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SURFACE TO MASS DEPENDENCE OF EFFECTIVE RESONANCE INTEGRALS FOR URANIUM 238 CYLINDERS

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SURFACE TO MASS DEPENDENCE OF EFFECTIVE RESONANCE INTEGRALS FOR URANIUM 238 CYLINDERS

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ABSTRACT

The effective resonance integral for U^{238} has been determined as a function of the surface to mass ratio for cylinders of 5/8" to 3" diameter. The measurements were calibrated by comparison with the published value of the effective resonance integral with infinite shielding. A linear approximation (in which the mass term must be decreased by a volume disadvantage factor correction) is

$$\left(\int \sigma_a \, dE/E\right)_{eff} = 9.0 \, (1 + 2.84 \, S/M) \text{ barns.}$$

This correction to the mass term was found to be given accurately by the simple diffusion theory volume disadvantage factor with $\overline{\mathcal{H}} = 0.42 \text{ cm}^{-1}$.

I. INTRODUCTION

Christy, Weinberg and Wigner (CP-2062) show the relationship between the resonance escape probability, p, and the effective resonance integral, $(\int \sigma_a \, dE/E)_{eff}$. To make possible a more accurate calculation of p, the effective resonance integral for U^{238} in cylindrical geometry has been measured as a function of the surface to mass ratio. This was accomplished by counting the U^{239} decay betas from a depleted metal sampling of the cylinders after irradiation in the Oak Ridge pile. This quantity has previously been measured by Creutz, Jupnik and Snyder (C-116) and by Mitchell, Brown, Pruett and Nering (CP-1676) and more recently by Muehlhause, Radkowsky, and Untermyer (ANL-4323) and Untermyer (ANL-4350). Creutz and Mitchell approximated the epi-thermal pile spectrum by slowing down cyclotron produced neutrons in a large block of graphite. Natural uranium as powdered metal and as uranium oxide was used. The experiments at Argonne were carried out by a pile oscillator technique using small natural uranium rods.

Until the present experiments were performed the evaluation of the effective resonance integral for U^{238} in the form of large cylinders required a considerable extrapolation of the existing data. In addition there were small uncertainties because of the neutron sources used, the presence of U^{235} and a considerable uncertainty in the disadvantage factor to be applied to correct for the reduction in neutron flux in the interior of the cylinders. It is because of the latter factor that the linear expression given in Section II of this report must be used with care. The present experiments represent direct measurements on large cylinders. A normal pile spectrum

(assumed to be 1/E) was used and the foils used to sample the volume of the cylinders had a negligible U^{235} contamination (approx. 1 part in 3,000 maximum).

II. THEORY

On the basis of early experiments Wigner (C-4) suggested that the activity of a sample of U^{238} exposed to unit flux for unit time could be expressed as the sum of a term proportional to the mass and a term proportional to the surface. This is roughly equivalent to assuming that the neutrons having resonant energies would be absorbed at the surface while the neutrons having energies between the resonances would be absorbed uniformly throughout the interior of the sample. The equivalence is not exact because the U^{238} atom do not have infinite mass and the neutrons will lose energy as they are scattered. The effective resonance integral may be written

$$\left(\int \sigma_a \, dE/E\right)_{eff} = C_1 + C_2 \frac{S}{M} \qquad (1)$$

Eq. (1) is not intended to apply when the sample is so thin that the resonant neutrons penetrate an appreciable fraction of the thickness nor when the sample is so thick that there is appreciable self shielding of the non-resonant neutrons. In particular Eq. (1) cannot be expected to hold for the larger of the cylinders used in this investigation.

Wigner (C-4) has given estimates of the mass and surface contributions in terms of average cross sections. A somewhat more elaborate theoretical analysis equivalent to evaluating C_1 and C_2 was carried out by Dancoff in two stages. In CP-1092 Dancoff assumed a level in U^{239} for a

neutron energy of 7 ev and a series of equally spaced levels starting at 30 ev. Using the available experimental data he derived plausible values of the partial widths and a range of values for the level spacing. With this model Dancoff and Ginsburg (CP-1589) calculated the absorption to be expected for a neutron flux incident upon spheres and cylinders. To guide their analysis they considered an infinite half plane of U^{238} and applied transport theory. Corresponding to the range of parameters in their model they obtained a range of possible values of $(\int \sigma_a \, dE/E)_{eff}$ which include the experimental points. It is now known (Havens and Rainwater A-3242) that resonances occur at 6.6, 22, and 40 ev.* However, these values are so close to those assumed by Dancoff and the possible range of nuclear parameters still so great that a recalculation is hardly justified.

