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Reactor Experimental Engineering Division

FORCED CONVECTION HEAT TRANSFER IN THERMAL ENTRANCE REGIONS

PART II

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L. D. Palmer

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FORCED CONVECTION HEAT TRANSFER IN THERMAL ENTRANCE REGIONS

PART II

Evaluation of the Mathematical Analysis presented in Part I, which pertained to Heat Transfer in the Thermal Entrance Region of Parallel Plates and Pipe Duct Systems, with Uniform Wall Temperatures, containing Flowing Liquid-Metals with Turbulent Velocity Profiles.

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SUMMARY

Mathematical solutions pertaining to convective heat transfer in thermal entrance regions of parallel plates and pipe duct systems with uniform wall temperatures containing flowing liquid-metals with turbulent velocity profiles (previously derived in Part I) are evaluated. The analyses were based on the postulate that all heat was transferred by means of the conduction mechanism and hence are valid only for the low Reynolds modulus range. A method of extending these conduction solutions for the case where the eddy diffusivity is small but not negligible compared to the molecular diffusivity (to higher Reynolds moduli) is proposed.

NOMENCLATURE

English Letters

a,	thermal molecular diffusivity of the fluid, ft^2/hr
b,	one half the distance between parallel plates, ft
B,	parameter in the turbulent velocity expression, ft/hr
c_n,	series coefficients in equation 7
c_p,	heat capacity of the fluid, BTU/lb ^oF
D,	diameter of the pipe, ft
h_{cx},	local unit thermal conductance, BTU/hr ft^2 ^oF
$J_{\frac{1}{m+2}}$	Bessel function of the first kind and order $\frac{1}{m+2}$
k,	thermal conductivity of the fluid, BTU/hr ft^2 (^oF ft)
m,	exponent in the turbulent velocity expression
t,	fluid temperature, ^oF
t_o,	initial fluid and wall temperature, ^oF
t_w,	duct wall temperature, ^oF
U,	mean fluid velocity, ft/hr
u,	fluid velocity at distance, y, from the duct wall, ft/hr
x, y,	Cartesian coordinates, ft

Greek Letters

α_n	roots of equation 6
γ	fluid density, lbs/ft^3
ϵ	thermal eddy diffusivity, ft^2/hr
ν	fluid kinematic viscosity, ft^2/hr
ρ	fluid mass density, lbs hr^2/ft^4

Terms

$$T_o = t_o - t_w$$

$$a = \frac{k}{\gamma c_p}$$

$$Z = \left(\frac{2}{m + 2}\right) \frac{(Y)}{F}^{\frac{m + 2}{2}}$$

$$Z_1 = \left(\frac{2}{m + 2}\right) \frac{1}{F}$$

$$F^2 = \frac{a}{bB}$$

$$z = \frac{Z}{Z_1}$$

Dimensionless Moduli

$$Nu_x = \frac{h_{cx}D}{k} \text{ or } \frac{h_{cx}\ 2b}{k}$$

$$Pr = \frac{\gamma c_p \nu}{k}$$

$$Re = \frac{2bU}{\nu} \text{ or } \frac{UD}{\nu}$$

$$Pe = Re \cdot Pr$$

$$X = \frac{x}{b}$$

$$Y = \frac{y}{b}$$

RECAPITULATION

A forced convection heat transfer analysis pertaining to thermal entrance regions in liquid metal systems was presented in Part I (reference 1). The analysis was based on the following postulates:

1. The established turbulent velocity profile is represented by the power law, $u = B\left(\frac{y}{b}\right)^m$.
2. The initial fluid and wall temperatures are t_o.
3. The uniform wall temperature for $x > o$ is t_w.
4. The thermal eddy diffusivity is small compared to the thermal molecular diffusivity.
5. Longitudinal heat conduction is small compared to transverse heat conduction and is neglected.
6. Fluid properties are invariant with temperature.

The fourth postulate implies that the thermal solution pertains to systems characterized by low Prandtl's moduli and low or intermediate Reynolds' moduli; the specific ranges of these moduli are presented in the Discussion section.

