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# DETECTION POSSIBILITY
# OF CIRCULAR POLARIZATION
# OF GAMMA RAYS



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## DETECTION POSSIBILITY OF CIRCULAR POLARIZATION OF GAMMA-RAYS

D. B. Beard and M. E. Rose

Biedenharn, Rose and Arfken[1] have derived the correlation between the polarization of capture gamma-rays and the polarization of the incident neutrons as a function of the angular momentum of the initial, compound, and residual nuclear states and the pole order of the emitted radiation. Since the emitted photons are circularly polarized, information about the angular momenta involved in the transitions can be furnished only by the polarization detection of circularly polarized gamma-rays. For nuclear gamma-ray energies, Compton scattering from aligned electrons would seem to offer the best chance of such detection.

If one uses the aligned electrons in magnetically saturated iron to determine the polarization of the emitted gamma-rays the average spin (in units $\hbar$) of the iron electrons along the axis of magnetization is only about .04 (from the experimentally determined magnetic moment of the iron atom, 2.22 Bohr magnetons, and a gyromagnetic ratio, 2). Unless the anisotropy in the Compton scattering is quite large, therefore, such an experimental determination of the polarization correlation of neutrons and gamma-rays will not be possible. In the hope that an apprciably larger factor might arise in the Compton scattering anisotropy, the Compton scattering of circularly polarized light was computed.

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1. L. C. Biedenharn, M. E. Rose, and G. B. Arfken, Phys. Rev. (to be published Aug. 1951) and ORNL 986.

Following the Feynman[2] formalism the matrix element for the Compton scattering process (in the laboratory system and in energy units of $m_e c^2$) is:

$$\bar{U}_f \not{e}_0 \frac{1}{\gamma_4 - \not{k} - 1} \not{e}\, U_i + \bar{U}_f \not{e} \frac{1}{\gamma_4 + \not{k}_0 - 1} \not{e}_0 U_i \tag{1}$$

where $\not{e}_0$ ( $= \sum_{\mu=1}^{4} \gamma_\mu e_{0\mu}$) and $\not{k}_0$ refer to the polarization and propagation vector, respectively, of the incident photon, $\not{e}$ and $\not{k}$ refer to the polarization and propagation vector of the scattered photon, and $U_f$ and $U_i$ refer to the final and initial electron states. Using projection operators for positive energy of the initial and final electron states and initial electron spin in the positive z-direction the square of the absolute value of the matrix element becomes:

$$\begin{aligned} d\sigma \sim \mathrm{Tr}\Big\{ & \frac{1}{k^2} \not{e}_0(\gamma_4 - \not{k} + 1) \not{e}(i\gamma_1\gamma_2 + 1)(\gamma_4 + 1) \not{e}(\gamma_4 - k + 1) \not{e}_0^*(\not{p}_f + 1) \\ & - \frac{1}{kk_0} \not{e}_0(\gamma_4 - k + 1) \not{e}(i\gamma_1\gamma_2 + 1)(\gamma_4 + 1) \not{e}_0^*(\gamma_4 + k_0 + 1) \not{e}(\not{p}_f + 1) \\ & - \frac{1}{kk_0} \not{e}(\gamma_4 + k_0 + 1) \not{e}_0(i\gamma_1\gamma_2 + 1)(\gamma_4 + 1) \not{e}(\gamma_4 - \not{k} + 1) \not{e}_0^*(\not{p}_f + 1) \\ & + \frac{1}{k_0^2} \not{e}(\gamma_4 + \not{k}_0 + 1) \not{e}_0(i\gamma_1\gamma_2 + 1)(\gamma_4 + 1) \not{e}_0^*(\gamma_4 + k_0 + 1) \not{e}(\not{p}_f + 1) \Big\} \end{aligned} \tag{2}$$

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2. R. P. Feynman, Phys. Rev. 76, 749 and 769 (1949).

where for the magnitude of a space vector, $\vec{k}$, we write k. We sum over the final polarization of the light beam, $\phi$, and after evaluating the trace we obtain*:

$$d\sigma \sim \frac{k_o}{k} + \frac{k}{k_o} - \sin^2\theta + (1 - \cos\theta)(\vec{k} + \vec{k}_o \cos\theta) \cdot \vec{S}\, P \qquad (3)$$

where $\theta$ is the scattering angle in the laboratory system, $\vec{S}$ represents the initial electron spin, and $P = +1, -1$ corresponding to right and left circular polarization, respectively, since our potential is $\vec{e}_o = 2^{-\frac{1}{2}}(\vec{e}_1 + i\, P\, \vec{e}_2)$ where $\vec{e}_1$, $\vec{e}_2$, and $\vec{k}_o$ form a right handed coordinate system. This is just the result obtained by Fano[3].

