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# A CALCULATION OF THE PROTON-ELECTRON ANGULAR CORRELATION FOR NEUTRON DECAY

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# ERRATA SHEET FOR ORNL-1444

## A CALCULATION OF THE PROTON-ELECTRON ANGULAR CORRELATION FOR NEUTRON DECAY

H. L. Reynolds L. C. Biedenharn, Jr.
D. B. Beard

The errors corrected here were typographical and did not affect the results of the paper.

| PAGE NO | LINE NO | ERRATA |
|---|---|---|
| 3 | 4, 6, 8 | *for E read* $\epsilon$ |
| 4 | Eq. 4 | should *read* $$\frac{d\theta_1}{d\theta_2} = \frac{\sin\theta_2}{\sin\theta_1}\left\{2\epsilon\cos\theta_2 \pm \frac{1+\epsilon^2\cos 2\theta_2}{\sqrt{1-\epsilon^2\sin^2\theta_2}} + \left[\sin\theta_2 \pm \frac{\epsilon\cos\theta_2\sin\theta_2}{\sqrt{1-\epsilon^2\sin^2\theta_2}}\right]\frac{d\epsilon}{d\theta_2}\right\}$$ |
| 4 | Eq. 5 | should *read* $$\frac{d\epsilon}{d\theta_2} = -\epsilon(\epsilon^2-1)\,\frac{\epsilon\sin\theta_2\sqrt{1-\epsilon^2\sin^2\theta_2} \pm \epsilon^2\sin\theta_2\cos\theta_2}{\frac{M}{\nu}\sqrt{1-\epsilon^2\sin^2\theta_2}\,(\epsilon\cos\theta_2 \mp \sqrt{1-\epsilon^2\sin^2\theta_2}) \pm (\epsilon^2-1)}$$ |
| 5 | 4, 5 | *for E read* $\epsilon$ |
| 5 | Eq. 6 | should *read* $$\frac{dN}{d\Omega_e\,d\Omega_p} = C\sqrt{E(E+2)}\,(E+1)\,(E_0-E)^2\,dE \cdot \left[1 + \frac{\sqrt{E(E+2)}}{E+1}\,n\left\{\frac{\sqrt{E(E+2)}}{E_0-E}\sin^2\theta_2 \mp \cos\theta_2\sqrt{1-\frac{E(E+2)}{(E_0-E)^2}\sin^2\theta_2}\right\}\right] \cdot \left[\frac{1+\frac{E(E+2)}{(E_0-E)^2}\cos 2\theta_2}{\sqrt{1-\frac{E(E+2)}{(E_0-E)^2}\sin^2\theta_2}} \pm 2\frac{\sqrt{E(E+2)}}{E_0-E}\cos\theta_2\right]$$ |

| PAGE NO. | LINE NO | ERRATA |
|---|---|---|
| 9 | Eq. 10 | should *read* $P(p)\ dp = K[1 + n)p^2 \times (E_t^2 - 1 - p^2)^2/(E_t^2 - p^2)^3$ etc. |
| 12 | 6 | *for* $E$ *read* $\epsilon$ |
| 13 | 8 | *for* $+ e \cos \theta$ *read* $\pm e \cos \theta$ |

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PHYSICS DIVISION

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# A CALCULATION OF THE PROTON-ELECTRON ANGULAR CORRELATION FOR NEUTRON DECAY*

H. L. Reynolds
L. C. Biedenharn, Jr.
D. B. Beard

OAK RIDGE NATIONAL LABORATORY

## Introduction

The various possible interactions in beta decay theory have been shown by Hamilton[1] to result in an electron-neutrino angular correlation of the form $1 + n \frac{v}{c} \cos \theta_1$, where v is the velocity of the electron and $\theta_1$ is the angle between the electron and neutrino momenta. The constant n lies between 1 and -1 depending upon the type of coupling between the lepton and nuclear fields and for pure interactions has the value: 1, 1/3, -1/3, -1 for polar vector (V), tensor (T), axial vector (A), and scalar (S) interactions respectively. The observation of an electron-neutrino correlation would assist in the determination of the type of beta decay coupling.

A practical experiment must, of course, infer the neutrino direction from observations of the recoil momentum of the residual nucleus. Therefore, it is of interest to calculate the angular correlations for the various interactions in terms of experimentally observed quantities. We have carried out such calculations for the case of neutron decay. Since this is the fundamental beta minus decay process, the angular correlation can be interpreted directly in terms of the interaction constants, uncomplicated by the problem of the size of the nuclear matrix elements.

