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A CRITICAL REVIEW OF THE LITERATURE ON PRESSURE DROP IN NONCIRCULAR DUCTS AND ANNULI

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A CRITICAL REVIEW OF THE LITERATURE ON PRESSURE DROP IN NONCIRCULAR DUCTS AND ANNULI

by

H. C. Claiborne

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SUMMARY

The usual correlation parameter for turbulent flow in noncircular geometries is the equivalent diameter. Since this parameter has no real theoretical basis, a question arose as to the generality and degree of applicability of the equivalent diameter for widely different geometries.

The literature was critically examined for pressure drop data on isothermal fluid flow in the turbulent regime through noncircular ducts, annuli and parallel flow systems, i.e., fluid flow parallel to the exterior surfaces of tube banks. The results were correlated on the basis of the conventional equivalent diameter and compared to the classic round pipe data. The results of 12 authors for 30 widely different geometries agreed with the round pipe data to within ± 15 per cent for Reynolds moduli greater than 5000 and ± 35 per cent for Reynolds moduli between 2100 and 5000.

In view of these results, it can be concluded that for practical purposes the equivalent diameter is a satisfactory correlation parameter for turbulent flow through noncircular ducts, annuli, and parallel flow systems.

INTRODUCTION

In order to ascertain the general validity of the equivalent diameter (4 times hydraulic radius) as the correlation parameter for turbulent flow in noncircular ducts and annuli, a compilation of the literature on pressure drop due to isothermal turbulent flow in noncircular geometries was made and critically examined. The pertinent data were correlated on the equivalent

diameter basis and compared with the classic round pipe data of Stanton and Pannell (ref. 15) and Blasius (ref. 4).

No data for unbaffled flow parallel to banks of tubes could be found in the literature. Since component parts of such a system approach geometric similarity to certain noncircular ducts, it was felt that the equivalent diameter should correlate parallel flow system as well. Subsequent experiments by Cohen, Fraas and La Verne (ref. 6) at Oak Ridge National Laboratory confirmed the supposition.

A theoretical evaluation of the pressure drop due to turbulent flow ($Re > 2100$) through a duct has not as yet been possible. It can be shown by analysis, however, that

$$\frac{D_e\, g}{\rho\, u^2}\left(\frac{dp}{dx}\right) = \Phi\left(\frac{D_e\, u}{\nu}\right) \tag{1}$$

where

D_e = equivalent diameter, 4 x hydraulic radius

Hydraulic radius = cross sectional area/wetted perimeter, which is equal to the diameter for a pipe.

u = mean velocity

ν = kinematic viscosity

g = gravitational conversion constant

ρ = density

$\left(\frac{dp}{dx}\right)$ = pressure gradient

$\Phi\left(\frac{D_e\, u}{\nu}\right)$ = function of Reynolds modulus

Let $$2f = \Phi\left(\frac{D_e\ u}{\nu}\right) \tag{2}$$

where f = Fanning friction factor. Therefore,

$$\left(\frac{dp}{dx}\right) = \frac{2\ f\ \rho\ u^2}{D_e\ g} \tag{3}$$

The analysis indicates that f is a unique function of the Reynolds modulus even though the exact nature of f is unknown. The relationship between f and Re for round pipes was determined by independent investigators using fluids whose densities varied 830 fold, viscosities 3,750 fold, velocities 1600 fold and pipe diameters 120 fold. The maximum deviation of any of the points from the mean curve was about 5 per cent. Such conclusive results definitely established f as a unique function of the Reynolds modulus for turbulent flow through pipes (ref. 17).

The corresponding relationship for noncircular ducts and annuli has not been as satisfactorily established. It would seem that the phenomenon of momentum transfer in noncircular ducts is more complex than in round pipes since it has been shown experimentally (ref. 11 and 12) that secondary flow occurs in the corners of noncircular ducts.