The differential and boundary equations for the parallel plates system are

$$B\left(\frac{y}{b}\right)^m \frac{\partial t}{\partial x} = a \frac{\partial^2 t}{\partial y^2} \tag{1}$$

$$t(x,o) = t_w \tag{2}$$

$$\frac{\partial t}{\partial y}(x,b) = o \tag{3}$$

$$t(o,y) = t_o \tag{4}$$

The resulting thermal solution in terms of the local Nusselt's modulus and Reynolds' and Prandtl's moduli is

$$Nu_x = \frac{2^{\frac{m+1}{m+2}}}{(m+1)\Gamma\left(\frac{1}{m+2}+1\right)} \frac{\sum_{n=1}^{\infty} c_n e^{-\frac{(m+2)^2}{m+1}\frac{\alpha_n^2}{\frac{Pe\ 2b}{x}}} \alpha_n^{\frac{1}{m+2}}}{\int_o^1 Y^{m+\frac{1}{2}} \sum_{n=1}^{\infty} c_n e^{-\frac{(m+2)^2}{m+1}\frac{\alpha_n^2}{\frac{Pe\ 2b}{x}}} J_{\frac{1}{m+2}}\left(\alpha_n Y^{\frac{m+2}{2}}\right) dY} \tag{5}$$

where α_n are the roots of[1]

$$\frac{1}{m+2} J_{\frac{1}{m+2}}(\alpha_n) + \alpha_n J_{\frac{1}{m+2}}(\alpha_n) = o \tag{6}$$

$$\text{and } c_n = 2T_o \frac{\int_o^1 z^{\frac{m+1}{m+2}} J_{\frac{1}{m+2}}(\alpha_n z)\,dz}{J^2_{\frac{1}{m+2}}(\alpha_n)} \tag{7}$$

where $z = \frac{Z}{Z_1}$

---

[1] Note that equation (6) was obtained from equation (18) in reference 1 by making the change of variable, $\alpha_n = Z_1 \beta_n$.

Convective heat transfer in the initial portion of a circular pipe entrance region may be obtained by utilizing the parallel plates solution modified by the pipe velocity profile. This approximation is quite good because in the initial portion of the entrance region the flow annuli near the pipe wall are the important heat transfer layers and may be treated as the flow layers of a parallel plates system. The solution for the initial portion of a pipe entrance region is

$$Nu_x = \frac{2\left(\frac{m+2}{2(m+1)}\right)^{\frac{1}{m+2}}}{(m+1)(m+2)\Gamma\left(\frac{1}{m+2}+1\right)} \frac{\sum_{n=1}^{\infty} c_n e^{-\frac{2(m+2)}{m+1}\frac{\alpha_n^2}{\frac{PeD}{x}}} \alpha_n^{\frac{1}{m+2}}}{\int_0^1 \left(Y^{m+\frac{3}{2}} - Y^{m+\frac{1}{2}}\right) \sum_{n=1}^{\infty} c_n e^{-\frac{2(m+2)}{(m+1)}\frac{\alpha_n^2}{\frac{PeD}{x}}} J_{\frac{1}{m+2}}\left(\alpha_n Y^{\frac{m+2}{2}}\right) dY} \tag{8}$$

The eigenvalues and series coefficients in equation (8) are defined by equations (6) and (7).

ASYMPTOTIC SOLUTIONS

For large values of $\frac{PeD}{x}$ the two series solutions (5) and (8) converge very slowly. Asymptotic solutions have been derived which greatly simplify the evaluation in that region. The boundary conditions for the asymptotic case are

$$t(x,o) = t_w \tag{9}$$

$$\lim_{x \to \infty} t(x,y) = t_w \tag{10}$$

$$t(o,y) = t_o \tag{11}$$

$$\lim_{y \to \infty} t(x,y) = t_o \tag{12}$$

The differential equation 1 may be solved by making the change of variable

$$S = c \frac{y}{x^p} \tag{13}$$

where p and c are constants to be determined. Upon substituting equation (13) into equation (1), the following total differential equation results

$$\frac{d^2t}{dS^2} = - S^{(m + 1)} \frac{dt}{dS} \tag{14}$$

where $p = \frac{1}{m + 2}$ and $c = \sqrt[m + 2]{\frac{B}{(m + 2)a\ b^m}}$

The solution of (14) for the boundary equations (9) to (12) is

$$\frac{t_w - t}{t_w - t_o} = \frac{\int_o^w e^{- w^{m + 2}} dw}{\int_o^\infty e^{- w^{m + 2}} dw} = \frac{\int_o^w e^{-w^{m + 2}} dw}{\Gamma\left(\frac{1}{m + 2} + 1\right)} \tag{15}$$

