UNCLASSIFIED

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES
3 4456 0360894 1

ORNL 1097
Physics
9N

TABLES OF COEFFICIENTS FOR INTERNAL CONVERSION ANGULAR CORRELATION

M. E. Rose
L. C. Biedenharn
G. B. Arfken

OAK RIDGE NATIONAL LABORATORY
CENTRAL RESEARCH LIBRARY
CIRCULATION SECTION
4500N ROOM 175
LIBRARY LOAN COPY
DO NOT TRANSFER TO ANOTHER PERSON
If you wish someone else to see this report, send in name with report and the library will arrange a loan.
UCN-7969 (3 9-77)

OAK RIDGE NATIONAL LABORATORY
OPERATED BY
CARBIDE AND CARBON CHEMICALS COMPANY
A DIVISION OF UNION CARBIDE AND CARBON CORPORATION
UCC
POST OFFICE BOX P
OAK RIDGE, TENNESSEE

UNCLASSIFIED

UNCLASSIFIED

ORNL 1097
This document consists of 17 pages. No. 9 of 223 copies, Series A.

Contract No. W-7405, eng 26

PHYSICS DIVISION

TABLES OF COEFFICIENTS FOR INTERNAL CONVERSION ANGULAR CORRELATIONS

M. E. Rose
L. C. Biedenharn
G. B. Arfken

Date Issued:

OAK RIDGE NATIONAL LABORATORY
operated by
CARBIDE AND CARBON CHEMICALS COMPANY
A Division of Union Carbide and Carbon Corporation
Post Office Box P
Oak Ridge, Tennessee

UNCLASSIFIED

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES
3 4456 0360894 1

ORNL 1097
PHYSICS

INTERNAL DISTRIBUTION

1. G. T. Felbeck (C&CCC)
2-3. Chemistry Library
4. Physics Library
5. Biology Library
6. Health Physics Library
7. Metallurgy Library
8-9. Training School Library
10-13. Central Files
14. C. E. Center
15. C. E. Larson
16. W. B. Humes (K-25)
17. W. D. Lavers (Y-12)
18. A. M. Weinberg
19. E. H. Taylor
20. E. D. Shipley
21. A. H. Snell
22. F. C. VonderLage
23. R. C. Briant
24. J. A. Swartout
25. S. C. Lind
26. F. L. Steahly
27. A. Hollaender
28. S. Bernstein
29. M. T. Kelley
30. K. Z. Morgan
31. J. S. Felton
32. A. S. Householder
33. C. S. Harrill
34. C. E. Winters
35. D. S. Billington
36. D. W. Cardwell
37. E. M. King
38. E. O. Wollan
39. C. B. Ellis
40. L. D. Roberts
41. R. N. Lyon
42. W. C. Koehler
43. W. K. Ergen
44. E. P. Blizard
45-56. M. E. Rose
57. L. C. Biedenharn
58. G. B. Arfken
59. M. J. Skinner

EXTERNAL DISTRIBUTION

60-71. Argonne National Laboratory
72. Armed Forces Special Weapons Project
73-78. Atomic Energy Commission, Washington
79. Battelle Memorial Institute
80. Brush Beryllium Company
81-84. Brookhaven National Laboratory
85. Bureau of Medicine and Surgery
86. Bureau of Ships
87. California Institute of Technology (Attn: R. F. Bacher)
88-91. Carbide and Carbon Chemicals Company (K-25 Plant)
92-95. Carbide and Carbon Chemicals Company (Y-12 Area)
96. Carnegie Institute of Technology (Attn: E. C. Creutz)
97. Columbia University (J. R. Dunning)
98. Columbia University (G. Failla)
99-103. E. I. du Pont de Nemours and Company
104-107. General Electric, Richland
108-111. Idaho Operations Office
112-113. Iowa State College
114. Kansas City Operations Branch
115. Kellex Corporation
116-117. Kirtland Air Force Base
118-121. Knolls Atomic Power Laboratory
122-124. Los Alamos Scientific Laboratory
125. Mallinckrodt Chemical Works
126. Massachusetts Institute of Technology (A. Gaudin)
127. Massachusetts Institute of Technology (A. R. Kaufmann)