RESULTS OF LITERATURE SURVEY

Noncircular Ducts

The results of the various investigators are shown in Table I and Figure 1. All the curves, with one exception, representing the experimental data are within ± 15 per cent of the Stanton-Pannell (ref. 15) and Blasius (ref. 4)

curve for smooth round pipe for Reynolds moduli greater than 5000. For Reynolds moduli between 2100 and 5000, the experimental curves are within ± 35 per cent of the round pipe curve.

The data of Nikuradse (ref. 11 and 12) are particularly remarkable in that one curve represents the results for seven different cross sections with a precision of ± 2 per cent.

The results of Washington and Marks (ref. 16) have the greatest disagreement with the round pipe data. This may be explained partially by the small length of calming section and that the air entered the test section through a nozzle or entrance cone. A possible reason for the appreciable increase of the friction factor with increase in distance between the long sides of the rectangular cross section was the heterogeneous character of the duct walls. The duct walls were made of smooth-rolled copper (5 in. wide for all ducts) separated at the edges by strips of transite that were 1/8, 1/4, and 9/16 in. for the three rectangular ducts. The transite strips were rougher than the copper plates and formed a greater percentage of the total wall surface as the distance between the copper plates was increased, thus partially accounting for the difference.

Annuli

The results of the various investigators are shown in Table II and Figure 2. All the curves, with one exception, are within ± 15 per cent of the round pipe curve. The high results (note the results agree well with the commercial pipe curve) of Carpenter et al (ref. 5) can possibly be attributed to artificial roughness created by thermocouples installed in the fluid stream at regular intervals.

TABLE I

RESULTS OF VARIOUS INVESTIGATORS FOR ISOTHERMAL FLOW IN NONCIRCULAR DUCTS

Investigator	Reference	Duct Shape	Aspect Ratio	Equivalent Diameter, cm.	Duct Dimensions, cm.	Calming Length, L/D_e
Allen and Grundberg	1	Rectangle	3.92	0.518	1.275 by 0.325	294
Cornish	7	Rectangle	2.92	0.602	1.178 by 0.404	50 and 111
Davies and White	8	10 Rectangles	168 to 37	0.0300 to 0.132	2.54 by 0.0151 to 0.0681	87 to 19
Lea and Tadros	10	2 Squares	1	sides, 0.678 and 1.561	---	---
Lea and Tadros	10	2 Rectangles	2.36 and 2.04	0.942 by 0.400 and 1.916 by 0.937	---	---
Nikuradse	11 and 12	Rectangle	3.50	1.24	2.8 by 0.8	---
Nikuradse	11 and 12	Equil. Triangle	----	1.56	side, 2.7	---
Nikuradse	11 and 12	Right Isosceles Triangle	----	0.874	hypoth., 2.1	119
Nikuradse	11 and 12	Acute Triangle	----	1.14	hypoth., 3.84	59
Nikuradse	11 and 12	Trapezoid	----	1.17	bases, 2.14 and 1.02 side, 1.61	91
Nikuradse	11 and 12	Circle with one notch	----	1.56	diameter, 2.3 notch, 0.75 by 1.05	61
Nikuradse	11 and 12	Circle with two notches	----	1.34	diameter, 2.3 notches, 0.59 by 0.88	78
Schiller	14	Rectangle	3.52	1.23	2.78 by 0.79	80
Schiller	14	Square	1	3.60	side, 1.800	28
Schiller	14	Equil. Triangle	----	1.545	side, 2.675	65
Washington and Marks	16	Rectangle	40	0.0203 ft.	5 by 1/8 in.	27
Washington and Marks	16	Rectangle	20	0.0397 ft.	5 by 1/4 in.	14
Washington and Marks	16	Rectangle	8.9	0.0842 ft.	5 by 9/16 in.	6

TABLE I (Con't.)