where $$w = \sqrt[m+2]{\frac{B}{(m+2)^2\, a\, b^m x}} \cdot y \tag{16}$$

It can be shown that for the parallel plates system, the Nusselt's modulus is equal to[2]

$$Nu_x = \frac{h_{cx}\, 2b}{k} = \frac{1}{\Gamma\left(\frac{1}{m+2} + 1\right)} \sqrt[m+2]{\frac{2^m\,(m+1)}{(m+2)^2}\;\frac{Pe\;2b}{x}} \tag{17}$$

and for the pipe system[3]

$$Nu_x = \frac{h_{cx}\, D}{x} = \frac{1}{\Gamma\left(\frac{1}{m+2} + 1\right)} \sqrt[m+2]{\frac{(m+1)}{2^{1-m}\,(m+2)}\;\frac{PeD}{x}} \tag{18}$$

---

[2] Note that the significant dimension chosen for the moduli in the parallel plates system is 2b not 4b.

[3] The difference in these two solutions lies in the fact that for a parallel plates system $B = (m + 1)U$, whereas for the pipe system $B = \frac{(m+1)(m+2)}{2} U$

EVALUATION OF SOLUTIONS

The exponent 'm' in the power velocity expression is somewhat dependent on Reynolds modulus. For the Reynolds' modulus range involved, a mean value of one seventh was choosen[4]. Values of the fractional Bessel functions were evaluated by interpolation and also with the aid of Legrange's formula (reference 2) and are graphed in Figure 1. Derivatives of the Bessel functions were obtained by the derivative relations and are also graphed in Figure 1. The eigenvalues, α_n, and series coefficients, c_n, were determined graphically and are presented in Table I. It was then possible to evaluate the parallel plates and pipe solutions presented above; the resulting Nusselt modulus solutions for the turbulent velocity profile are graphed in Figures 2 and 3. The solutions for the parabolic velocity profile and the uniform velocity profile have also been graphed for the purpose of comparison.

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[4] Note that these solutions are not sensitive to the exponent variation that exists.