128-130. Mound Laboratory
131. National Advisory Committee for Aeronautics
132. National Bureau of Standards (R. D. Huntoon)
133. Naval Medical Research Institute
134-135. Naval Radiological Defense Laboratory
136. New Brunswick Laboratory
137-139. New York Operations Office
140. North American Aviation, Inc.
141. Patent Branch, Washington
142. RAND Corporation
143. Sandia Corporation
144. Santa Fe Operations Office
145. Savannah River Operations Office (Wilmington)
146-160. Technical Information Service, Oak Ridge
161-162. U. S. Geological Survey (T. B. Nolan)
163-164. U. S. Public Health Service
165. University of California at Los Angeles
166-170. University of California Radiation Laboratory
171-172. University of Rochester
173. University of Washington
174. Western Reserve University
175-178. Westinghouse Electric Corporation
179. Yale University
180-183. Atomic Energy Project, Chalk River
184. Chief of Naval Research
185. H. K. Ferguson Company
186. Harshaw Chemical Company
187. Isotopes Division (Mr. McCormick)
188-189. Library of Congress, Acquisition Department (J. W. Cormn)
190. National Bureau of Standards
191. National Research Council, Ottawa
192. Naval Research Laboratory
193-194. Nevis Cyclotron Laboratories
195-196. Oak Ridge Institute of Nuclear Studies
197-206. United Kingdom Scientific Mission (M. Greenhill)
207. USAF, Eglin Air Force Base (Technical Library)
208. USAF, Wright-Patterson Air Force Base (Rodney Nudenberg)
209-213. USAF, Wright-Patterson Air Force Base (CADO)
214. U. S. Army, Army Field Forces (Captain James Kerr)
215. U. S. Army, Atomic Energy Branch (Lt. Col. A. W. Betts)
216. U. S. Army, Director of Operations Research (Dr. Ellis Johnson)
217. U. S. Army, Office, Chief of Ordnance (Col. A. R. Del Campo)
218-219. U. S. Army, Office of the Chief Signal Officer (Curtis T. Clayton through Major George C. Hunt)
220-222. U. S. Army, Technical Command (Col. J. H. Rothschild Attention: Technical Library)
223. UT-AEC Agricultural Research Program (Charles S. Hobbs)

TABLES OF COEFFICIENTS FOR INTERNAL CONVERSION ANGULAR CORRELATIONS

M. E. Rose, L. C. Biedenharn and G. B. Arfken

The angular correlation function, $W(\vartheta)$, in a double cascade in which one or both transitions involves the emission of a conversion electron can be obtained from any standard correlation function by insertion of appropriate factors in the Legendre polynomial expansion of $W(\vartheta)$. We adopt the γ-γ correlation as the standard one,

$$W_{\gamma\text{-}\gamma}(\vartheta) = \sum_{\nu=0}^{\nu_m} A_\nu \, P_\nu(\cos\vartheta) \tag{1}$$

$\frac{1}{2}\nu_m$ = maximum integer of the set L_1 L_2 J (or $J - \frac{1}{2}$) where L_1 and L_2 are the angular momenta of the two quanta and J is the angular momentum of the intermediate state. In (1) ν takes on even values only. In (1) and below we normalize to $A_0 = 1$ and, in general, coefficient of $P_0 = 1$. The coefficients A_ν have been tabulated for all cases of interest.[1] To get the γ-x correlation we replace (1) by

$$W_{\gamma\text{-}x} = \sum_\nu A_\nu \, b_\nu(x) \, P_\nu(\cos\vartheta) \tag{2}$$

For $x \equiv \alpha$- particle

$$b_\nu(\alpha) = \frac{2L(L+1)}{\nu(\nu+1) - 2L(L+1)} \tag{3}$$

---

1. S. P. Lloyd, Phys. Rev. 83, 716 (1951).
   D. L. Falkoff and G. E. Uhlenbeck, Phys. Rev. 79, 323 (1950).
   L. C. Biedenharn and M. E. Rose, ORNL report No. 1098.

where L is the orbital angular momentum of the α -particle. If x is a conversion electron

$$W_{\gamma-e} = \sum_{\nu=0}^{\nu_m} A_\nu \; b_\nu(e) \; P_\nu(\cos\vartheta) \tag{4}$$

The coefficients $b_2(e)$ are tabulated below for the K-shell conversion, for 12 values of Z in the range $10 \leqslant Z \leqslant 96$, for transition energies k mc^2 in the range $0.3 \leqslant k \leqslant 5.0$ and for the first five electric and first five magnetic multipoles.[2] The values of b_ν ($\nu > 2$) are easily obtained from

$$b_\nu - 1 = \frac{\nu(\nu+1)\left[L(L+1)-3\right]}{3\left[2L(L+1)-\nu(\nu+1)\right]} \; (b_2 - 1)$$

for both parities. As explained, $b_0 = 1$ in all cases. For transitions in which the γ -ray is mixed and the conversion transition is pure we follow the same procedure. Of course, in this case the A_ν are not the same as for the pure γ -ray transitions (see S. P. Lloyd[1]).