RESULTS OF VARIOUS INVESTIGATORS FOR ISOTHERMAL FLOW IN NONCIRCULAR DUCTS

Investigator	Test Section Length, L/D_e	Duct Material	Test Fluid	Maximum Re.	Max. Dev. of Data from Mean Curve, %	Dev. from Round Pipe Curve for Re >5000, %
Allen and Grundberg	294	Smooth brass	water	10,000	± 5	- 15
Cornish	111 and 60	Smooth brass	water	64,000	± 3	0 to -3
Davies and White	127 to 29	Smooth brass	water	4,600	± 3	---
Lea and Tadros	----	brass	water	20,000	± 3	0 to -10
Lea and Tadros	----	brass	water	7,100	± 2	0 to -10
Nikuradse	----	brass	water	100,000	± 2	-7 to +3
Nikuradse	----	brass	water	190,000	± 2	-7 to +3
Nikuradse	69	brass	water	100,000	± 2	-7 to +3
Nikuradse	84	brass	water	160,000	± 2	-7 to +3
Nikuradse	49	brass	water	170,000	± 2	-7 to +3
Nikuradse	38	brass	water	120,000	± 2	-7 to +3
Nikuradse	45	brass	water	110,000	± 2	-7 to +3
Schiller	80	brass	water	60,000	± 8	-1 to 0
Schiller	28	brass	water	36,000	± 10	0 to -3
Schiller	65	brass	water	228,000	± 5	-6 to -3
Washington and Marks	197	Copper and transite	air	28,000	± 5	-15 to -11
Washington and Marks	101	Copper and transite	air	39,000	± 4	-6 to -4
Washington and Marks	48	Copper and transite	air	10,000	± 5	+22 to +10