The discussion by Christy, Weinberg and Wigner (CP-2062) clearly emphasizes the importance of the neutron distribution. To take account of the decrease in the neutron flux inside the cylinders, Eq. (1) may be rewritten

$$(\int \sigma_a \, dE/E)_{eff} = \frac{C_1}{F} + C_2 \frac{S}{M} \qquad (2)$$

The factor F is the usual volume disadvantage factor. It may be defined as the surface flux divided by the average neutron flux inside the cylinder.

---

* A recent report from Argonne (ANL-4544) tentatively places the resonances at 7.1 ± 0.4, 24 ± 2, and 47 ± 6 ev. No other pronounced resonances were observed up to a few kev.

Simple diffusion theory gives

$$F = \frac{\bar{\varkappa}\, r\, I_0(\bar{\varkappa}\, r)}{2\, I_1(\bar{\varkappa}\, r)} \tag{3}$$

where $\bar{\varkappa}$ is an averaged inverse diffusion length and r is the radius of the cylinder. I_0 and I_1 are the Bessel functions of imaginary argument. A plot of F vs $\bar{\varkappa}\, r$ appears in CL-697, Chap. IV E. It is well known that diffusion theory must not be taken too literally for cases such as this. Nevertheless since F is not needed to apply the data, since $\bar{\varkappa}$ is known only approximately, and since use of transport theory would involve prohibitive computation, Eq. (3) is used as a correction factor. The deviation from the linear approximation, Eq. (1) is clearly shown in Fig. 2.

III. EXPERIMENTAL PROCEDURE

1. Cylindrical geometry

In the present experiment uranium cylinders of diameters 5/8" to 3" were exposed in cadmium to the neutron flux in the Oak Ridge pile. The aim was to measure a sampling of the total activity of the cylinder. For this purpose foils depleted to at least 1 part in 3,000 were inserted in milled slots in the cylinders in such a way that the uranium mass in the foil was distributed in the same way between surface and volume as in the cylinder. The foils were clamped tightly between two portions of the cylinder to prevent leakage of easily absorbed neutrons into the cylinder interior. To avoid end-effect errors the cylinders extended axially at least 6" on each side of the foil. Fig. 1 is a sketch of the method of inserting the foil with adjacent

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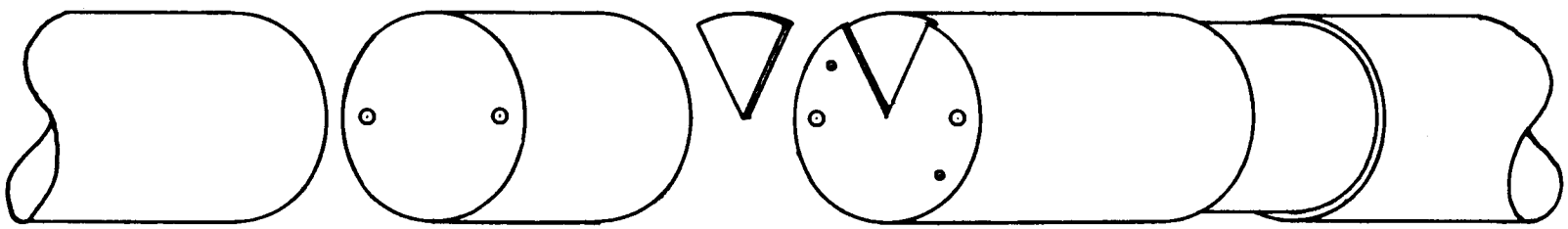

FIG. 1

Showing an exploded view of uranium cylinder and foil. In the final assembly the central small cylinder was accurately located with two pins and then screwed tightly to the cylinder on the right with the uranium foil in its slot; and the left cylinder was pushed tightly against the central one. A second uranium monitor was backed by a twenty mill indium foil and held by fine nickel wire to the periphery of the uranium cylinder on the right.

portions of the cylinder pulled apart. After activation in the pile the activity of the foils were counted without chemical separation. The cylinders were of natural uranium.

The choice of foil thickness represented a compromise. First, in order to obtain a successful fit, it was necessary to have the foil sufficiently thick so that it clamped flat in the assembly and so that machining operations on the cylindrical surface of the assembly with foil in place resulted in a smooth surface on the foil edge making it an indistinguishable continuation of the cylindrical surface. Second, the foil thickness was made large compared to the range of recoil fission fragments from the surface of the natural uranium in the cylinders. On the other hand increasing the foil thickness beyond a certain point was useless because of the absorption of the foil for uranium 239 betas. Foils of $3\frac{1}{2}$ mils or about 160 mg/cm^2 thickness were chosen. These fitted well into the cylinders; the thickness was roughly 20 times the range of recoil fission fragments; and the self absorption was such that in our counting arrangement the betas from the under surface of the foil were cut down an estimated factor of 10 compared to those of the top layer.

