j} |\alpha_{ij}|, \qquad N(A) = \left[\sum_{i,j} \alpha_{ij}^2 \right]^{1/2},$$

with analogous definitions for $b(x)$ and $N(x)$, and let

$$M(A) = \max_{N(x)=N(y)=1} |x^T A y|, \qquad m(A) = \min_{N(x)=N(y)=1} |x^T A y|.$$

It is known that if λ_1^2 and λ_n^2 are the largest and smallest proper values of $A^T A$, then

$$M(A) = \lambda_1, \qquad m(A) = \lambda_n,$$

$$M(A^{-1}) = \lambda_n^{-1}, \qquad m(A^{-1}) = \lambda_1^{-1}.$$

Let the machine operate in the base β (normally = 10 or 2) to ρ figures; let, for every p,

$$\beta^{-\rho'} b(r_p) \leq b(r_{p+1}) \leq \beta^{-\rho} b(r_p), \qquad \beta^{-\rho'} N(r_p) \leq N(r_{p+1}) \leq \beta^{-\rho} N(r_p)$$

where $r_0 = y$; and let all elements of the matrices and vectors lie in the range from -1 to +1. The operations which yield x_0 from y are equivalent to the multiplication by y of an approximate inverse C, $x_0 = Cy$. Hence $x_p = Cr_p$.

If at any step the possible round-off error can exceed the magnitude of the correction, the step provides no dependable improvement. It may be seen inductively that $N(x_p) \geq \beta^{-p\rho'} m(C) N(y)$. Hence the iteration should stop when

$$n^{3/2} \beta^{p\rho'-\sigma} [1 + M(A)] M^{p-1}(A) M^p(C) \geq 2m(C) N(y).$$

Since C is an approximate inverse, approximately

$$M(C) = 1/m(A), \qquad m(C) = 1/M(A).$$

Hence the criterion may be stated in the form

$$n^{3/2} \beta^{p\rho'-\sigma} [1 + M(A)] [M(A)/m(A)]^p \geq 2N(y),$$

or

$$n^{3/2} \beta^{p\rho'-\sigma} [1 + M(A)] [M(A) M(C)]^p \geq 2N(y).$$

Finding $M(A)$ and $m(A)$ involves solving for a largest and smallest proper value. However, for any matrix $N(A) \geq M(A)$, so that N can be used to replace M in the last statement of the criterion.

Problem: COUNTER STATISTICS

Participating Members of Panel: G. E. Albert, Lewis Nelson

References: G. E. Albert and M. L. Nelson, "Counter Statistics," *Mathematics Panel Quarterly Progress Report for Period Ending January 31, 1951*, ORNL-979, pp. 10-16 (Mar. 30, 1951).

Background and Status: Work on the types I and II counters reported in the reference was temporarily discontinued in favor of study of a rate meter. This study was requested by Henri Levy of the Chemistry Division who needs a criterion for deciding whether "unusual" behavior can be attributed to instrument malfunction.

In this study a rate meter is regarded as a linear instrument whose output at time T is $q(T - t)$ in response to arrival at time t of a particle from the sample. $q(x)$ is known and vanishes for negative x. The particular case at hand is an R-C circuit, the charge on whose condenser is increased by a constant amount when a particle reaches the meter. In this case $q(x)$ is taken to be

$$x < 0, \qquad q(x) = 0; \qquad 0 < x, \qquad q(x) = \exp(-\lambda x) \tag{1}$$

where the circuit time constant λ^{-1} is of the order of 10 to 30 min.

Let t_j be the time of arrival of the jth particle, and define ξ_j by $t_j = t_{j-1} + \xi_j$ with $\xi_1 = t_1$. We suppose t_{j-1} and ξ_j are independent random variables and that each ξ_j is distributed by

$$P(\xi_j \leq \xi) = 1 - \exp(-a\xi) \qquad 0 \leq \xi .$$

If N is the number of particles which arrive prior to some fixed time T, it follows that N is distributed by

$$P(N = k) = \frac{(aT)^k}{k!} \exp(-aT) \qquad k = 0, 1, 2, \ldots . \tag{2}$$

The constant a is the quantity being measured by the meter, that is, the rate of arrival of particles at the meter.

At time T the output of the meter is proportional to the random variable

$$Q(T) = \sum_{j=0}^{N} q(T - t_j) \tag{3}$$

which is a random sum of random variables. It is of importance to study the distribution $F(x;\ T) = P[Q(T) \leq x]$. We have succeeded in finding its Laplace

transform from which each moment can be computed. We proceeded thus:

Letting T be fixed and abbreviating $Q(T)$ by Q, we can write

$$E[\exp(\theta Q)] = \sum_{k=0}^{\infty} P(N = k)\, E\{\exp[\theta(Q|N = k)]\} . \tag{4}$$

To compute the individual terms we express Q in terms of the ξ_j, and determine the distribution of $(\xi_1, \xi_2, \ldots, \xi_k)$ under the condition $0 \le \xi_1 + \ldots + \xi_k \le T$ and $\xi_1 + \ldots + \xi_{k+1} > T$, i.e., under the condition $N = k$. From Eq. (3),

$$\{Q|N = k\} = \sum_{j=0}^{k} q(T - \xi_1 - \xi_2 - \ldots - \xi_j). \tag{3'}$$

Suppose $k = 1$; then

$$P(\xi_1 \le x,\ \xi_1 \le T < \xi_1 + \xi_2) = P(\xi_1 \le T < \xi_1 + \xi_2)P(\xi_1 \le x|N = 1) . \tag{5}$$

By direct computation we find

$$P(\xi_1 \le x,\ \xi_1 \le T < \xi_1 + \xi_2) = ax \exp(-aT) \qquad 0 \le x \le T$$

which with Eq. (5) gives

$$P(\xi_1 \le x|N = 1) = x/T \qquad 0 \le x \le T .$$

By a similar computation we find for $N = k$ and $0 \le x_1 + x_2 + \ldots + x_k \le T$,

$$P(x_1 \le \xi_1 < x_1 + \Delta x_1, \ldots, x_k \le \xi_k \le x_k + \Delta x_k|N = k) = \frac{k!}{T^k} \prod_{j=1}^{k} \Delta x_j . \tag{6}$$