For conversion-conversion correlation we insert two b-factors, one for each conversion electron

$$W_{e-e} = \sum_{\nu=0}^{\nu_m} A_\nu \; b_\nu(e_1) \; b_\nu(e_2) \; P_\nu(\cos\vartheta) \tag{5}$$

The justification for this is simply that after the first conversion the K-shell fills up by X-ray or Auger-electron emission very rapidly compared

---

2. The formal part of the calculation has been described in a paper submitted to the Physical Review.

to the lifetime of the intermediate nuclear state in all practical cases. The filling up time is also too short to allow appreciable precession of the nuclear moment, in the intermediate state, by hfs coupling.

For α -conversion correlation we use

$$W_{\alpha-e} = \sum_{\nu=0}^{\nu_m} A_\nu \, b_\nu(e) \, b_\nu(\alpha) \, P_\nu(\cos\vartheta) \tag{6}$$

The authors are indebted to Dr. C. L. Perry, Mr. C. Perhacs and Mrs. Nancy Dismuke of the Mathematics Panel for performing the computations.

TABLE I: b_2 Coefficients for Z = 10

k =	0.3	0.5	1.0	1.8	3.0	5.0
E_1	- 1.8624	- 1.6544	- 1.0000	- 0.1757	0.4157	0.7506
E_2	1.9611	1.9015	1.7023	1.4281	1.2176	1.0942
E_3	1.3209	1.3020	1.2376	1.1458	1.0737	1.0316
E_4	1.1699	1.1600	1.1264	1.0777	1.0389	1.0163
E_5	1.1069	1.1007	1.0796	1.0489	1.0243	1.0100
M_1	0.9288	0.9551	0.9769	0.9875	0.9930	0.9963
M_2	1.0291	1.0186	1.0100	1.0058	1.0035	1.0020
M_3	1.0115	1.0073	1.0040	1.0024	1.0015	1.0010
M_4	1.0070	1.0045	1.0024	1.0015	1.0009	1.0006
M_5	1.0050	1.0032	1.0017	1.0010	1.0007	1.0004

Note: E_L, M_L designate electric, magnetic 2^L-pole transitions respectively.

TABLE II: b_2 Coefficients for Z = 20

k =	0.3	0.5	1.0	1.8	3.0	5.0
E_1	- 1.8539	- 1.6419	- 1.0000	- 0.2077	0.3730	0.7188
E_2	1.9527	1.8850	1.6717	1.4004	1.2044	1.0913
E_3	1.3167	1.2940	1.2219	1.1290	1.0632	1.0270
E_4	1.1670	1.1546	1.1159	1.0661	1.0311	1.0126
E_5	1.1047	1.0966	1.0718	1.0403	1.0184	1.0071
M_1	0.7685	0.8430	0.9119	0.9488	0.9696	0.9830
M_2	1.0931	1.0637	1.0369	1.0226	1.0144	1.0087
M_3	1.0361	1.0248	1.0145	1.0091	1.0061	1.0038
M_4	1.0214	1.0149	1.0087	1.0055	1.0036	1.0024
M_5	1.0149	1.0104	1.0061	1.0038	1.0026	1.0017

TABLE III: b_2 Coefficients for Z = 30

k =	0.3	0.5	1.0	1.8	3.0	5.0
E_1	- 1.8452	- 1.6292	- 1.0000	- 0.2411	0.3260	0.6815
E_2	1.9411	1.8631	1.6338	1.3679	1.1887	1.0874
E_3	1.3105	1.2827	1.2018	1.1099	1.0518	1.0225
E_4	1.1625	1.1466	1.1022	1.0531	1.0232	1.0090
E_5	1.1009	1.0903	1.0615	1.0308	1.0126	1.0045
M_1	0.5810	0.6951	0.8130	0.8830	0.9261	0.9560
M_2	1.1646	1.1206	1.0755	1.0491	1.0328	1.0210
M_3	1.0623	1.0460	1.0292	1.0193	1.0132	1.0091
M_4	1.0362	1.0271	1.0172	1.0114	1.0079	1.0054
M_5	1.0246	1.0186	1.0119	1.0079	1.0055	1.0038