It was required that the foils have the same mass distribution between surface and volume elements as the cylinders they represented: that is to say an edge of the foil was cut to coincide with an arc of the cylinder surface and the foil area was tapered uniformly toward the cylinder axis so that the mass between $\underline{r}$ and $\underline{r} + \underline{dr}$ of the foil was proportional to r. The foils were thus pie shaped, the outer arc coinciding with the cylinder surface. The apex angles of the foils increased from the larger to the smaller cylinders

so that the uranium activities were comparable between foils. At least one of the U^{238} resonances is known to be high (approx. 10,000 barns) resulting in extremely shallow penetration at the cylinder surface ($\sim$1 mil). This meant that the edge of the foil had to be shaped with precision to be a continuation of the cylinder surface. This was accomplished by machining and hand finishing the cylinder surface with the foil in place. The pie shaped depression ~~is~~ in the end of one cylindrical section, which formed a groove for the foil in the assembly, and the flat portion of the adjacent cylindrical section, which clamped it, were both milled flat to a tenth mil or better so that the easily absorbed neutron component could not penetrate through cracks to surfaces of the foil other than the foil edge. In fitting the foil to the cylinder for a run the fractional error introduced for the soft neutron component was at worst, in case the foil extended out of the groove, just the ratio of the areas of uncovered side surface to cylindrical edge surface, or, in case the foil was somewhat retracted, the ratio of shadowed edge surface to total edge surface (solid angle considerations being in the direction of cutting down the error). Assembly was inspected under a binocular microscope. It is believed that a five percent error due to extension or retraction could have been caught. Actually successive runs repeated within 2% of the average at worst and usually within 1%. For the less easily absorbed neutrons with penetration greater than about a mil the fit was considerably less critical.

To measure the U^{239} beta activity induced in the foils, a counter of the beta-proportional type was used. This choice was made because of the necessity of counting over a wide range of counting rates up to 10^5 beta counts

per minute. The beta proportional counter had the advantage of low resolving time (5μ sec. due to slowest circuit element in electronic equipment). Corrections for resolving time at the highest counting rates were less than 1% and could be neglected. The counter was an all glass cylindrical Eck and Krebs counter with thin silvered glass wall filled at one half atmosphere pressure with 90% argon and 10% methane. The amplifier was an A-1 with non-linear circuit elements introduced to avoid saturation by large pulses. As a result of this non-linearity, the counting-rate vs. counter-voltage curve showed a plateau in the proportional region with a 3% rise in 100 volts. The counter voltage stabilization was good to better than $\pm$ 10 volts over an indefinite period and the counter showed a constant response to an uranium standard source within statistical limits over the period of several months required for the measurements. Resolving time was checked by the use of two sources counted individually and together.

Since the counting system was required to measure the activity of the foil as a whole it was necessary that no portion of any foil should be counted preferentially because of counter geometry. The counter was placed with axis crosswise in the upper portion of a vertical cylindrical lead shield 5 inches inside diameter and 20 inches inside height. The foils were placed on a shelf 22 cm below the counter axis. Thus, since the foils all fitted in a spot on the counting shelf 1 3/4 inches in diameter, the variation of the inverse square distance to the counter was negligible over the foil area. Aluminum baffles were placed between the counting shelf and counter so that no electrons could reach the counter from the foil after being scattered

once from the shield walls. The walls themselves were lined with aluminum to cut down scattering. Scattering from the walls could cause preferential counter response to betas emerging from the foil edges at an angle to the vertical, since these betas would not have traversed as great a distance in uranium as electrons from a corresponding depth in the body of the foil. The air path and counter wall totaled 40 mg/cm^2 of absorption path.

During the course of the experiment it was necessary to compare the activities of foils differing in thickness. Since the self absorption for the betas made the effective foil thickness less than the actual thickness, it was not enough to weigh the foils. The foils to be compared were exposed together in the pile in a shallow box of uranium. The easily absorbed neutrons which would activate thin surface layers of the foils were thus absorbed by the box and the enclosed foils were uniformly activated throughout their volume. Comparing the foil activities in our counter geometry then gave a relative measure of the effective number of uranium atoms in the foils taking account of the penetration of the betas involved in the experiment.

Contamination activities of a few percent with half lives longer and shorter than 24 minutes were observed in the decay of the uranium foils. These were attributed to fast neutron fission in U^{238} and to fission fragment recoils from the natural uranium in the cylinders. To correct for the long period contamination a constant background representing about 1% of the initial activity was subtracted, the criterion for the amount subtracted being fit to a 23.6 minuted decay curve between 3 and 5 half lives after the end of the bombardment. No correction for short period activity was made but no points were used in calculating the activity which were taken within the first 30

minutes after bombardment, our experimental procedure in removing the foils requiring about 20 minutes. From the magnitude of the short and long half life contamination activities it was estimated that contamination of 24-min. half life would represent about 5% of the initial activity. In the relative measurements which compared the activities of foils in different cylinders, the effect of an impurity representing a constant percentage of initial activity would drop out in taking ratios.