Using this distribution and Eq. (3′) gives

$$E\{\exp[\theta(Q|N = k)]\} = \frac{k!}{T^k} \int_{S_k} \prod_{j=1}^{k} G(T - \xi_1 - \ldots - \xi_j;\ \theta)\, d\xi_1 \ldots d\xi_k \tag{7}$$

where S_k is the simplex $0 \leq \xi_1 + \ldots + \xi_k \leq T$ and $G(x;\ \theta) = \exp\ [\theta q(x)]$. Transforming the variables of integration by

$$Tx_j = T - \xi_1 - \ldots - \xi_j \qquad j = 1,\ 2,\ \ldots,\ k$$

reduces the right hand member of Eq. (7) to

$$k! \int_0^1 G(Tx_1;\ \theta)\ dx_1 \int_0^{x_1} G(Tx_2;\ \theta)\ dx_2 \ldots \int_0^{x_{k-1}} G(Tx_k;\ \theta)\ dx_k. \qquad (7')$$

Now define

$$H_1(Tx;\ \theta) = \int_0^x G(Tx_1;\ \theta)\ dx_1$$

and

$$H_k(Tx;\ \theta) = \int_0^x G(Tx_1;\ \theta)\ dx_1 \int_0^{x_1} G(Tx_2;\ \theta)\ dx_2 \ldots \int_0^{x_{k-1}} G(Tx_k;\ \theta)\ dx_k,$$

from which for $k = 2,\ 3,\ \ldots$,

$$H_k(Tx;\ \theta) = \int_0^x H_{k-1}\ (Tx_1;\ \theta)\ \ G(Tx_1;\ \theta)\ dx_1. \qquad (8)$$

But from the definition of $H_1(Tx;\ \theta)$,

$$H_1(0;\ \theta) = 0 \qquad H_1^*\ (Tx;\ \theta) = G(Tx;\ \theta)$$

where * means differentiation as to x. Applying this to Eq. (8) when $k = 2$ yields

$$H_2(Tx;\ \theta) = \int_0^x H_1(Tx_1;\ \theta)\ H_1^*\ (Tx_1;\ \theta)\ dx = \frac{1}{2!}\ [H_1(Tx;\ \theta)]^2$$

and an easy induction leads to

$$H_k(Tx;\ \theta) = \frac{1}{k!}[H_1(Tx;\ \theta)]^k$$

Substituting from this into Eqs. (7′), (7), and (4), recalling Eq. (2), we find

$$E\{\exp\ [\theta Q(T)]\} = \sum_{k=0}^{\infty} \exp\ (-aT)\ \frac{(aT)^k}{k!}[H_1(T;\ \theta)]^k = \exp\ \{-aT[1 - H_1(T;\ \theta)]\} \qquad (9)$$

Differentiating Eq. (9) n times as to θ and then setting $\theta = 0$ gives the nth moment of the distribution of $Q(T)$. The first three *central* moments are

$$E\{Q(T)\} = \frac{a}{\lambda}\ \{1 - \exp\ (-\lambda T)\} = m,$$

$$E\{[Q(T) - m]^k\} = \frac{a}{k\lambda}\{1 - \exp\ (-k\lambda T)\} \qquad k = 2,\ 3.$$

From this it is clear that the function $\lambda Q(T)$ will have mean approximately a for the large T. The variance of $zQ(T)$ is approximately

$$\sigma^2 = \lambda a/2.$$

The explicit distribution of λQ has not been found but it is known that that distribution is not Gaussian. Pending further study, Tchebycheff's inequality,

$$P_r\,[|\lambda Q - a| > c\sigma] \leq \frac{1}{c^2}$$

for arbitrary positive c may be used to test the significance of deviations of λQ from its mean a. For example, choosing $c = 3$,

$$P_r\,[|\lambda Q - a| > \frac{3}{2}\lambda a] \leq \frac{1}{9}.$$

This estimate of probability is very likely much too large since no knowledge of the distribution of Q beyond the first and second moments has been used.

Some work has been done and more will be done on the significance of deviations of the

$$I = \lambda \int_{T_1}^{T_2} Q(T)\, dT$$

from its mean $\alpha(T_2 - T_1)$.

Problem: **TO INVESTIGATE THE STABILITY OF THE REACTOR SIMULATOR**

Origin: Dr. J. Palmer, Instrument Department, Engineering and Maintenance Division

Participating Members of Panel: A. S. Householder, W. C. Sangren, B. M. Drucker

Background: The mathematical problem consists in investigating the stability of two systems of differential equations. The systems are

$$n'(t) = F_1 n(t) - G \sum_{i=1}^{6} \lambda_i C_i'(t) - H_1 n(t)\, v(t)$$

$$C_i'(t) = -\lambda_i C_i(t) + E_i n(t) \qquad (i = 1, \ldots, 6) \tag{1}$$

$$v'(t) = n(t) - n_0$$

and

$$n' = F_1 n - G \sum_{i=1}^{6} \lambda_i C_i' - H_2 n\theta$$

$$C_i' = -\lambda_i C_i + E_i n \qquad (i = 1, \ldots, 6) \tag{2}$$

$$v' = n - n^0$$

$$\theta' = \theta_1$$

$$\theta_1' = -A\theta_1 - B\theta + Cv ,$$

where primes denote differentiation with respect to the time t and where A, B, C, F, G, H_1, H_2, E_i, λ_i, and $n^0 = n(0)$ are constants. If the right sides of systems (1) and (2) set equal to zero are solved, the "stability" points are obtained. In both cases there proves to be but one stable point. In order to test for stability in the neighborhood of the stable point we can consider in these neighborhoods the approximating linear systems

$$
\begin{aligned}
x_8' &= G\sum_{i=1}^{6}\lambda_i x_i - G\sum_{i=1}^{6}E_i x_8 - H_1 n^0 x_7 \\
x_i' &= -\lambda_i x_i + E_i x_8 \qquad (i = 1, \ldots, 6) \\
x_7' &= x_8
\end{aligned}
\tag{1'}
$$

and

$$
\begin{aligned}
x_8' &= G\sum_{i=1}^{6}\lambda_i x_i - G\sum_{i=1}^{6}E_i x_8 - H_2 n^0 x_9 \\
x_i' &= -\lambda_i x_i + E_i x_8 \qquad (i = 1, \ldots, 6) \\
x_7' &= x_8 \\
x_9' &= x_{10} \\
x_{10}' &= -Ax_{10} - Bx_9 + Cx_7
\end{aligned}
\tag{2'}
$$

where $x_i = C_i - C_i^0$ $(i = 1, \ldots, 6)$, $x_7 = v - v^0$, $x_8 = n - n^0$, $x_9 = \theta - \theta^0$, $x_{10} = \theta_1 - \theta_1^0$, and the superscript 0 indicates the value of the variables at the stable point. Since the equations have constant coefficients, the character of the solutions can be determined by finding the characteristic numbers δ_i of the determinant $|A - \delta I|$ where A is the matrix of coefficients of the right-hand sides of system (1′) or (2′). If the real part of all the δ_i's in either system are negative, then that system can be considered stable.