Since we are concerned with only the anisotropy due to incident gamma-ray polarization it is useful to rewrite equation (3) so as to express the intensity of the scattered radiation at constant scattering angle, $\theta$, as a function of the rotation of the k, $k_o$ plane about the fixed $k_o$, S plane:

$$d\sigma \sim \frac{k_o}{k} + \frac{k}{k_o} - \sin^2\theta + (1 - \cos\theta)\Big[(k + k_o)\cos\alpha\, \cos\theta + k\sin\theta\, \sin\alpha\, \cos\varphi\Big] S\, P \qquad (4)$$

where S is the magnitude of the average electron spin, $\alpha$ is the fixed angle

3. U. Fano, J. Opt. Soc. 39, 859 (1949).

*To obtain the differential scattering cross-section this square of the absolute value of the matrix element must be multiplied by

$$\frac{1}{4}(e^2)^2 \left(\frac{k}{k_o}\right)^2 d\Omega.$$

between the incident photon propagation vector and the spin direction of the target electron, and $\varphi$ is the angle between the k, $k_o$ plane and the S, $k_o$ plane.

We can now use this result to calculate the net azimuthal anisotropy observable from the gamma-ray polarization caused by the initial polarization of the neutrons. Biedenharn, Rose and Arfken[1] obtain for the probability, $W(P_o, k_o, P)$, computed for unmixed $2^L$ pole radiation that a neutron with polarization $\vec{P}_o$ will lead to the emission of a photon with propagation vector $\vec{k}_o$, circular polarization P, and of multipole order L, the result:

$$W(P_o, k_o, P) \sim 1 + \left| P_o \right| P \frac{J'(J'+1) - L(L+1) - J(J+1)}{L(L+1)(2J_o+1)} \cos \omega \tag{5}$$

where $\omega$ is the angle between $\vec{k}_o$ and the incident neutron polarization, $\vec{P}_o$, and $J_o$, J, and J' are the angular momenta of the target, compound, and final nuclei respectively. For simplicity we set $W = I_o + I_1 P \cos \omega$ and obtain after summing over P for the azinuthal anisotropy of capture gamma-rays produced by polarized neutrons and scattered by magnetized iron:

$$d\Sigma = I_o J_o + I_1 J_1 \cos \omega \cos \varphi \; S \tag{6}$$

where $J_o = \frac{k_o}{k} + \frac{k}{k_o} - \sin^2 \theta + (1 - \cos \theta)(k + k_o) \cos \alpha \; \cos \theta \; S$

and $J_1 = (1 - \cos \theta) k \sin \theta \; \sin \alpha$ .

Clearly the maximum azimuthal anisotropy for a particular value of $I_1/I_o$ will occur if we set $\vec{S}$ perpendicular to $\vec{k}_o$ to obtain:

$$J_o = \frac{k_o}{k} + \frac{k}{k_o} - \sin^2 \theta \tag{7}$$

$$J_1 = (1 - \cos \theta)\, k \sin \theta \tag{8}$$

Taking the derivative of $J_1$ with respect to $\cos \theta$ and remembering that from momentum conservation $k = k_o \left[ 1 + k_o(1 - \cos \theta) \right]^{-1}$ we find that the maximum anisotropy will occur at:

$$\cos \theta_{max} = \frac{1}{k_o} \left\{ 1 + \frac{k_o}{2} - \sqrt{\left(1 + \frac{k_o}{2}\right)^2 + k_o} \right\} \tag{9}$$

A graph of $\cos \theta_{max}$ as a function of $k_o$ is plotted in figure 1.

Substituting this value of $\cos \theta$ into the equation for $J_1$, equation (8), we obtain:

$$J_{1_{max}} = k_o \frac{\sqrt{1 - \cos^2 \theta_{max}}\,(1 - \cos \theta_{max})}{1 + k_o(1 - \cos \theta_{max})} \tag{10}$$

$J_{1_{max}}$ has been plotted as a function of $k_o$ in figure 2. One observes that the maximum value of $J_1$ approaches 1 for large $k_o$.