All U^{239} activities had to be referred to unit flux of pile neutrons. With a given pile power the flux at the point of exposure varied from one run to another because distribution in the pile of material with high absorption cross section was not always the same and because the local depression of the flux by the uranium itself depended on cylinder size. The activities of standard indium monitors exposed with the cylinders were taken as a measure of the flux. The standard monitors were from 0.5 to 1 mg/cm^2 thick obtained by evaporating indium on a thin inert foil backing (aluminum of high purity). Since the 1.44 ev indium resonance is known not to overlap any resonance in U^{238}, indium foils of this thickness could be used in contact with the uranium foils when necessary without mutual shielding. An additional practical advantage was the absence of impurity activities since the indium activity was due almost entirely to the high cross section at the 1.44 ev peak.

Although there is no resonance in the U^{238} cross section in the 1.44 ev region, this cross section is high enough that the large amounts of uranium used in the experiment would have a small shielding effect on the monitors. On the other hand the theory of the experiment required that the monitors should not introduce a compensation for uranium self-shielding effects.

The monitors were therefore made to respond to neutrons incident on the cylindrical surface from the outside, or graphite side, only by placing a heavy indium foil several hundred milligrams/cm^2 thick between the monitor and cylindrical uranium surface. This backing, or shield, foil was opaque to 1.44 ev neutrons and therefore shielded the monitor from neutrons which had traversed uranium. Thus the flux of 1.44 ev neutrons incident from the graphite onto several square centimeters of cylindrical surface was taken as a measure of the pile neutron flux.

In order to prevent shielding of the uranium foil by the heavy indium shield foil, the monitor arrangement was placed a fixed distance measured parallel to the cylinder axis from the uranium foils, so that there was one inch axial clearance between the uranium foil and the nearest edge of the indium. See Fig. 1.) In addition the indium was placed on the opposite end of a diameter from the exposed edge (region of largest mass) of the uranium foil. The monitor was always positioned in the direction of the center of the pile from the uranium foil. Accuracy in comparing cylinders only required that the slope in the pile flux along the cylinder axis be constant over a length of about two inches in the region of exposure, a condition which was found to be fulfilled within the approximately one percent accuracy of the experiment. As an additional precaution, the region of exposure was chosen approximately midway between slugs in the pile lattice. The cylinders themselves were chosen 1 ft. long in order to avoid end effects over the span of several inches midway along the length in which monitor and uranium foils were placed.

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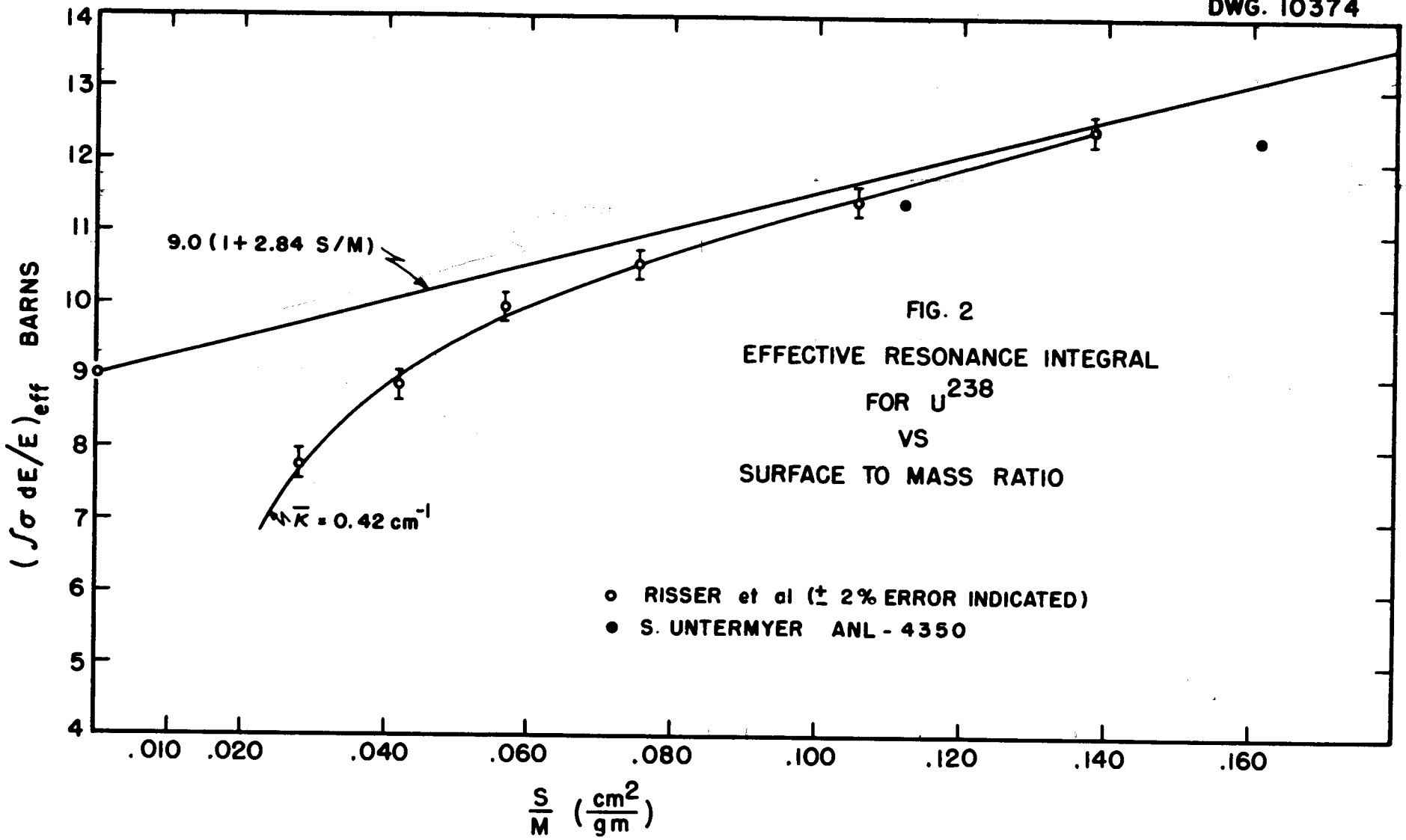