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* This topic was previously discussed briefly in ORNL Physics Division Quarterly Report Number 1164.

The advent of high intensity neutron beams has made feasible experimental studies of the neutron decay[2]. It is hoped that the angular correlation experiment can be carried out in the future. Besides application to such an experiment, the calculations to be described below are of interest in the neutron decay experiment now in progress at this laboratory since the proton-electron correlation enters into the determination of the proton collection efficiency.

## Calculation

The notation we shall use throughout is

$\theta_1$ the angle between the electron and neutrino momenta

$\theta_2$ the angle between the electron and recoil proton momenta

$\vec{e}$ the electron momentum

$\vec{\nu}$ the neutrino momentum

$\vec{p}$ the proton momentum

$$\epsilon = \frac{e}{\nu}$$

$E_0$ the maximum kinetic energy available to the system

$E$ the kinetic energy of the electron

Energy units of $m_e c^2 = 0.511$ mev are used throughout.

The intensity of electrons d N per solid angle of the electron $d\Omega_e$ and solid angle of the neutrino $d\Omega_\nu$ is given as:

$$\frac{d\,N}{d\Omega_e\ d\Omega_\nu} = c\ e^2\ de\ (E_0 - E)^2\ (1 + \beta n \cos\theta_1) \qquad (1)$$

where C is the normalization constant. Relativistic corrections to this expression are of negligible importance for the neutron decay[3].

The introduction of the proton-electron angle $\theta_2$ in place of $\theta_1$ is straighforward. From momentum conservation one determines that

$$\cos\theta_1 = -\epsilon \sin^2\theta_2 \pm \cos\theta_2\sqrt{1-\epsilon^2\sin^2\theta_2} \qquad (2)$$

The two signs in front of the square root in equation 2 are essential. The sign is to be taken as follows:

$E < 1$: $\theta_1$ is a single valued function of $\theta_2$, and the sign of the square root is negative.

$E > 1$: $\theta_1$ is a double valued function of $\theta_2$, and consequently both signs enter.

$E = 1$: here only the positive sign enters.

The double valuedness of $\theta_1$ for $\epsilon > 1$ can be readily seen from the momentum diagrams in figures 1-a and 1-b.

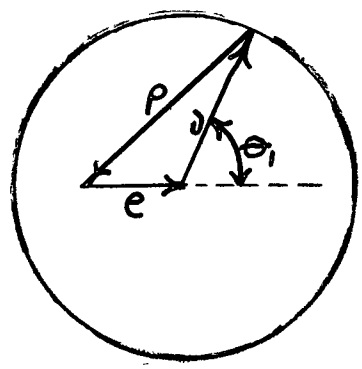


Figure 1-a

Momentum diagram for neutron decay when $\epsilon < 1$

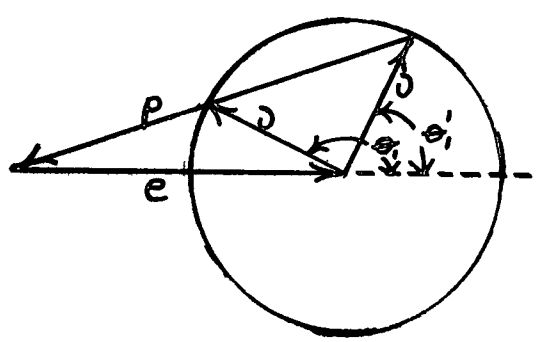


Figure 1-b

Momentum diagram showing both possibilities for neutron decay when $\epsilon > 1$

In summary:

$\epsilon < 1$: use lower sign

$\epsilon = 1$: use upper sign

$\epsilon > 1$: use both signs

To obtain the Jacobian for the transformation from the solid angle involving $\theta_1$ to that of $\theta_2$ we differentiate equation 1 with respect to $\theta_2$ and obtain

$$\frac{d\theta_1}{d\theta_2} = \frac{\sin\theta_2}{\sin\theta_1}\left(2\epsilon\cos\theta_2 \pm \frac{1 \pm \epsilon^2\cos^2\theta_2}{\sqrt{1-\epsilon^2\sin^2\theta_2}} \pm \left[\sin\theta_2 \pm \frac{\epsilon\cos\theta_2\sin\theta_2}{\sqrt{1-\epsilon\sin^2\theta_2}}\right]\frac{\partial\epsilon}{\partial\theta_2}\right) \quad (4)$$

