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A DETERMINATION OF THE DIFFUSION LENGTH OF THERMAL NEUTRONS IN THE X-10 STANDARD GRAPHITE PILE

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DATE ISSUED

JUL 30 1952

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In connection with the restandardization of the Oak Ridge Standard Pile, it seemed of interest to measure the diffusion length of thermal neutrons in the pile. For this purpose two sets of experimental data were taken.

In the first set, a 1/2 gm RaBe neutron source was placed on the axis of the pile. Indium foils, about 5.5 mils thick and 4 x 6.35 cm in area, were irradiated on the axis of the pile at various distances from the source both in aluminum and 30-mil cadmium covers. The activated foils were placed inside brass cylinders and were counted on two glass-walled Geiger tubes. Both the side of the foil toward the source and the side away from the source during exposure were placed next to the Geiger counter and counted. The average of the two saturated activities thus obtained was used in the calculations. In each case the difference between the aluminum-covered and cadmium-covered saturated activities was used as the saturated activity for the point in question. Points at distances from 74 cm to 135 cm from the neutron source were used in calculating the diffusion length.

In the second set of measurements, two 1/2 gm RaBe sources were placed on a line perpendicular to the axis of the pile, each source being 1/4 of the width of the pile (the pile is square in cross section) from the axis. Foils were again exposed on the axis of the pile and counted as described in the first set of measurements.

Assuming that Fermi age theory[1] and first-order diffusion theory[2] adequately describe the transport of fast and thermal neutrons, respectively, in graphite, the equation representing the slowing-down density $q(\bar{r}, t)$ at age t

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(1) Marshak, R. E., Rev. Mod. Phys. 19, 3, 214-216, (1947).
(2) Marshak, Brooks and Hurwitz, Nucleonics, May-August, (1949).

(the number of neutrons per unit volume at $\bar{r}$ passing the age t per unit time) is

$$\nabla^2 q(\bar{r}, t) = \frac{\partial q}{\partial t} - S(\bar{r}, t) \tag{1}$$

and the thermal neutron flux $\phi(\bar{r})$ is given by

$$\nabla^2 \phi(\bar{r}) - \frac{1}{L^2}\phi(\bar{r}) = -\frac{3}{\lambda_t} q(\bar{r}, t_0) \tag{2}$$

where $S(\bar{r}, t)$ is the source term representing the distribution of fast neutron sources throughout the medium, t_0 is the age corresponding to thermal energy, L and λ_t are the diffusion length and transport mean free path, respectively, of thermal neutrons in graphite. The age t is given by

$$t = \int_E^{E_0} \frac{\lambda^2(E)}{3\xi[1-\overline{\cos\theta}]} \frac{dE}{E} \tag{3}$$

where E is the neutron energy, E_0 is the initial energy, ξ is the average logarithmic energy loss per collision in graphite, $\overline{\cos\theta}$ is the average cosine of the angle of deflection in one collision, and $\lambda(E)$ is the scattering mean free path in graphite.

We assume that both ϕ and q vanish at a distance of $0.71\,\lambda_t$ beyond the actual boundaries of the graphite pile[3]. The pile is square in cross section with an effective side a of 165.9 cm, extends an effective distance h of 198.1 cm above the source plane and is assumed to extend to infinity below the source plane. The origin of coordinates is taken at a top extrapolated corner of the pile with the z axis pointing downward so that q and ϕ must vanish on the planes x = 0, a; y = 0, a; $z = 0, \infty$.

(3) Marshak, R. E., Rev. Mod. Phys. 19, 3, 218 (1947).

We use the Fourier Transform to solve this problem[4]. Equation (1) is multiplied by $\sqrt{\frac{2}{\pi}} \sin \xi z \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a}$

and integrated through the volume of the pile, i.e.,

$$\sqrt{\frac{2}{\pi}} \int_0^a dx \int_0^a dy \int_0^\infty dz \sin \xi z \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a}$$

$$\times \left[\nabla^2 q(x,y,z,t) - \frac{\partial q}{\partial t} + S(x,y,z,t) \right] = 0$$

and gives

$$\frac{d\bar{q}}{dt} + \left(\xi^2 + \frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{a^2} \right) \bar{q}(\xi, m, n, t) = \bar{S}(\xi, m, n, t) \quad (4)$$

where

$$\bar{q}(\xi, m, n, t) = \sqrt{\frac{2}{\pi}} \int_0^a dx \int_0^a dy \int_0^\infty dz \; q(x,y,z,t) \sin \xi z \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a}$$

$$\bar{S}(\xi, m, n, t) = \sqrt{\frac{2}{\pi}} \int_0^a dx \int_0^a dy \int_0^\infty dz \; S(x,y,z,t) \sin \xi z \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a}.$$

Equation (4) may be solved immediately to give

$$\bar{q}(\xi, m, n, t) = \int_0^t \bar{S}(\xi, m, n, \tau) \exp\left\{ -\left(\xi^2 + \frac{\pi^2 m^2}{a^2} + \frac{\pi^2 n^2}{a^2} \right)(t - \tau) \right\} d\tau \quad (5)$$

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(4) Sneddon, I. N., Fourier Transforms, McGraw-Hill (1951).