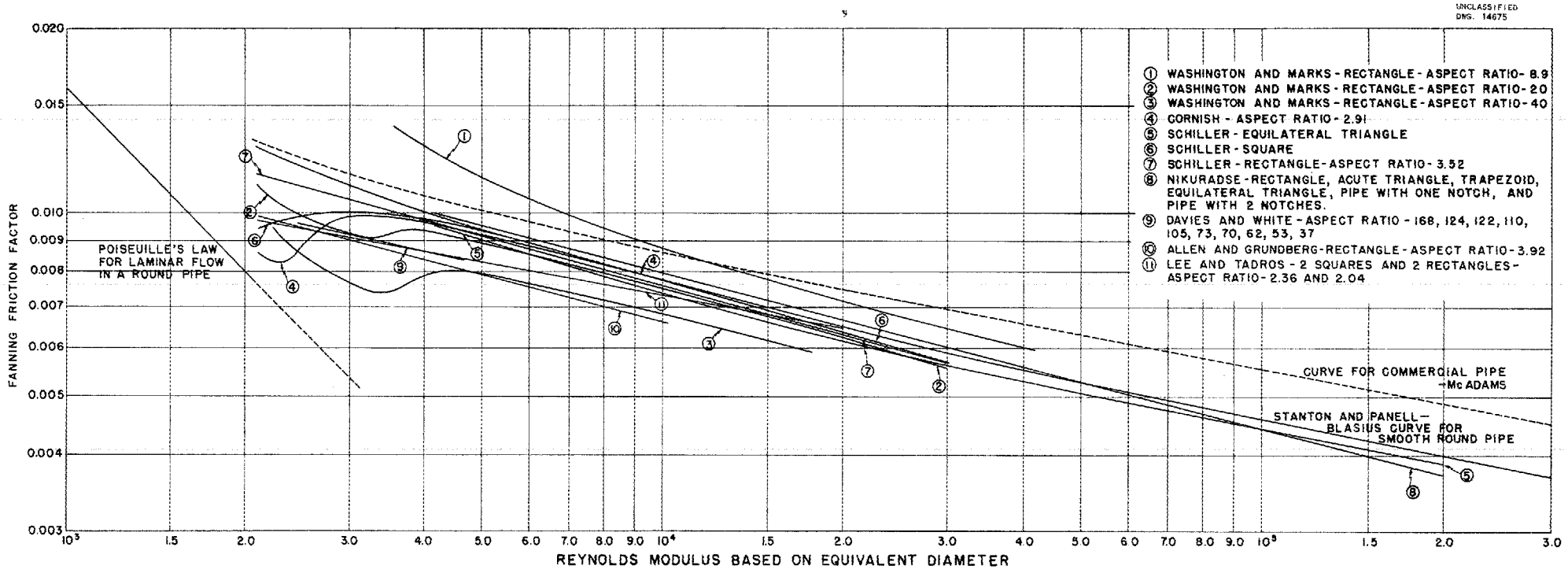

FIG. 1

FRICTION FACTORS FOR ISOTHERMAL FLOW IN STRAIGHT NONCIRCULAR DUCTS

TABLE II

RESULTS OF VARIOUS INVESTIGATORS FOR ISOTHERMAL FLOW IN ANNULI

Investigator	Ref.	Diam. Ratio	O.D. of Inner Pipe	Equiv. Diameter	Calming Length L/D_e	Test Section Length, L/D_e	Pipe Material	Test Fluid	Max. Re.	Max. Dev. of Data from Mean Curve, %	Dev. from Round Pipe Curve for Re $\geq$ 5000,%
Becker	3	1.04 and 1.07	2.08 and 2.120 cm.	0.084 and 0.124 cm.	17 and 11	159 and 235	polished brass	water	7,000	± 5	0 to -7
Carpenter et al	5	1.334	0.625 in.	0.209 in.	230	201	copper	water	9,600	± 1	+11 to +15
Kratz et al	9	1.252	0.1383 ft.	0.00833ft.	540	1200	wrought iron	water and $CaCl_2$ brine	17,500	± 8	-11 to -10*
Kratz et al	9	1.581	0.1083 ft.	0.01604ft.	281	624	wrought iron	water and $CaCl_2$ brine	20,500	± 5	+1 to +5*
Kratz et al	9	1.980	0.0883 ft.	0.02085ft.	216	480	wrought iron	water and $CaCl_2$ brine	31,300	±12	-10 to -5*
Rothfus	13	6.156	0.5000 in.	2.578 in.	93	56	brass tubing and pipe	air	25,000	±5	+17 to +22
Rothfus	13	1.539	2.000 in.	1.078 in.	223	134	brass tubing and pipe	air	14,500	±9	+6 to +9
Winkel	18	1.200	2.5 cm	0.500 cm	---	---	---	water	13,500	±7	-1 to +1

* referred to commercial pipe curve given by McAdams (ref. 17) since wrought iron is relatively rough compared to smooth tubing.