TABLE IV: b_2 Coefficients for Z = 40

k =	0.3	0.5	1.0	1.8	3.0	5.0
E_1	- 1.8361	- 1.6160	- 1.0000	- 0.2759	0.2743	0.6375
E_2	1.9246	1.8333	1.5863	1.3298	1.1703	1.0824
E_3	1.3008	1.2662	1.1764	1.0885	1.0400	1.0174
E_4	1.1549	1.1346	1.0849	1.0392	1.0156	1.0058
E_5	1.0944	1.0804	1.0485	1.0209	1.0075	1.0024
M_1	0.4009	0.5359	0.6898	0.7908	0.8583	0.9095
M_2	1.2298	1.1787	1.1205	1.0831	1.0586	1.0398
M_3	1.0810	1.0668	1.0454	1.0317	1.0228	1.0160
M_4	1.0486	1.0387	1.0266	1.0186	1.0136	1.0096
M_5	1.0325	1.0262	1.0182	1.0128	1.0093	1.0067

TABLE V: b_2 Coefficients for Z = 54

k =	0.3	0.5	1.0	1.8	3.0	5.0
E_1	- 1.8231	- 1.5966	- 1.0000	- 0.3273	0.1936	0.5624
E_2	1.8868	1.7698	1.4983	1.2660	1.1402	1.0737
E_3	1.2764	1.2293	1.1303	1.0564	1.0241	1.0113
E_4	1.1347	1.1068	1.0546	1.0200	1.0067	1.0027
E_5	1.0765	1.0579	1.0269	1.0085	1.0022	1.0007
M_1	0.1917	0.3244	0.4961	0.6231	0.7174	0.7974
M_2	1.3024	1.2518	1.1863	1.1390	1.1054	1.0778
M_3	1.1094	1.0919	1.0686	1.0514	1.0393	1.0295
M_4	1.0613	1.0521	1.0393	1.0297	1.0228	1.0172
M_5	1.0403	1.0346	1.0264	1.0200	1.0154	1.0118

TABLE VI: b_2 Coefficients for Z = 64

k =	0.3	0.5	1.0	1.8	3.0	5.0
E_1	- 1.8129	- 1.5819	- 1.0000	- 0.3661	0.1293	0.4972
E_2	1.8400	1.6984	1.4152	1.2131	1.1162	1.0668
E_3	1.2444	1.1876	1.0905	1.0344	1.0149	1.0085
E_4	1.1082	1.0768	1.0311	1.0088	1.0027	1.0019
E_5	1.0540	1.0354	1.0122	1.0025	1.0006	1.0010
M_1	0.0800	0.1954	0.3566	0.4842	0.5837	0.6736
M_2	1.3410	1.2952	1.2312	1.1817	1.1446	1.1131
M_3	1.1220	1.1064	1.0840	1.0660	1.0527	1.0415
M_4	1.0677	1.0596	1.0474	1.0375	1.0299	1.0236
M_5	1.0441	1.0392	1.0316	1.0251	1.0201	1.0159

TABLE VII: b_2 Coefficients for Z = 72

k =	0.3	0.5	1.0	1.8	3.0	5.0
E_1	- 1.8041	- 1.5693	- 1.0000	- 0.3986	0.0735	0.4368
E_2	1.7801	1.6155	1.3348	1.1678	1.0968	1.0615
E_3	1.2039	1.1423	1.0572	1.0194	1.0096	1.0074
E_4	1.0772	1.0480	1.0147	1.0031	1.0017	1.0027
E_5	1.0311	1.0171	1.0038	1.0004	1.0009	1.0021
M_1	0.0152	0.1104	0.2520	0.3687	0.4615	0.5460
M_2	1.3647	1.3242	1.2645	1.2158	1.1781	1.1455
M_3	1.1296	1.1160	1.0950	1.0773	1.0637	1.0517
M_4	1.0716	1.0645	1.0533	1.0436	1.0358	1.0292
M_5	1.0464	1.0422	1.0352	1.0289	1.0238	1.0195