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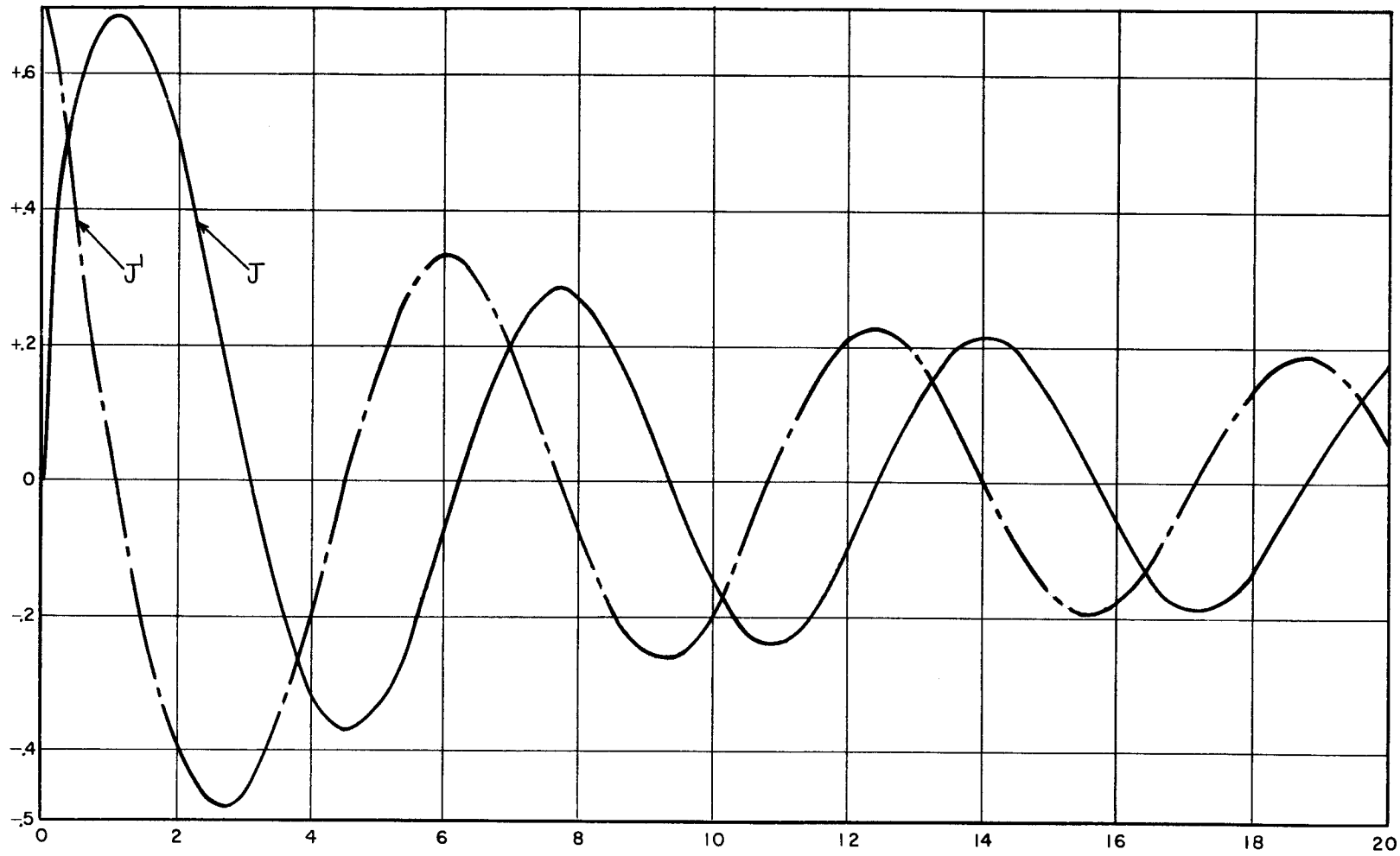

Fig. 1 The Bessel Function $J_{0.466}$ and Its Derivative

TABLE I

n	α_n	c_n
1	1.51	1.9485
2	4.68	1.1981
3	7.79	.8493
4	10.93	.7986
5	14.05	.5685
6	17.23	.5854
7	20.38	.4703
8	23.52	.5130
9	26.66	.5138
10	29.80	.5023
11	32.94	.4042
12	36.08	.3333