Status: It remains to determine the characteristic numbers δ_i. Although there exists a criterion (Hurwitz) for determining whether a polynomial has only zeros with negative real parts, it involves much computation and will be used only as a last resort.

Problem: **SCATTERING FROM A SCREENED COULOMB POTENTIAL**

Origin: Dr. S. Tamor, Physics Division

Participating Members of Panel: C. L. Perry, W. C. Sangren, K. A. Pflueger, B. S. McGill

Reference: *Mathematics Panel Quarterly Progress Report for Period Ending January 31, 1951*, ORNL-979, p. 41 (Mar. 30, 1951).

Background: The function $\theta(A, E)$ was evaluated for 27 pairs of A and E. This was completed by hand machine computation using the tabulations of the integrand calculated on the IBM equipment.

For each of the three values of E, $\lambda \equiv 1/A$ is then a function of θ. Likewise the cross-section $\sigma(\theta) = \frac{\lambda}{\sin\theta}\frac{d\lambda}{d\theta}$ is a function of θ. The 27 values of $\sigma(\theta)$ were calculated by hand machine using a three-point differentiation formula for the evaluation of $d\lambda/d\theta$. Graphs were made of λ vs. θ and $\sigma(\theta)$ vs. θ.

Status: Completed.

Problem: **BASIC STUDIES IN THE MONTE CARLO METHOD**

Participating Member of Panel: G. E. Albert

Reference: G. E. Albert, "Basic Studies in the Monte Carlo Method," *Mathematics Panel Quarterly Progress Report for Period Ending January 31, 1951*, ORNL-979 (Mar. 30, 1951).

Status: A first draft has been completed of an informal memorandum on the solution of certain types of Fredholm integral equations by the Monte Carlo method. The memorandum explains and generalizes the theory of the procedures set forth in the papers mentioned in the reference. Rules of procedure for estimating the solutions of specific integral equations are not given. Rather, the aim of the memorandum is to open up lines of research for the development of such procedures.

Problem: **ANGULAR CORRELATION BETWEEN CONVERSION ELECTRONS AND GAMMA RAYS**

Origin: Dr. M. E. Rose, Physics Division

Participating Members of Panel: N. M. Dismuke, C. Perhacs

Background: The first step in the calculation is to find the value of the ratio

$$\frac{\Gamma(\gamma_l + i\beta)}{|\Gamma(\gamma_l + i\beta)|},$$

where

$$\gamma_l = \sqrt{l^2 - \alpha^2 z^2}$$

$$\beta = \frac{\alpha z W}{p}; \qquad p = \sqrt{W^2 - 1} > 0 \quad \text{and} \quad W = k + \sqrt{1 - \alpha^2 z^2}.$$

The values of l, z, and k are:

l = 1, 2, 3;

z = 10, 20, 30, 40, 54, 64, 72, 78, 83, 88, 92, 96;

k = 0.3, 0.5, 1.0, 1.8, 3.0, 5.0.

Status: The argument values, γ_l and β, have been computed and interpolation from the National Bureau of Standards table of the gamma function for complex argument is in progress.

Problem: **MONTE CARLO ESTIMATE OF AGE AND COLLISION DISTRIBUTION IN TISSUE**

Origin: Dr. W. S. Snyder and Dr. J. Neufeld, Health Physics Division

Participating Member of Panel: M. R. Arnette

Reference: See previous quarterly reports (numbers will be found on p. 3 of this report).

Status: The IBM results for the collision densities obtained at Y-12 were computed with the position components of the *n*th collision corresponding to the energy after that collision instead of the energy before the collision. The problem is now in progress on the IBM machines at K-25, and new history

cards have been made with the correct energy. A new collision density distribution for the 10-Mev neutrons is planned. Also, collision density calculations are underway for all neutrons with initial energies between 3.7 and 6.06 Mev in that the new position components in the z direction are ready to be computed. To obtain the new z components, the old particle path from the point of the first collision in the lower energy interval will be rotated and translated to obtain a collimated beam at the origin for the lower energy source. These distributions are to be given by cell number; that is, the cells will be numbered 1 through 1020 for the 10-Mev neutrons, with the same energy intervals as before and with the z in steps of 1 cm between 0 and 30 cm.

Problem: **BETA DECAY (FERMI FUNCTIONS)**

Origin: Dr. M. E. Rose, P. R. Bell, Physics Division

Participating Members of Panel: J. H. Fishel, N. D. Given, K. A. Pflueger, N. M. Dismuke

Status: Values of $L_\nu \Big/ \left[\frac{F_\nu}{F_0}\right]$, $M_\nu \Big/ \left[\frac{F_\nu}{F_0}\right]$, $N_\nu \Big/ \left[\frac{F_\nu}{F_0}\right]$ are being calculated on the K-25 IBM machines by J. H. Fishel. Coding and board wiring have been completed and calculation is in progress.

The following checks were made on the ratios F_ν/F_0:

1. Eighteen selected points were calculated by hand.
2. Fifteen points chosen at random were calculated by hand.
3. The ratios were differenced with respect to z through fourth differences.

All hand-calculated points showed better agreement with machine calculations than 0.1%. The difference table indicated that errors as large as 1% do not exist.

The time this quarter devoted to calculating the corrections to the field factors for finite size of the nucleus has been spent checking some odd results found earlier for the positron case. The difficulty has not yet been resolved.

Problem: **RaE BETA SPECTRUM**

Origin: Dr. M. E. Rose, Dr. D. K. Holmes, Physics Division

Participating Members of Panel: A. S. Householder, C. L. Perry, W. C. Sangren, N. D. Given, N. M. Dismuke

Background: See previous quarterly report, ORNL-979. Twelve theoretical formulations of the correction factor $C_{nx}(W)$ were investigated in the attempt to fit the experimental data for the RaE beta spectrum. The formulas for C_{nx} [E. Greuling, *Phys. Rev.* **61**, 568 (1942)], where n is the order of forbiddenness and x is the interaction type, are denoted by $n = 1, 2, 3$ and $x = A$ (axial), S (scalar), T (tensor), and V (polar-vector). For each case C_{nx} is a function of energy, W.