FIG. 2

EFFECTIVE RESONANCE INTEGRAL

FOR U^{238}

VS

SURFACE TO MASS RATIO

The difference in half lives of monitor and sample makes it possible to have errors in the final uranium/indium activity ratios due to errors in timing the exposure and due to time variations of pile flux during exposure. Ideally exposure time should be short compared to both half lives. As a compromise, an exposure time of 12 minutes (one half the shorter half life) was chosen. Calculations show that a 10% amplitude fluctuation in pile flux during exposure for 10% of the exposure period resulted in less than 0.5% error in the uranium/indium activity ratio. Timing from the end of bombardment was, however, critical. The zero of time was the instant of throw of the emergency trip mechanism used to shut down the pile at the end of the exposure. All samples received the subsequent small exposure during the decay of the pile flux.

The foil activities, calibrated in barns per U^{238} atom (c.f. Section IV); are plotted in Fig. 2 as a function of the cylinder surface to mass ratio. An error of $\pm$ 2% is indicated. For comparison the Argonne data (ANL-4350) are included.

2. Plane geometry

In order to calibrate the data and to obtain an experimental check on the mass-dependent term in the resonance integral, typical foils from the cylinder experiments were exposed to the pile flux completely shielded by natural uranium. A one dimensional arrangement was used in which the foils were sandwiched between plates of uranium of varying thickness. By stacking plates of increasing thickness the decrease in activity of the foil with increase in mass of the uranium shield could be studied.

For small amounts of shielding material the depleted U^{238} foils were held between thin flexible rolled sheets of natural uranium with edges clamped so that no neutron leakage between the sheets could take place. For shielding by 1/16 inch or more, flat plates of natural uranium were made by milling, and the depleted foils were placed in a shallow tray-like depression milled in one plate and held by a flat milled boss on the opposite plate which fitted into the tray. The boss cleared the foil so that the plates fitted snugly over their entire flat surface. There was thus no line-of-sight access of neutrons to the foil without passage through uranium. In order to make the neutron filtering truly one-dimensional the area of the plates was large enough that the foil edges were shielded by a uranium thickness several times greater than the shielding perpendicular to the flat face of the foil. A schematic diagram of the foil location is shown in cross section in Fig. 3, $\underline{t}$ designates the thickness of uranium shielding.