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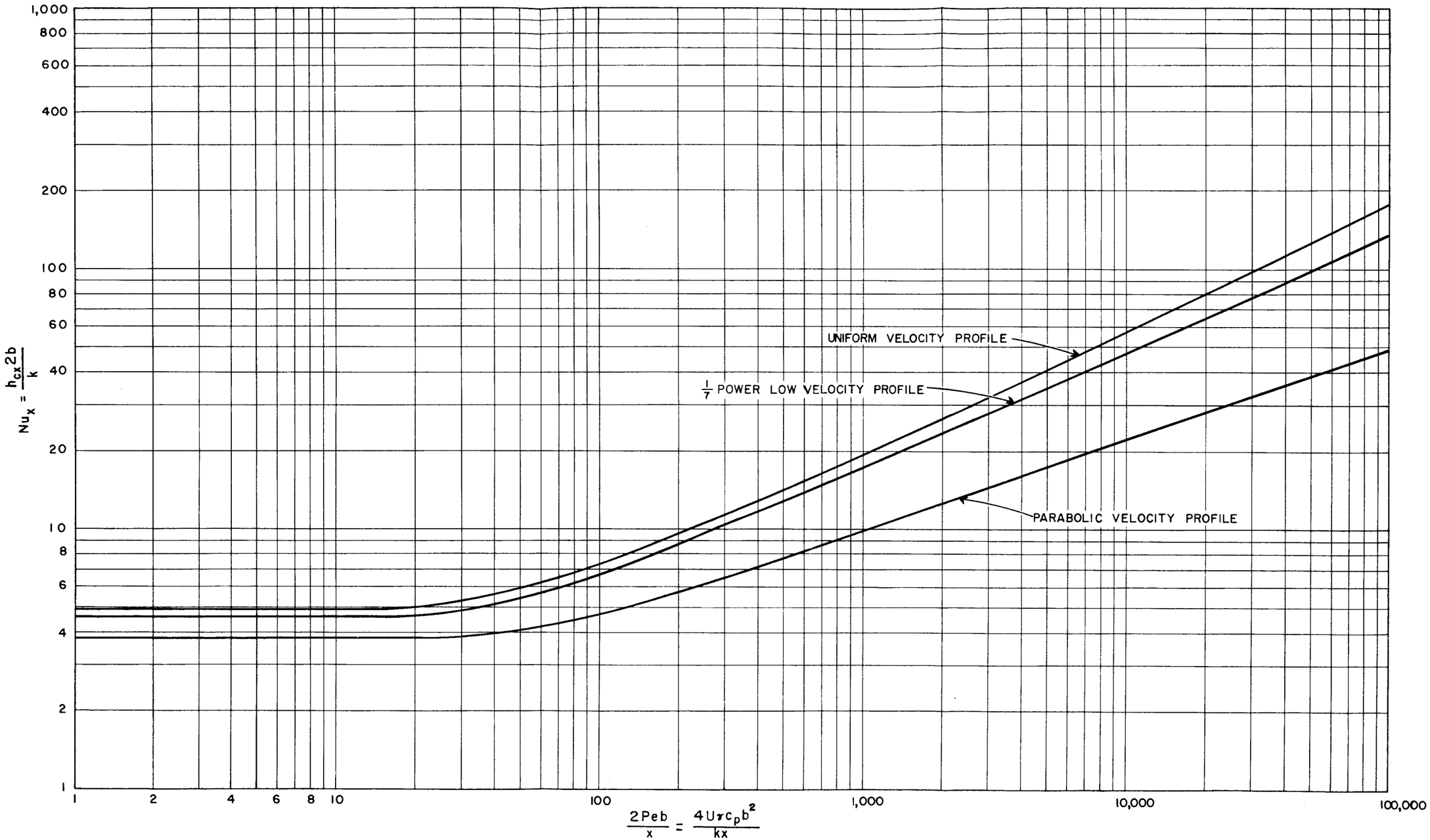

Fig. 2 Conduction Solutions for Three Different Fluid Velocity Profiles Between Parallel Plates with a Uniform Wall Temperature Entrance Region