The factors depend on $L_\nu(W)$, $M_\nu(W)$, $N_\nu(W)$ for $\nu = 0, 1, \ldots, n$, on integral powers of $q = W_0 - W_l$, and on some W independent coefficients. All these parts were calculated in duplicate. Finally, summations over ν were required; these were done in duplicate for three of the 11 q values for each C_{nx}.

Of the 12 sets of $C_{nx}(W)$, six were calculated, normalized, and compared with the experimental data. The other six required fitting, i.e., a, b, and c were required which gave the best fit to experimental data where

$$C_{nx} = a\phi_1(W) + b\phi_2(W) + c\phi_3(W)$$

$$a > 0, \qquad b > 0, \qquad -2\sqrt{ab} \leq c \leq 2\sqrt{ab}$$

Fittings were carried out as follows:

1. The best fit using the method of least squares was obtained ignoring the conditions on a, b, and c.
2. For cases in which a, b, and c fell outside the region of interest (since it is known that in this case the best fit will fall on the boundary) the set of a, b, and c on the boundary which gave the best fit was sought graphically by systematically investigating the boundary.
3. Fittings obtained in this way were checked by using an explicit formula for finding the required a, b, and c given by Dr. Rose.

None of the 12 formulations adequately represented the experimental data.

Status: Completed.

Problem: **MONTE CARLO ESTIMATE OF AGE IN H_2O FOR 10-Mev NEUTRONS**

Origin: E. P. Blizard, Physics Division

Participating Members of Panel: A. S. Householder, M. R. Arnette, N. M. Dismuke

Background: See ORNL-888. The age in water for neutrons slowing from 10 Mev to 1 ev was obtained, from the histories found in the Monte Carlo tissue problem, by weighting to correct the proportions of H to O. The weighting assumed N and C collisions were O collisions. The value obtained is

$$\tau \text{ (10 Mev, 1 ev)} = 177 \text{ cm}^2$$

with a standard deviation of 5 cm^2. The ratio of mean time between collisions for water to that for tissue ranges from 1.005 for 10-Mev neutrons to 0.922 for 1-ev neutrons. Since large mean free paths are in the high-energy region for both water and tissue, probably no correction for time scale need be made. The value obtained in MonP-219* was 183 cm^2. In this report the age as a function of energy was obtained and averaged over the fission spectrum, with a result, $\bar{\tau}$, in close agreement with the experimental value.

Status: Completed.

Problem: **INTERPOLATION ON COMPUTED VALUES OF INTERNAL CONVERSION COEFFICIENTS (K-SHELL)**

Origin: Dr. M. E. Rose, Physics Division

Participating Members of Panel: J. H. Fishel, K. A. Pflueger, B. S. McGill, N. D. Given, C. L. Perry

References: See the following quarterly progress reports: Physics Division, ORNL-228 (for period ending November 1948); Mathematics Panel, ORNL-345 (for period ending February 1949), ORNL-408 (for period ending July 1949), ORNL-726 (for period ending April 1950), ORNL-818 (for period ending July 1950), ORNL-888 (for period ending October 1950), ORNL-979 (for period ending January 1951).

Background and Status: The K-shell interpolations were completed during this quarter. In the region $z < 70$ or $k > 0.6$ the interpolation scheme used was the one described in ORNL-979, pp. 38-40. In the region $z \geq 70$ and $k \leq 0.6$

*For derivation of the formula used in MonP-219 and comments on its goodness see L. W. Nordheim, G. Nordheim, and H. Soodak, CP-1251; R. E. Marshak, MT-17; A. M. Weinberg (Pile Neutron Physics), 48-9-128, p. 39.

the value of the ratio of the exact values of the internal conversion coefficient to the Dancoff and Morrison approximate value (neglects binding) differs greatly from 1. For this reason the interpolations were done directly on log α_l and log β_l. The interpolations in both regions were done in the two sequences, k then z, and, z then k, of single variable interpolations. The interpolated values of the internal conversion coefficients were tabulated in 26 tables, one for each of the following atomic numbers: z = 10, 15, 20, 25, 30, 35, 40, 44, 48, 51, 54, 58, 61, 64, 67, 70, 72, 75, 78, 81, 83, 86, 88, 90, 92, and 96.

In addition to the tables the Mathematics Panel also completed 26 graphs of the interpolated values of the internal conversion coefficients. Each graph is for one atomic number and contains the ten coefficients α_l, β_l (l = 1, 2, 3, 4, 5) plotted as functions of the energy k. These graphs and tables will be published as an ORNL-1023.

Problem: CALCULATION OF RACAH COEFFICIENTS FOR THE ANGULAR DISTRIBUTION IN NUCLEAR REACTIONS

Origin: Drs. M. E. Rose, L. C. Biedenharn, Physics Division

Participating Members of Panel: B. S. McGill, R. Crook, C. L. Perry

Reference: *Physics Division Quarterly Progress Report for the Period Ending March 25, 1951,* ORNL-1005

Background and Status: The Mathematics Panel is calculating the Racah coeffients $W(l_1, J_1, l_2 J_2; S, L)$ for S = 1, 3/2, 2 and L = 0, 1, 2, ..., 8. The calculations for S = 1 have been completed and are being checked. The calculations may be extended to include S = 5/2 and 3.

Problem: HARMONIC ANALYSIS

Origin: B. S. Borie, Jr., Metallurgy Division

Participating Members of Panel: W. C. Sangren, J. H. Fishel, C. L. Perry

Reference: *Mathematics Panel Quarterly Progress Report for the Period Ending January 31, 1951,* ORNL-979.

Background and Status: Mr. Borie asked the Mathematics Panel to evaluate 248 Fourier integrals similar to those mentioned in the above report. Since the

NEPA digital computer has been inoperative, the Panel investigated evaluating the integrals on IBM and other computers. Several methods of programming the evaluation of the integrals were evaluated. The differential analyzer and cinema integraph are both better suited to this problem than the IBM computers. The integrals are now being evaluated on the Wayne University differential analyzer.

Problem: DETERMINATION OF THE DIFFUSION LENGTH OF THERMAL NEUTRONS IN THE STANDARD GRAPHITE PILE

Origin: Dr. R. H. Ritchie, Health Physics Division; E. D. Klema, Physics Division

Participating Members of Panel: K. A. Pflueger, C. L. Perry

References: *Preliminary Material for Manual on Graphite Testing,* CL-573; *A Review of Graphite Testing Procedures and Their Applications to the BNL Reactor,* BNL-77.