To make the results of this experiment comparable to that of the cylindrical geometry, the same thin indium monitors with heavy indium backing were placed on an outside surface of the plates. These were placed so as not to shield the uranium foil, i.e. displaced laterally the same distance toward the center of the pile as in the cylindrical case (see Fig. 2). The plates were large enough in area so that the monitors could be placed as far from the plate edges as the uranium foil, the monitor and foil thus being arranged symmetrically with respect to the plate edges. In addition to the outside monitor, a thin indium monitor was included with the uranium foil. Since the lowest resonances of U^{238} is 5.5 ev above the indium 1.44 ev resonance,

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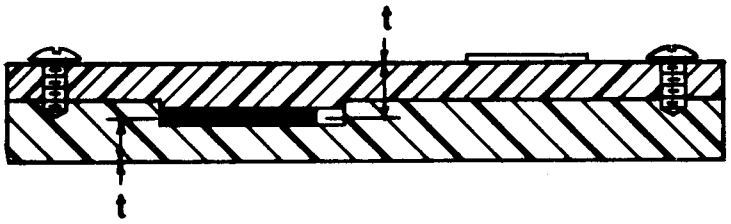

FIG. 3

Flat plates for measuring the volume activity. Uranium and indium monitors were placed in a slot in the lower uranium plate and a top uranium plate with a corresponding projection was tightly screwed to the lower plate. A second uranium monitor with an indium backing was held by fine nickel wire.

the inside monitor activity was taken to be proportional to the flux of neutrons of energy lying between the uranium resonances, i.e. those contributing to the mass term only of the resonance integral. The depression of the uranium foil activity with respect to the outside monitor was taken as a measure of the absorption of the entire neutron spectrum by the uranium shield; when corrected for depression of the inside monitor activity, the depression of the uranium activity was taken as a measure only of the absorption due to the uranium resonances themselves. Thus an independent estimate could be obtained of the relative importance of surface and mass absorption terms in the resonance integral.

In addition to giving this value of the mass term which was used to calibrate the cylinder data, (Section IV), the shielded foil data may be used to calculate directly the effective resonance integrals of the cylinders or of any other sufficiently simple geometrical shape. The absorption at the center of a plane slab 2 $\underline{x}$ cm thick is

$$A_1(x) = \int \sigma_a(E) \frac{1}{\cosh \mathcal{H}(E)x} \frac{dE}{E} \qquad (4)$$

with a limiting value $A(0) \equiv \int \sigma_a(E)\, dE/E = 240$ barns. The function $\mathcal{H}(E)$ varies with E though its dependence upon $\sigma_a(E)$. Interpolation for small x may be carried out by use of Bethe's expression for absorption of resonance neutrons (Rev. Mod. Phys. 9, 69 (1937) p. 148). The effective resonance integral for the cylinder of radius $\underline{r_o}$ will be given by

$$\left(\int \sigma_a \, dE/E \right)_{eff} = \int_0^{r_o} \int \sigma_a(E) \, \frac{I_o(\mathcal{H} r)}{I_o(\mathcal{H} r_o)} \, \frac{dE}{E} \, \frac{2\pi r \, dr}{\pi r_o^2} \qquad (5)$$

One can assume an arbitrary functional form for $\sigma_a(E)$ and using Eq. (4) determine its parameters. Then by Eq. (5) the effective resonance integrals can be calculated. Since the transformation from the plane slab data to the cylinder data is mainly one of geometry the result is at least to the first order independent of the form assumed for $\sigma_a(E)$ and of errors of simple diffusion theory. The results of such a calculation agreed with the experimental curve (Fig. 2) to within 2%. It is worth noting that in cases where slabs are available but cylinders or spheres would be difficult to machine, this admittedly rough procedure provides a simple method of determining the effective resonance integrals.

IV. ANALYSIS

1. Calibration

All the counting measurements in this experiment as described in Section III-1, were relative measurements. Because of the unknown counter efficiency and more important, because of the unknown self absorption of the U^{239} beta decays in the thick foils no absolute counting was attempted. It proved possible to make a reasonable calibration of the cylinder data by making use of the plane geometry runs. As described in Section III-2 the foils used in the cylinder experiments were irradiated when shielded by slabs of natural uranium up to 1" thickness. A plot of foil activity versus thickness of shielding appears in Fig. 4. The activity, relative to an inside