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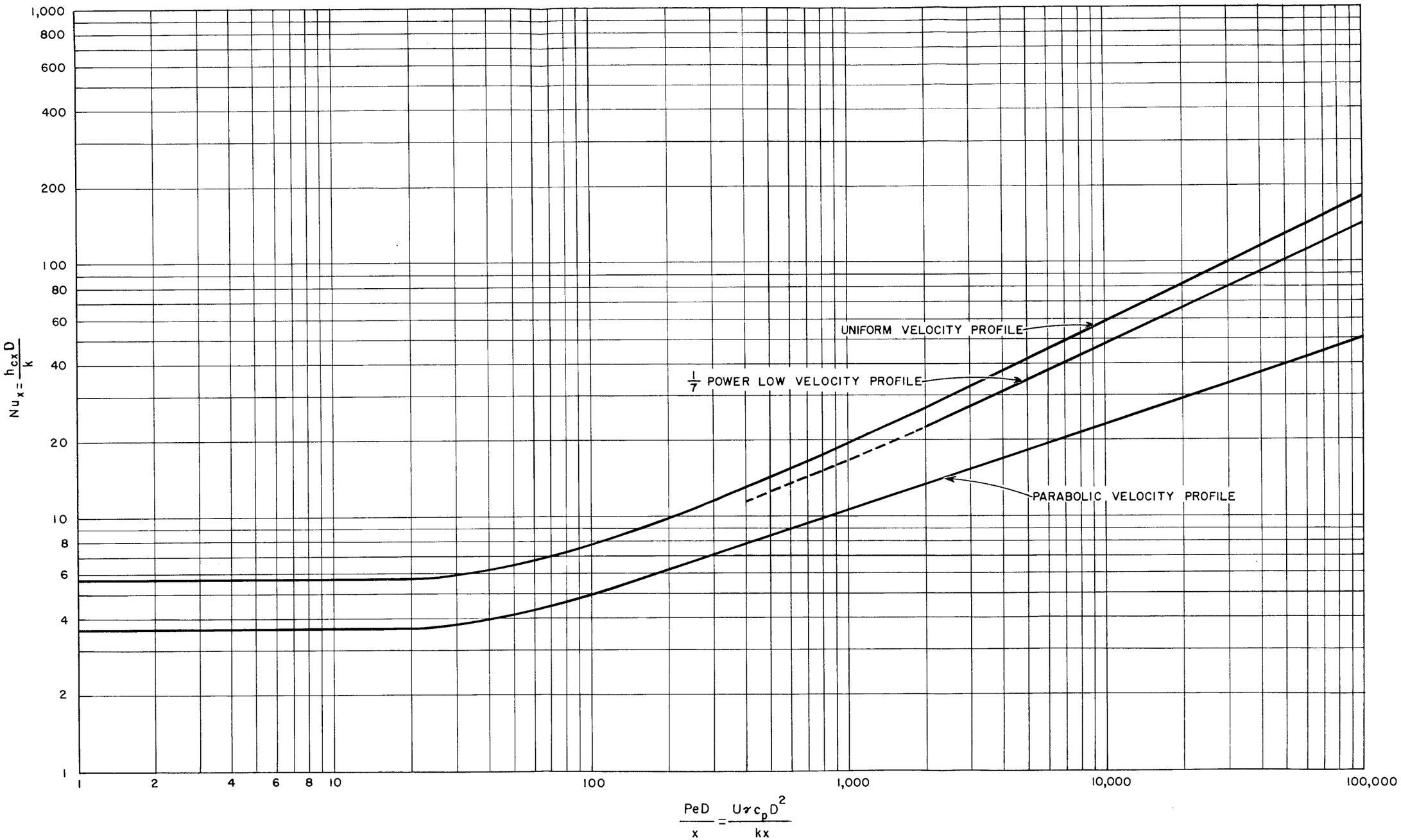

Fig. 3 Conduction Solutions For Three Different Fluid Velocity Profiles in a Pipe with a Uniform Wall Temperature Entrance Region

DISCUSSION

The analyses which have been presented pertain to systems where all heat is transferred via conduction. This is very nearly the case when low Prandtl's and Reynolds' moduli are involved. It is possible to use conduction solutions for cases where the eddy diffusivity is small but not negligible compared to the molecular diffusivity. If the mean $\frac{\epsilon}{\nu}$ ratio in a duct system is as much as, say 20 percent, of the $\frac{a}{\nu}$ or $\frac{1}{Pr}$ ratio it is possible to use an equivalent diffusivity[5] in the computation (see Figure 4); under this condition the error in the heat transfer prediction would be only a fraction of the twenty percent. Figure 5 shows the useful Reynolds'-Prandtl's modulus range of the solutions under consideration. For each Prandtl's modulus there is plotted a Reynolds modulus below which the mean $\frac{\epsilon}{\nu}$ is less than 20 percent of $\frac{a}{\nu}$ or $\frac{1}{Pr}$. The significant dimension in the Reynolds' moduli given in Figures (4) and (5) is D for a pipe system and 4b for a parallel plates system.

Note that the one seventh power law solutions fall closer to the uniform velocity solutions than the parabolic velocity solutions. The greatest difference between the three types of solutions occurs at large values of $\frac{PeD}{x}$ as expected. The seventh power law solution for the pipe system was not graphed for low values of $\frac{PeD}{x}$ because equation (8) is not valid there.

An average Nusselts modulus in the interval $o < x < L$ can be obtained by performing a simple integration. For example the average Nusselts modulus for a parallel plates system is

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[5] The equivalent diffusivity would be equal to the molecular diffusivity plus the mean eddy value for the system.