Although the maximum relative anisotropy was not computed, approximately the same thing may be observed if we study the expression for the isotropic term, $J_o$, equation (7), $J_{o_{max}} = k_o/k_{max} + k_{max}/k_o - \sin^2 \theta_{max}$. Where one can see that $J_{o_{max}}$ is always greater than 1 and is indeed approximately $1 + k_o$ for all values of $k_o$. To determine what relative anisotropy was possible $\frac{J_{1_{max}}}{J_{o_{max}}}$ has been plotted as a function of $k_o$ in figure 3. One can see that the maximum relative anisotropy is around 30% at $k_o \sim 1/2$ mev decreasing to 20% at $k_o \sim 1.5$ mev, 10% at 4 mev, and to 5% for 8 mev gamma-rays.

One cannot expect much help, therefore, from the Compton scattering anisotropy factor in obtaining a large observable anisotropy. In view of the small average spin of iron electrons (S = .043), magnetized iron does not offer much hope for the experimental determination of the polarization of circularly polarized gamma-rays. For using magnetized iron the maximum observable anisotropy in the Compton scattering of circularly polarized photons is at best only one per cent.

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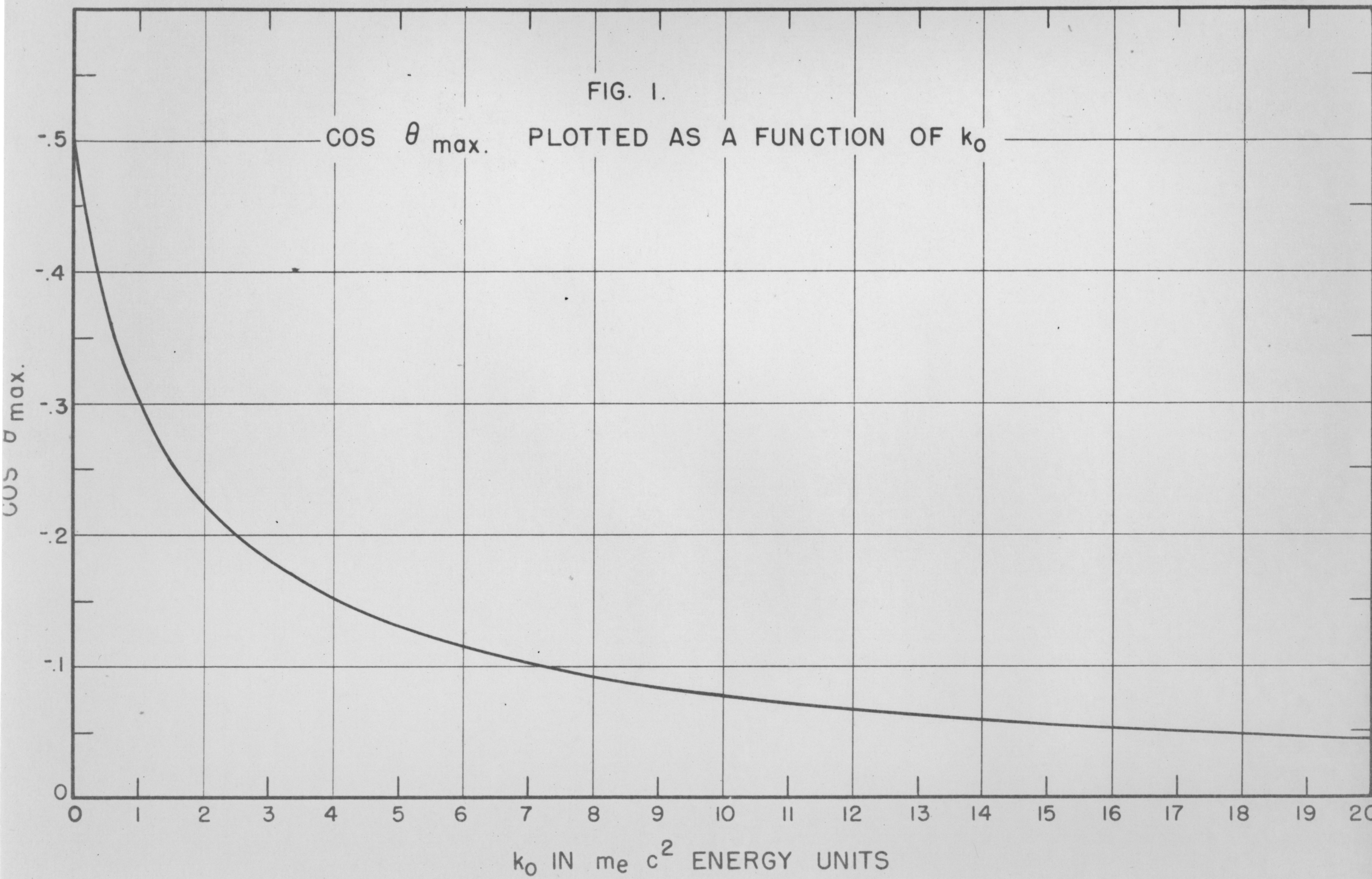


FIG. 1.