TABLE VIII: b_2 Coefficients for Z = 78

k =	0.3	0.5	1.0	1.8	3.0	5.0
E_1	- 1.7968	- 1.5592	- 1.0000	- 0.4240	0.0288	0.3861
E_2	1.7133	1.5325	1.2672	1.1339	1.0832	1.0584
E_3	1.1621	1.1023	1.0344	1.0111	1.0075	1.0082
E_4	1.0499	1.0269	1.0059	1.0013	1.0024	1.0042
E_5	1.0149	1.0065	1.0007	1.0007	1.0022	1.0036
M_1	- 0.0189	0.0589	0.1808	0.2838	0.3652	0.4365
M_2	1.3789	1.3427	1.2874	1.2408	1.2039	1.1717
M_3	1.1343	1.1220	1.1025	1.0855	1.0719	1.0601
M_4	1.0739	1.0676	1.0573	1.0479	1.0403	1.0336
M_5	1.0478	1.0440	1.0376	1.0316	1.0266	1.0222

TABLE IX: b_2 Coefficients for Z = 83

k =	0.5	1.0	1.8	3.0	5.0
E_1	- 1.5499	- 1.0000	- 0.4460	- 0.0103	0.3399
E_2	1.4467	1.2083	1.1073	1.0734	1.0569
E_3	1.0680	1.0188	1.0071	1.0078	1.0100
E_4	1.0125	1.0016	1.0016	1.0041	1.0062
E_5	1.0014	1.0003	1.0020	1.0040	1.0053
M_1	0.0247	0.1279	0.2166	0.2853	0.3407
M_2	1.3562	1.3051	1.2609	1.2255	1.1944
M_3	1.1265	1.1083	1.0922	1.0789	1.0673
M_4	1.0698	1.0603	1.0514	1.0439	1.0373
M_5	1.0453	1.0394	1.0338	1.0289	1.0245

TABLE X: b_2 Coefficients for Z = 88

k =	0.5	1.0	1.8	3.0	5.0
E_1	- 1.5404	- 1.0000	- 0.4687	- 0.0514	0.2899
E_2	1.3461	1.1503	1.0841	1.0661	1.0573
E_3	1.0367	1.0081	1.0059	1.0097	1.0129
E_4	1.0033	1.0007	1.0036	1.0068	1.0088
E_5	1.0001	1.0018	1.0044	1.0064	1.0075
M_1	- 0.0012	0.0820	0.1548	0.2086	0.2457
M_2	1.3678	1.3214	1.2803	1.2468	1.2176
M_3	1.1302	1.1137	1.0985	1.0858	1.0745
M_4	1.0718	1.0630	1.0547	1.0475	1.0410
M_5	1.0465	1.0411	1.0358	1.0312	1.0269

TABLE XI: b_2 Coefficients for Z = 92

k =	0.5	1.0	1.8	3.0	5.0
E_1	- 1.5321	- 1.0000	- 0.4875	- 0.0856	0.2470
E_2	1.2581	1.1081	1.0696	1.0631	1.0596
E_3	1.0172	1.0044	1.0081	1.0128	1.0166
E_4	1.0004	1.0022	1.0064	1.0097	1.0114
E_5	1.0015	1.0043	1.0069	1.0086	1.0095
M_1	- 0.0160	0.0511	0.1105	0.1522	0.1743
M_2	1.3758	1.3335	1.2951	1.2636	1.2362
M_3	1.1330	1.1179	1.1036	1.0912	1.0805
M_4	1.0731	1.0651	1.0572	1.0503	1.0440
M_5	1.0473	1.0424	1.0374	1.0329	1.0287

TABLE XII: b_2 Coefficients for Z = 96

k =	0.5	1.0	1.8	3.0	5.0
E_1	- 1.5231	- 1.0000	- 0.5069	- 0.1213	0.2013
E_2	1.1701	1.0734	1.0603	1.0634	1.0643
E_3	1.0050	1.0042	1.0115	1.0173	1.0207
E_4	1.0013	1.0057	1.0103	1.0132	1.0146
E_5	1.0046	1.0075	1.0098	1.0112	1.0117
M_1	- 0.0253	0.0258	0.0719	0.1019	0.1106
M_2	1.3828	1.3446	1.3093	1.2801	1.2548
M_3	1.1354	1.1214	1.1081	1.0965	1.0862
M_4	1.0744	1.0670	1.0596	1.0531	1.0471
M_5	1.0480	1.0435	1.0389	1.0346	1.0306