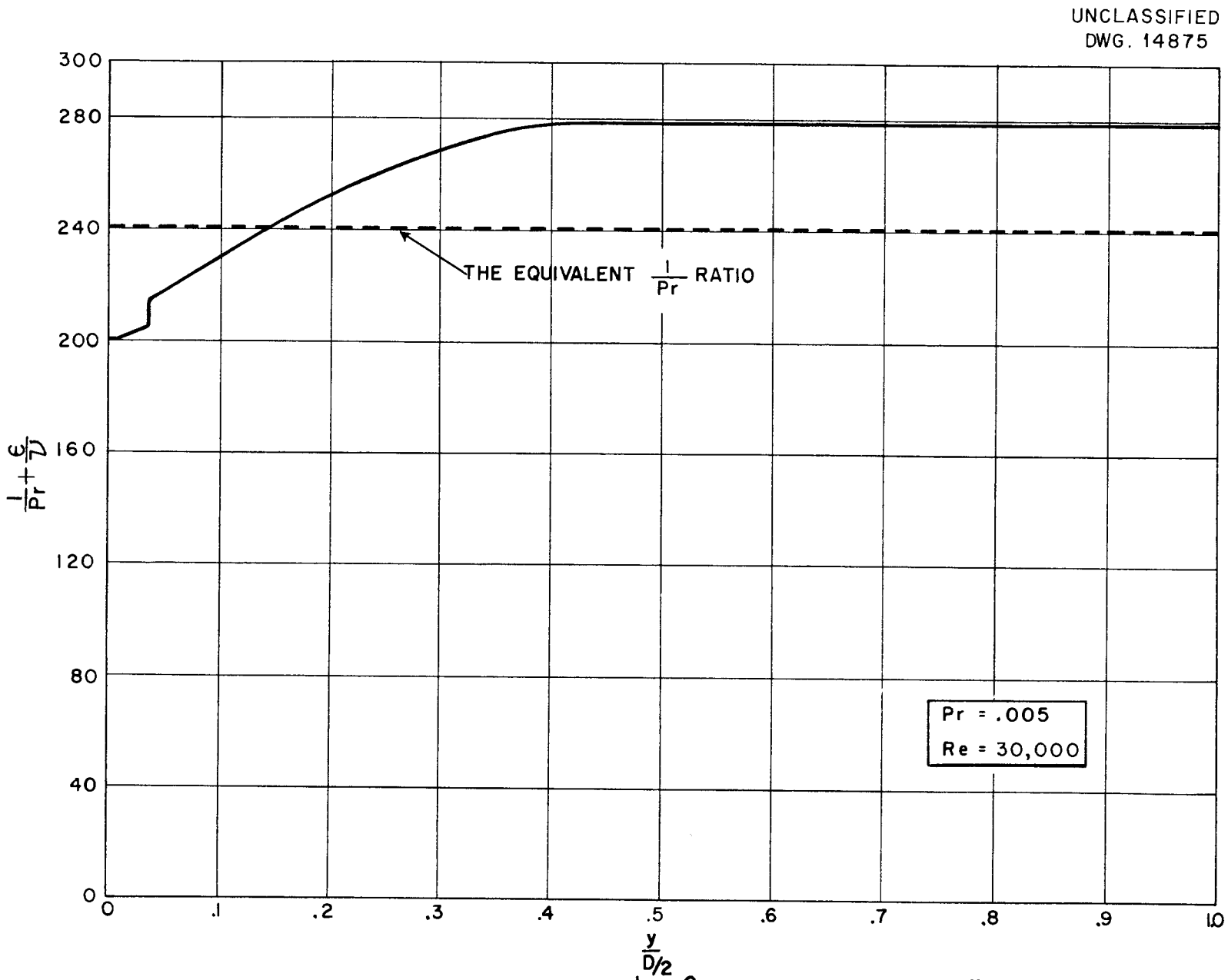

Fig. 4 The Quantity $\frac{1}{Pr}+\frac{\epsilon}{\nu}$ as a Function $\frac{y}{D/2}$