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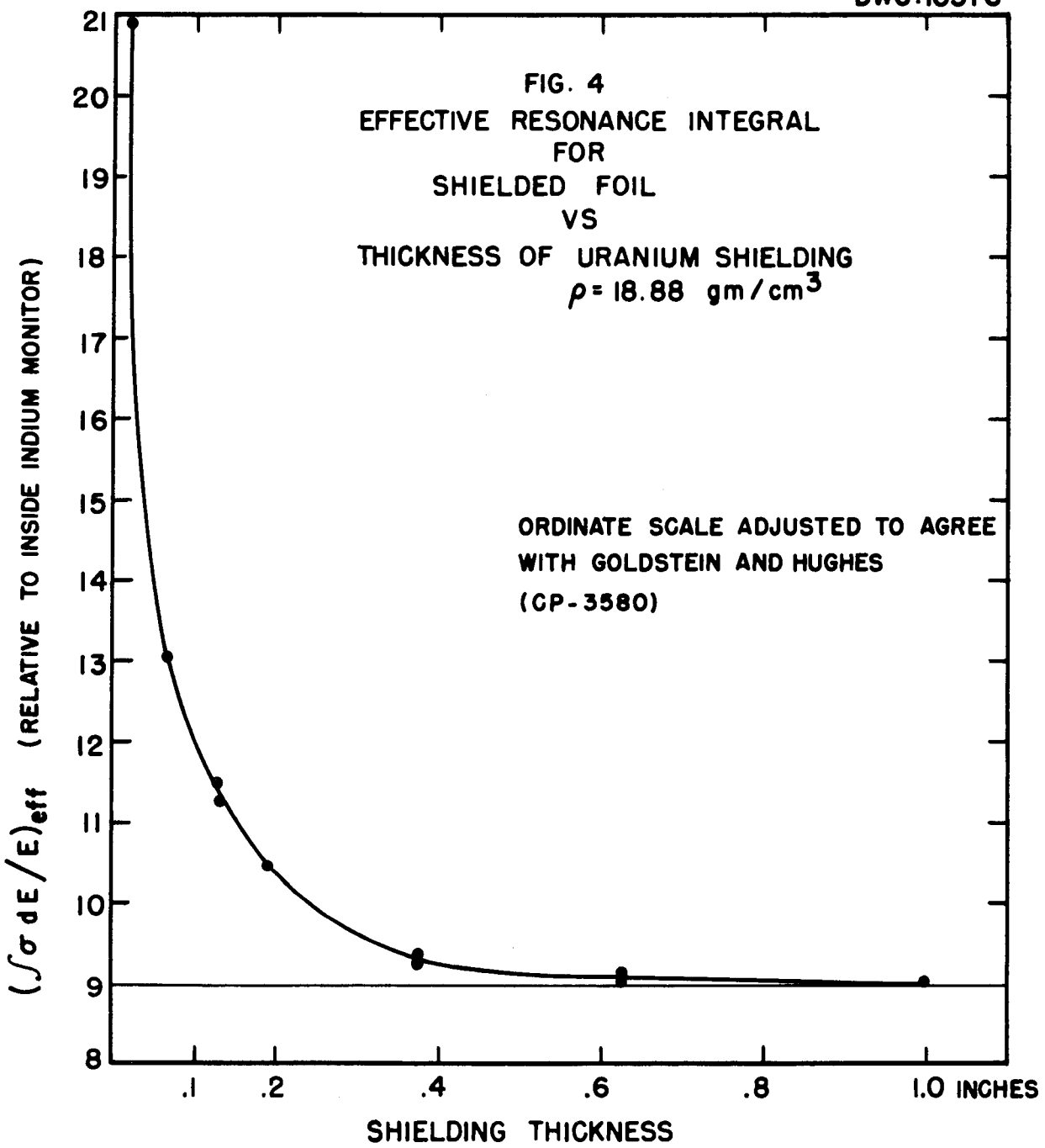

In monitor, reaches a constant lower bound at about 5/8" shielding thickness. This is interpreted as an equilibrium condition of the flux such that any additional uranium shielding will not produce any further distortion of the neutron spectrum. Beyond this point any filtering of the neutron flux by the U^{238} resonances is just smoothed out by the U^{238} scattering. This limit is identified with the mass absorption term, (C_1), in Eq. (1). This lower bound has been determined by Goldstein and Hughes (CP-3580) from their experiments on uranium and uranium oxide as 9.0 barns, with an estimated experimental uncertainty of ±2%. Their value for the cross section has been adopted to calibrate the cylinder data. Any change in this "shielded foil cross section" will appear as a constant multiplying factor in the effective resonance integral.

It had been hoped to relate these effective resonance integrals to the published values of the U^{238} thermal absorption cross section and to the true resonance integral. Experimental difficulties associated with preparing and activating thin films prevented high precision determinations. However, rough confirmation of the 9.0 barn shielded activity was obtained by calibrating the effective resonance integral of the In monitors with the known resonance integral of Au and relating the shielded activity to the activity due to a thermal neutron flux. The effective resonance integral was calculated from

$$\left(\int \sigma_a \, dE/E\right)_{eff\ U^{238}} = \sigma_a(Th)\ U^{238} \left(\frac{A_{In}}{A_U}\right)_{Th} \times \left(\frac{A_{Au}}{A_{In}}\right)_{Th} \times \frac{\left(\int \sigma_a \, dE/E\right)_{Au}}{\sigma_a(Th)_{Au}} \times$$

$$\times \left(\frac{A_{In}}{A_{Au}}\right)_{1/E} \times \left(\frac{A_U}{A_{In}}\right)_{1/E\ shielded} \qquad (6)$$