Background and Status: The diffusion length of thermal neutrons was calculated using the separated variables solution to the diffusion equation for a source in an infinitely long rectangular column. The sources considered were (1) a single source and (2) a double source. For the single source the series solution was approximated by the sum of terms to and including the fifth harmonic. For the double source the series solution was approximated by the sum of terms to and including the seventh harmonic. End correction terms were also calculated to compensate for the finite length of the pile. The computations have been completed.

Problem: DETERMINATION OF STOPPING POWERS FOR IONS

Origin: Drs. W. S. Snyder and J. Neufeld, Health Physics Division

Participating Member of Panel. K. A. Pflueger

References: *Mathematics Panel Quarterly Progress Report for the Period Ending January 31, 1951,* ORNL-979, pp. 42-43; *Ionization and Excitation Losses of Charged Particles of Intermediate Energies,* ORNL-884.

Background and Status: The rate of energy loss of an ion (z_1, m_1) moving in a medium with one nucleus (z_2, m_2) per cubic centimeter in Mev/cm is given by

$$-\frac{1}{N}\frac{dE}{dx} = \frac{4\pi z_1 e^4}{1.602 \times 10^{-6} mV^2} \sum_{r=1}^{z_2} \frac{u_r}{V} K_0\left(\frac{u_r}{V}\right) K_1\left(\frac{u_r}{V}\right) +$$

$$+ \frac{2\pi z_1{}^2 e^4}{1.602 \times 10^{-6} mV^2} \sum_{r=1}^{z_2} \left[0, \ln \frac{2mV^2}{I_r \times 1.602 \times 10^{-12}}\right]$$

(see ORNL-884, pp. 9-10, formulas 11 and 16), where m is the electron rest mass; e the electron charge; $[a, b]$ = larger of a and b; I_r = ionization energy of the rth orbital electron of the z_2 nucleus; $u_r = (2I_r \cdot 1.602 \cdot 10^{-2}/m)^{1/2}$; and K_0 and K_1 are modified Bessel functions of the second kind. The Mathematics Panel is evaluating $(-1/N)(dE/dx)$ for the range $0.1 \leq W \leq 44.5$ where $W = hV/2\pi e^2$; z_1 = 1, 2, 6, 7, 8; and z_2 = 1, 6, 7, 8, tissue.

Problem: ESTIMATION OF TOTAL ABSORPTION BY THE BLOOD OF RADIOCALCIUM ADMINISTERED ORALLY IN DAIRY CATTLE

Origin: Dr. S. L. Hansard, Dr. C. L. Comar, UT-AEC Agricultural Research Program

Participating Members of Panel: A. W. Kimball, C. L. Perry, B. S. McGill, G. J. Atta

Background: This problem was first introduced in the previous quarterly report (ORNL-979, p. 25) and is described in detail there.

Status: Preliminary computations on the first few experiments indicated the need for more empirical data, which have just now been received. Work on the new data will begin shortly.

Problem: **COMPARISON OF MUTATIONS IN YEAST INDUCED BY ULTRAVIOLET IRRADIATION**

Origin: Dr. S. Pomper, Biology Division

Participating Members of Panel: A. W. Kimball, B. S. McGill, G. J. Atta

Background: This is the conclusion of the analysis outlined in the previous quarterly report (ORNL-979, p. 24).

Status: The analysis concerns the results of 16 experiments investigating the effect of ultraviolet irradiation on adenine and uracil mutation rates in both diploid and haploid organisms. In three of the experiments some cultures were exposed to ultrasonic treatment before exposure to ultraviolet radiation. In addition to a control group, exposures of 1, 2, 4, 8, and 16 min were employed, although not all exposure groups were included in each experiment. Mutation rates were computed by reference to complete agar cultures run simultaneously with the adenine- and uracil-deficient cultures.

The analysis was based on the square roots of the mutation rates, and rectilinear plots of several experiments indicated that the transformed rates were linearly related to exposure time. Furthermore, repeated experiments with the same treatment combinations exhibited similar slopes, although slight differences in conditions from one experiment to another resulted in average mutation rates at different levels. A slope and an intercept for each experiment were estimated by the method of maximum likelihood, which in this case is identical with the method of least squares.

The effects of primary interest were examined on the basis of the estimated average slopes. For uracil, the slopes were

	DIPLOID	HAPLOID
No ultrasonic	0.218 (b_1)	0.281 (b_2)
Ultrasonic	0.104 (b_2)	0.124 (b_4)

The principal effects were tested for significance by forming the appropriate linear combination of the b's and using the familiar Student's t statistic. The results are as follows:

EFFECT	COMBINATION	t	P
Haploid-diploid	$b_1 + b_3 - b_2 - b_4$	3.05	<0.01
Ultrasonic—no ultrasonic	$b_1 + b_2 - b_3 - b_4$	9.89	<0.01
Interaction	$(b_1 - b_2) - (b_3 - b_4)$	1.55	0.13

It seems clear that the effect of ultraviolet radiation on haploids is more pronounced than it is on diploids, a conclusion which is consistent with previous observations. It is also clear that ultrasonic treatment reduces the slopes about 50%. Both these comparisons are significant at the 1% level. However, the interaction comparison, which is not statistically significant, indicates that the effect of ultrasonic treatment is about the same on haploids as it is on diploids, so that the effect is not specific for either type of organism.

For adenine the slopes were

	DIPLOID	HAPLOID
No ultrasonic	0.587 (b_1)	0.619 (b_2)
Ultrasonic		0.278 (b_4)

No experiments with ultrasonics were performed with the adenine mutant, so that no estimate of b_3 is available. In this case the comparisons are

EFFECT	COMBINATION	t	P
Haploid-diploid	$b_1 - b_2$	0.91	0.37
Ultrasonic—no ultrasonic	$b_2 - b_4$	7.91	<0.01

Here there appear to be no differences between the haploid and diploid organisms, although the same pronounced effect of ultrasonic treatment is observed. Because no estimate of b_3 is available, the test for interaction cannot be carried out.