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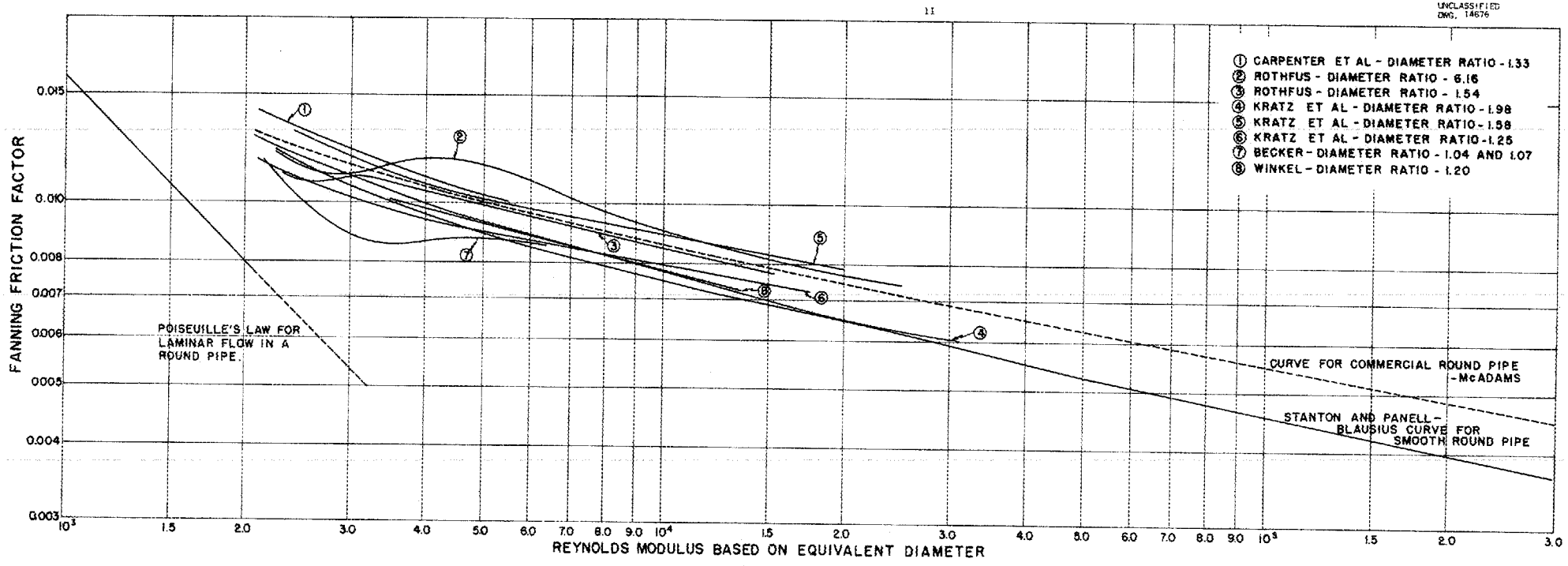

FIG. 2

FRICTION FACTORS FOR ISOTHERMAL FLOW IN STRAIGHT ANNULI

The results by Rothfus (ref. 13) show the largest deviation (+6 to +22 per cent) from the round pipe curve. This might possibly be explained as the result of spacers in the annuli, except that no appreciable difference in the data could be detected when using different sized spacers (0.036, 0.063, and 0.094 in. across).

It is interesting to note that the investigators, Rothfus and Washington and Marks, who found the greatest deviation from the round pipe data used air as the test fluid and all the others used water.

Atherton (ref. 2) measured pressure drops in three annuli using air, water and fuel oil for Reynolds moduli up to 100,000. The data were so widely scattered that they were not considered in this survey. Apparently the technique used was not as good as the other investigators who obtained good precision of results.

Parallel Flow to Tube Banks

Pressure drop data for unbaffled flow parallel to tube banks could not be found in the literature.

Cohen et al (ref. 6) have recently measured pressure drop for parallel flow to tube banks for several geometries. The results are to be published soon in form of an Oak Ridge National Laboratory report. The pertinent results of these experiments are shown in Figure 3. In general, the data are slightly lower than the Stanton and Pannell-Blasius data for round pipe. The maximum deviation of an experimental point is 22 per cent low in the turbulent regime. The data, however, are in good agreement with the calibration run on a 1.1 in. glass pipe. Again it is noted that the maximum deviation occurs when the test fluid is air.