Here the A's refer to measured activities, the σ's to known cross sections. Because of accumulation of errors it was not possible to do more than obtain a rough confirmation (to $\pm$ 1 barn) of Goldstein and Hughes' value. The cross sections used were

$$\sigma_a(\mathrm{Th})\ U^{238} = 2.59\ \mathrm{b}^*$$

$$\sigma_a(\mathrm{Th})\ \mathrm{Au} = 93\ \mathrm{b}$$

$$\left(\int \sigma_a\ dE/E\right)_{Au} = 1300\ \mathrm{b}.$$

2. Results

The linear approximation to the effective resonance integral Eq. (1) is known for zero surface from the lower bound of the shielded foil curve. A second point may be fixed by assuming $\overline{\mathcal{H}}$ in Eq. (2) to be 0.4 cm^{-1} and fitting to the point at $\underline{r}$ = 0.767 cm ($\underline{S/M}$ = 0.1382). It is readily seen that at small $\underline{r}$, $\underline{F}$ is quite insensitive to the exact value assumed for $\overline{\mathcal{H}}$. This gives as a linear approximation

$$\left(\int \sigma_a\ dE/E\right)_{eff} = 9.0\ (1 + 2.84\ S/M)\ \text{barns}. \qquad (7)$$

The coefficient, 9.0, has been taken from the report of Goldstein and Hughes (CP-3580). It was not determined by the experiment being reported here.

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*Dr. H. Pomerance informs us that recent measurements suggest a value of 2.83 barns.

Using the simple diffusion theory disadvantage factor, F, Eq. (3), and $\overline{\mathcal{H}} = 0.42\ cm^{-1}$, Eq. (2) was found to give a good description of the experimental points as shown in Fig. 2. This may be written

$$\left(\int \sigma_a\, dE/E\right)_{eff} = 9.0\ (F^{-1} + 2.84\ S/M)\ \text{barns} \qquad (8)$$

where $F = F(0.42\ r)$. The value $\overline{\mathcal{H}} = 0.42\ cm^{-1}$ is in close agreement with that of Christy, Weinberg, and Wigner who had estimated it to be about $0.4\ cm^{-1}$. Rough experimental confirmation may be found in the measurements of Snell et al on a uranium sphere (CP-493) who find that for iodine resonance neutrons (20 to 40 ev) $\mathcal{H} = 0.32$ while for thermal neutrons $\mathcal{H} = 0.65$. Snell points out that because of the phenomenon of "hardening" or selective absorption of the low energy neutrons $\mathcal{H}$ is really an average and that it will probably be a slowly varying function of lump size.

It is sometimes of interest to have a mean value of the logarithmic energy interval, $\overline{\ell n(E'/E)}$, defined by

$$\left(\int \sigma_a\, dE/E\right)_{eff} = \overline{\sigma_a}\ \overline{\ell n(E'/E)}$$

with $\overline{\sigma_a}$ given by

$$\overline{\mathcal{H}}^2 = 3\ N^2\ \sigma_{total}\ \sigma_a\left(1 - \frac{4}{5}\ \frac{\sigma_a}{\sigma_{total}}\right).$$

Using $\overline{\mathcal{H}} = 0.42\ cm^{-1}$

$\rho = 18.88$ gm/cc

$\sigma_s = 8.2$ barns and

$\left(\int \sigma_a\, dE/E\right)_{eff} = 9.0$ barns

one readily obtains $\overline{\sigma_a} = 2.93$ barns and $\overline{\ell n(E'/E)} = 3.07$.

Based on their cyclotron experiments Creutz, Jupnik, and Synder (CP-116) obtained

$$\left(\int \sigma_a \, dE/E \right)_{eff} = 9.25 \, (1 + 2.67 \, S/M). \tag{9}$$

To fit data from pile oscillation experiments on thin rods Untermyer (ANL-4350) proposed

$$\left(\int \sigma_a \, dE/E \right)_{eff} = 9.23 \left(1 + \frac{2.50}{\frac{M}{S} + 0.10}\right) . \tag{10}$$

It is seen that Eq. (6) is in good agreement with these results. The deviation from the linear approximation due to the reduction of the neutron flux in the interior of the cylinders is apparent in Fig. 2. It is perhaps of interest to note that a reduction in the value of the effective resonance integral with infinite shielding of some 4% will bring Eq. (6) into almost exact agreement with the Argonne experimental values for their largest uranium rods. It should be emphasized, however, that while a redetermination of the shielded foil cross section may modify the coefficient of the mass term (9.0) in Eq. (6), the values of $\overline{\varkappa}$ (0.42) and of the slope-intercept ratio (2.84) will not be affected.

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