In this analysis the equation

$$\sqrt{y} = a + bt$$

was fitted to the data from each experiment, where y = mutation rate and t = length of exposure to ultraviolet radiation. In some cases the observed points fall almost exactly on a straight line. If the linearity of response is real, this implies that

$$y = a^2 + 2abt + b^2t^2.$$

In much of the literature on radiation genetics responses of this nature are interpreted as evidence of a two-hit phenomenon, whereas a linear relation between response and dose is taken to mean that a single-hit phenomenon is at work. Because of the ambiguity in arguing from an empirically fitted curvilinear response, which depends in large measure on the particular function chosen (and there is almost an unlimited number from which to choose), to the fundamental laws governing the radiation effect, any conclusion which goes beyond the postulation of *some* multiple-hit phenomenon is open to serious question. The most we can hope to do in biological experiments is to classify a radiation effect either as a single-hit or as a multiple-hit phenomenon. In these experiments, therefore, there is ample evidence to indicate that a multiple-hit radiation effect is at work, but any conclusion beyond this can certainly not be supported, at least not by appeal to statistical inference.

Problem: ESTIMATION OF DIFFERENCE IN ACTIVATION ENERGIES OF C^{12} and C^{14}

Source: Dr. G. A. Ropp, A. J. Weinberger, Chemistry Division

Participating Members of Panel: A. W. Kimball, C. Perhacs, G. J. Atta

Background: There is recent evidence to indicate the existence of isotope effects in the reactions of isotopic carbon-labeled organic compounds, and such effects are important because of the doubt which they cast on conclusions drawn from tracer experiments in chemical and biological systems. In these experiments the effect of temperature on the magnitude of a simple isotope effect was investigated.

Tracer-labeled formic-C^{14} acid was dehydrated in excess 95% sulfuric acid at four temperatures. At each temperature the evolved carbon monoxide was passed through a small ionization chamber where its instantaneous specific activity was measured and automatically plotted as a function of time by a Brown recorder used with a vibrating-reed electrometer. In each case the first-order reaction rate constant, k_{12}, was determined simultaneously by measurement of the change of the total gas volume with time.

At each temperature the difference, $k_{12} - k_{14}$, in the reaction rate constants for the two species was determined from the slope of the linear plot of the logarithm of the specific activity against time, and the isotope effect was calculated as $100(k_{12} - k_{14})/k_{12}$. By the method of least squares,

a straight line was fitted to each set of data, and the estimates of the slopes and their variances were combined to determine the difference in activation energies, ΔE. The problem was to find a variance for ΔE.

Since it was virtually impossible to find the exact variance of ΔE, the following approximate method* was employed: Let ξ be a continuous random variable, and let $f(\xi)$ be another random variable dependent only on ξ, and satisfying the conditions stated in the reference. The variance of $f(\xi)$ is defined to be

$$E\{f(\xi) - E[f(\xi)]\}^2$$

where the expected values are taken over the probability distribution of ξ. We can expand $f(\xi)$ in a Taylor's series about $E(\xi)$ as follows:

$$f(\xi) = f[E(\xi)] + [\xi - E(\xi)]\ f'\ [E(\xi)] + \frac{[\xi - E(\xi)]^2}{2!}\ f''[E(\xi)] + \cdots \quad .$$

Ignoring all nonlinear terms in ξ, we have

$$f(\xi) - f[E(\xi)] \approx [\xi - E(\xi)]\ f'[E(\xi)] \quad .$$

Now if $E[f(\xi)] = f[E(\xi)]$, we may write

$$V[f(\xi)] = E\{f(\xi) - E[f(\xi)]\}^2 \approx E\{[\xi - E(\xi)]\ f'[E(\xi)]\}^2$$

as an approximation to the variance of $f(\xi)$. When two random variables are involved, a Taylor's series expansion in two variables is employed, and the calculation proceeds in a similar fashion.

The calculation of ΔE in the present experiments involved taking ratios of random variables and logarithms of random variables. The approximate

*Proved in detail in *Techniques of Statistical Analysis*, ed. by Eisenhart, Hastay, and Wallis, pp. 409-410, McGraw-Hill, New York, 1947.

method just described leads to the following variance formulas:

$$V\left(\frac{x}{y}\right) \approx \left(\frac{x}{y}\right)^2 \left\{\frac{V(x)}{x^2} + \frac{V(y)}{y^2} - 2\frac{Cov\ (x,y)}{xy}\right\}$$

$$V(\log x) \approx \frac{V(x)}{x^2}$$

The first formula is used widely in bioassay calculations.

The final estimate of the difference, ΔE, in the Arrhenius activation energies for the two species was 183 calories per mole, with a standard error of 9 calories per mole. If, because of the approximations involved in the calculation of the standard error, it should be in error as much as 50 or even 100%, the results are definite enough to establish the existence of this simple isotope effect.

The paper describing this experiment in detail has been submitted for publication to the *Journal of the American Chemical Society*.

Problem: DESIGN AND ANALYSIS OF EXPERIMENT ON REMOVAL OF RADIOACTIVITY FROM WATER

Origin: Dr. C. P. Straub, Health Physics Division

Participating Members of Panel: A. W. Kimball, G. J. Atta

Background and Status: The experimenters are interested in determining the conditions under which radioactivity in waste water can be removed most efficiently. Several factors may influence the removal, and it was desired in a preliminary experiment to investigate four of them. A factorial design including the four factors, each at three levels, was constructed, and, after proper randomization, was used in the execution of the experiment. The four factors were initial *p*H, initial activity, turbidity, and coagulant.

The quantity measured was residual activity, and readings were taken at 2 and 24 hr after processing. Separate analyses were performed for each set of measurements, and for all practical purposes the results were identical for the two time periods.

On the assumption that the original data followed a Poisson distribution, the transformation

$$y = \sqrt{x} + \sqrt{x + 1} \quad ,$$

shown recently by Freeman and Tukey [*Ann. Math. Stat.* **21**, 610 (1950)] to be most desirable for stabilizing variance, was applied to each measurement. The analyses of variance on which the conclusions are based were performed with the transformed variates.

The results of the experiment indicate that greater removal is effected when the water inflow is either neutral or alkaline, although it seems that nothing is gained by making the water excessively alkaline. It is clear that turbidity and coagulant are positively correlated with removal, and that possibly greater removal could be achieved at levels of coagulant and turbidity greater than those used in the present experiment. In general, percentage removal seems to lie between 50 and 60% with a tendency toward greater relative removal at higher activity levels, which is to be expected. The first-order interaction between *p*H and coagulant was statistically significant, and this effect was traced to the fact that the addition of increasing amounts of coagulant to the acid water made the water neutral or alkaline and thereby increased removal. No such effect was observed at the neutral and alkaline *p*H levels since maximum removal with respect to *p*H had already been achieved. The first-order interaction between turbidity and coagulant was also statistically significant. In this case very poor removal was effected when neither of the two factors was added. However, with some turbidity present, additional amounts of coagulant increased removal proportionately at the two levels of turbidity used. This observation merely draws attention to the need for both factors in the processing.