Performing a similar transformation on equation (2) we find

$$\bar{\phi}(k,m,n) = \frac{3}{\lambda_t} \frac{\int_0^{t_0} \bar{S}(k,m,n,\tau) \exp\left\{-\left(k^2+\frac{\pi^2 m^2}{a^2}+\frac{\pi^2 n^2}{a^2}\right)(t_0-\tau)\right\} d\tau}{\left(k^2+\frac{\pi^2 m^2}{a^2}+\frac{\pi^2 n^2}{a^2}+\frac{1}{L^2}\right)} \tag{6}$$

Now we take the source term to be $\delta\left(x-\frac{a}{2}\right)\delta\left(y-\frac{a}{2}\right)\delta(z-h)$ and suppose that the energy spectrum of the source may be represented by a finite number of discrete lines of strengths f_i and ages to thermal energy of $t_i = t_0 - \tau_i$ $(i = 1, 2, \cdots p)$. Then the integral becomes a sum and

$$\bar{\phi} = \frac{3}{\lambda_t}\sqrt{\frac{2}{\pi}} \sum_{i=1}^{p} f_i \exp\left\{-\left(k^2+\frac{\pi^2 m^2}{a^2}+\frac{\pi^2 n^2}{a^2}\right)t_i\right\} \frac{\sin kh \sin\frac{m\pi}{2}\sin\frac{n\pi}{2}}{\left(k^2+\frac{\pi^2 m^2}{a^2}+\frac{\pi^2 n^2}{a^2}+\frac{1}{L^2}\right)} \tag{7}$$

and by the inversion theorem

$$\phi = \frac{3}{\lambda_t}\frac{2}{\pi}\frac{4}{a^2}\sum_{i=1}^{p}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_i \sin\frac{m\pi}{2}\sin\frac{n\pi}{2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{a}$$
$$\times \int_0^\infty dk \sin kh \sin kz \frac{\exp\left\{-\left(k^2+\frac{\pi^2 m^2}{a^2}+\frac{\pi^2 n^2}{a^2}\right)t_i\right\}}{\left(k^2+\frac{\pi^2 m^2}{a^2}+\frac{\pi^2 n^2}{a^2}+\frac{1}{L^2}\right)} . \tag{8}$$

Performing the inverse Fourier transformation in the z coordinate by using the convolution theorem for the Fourier sine transformation[5] we find

$$\phi = \frac{3}{\lambda_t}\frac{1}{a^2}\frac{1}{\sqrt{\pi}}\sum_{i=1}^{p}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{f_i}{\sqrt{t_i}} \exp\left\{-(m^2+n^2)\frac{\pi^2 t_i}{a^2}\right\}\sin\frac{m\pi}{2}\sin\frac{m\pi x}{a}\sin\frac{n\pi}{2}\sin\frac{n\pi y}{a}$$
$$\times b_{mn}\int_0^\infty\left[e^{-(\eta-h)^2/4t_i} - e^{-(\eta+h)^2/4t_i}\right]\left[e^{-|z-\eta|/b_{mn}} - e^{-(z+\eta)/b_{mn}}\right]d\eta \tag{9}$$

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(5) Ibid, p. 24.

Carrying out the indicated integration, writing $t_i = \frac{r_i^2}{4}$, and moving the origin of z so as to coincide with the source plane and so that the boundary is at z = h, we find eventually

$$\phi = \frac{3}{\lambda_x}\frac{1}{a^2}\sum_{i=1}^{p}\sum_{m,n} f_i\, e^{r_i^2/4L^2} \sin\frac{m\pi}{2}\sin\frac{m\pi x}{a}\sin\frac{n\pi}{2}\sin\frac{n\pi y}{a}\, b_{mn}$$

$$\times\left\{e^{-z/b_{mn}}\operatorname{erfc}\left(-\frac{z}{r_i}+\frac{r_i}{2b_{mn}}\right)+e^{z/b_{mn}}\operatorname{erfc}\left(\frac{z}{r_i}+\frac{r_i}{2b_{mn}}\right)\right. \tag{10}$$

$$\left.-e^{-(2h-z)/b_{mn}}\operatorname{erfc}\left(-\frac{2h-z}{r_i}+\frac{r_i}{2b_{mn}}\right)-e^{(2h-z)/b_{mn}}\operatorname{erfc}\left(\frac{2h-z}{r_i}+\frac{r_i}{2b_{mn}}\right)\right\}$$

where $\frac{1}{b_{mn}^2} = (m^2+n^2)\frac{\pi^2}{a^2}+\frac{1}{L^2}$, $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-w^2}dw$