where $\frac{d\epsilon}{d\theta_2}$ is determined from energy-momentum relations to be:

$$\frac{d\epsilon}{d\theta_2} = -\epsilon(\epsilon^2-1)\frac{\epsilon\sin\theta_2 \quad 1 - {}^2\sin^2\theta_2 \pm {}^2\sin\theta_2\cos\theta_2}{\frac{M}{\partial}\sqrt{1-\epsilon\sin^2\theta_2}\,(\epsilon\cos\theta_2 \mp \sqrt{1-\epsilon^2\sin^2\theta_2}) \pm (\epsilon^2-1)} \quad (5)$$

M is the mass of the recoil proton, and the velocity of light is one in our units.

The term given in equation 5 vanishes as $M \to \infty$. We shall use the approximation that $M = \infty$, in the following, since it is demonstrated in the appendix that the resulting error is negligible. Specifically, the values obtained by this approximation differ by not more than 1/2 of 1% from the exact values.

Equation 1 becomes, using equation 4 to change from the number electrons per neutrino solid angle, $d\Omega_\nu$, to the recoil nucleus solid angle, $d\Omega_p$, and assuming that $M = \infty$:

$$\frac{dN}{d\Omega_e\, d\Omega_p} = C\sqrt{E(E+2)}\;(E+1)\,(E_o - E)^2\, dE \;\; X$$

$$\left[1 + \frac{E(E+2)}{E+1} \cdot n\left(\frac{\sqrt{E(E+2)}}{(E_o - E)}\sin^2\theta_2 \mp \cos\theta_1\sqrt{1 - \frac{E(E+2)}{(E_o - E)^2}}\right)\right] X \qquad (6)$$

$$\left[\frac{1 + \frac{E(E+2)}{(E_o - E)^2}}{\sqrt{1 - \frac{E(E+2)}{(E_o - E)^2}}} \mp 2\frac{\sqrt{E(E+2)}}{E_o - E}\cdot\cos\theta_2\right]$$

The lower sign is to be used for $E < 1$, and both sigrs are to be used for $E > 1$.

The total number of electrons of all energies emitted at an angle $\theta_2$ with the direction of the recoil nucleus, $G(\theta_2)$, may be found by intergrating equation 6 over the energy. Not all electron energies less than $E_o$ are possible as is obvious from momentum conservation relations and Figures 1-a and 1-b. To determine the limits of integration we consider the possible values of $\theta_2$. From conservation of momentum:

$$\cos\theta_2 = -\frac{\epsilon + \cos\theta_1}{\sqrt{1 + \epsilon^2 + 2\epsilon\cos\theta_1}} \qquad (7)$$

$$\begin{aligned} \epsilon < 1&: \; 0 \leq \theta_2 \leq \pi \\ \epsilon = 1&: \; \tfrac{\pi}{2} \leq \theta_2 \leq \pi \\ \epsilon > 1&: \; \theta\,\mathrm{min} \leq \theta_2 \leq \pi \end{aligned} \qquad (8)$$

where $\theta_{min}$ is obtained by minimizing $\cos\theta_2$ with respect to $\cos\theta_1$

$$\theta_{min} = \cos^{-1}\left(-\sqrt{1 - 1/\epsilon^2}\right) \text{ and, therefore, } \epsilon \leq \frac{1}{\sin\theta_2}$$

$$\text{For } \epsilon = 1,\ E_1 = \frac{E_o^2}{2(E_o + 1)};\ \text{for } \epsilon = \frac{1}{\sin\theta_2},\ E_m = \tan^2\theta_2\left(1 + \frac{E_o}{\sin^2\theta_2} - \sqrt{\frac{E_o^2}{\sin^2\theta_2} + 2\frac{E_o}{\sin^2\theta_2} + 1}\right) \qquad (9)$$

Equation (6) was integrated over the variable E numerically. The result $G(\theta_2)$ is the non-normalized probability of observing an electron per unit solid angle at an angle $\theta$ with the proton resulting from the decay of a neutron. The results are plotted in Figure 2 with the values presented in Table 1.

## Discussion

The strong back-to-back correlation shown in Figure 2 is readily seen to be a consequence of conservation of momentum. One notes that all electrons of energies greater than .463 can be emitted only at an angle $\theta_2 > \pi/2$.