The experimental data and the corresponding statistical analyses are quite extensive, and for this reason have not been presented here. The entire experiment will be described in detail in the near future in a Health Physics Division intradepartmental memorandum entitled *Preliminary Report of*

Progress: Removal of Fission Products by Coagulation and Turbidity in Jar Tests. It is planned eventually to incorporate the results in an ORNL report.

Problem: TEMPORAL EFFECTS OF IRRADIATION ON CELL DIAMETERS

Origin: Nyra Harrington, Biology Division

Participating Members of Panel: J. Moshman, G. J. Atta

Background: Samples of control and irradiated cells were maintained in a culture for periods of 10, 12, 14, and 16 hr after the treated cells were irradiated. It was desired to examine the effects of the various time intervals on the cell diameters.

Submitted for analysis were diameter readings of groups of 20 each for control and treated cells for each of the four time periods. The data were examined by the analysis of variance in two orthogonal breakdowns. The first comparison partitioned the sources of variation into:

1. Irradiated vs. control (IC)
2. Time (T)
3. Interaction ($IC \times T$)
4. Error (E)

and the second into:

1. Irradiated vs. control (IC)
2. Control (C)
3. Irradiated (I)
4. Error (E)

Sources 1 and 4 were the same for both cases.

It was found that the IC sum of squares was highly significant in comparison with the error term, revealing a true difference between the means of the control and irradiated groups. Similarly, highly significant differences were obtained between the four time periods, but the interaction was negligible, indicating that the time had no appreciable effect on control-irradiated differences.

From the second partition it was found that the control cells themselves differed significantly ($P < 0.01$) from each other from one time period to another and so did the irradiated ($P < 0.05$). This phenomenon was to be expected in view of the fact that the cells continued to be viable and so continued to grow.

Status: Completed.

Problem: A SIMPLE CRITERION FOR TESTING AN OUTLYING OBSERVATION

Participating Members of Panel: J. Moshman, G. J. Atta

Background: A previous progress report of this Panel (ORNL-888, p. 13) described a rough test which could be used to examine one of four readings as a possible anomalous observation. The test so described has been sharpened and extended.

Greater precision has been obtained by equating the expected value of the range, R_n, of n observations to $C_{1,n}\sigma$ instead of $d_n\sigma$. The factor d_n is the expected value of the range in repeated samples of n, each from a normal population with unit variance. The more precise value, $C_{1,n}$, is the appropriate scale factor for one sample of size n. If, in general, $C_{s,n}$ is the scale factor for s samples of n each, it is true that

$$\lim_{s \to \infty} C_{s,n} = d_n \quad .$$

Tables of critical values of the test ratio

$$g_n = \frac{R_n}{R_{n-1}}$$

for testing whether one observation of a sample of n should be rejected have been computed for $n = 3(1)15(5)25$.

Status: A more comprehensive discussion of the theory and tables of critical values of g_n are being prepared for publication as a project report.

Problem (continuation): DETERMINATION OF FAST-NEUTRON FLUX IN ORNL PILE

Origin: Dr. D. K. Holmes, Physics Division

Participating Member of Panel: J. Moshman

Participating Member of K-25 Central Statistical Laboratory: E. B. Carter

Background and Status: The problem as described in previous progress reports of this Panel (see p. 3 for ORNL numbers) is presently in the machine stage of computation. Prior to the actual calculation of complete neutron histories preliminary card decks have been prepared with sets of random direction cosines for each collision, random radial starting points in the fuel rods in accordance with their known Bessel function distribution, and random resulting energies resulting from inelastic collision with uranium atoms. Actual histories should be commenced in May.

In the event that it is seen that the relative extreme accuracy striven for would be impractical in terms of time requirements, certain modifications are planned which should expedite the calculations at what is hoped will not be too great a sacrifice in accuracy. These modifications include the substitution of a small number (8 to 16) of energy levels with cross-sections averaged over the level instead of the continuous energy spectrum as now set up and the use of only the 20 possible scattering angles of a regular icosahedron instead of the current three-digit random direction cosines.

Problem: HEAT EFFECT ON CELL EXTINCTION

Origin: Dr. M. E. Gaulden, Nyra Harrington, Biology Division

Participating Members of Panel: J. Moshman, C. Perhacs, G. J. Atta

Background: Eighty-one grasshopper cells were prepared from seven embryos with methyl green and Feulgen stains. Half the cells were treated with heat, and the remainder were untreated. Quantities measured were:

R, the radius of the cell nucleus,
C, aperture radius,
I_0, light absorption of the background, and
I_n, light absorption of the nucleus.

The measurement of interest was E, the extinction of the entire cell, where

$$E = \frac{C^2R^3}{R^3 - (R^2 - C^2)^{3/2}}(\log I_0 - \log I_n) \quad .$$

It was desired to determine the effect of heat on cell extinction.

Each cell had five or six groups of readings of the four variables. The quantity E was computed on IBM machines for each of the readings and extinctions were averaged for each cell.

It was then observed that variances of the extinctions varied with mean extinction and a least-squares fit revealed that the variance s^2 could be adequately described by the relation

$$s^2 = 0.02656(1.172)^E \quad .$$

Accordingly, the variables E were transformed to a new series of variables,

$$y = \frac{2}{0.1587\sqrt{0.02656(1.172)^E}} \quad ,$$

to stabilize the variance.

An analysis of variance was performed on the transformed variates in which the embryo variation was accepted as the proper error term. The analysis revealed that there was no significant difference due to the heat effect, although the Feulgen stain had a significantly ($P << 0.01$) greater extinction than the methyl green. The interaction between heat and stain was also not significant.

Status: Completed.

Problem: ISODOSE CURVES FOR BETA EMITTERS

Origin: Dr. Marshall Brucer, ORINS Medical Division

Participating Members of Panel: J. Moshman, G. E. Albert

Background: In the treatment of neoplasia with radioactive isotopes that are predominantly beta emitters, by means of radioautographs very irregular patterns of ionization have been observed to exist. It is believed that if concentric isodose curves can be theoretically computed in tissue from known sources, a method may be found of introducing the isotope in the tumor so that maximum therapeutic effect may be obtained with minimum damage to adjacent healthy tissue.