It was assumed, following Fermi and Anderson[6], that the energy spectrum of a RaBe source may be represented by three discrete energies having corresponding gaussian ranges in graphite to indium resonance energy (1.44 ev) of 28, 40, and 60 cm. The ranges, r_i, to thermal energy were taken to be 32.5, 43.3, and 62.2 cm, respectively[7].

In order to obtain a value for the diffusion length L, the experimental data on the variation of ϕ with distance along the axis of the pile were corrected by an iterative process to an exponential variation having the form of the first harmonic term, $e^{-z/b_{11}}$. A trial value of b_{11} (and consequently L) was obtained by fitting the experimental points with an exponential. Each point was then

(6) CPA-6 Classified document.
(7) CF-2804 Classified document.

multiplied by a correction factor $C(\zeta, L)$ given by

$$C(\zeta, L) = 2b_{11} e^{-\zeta/b_{11}} \sum_{i=1}^{3} f_i e^{r_i^2/4L^2} \Big/ \Big\{ \sum_{m,n} \sum_{i=1}^{3} f_i e^{r_i^2/4L^2} b_{mn} \Big[e^{-\zeta/b_{mn}} \operatorname{erfc}\Big(-\frac{\zeta}{r_i} + \frac{r_i}{2b_{mn}}\Big)$$

$$+ e^{\zeta/b_{mn}} \operatorname{erfc}\Big(\frac{\zeta}{r_i} + \frac{r_i}{2b_{mn}}\Big) - e^{-(2h-\zeta)/b_{mn}} \operatorname{erfc}\Big(-\frac{2h-\zeta}{r_i} + \frac{r_i}{2b_{mn}}\Big) - e^{2h-\zeta/b_{mn}} \operatorname{erfc}\Big(\frac{2h-\zeta}{r_i} + \frac{r_i}{2b_{mn}}\Big)\Big]\Big\}$$

A new value of b_{11} was obtained from the corrected data and the process was repeated until there was no change in b_{11}. The end correction terms in the denominator (those containing the height, h) were significant only in the first harmonic term.

For the case in which a pair of point sources is located at $x = a/4, \frac{3a}{4}$; $y = a/2$, the correction factor $C(\zeta, L)$ is identical with that for the single source except that the chief harmonics (m, n) = (1, 3), (3, 1), (1, 5), (5, 1), etc., are suppressed. A value of L was obtained using the modified $C(\zeta, L)$ from the double-source data.

The strengths of the three groups were obtained experimentally. Indium resonance activations on the axis of the pile at various distances from the source were obtained from cadmium-covered foil data. Again both sides of the indium foils were counted. A correction factor of 1.14 was applied for the absorption of the resonance neutrons by the 30-mil cadmium covers, and a correction for the self-absorption of the indium foils was made by decreasing the geometrical distances to the source by 6 mm. Both of the above corrections were made following CP-2804.

These data were fitted by the least squares method with the sum of three gaussians having ranges of 28, 40, and 60 cm. No end corrections were made since all experimental points were taken well away from the end of the pile. Following

a previous experiment at Oak Ridge, the least squares equations were weighted so as to give at each distance the same apparent saturated activity, namely about 1/2 of that of the point nearest the source (13 cm).

The results of the mathematical analysis are as follows: The L obtained in the single-source experiment from the experimental data with no corrections was 46.6 cm. Assuming a point source of thermal neutrons and applying the harmonic correction[8] appropriate to this source changed this to a value of 54.3 cm, and taking both the harmonic correction and the source energy spectrum into account as discussed above yielded a final value of L of 49.8 cm. Similarly, in the double-source experiment the corresponding three values of L obtained were 52.3, 53.3, and 49.9 cm, respectively.

A recent measurement of the diffusion length of thermal neutrons in graphite using the "cadmium shutter" technique[9] has been made at Hanford. In the report describing these results, the authors state that the extra nuisance of the cadmium shutter technique is as much trouble as the extra complexity of the fast source equations. We have found that our measurements as described above using these equations lead to consistent results and that the present method is entirely feasible for the measurement of diffusion length.

We should like to express our appreciation to the members of the Mathematics Panel, C. L. Perry, Kathryn Graw, Nancy Given, and Virginia Carlock, who were responsible for the numerical calculations.

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(8) H W 21793, p. 16 (Classified Document).
(9) H W 21793