Figure 2 suggests that the value of n could be determined by observing the relative number of electrons which occur at two angles. The optimum experimental angles would probably be $180^\circ$ and $145^\circ$. It can be seen that the discrimination for different values of n would be good even for a fairly poor angular resolution.

In such an experiment a low energy cut off would be applied to the protons because the time required for the very slow protons to reach a counter would exceed the practical resolving time which could be used. A practical low energy cut off would probably be of the order of 20 electron volts. We have found with a rough calculation that such a cut off would require a correction of only a few per cent to the curves given in Figure 2.

Table I

Number of Electrons Per Unit Solid Angle $G(\theta_2)$ at Various Angles with Respect to the Recoil Proton

| $\theta_2$ | $G(\theta_2)$ |
|---|---|
| 0° | 0.0987 - 0.0557n |
| 30° | 0.142 - 0.0626n |
| 60° | 0.212 - 0.0881n |
| 90° | 0.448 - 0.152n |
| 120° | 1.349 - 0.269n |
| 140° | 3.515 - 0.194n |
| 160° | 10.85 + 1.47n |
| 170° | 19.57 + 4.72n |
| 180° | 30.27 + 9.85n |

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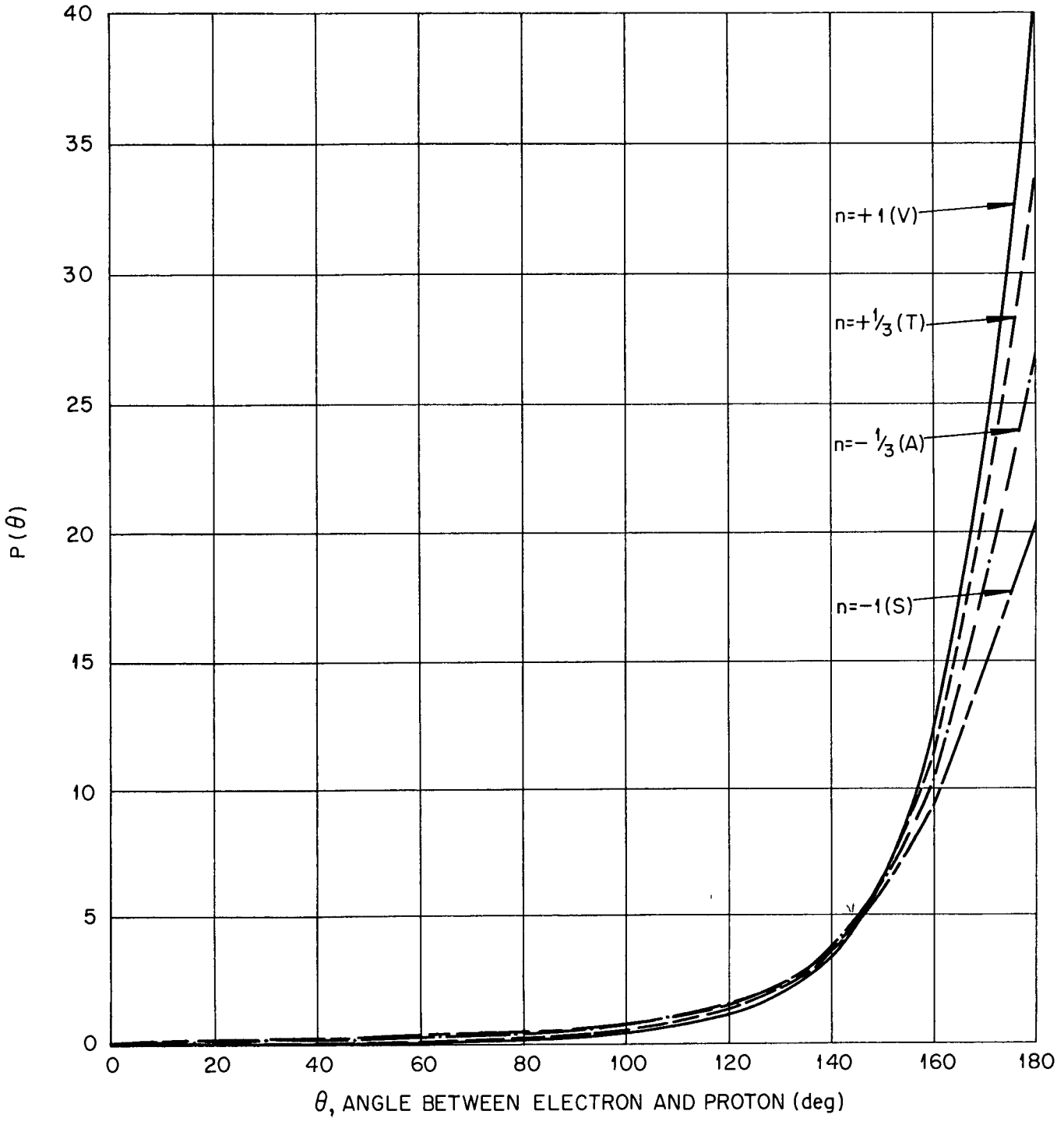