Status: The problem has just recently been proposed. It is believed that a Monte Carlo procedure may advantageously be employed if there exists sufficient knowledge of the tissue material cross-sections to beta rays. An investigation is underway into the availability of the necessary physical data to determine the feasibility of Monte Carlo approach.

Problem: TESTING A STRAGGLER MEAN IN A TWO-WAY CLASSIFICATION USING THE RANGE

Participating Member of Panel: J. Moshman

Background: There have recently appeared in the statistical literature several methods, based on the range, supplanting classical analogs which require a far more cumbersome calculating procedure, at the cost of a (usually) moderate degree of efficiency. For example, Lord [*Biometrika* **34**, 41 (1947)] proposed a substitute for the Student t test for examining whether the sample came from a normal population with mean ξ, which was of the form

$$u = \frac{|\bar{x} - \xi| d_n \sqrt{n}}{w_n}$$

where

$\bar{x}$ is the sample mean,

n is the sample size,

w_n is the sample range, and

d_n is the expected value of the range in repeated samples of size n from a normal population with unit variance.

More recently Hartley [*Biometrika* **37**, 241 (1950)] applied the range to a test of homogeneity of rows or columns in a two-way layout in the analysis of variance.

A similar attack provides a test for deciding whether there exists a straggler from some or all of a collection of row or column means in a two-way classification. The procedure used applies the methodology of Hartley and Patnaik [*Biometrika* **37**, 78 (1950)] to a statistic for a similar test whose distribution was originally found by Nair [*Biometrika* **35**, 118 (1948)].

Status: A detailed paper describing the theory, providing tables of critical values, and giving examples is being prepared as a project report and for submission to the *Annals of Mathematical Statistics.*

Problem: **PUNCHED-CARD PROTOCOL FOR PATHOLOGY AND PHYSIOLOGY SECTION**

Origin: Drs. J. Furth and M. C. Woods, Biology Division

Participating Member of Panel: J. Moshman

Background: In view of the numerous animals used for breeding purposes and various physiological experiments, it was deemed advisable to consider the merits of installing a punched-card protocol. A procedure was devised for recording the following information on a single punch card:

1. Identification
2. Sex
3. Age at treatment
4. Treatment
5. Cataracts
6. Leukemia
7. Tumors
8. Other diagnoses
9. Age at death
10. Cause of death
11. Disposition of animal

Status: The procedure is being employed on a C^{14} tracer experiment using mice with the idea of removing whatever difficulties may be revealed.

Problem (continuation): OPERATION GREENHOUSE

Origin: Dr. J. Furth, Biology Division

Participating Members of Panel: J. Moshman, J. W. Tukey

Background: The tentative analysis has been broken down as follows:

1. Lethality. It is planned to fit probit curves to the lethality data for the various gamma stations. Slopes will be compared for zero- and first-order interactions for the various variables. If the slopes differ, LD_{25}'s, LD_{50}'s, and LD_{75}'s will be similarly compared.

2. Longevity. Survival times will be compared when broken down by treatment, age, weight, and sex.

3. Weight. Representative weight curves will be obtained for various dosages. Weights will be compared by analysis of covariance on original weight.

4. Neoplasia. Tumors will be examined by time of onset for general incidence and specifically for occurrence in certain designated physiological systems. At the various stations differences will be looked for in relative proportion of malignant vs. benign tumors, size of neoplasm, and regression with respect to age, weight, and sex.

5. Leukemia. General incidence, grade, and time on onset of leukemia will be examined in relation to the several variables in the study.

6. Cataracts. The gamma-shielded animals will be examined for incidence, location, and grade of cataracts as well as the relation of cataracts to age, weight, and sex.

Status: When animals are received they will be assigned at random to prepared cages. Except for the lethality study the analysis will await the death of most of the survivors. In the intervening period punch cards will be prepared with the required data exclusive of autopsy findings and time of death.

Problem: EFFECT OF TIME LAG AFTER IRRADIATION ON CHROMOSOME BREAKS

Origin: Dr. M. E. Gaulden, Biology Division

Participating Member of Panel: J. Moshman

Background: It was desired that cells be examined at various periods after irradiation to determine their homogeneity with respect to the observed number of resulting chromosome fragments and single and double breaks.

From the irradiated cells, those undergoing division were chosen for the study inasmuch as the chromosomes of these cells were pulled toward their respective poles and any fragments could be counted relatively easily. Chromosomes were counted in these cells, and this number served as a base for further calculations.

The time periods under study were 22, 44, 154, 198, 270, and 374 min. The indices examined were:

1. Cells with fragments as fraction of total cells in division.
2. Proportion of single breaks among total chromosomes counted.
3. Proportion of double breaks among total chromosomes counted.
4. Difference between single and double break proportions.
5. Sum of single and double break proportions.

The method of analysis consisted of a χ^2 analysis for homogeneity of the proportions in each index for the six time periods. In none of the cases could all six periods be considered random fluctuations about a single mean proportion. The time periods were then broken down into smaller homogeneous groups. It may be added that the frequency of breaks increased with the length of the time after irradiation.

Status: The data are under study for their biological significance in view of current knowledge.

Program: **AGE IN H_2O, D_2O MIXTURES**

Origin: L. C. Noderer, Physics Division

Reference: ORNL-634, ORNL-726.

Status: Inactive.

Problem: **QUANTUM MECHANICS INTEGRATIONS**

Origin: F. G. Prohammer, Physics Division

Status: Inactive.

Problem: **ANALYSIS OF β-RAY SPECTROSCOPY**

Origin: P. R. Bell, Physics Division

Reference: *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1949* (ORNL-408); for *July 31, 1950* (ORNL-818); and for *January 31, 1951* (ORNL-979).

Status: Inactive.

Problem: **HEAT TRANSFER BY A FLUID FLOWING TURBULENTLY IN A CYLINDRICAL TUBE**

Origin: Drs. R. V. Bailey, H. F. Poppendiek, Reactor Technology Division

Reference: *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1950* (ORNL-818); and for *October 31, 1950* (ORNL-888).

Status: Inactive.

Problem: **CRYSTAL ANALYSIS**

Origin: Dr. H. Levy, Chemistry Division; B. S. Borie, Jr., Metallurgy Division

Status: Pending.

Problem: **DIFFUSION LENGTH**

Origin: Dr. R. H. Ritchie, Health Physics Division; E. D. Klema, Physics Division

Status: Pending.