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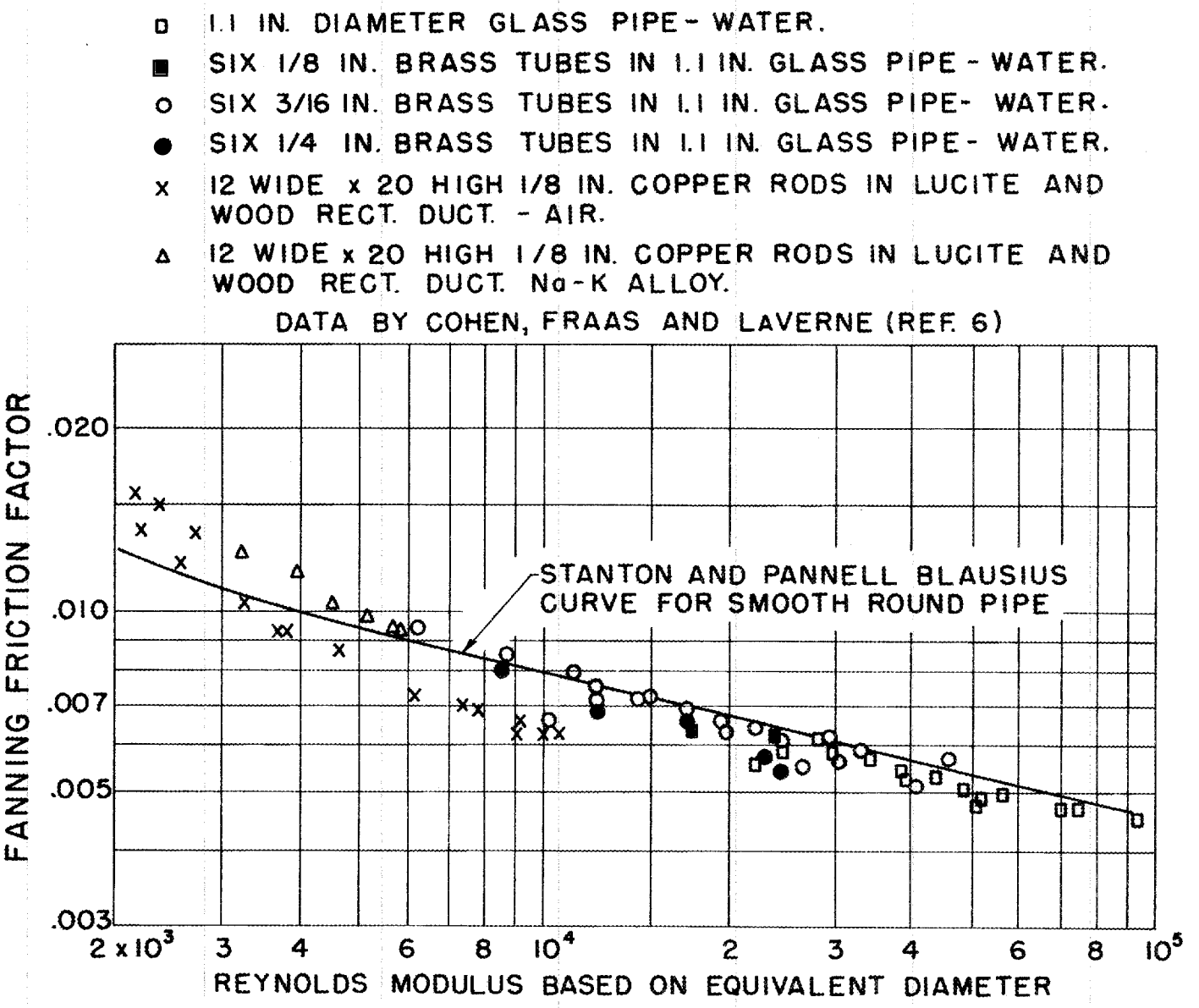

FIG. 3

FRICTION FACTORS FOR ISOTHERMAL FLOW PARALLEL TO BANKS OF TUBES

CONCLUSIONS

In view of the ± 15 per cent agreement with the round pipe data, it can be concluded that for practical purposes the equivalent diameter is a satisfactory correlation parameter for pressure drop due to established turbulent flow in noncircular ducts, annuli and parallel flow systems.

The differences existing for the various geometries indicate that the correlation is only approximate and that the friction factor is not a unique function of the Reynolds modulus as in the case of turbulent flow in pipes. The contributory causes for the difference are probably the complexity of momentum transfer created by the existence of secondary flow in corners of noncircular ducts and the non-linearity of the shear distribution in noncircular geometries and annuli. In view of the relatively good correlation, however, it must be concluded that these effects are small.

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