Fig 2 Angular Correlation of Electron and Proton for Neutron Decay

There would also be an experimental low energy cut off applied to the electrons because of the finite thickness of counter windows. The effects of this cut off can be easily determined if required.

If an energy as well as angular selection were placed on the electrons, the results would be much more sensitive to the value of n. However, the experimental problem will be one of obtaining an observable counting rate. Any lowering of counting rate because of additional selection would be impractical.

Information on the type of coupling in beta decay can also be obtained experimentally by observing the recoil proton momentum spectrum.[4] The differential momentum spectrum can be obtained from Equation (1) and can be expressed as:

$$\begin{aligned} P(P)\, dp = K \Big[ & (1 \pm n)\, p^2 \times (E_t^2 - 1 - p) \,/\, (E_t^2 - p^2)^3 \\ & \times \left(p^4 - p^2(4 E_t^2 - 1) \pm 3 E_t^2 (E_t^2 \pm 1)\right) \\ & - 6 n p^2 (E_t^2 - 1 - p^2)^2 \,/\, (E_t^2 - p^2) \Big] dp \end{aligned} \tag{10}$$

where $E_t$ is the total available energy including the electron rest energy. Figure 3 gives a plot of the expected momentum distribution for various values of n. The measurement of this spectrum may provide fewer experimental problems than the measurement of the angular correlation.

We wish to thank Mr. C. Hubbard of the ORNL Mathematics Panel for his assistance with the computations.