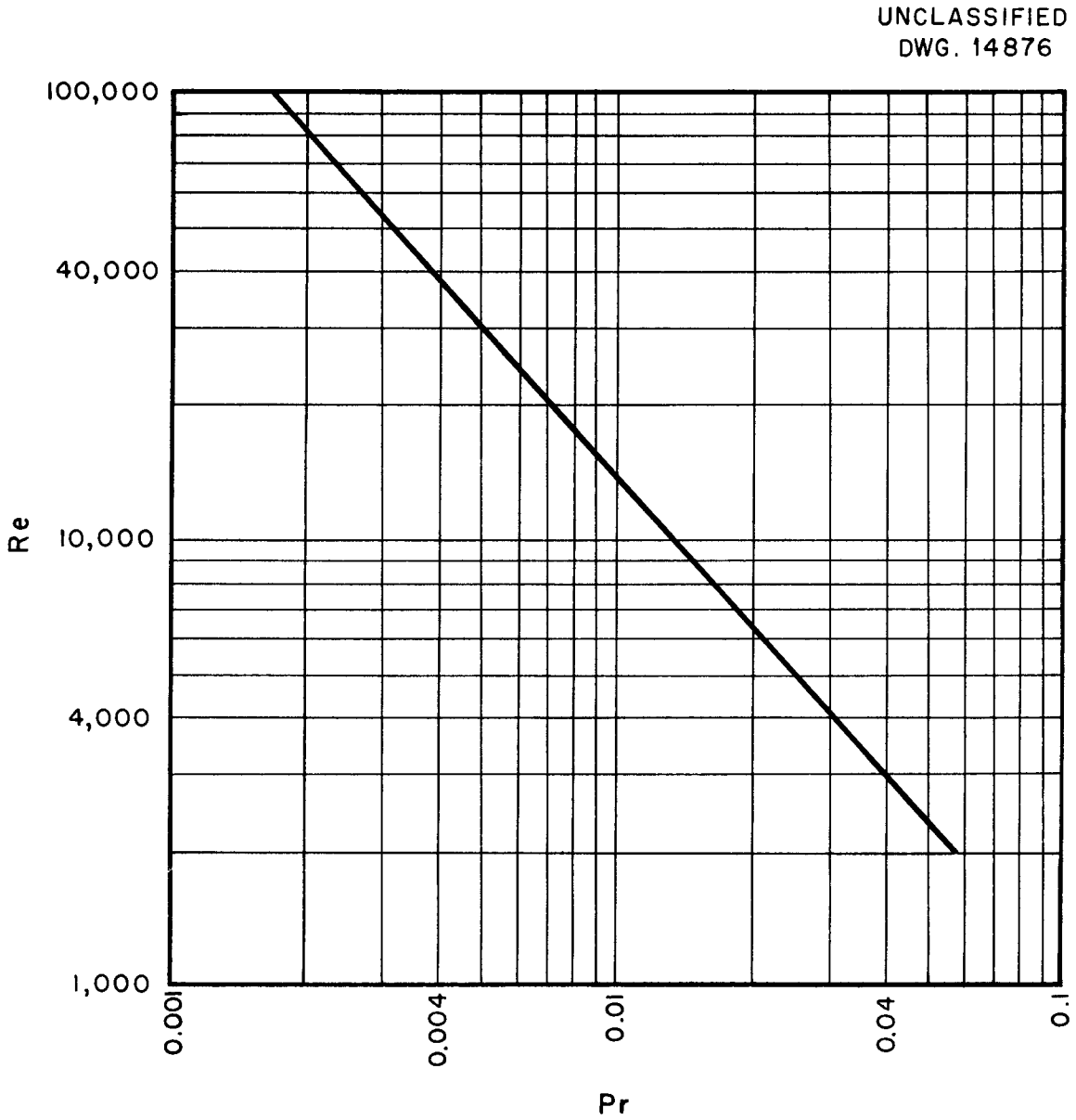

Fig. 5 Useful Reynolds'-Prandtl's Modulus Range When Using an Equivalent Diffusivity

$$Nu_{ave} = \frac{2b}{L} \int_{0}^{\frac{L}{2b}} Nu_{\left(\frac{x}{2b}\right)} \, d\left(\frac{x}{2b}\right)$$

Several other thermal entrance solutions for liquid metal systems have been derived and are to be presented in subsequent reports. One analysis differs from the one presented here in that the wall temperature varies linearly with axial distance. Other analyses pertain to high Reynolds modulus systems where the effect of the radial eddy diffusivity variations must be included; uniform wall temperature and uniform wall heat flux cases have been studied.

Experimental liquid-metal heat transfer investigations are currently being conducted by Messrs. W. B. Harrison and H. C. Claiborne which are expected to yield pertinent information on entrance heat transfer. Mr. Harrison has also aided in the evaluation of the solutions presented here.

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