COS $\theta_{max.}$ PLOTTED AS A FUNCTION OF $k_o$

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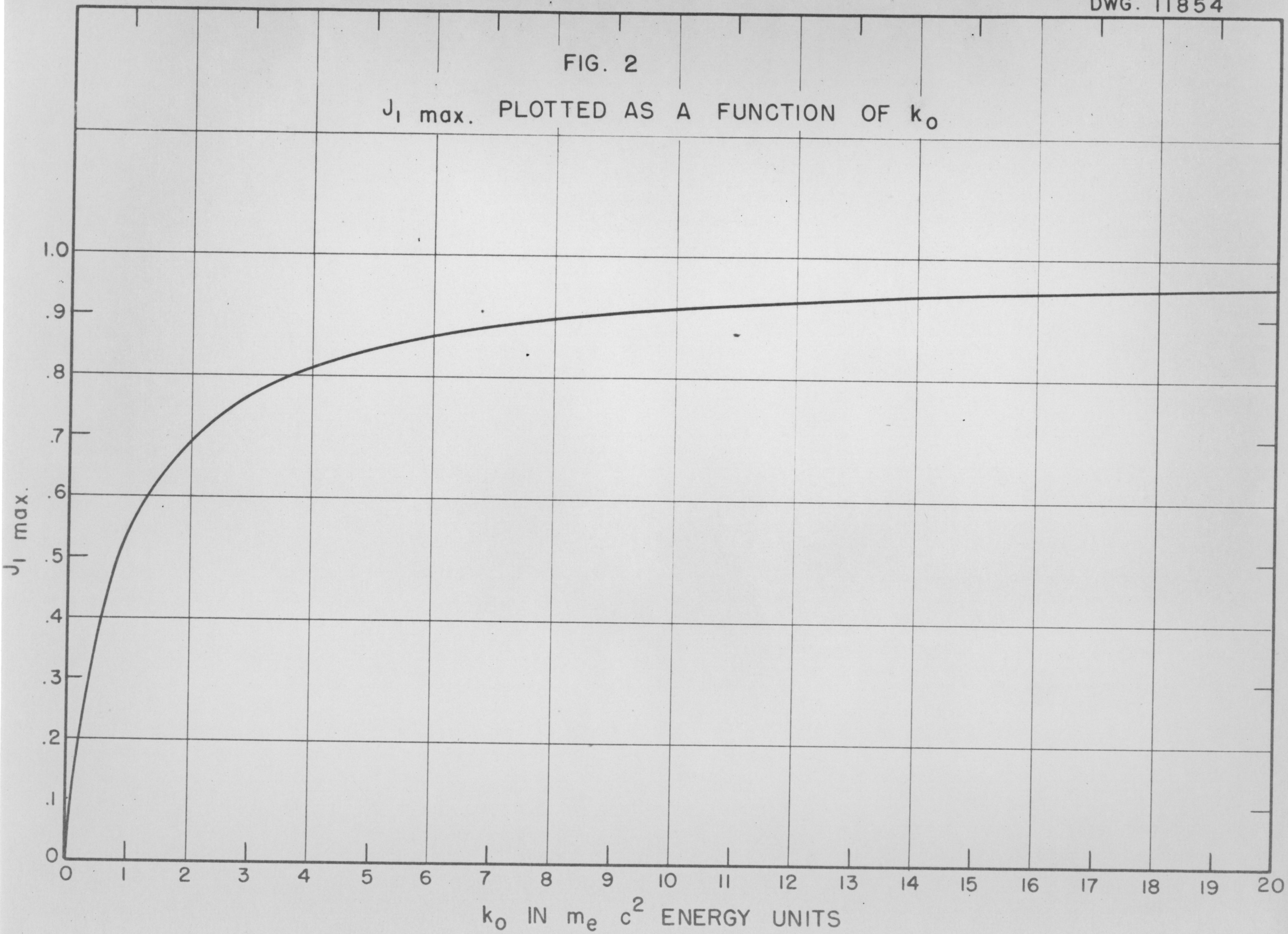
FIG. 2

$J_1$ max. PLOTTED AS A FUNCTION OF $k_o$



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