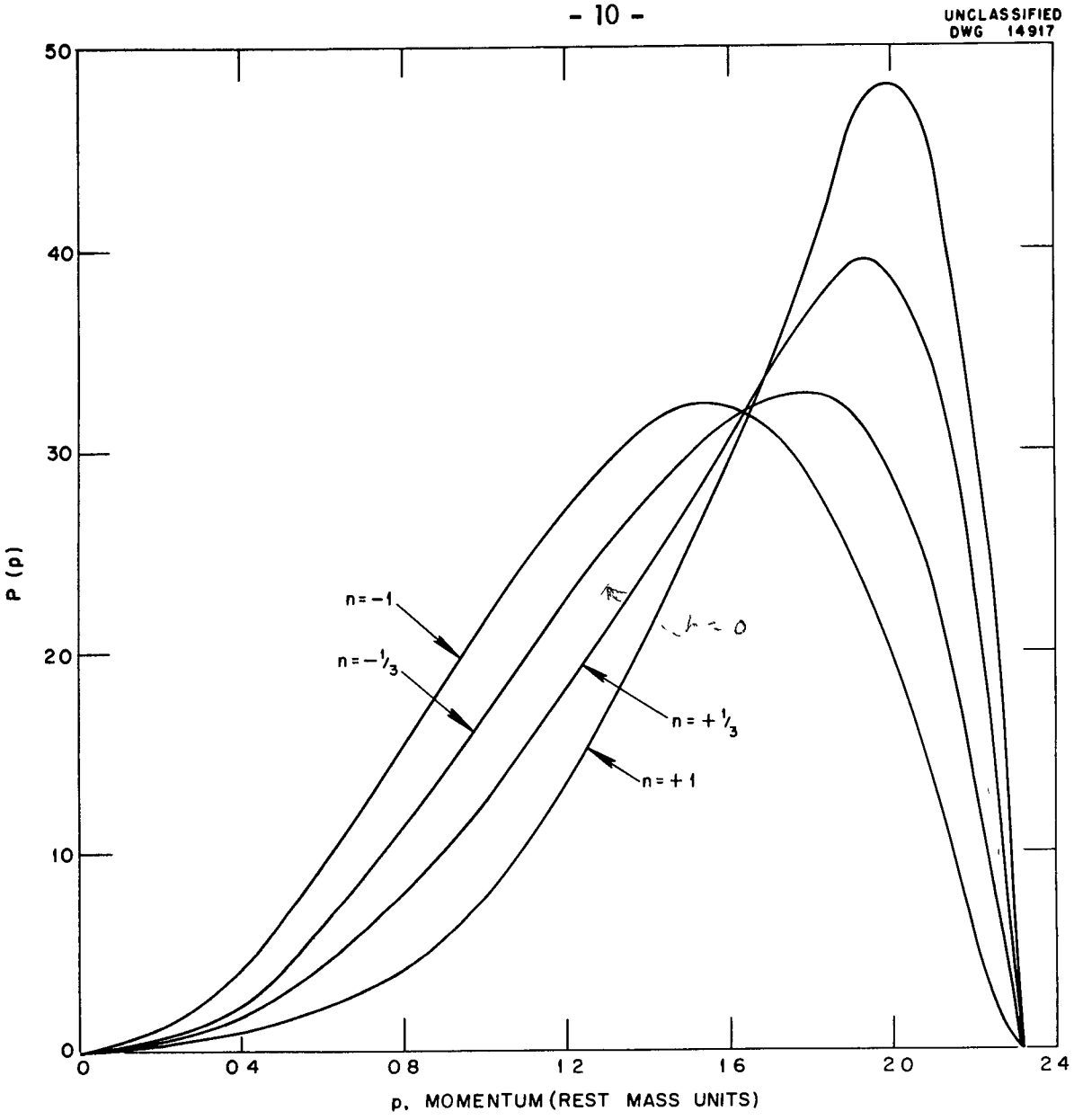


Fig. 3. Differential Momentum Spectrum for Recoil Protons from Neutron Decay

References:

1. D. R. Hamilton, Phys. Rev. 71, 456 (1947)

2. Robson, Phys. Rev. 83, 349, (1951) Snell, Pleasonton, McCord, Phys. Rev. 78, 310, (1950)

3. Kofoed - Hansen, Phil. Mag. 42, 1411, (1951)

4. Kofoed - Hansen, Phys. Rev. 74, 1785, (1948)

# APPENDIX

We have investigated the error which arose in the assumption that the mass of the recoil particle was very large. The Jacobian (Equation 4) can be separated into an approximate term which was used in the previous integration, and an error term involving $\frac{d\epsilon}{d\theta_2}$. The $\frac{d\epsilon}{d\theta_2}$ term is negligible because of the large magnitude of $\frac{M}{\delta}$ except where the coefficient of $\frac{M}{\delta}$ is small as it is at $E = 1$ or $\frac{1}{\sin\theta_2}$. For $\epsilon < 1$, $\epsilon \not\gtrless \frac{1}{\sin\theta_2}$. Since the lower sign is always taken in this region, the coefficient of $\frac{M}{\delta}$ is never small; and the $\frac{d\epsilon}{d\theta_2}$ term can be neglected. When the upper sign is taken, as $\epsilon \rightarrow 1$ for $\epsilon > 1$, we observe the limit of the denominator to be: $\frac{1}{2}\frac{M}{\delta} \not\sim 1$ which is still large. In this case $\frac{d\epsilon}{d\theta_2}$ can be neglected. Finally we examined $\frac{d\theta_1}{d\theta_2}$ as $\epsilon \rightarrow \frac{1}{\sin\theta_2}$. The approximate expression becomes infinite at this point, but the $\frac{d\epsilon}{d\theta_2}$ term causes the whole Jacobian to remain finite. Therefore, $\frac{d\epsilon}{d\theta_2}$ is non-negligible only when

$$\frac{1}{\sin\theta_2} - \epsilon \leq \frac{1}{\frac{(M)^2}{(\delta)^2}\sin\theta_2} = \frac{1}{(2400)^2\sin\theta_2} \tag{11}$$

Although this term affects the Jacobian over an extremely small region of integration, it was not obvious that its contribution to the integral was negligible. Therefore, we investigated its contribution in the interval between the upper limit of integration $\epsilon = \frac{1}{\sin\theta_2}$ and a lower limit expressed in Equation (11). The error term is zero at 180° and 90°. We, therefore, chose an intermediate angle where the integral was evaluated over the small interval given above, using both the approximate and the error terms in the Jacobian.

In principle the calculation can be done by a direct numerical integration. However, it was found that some care had to be taken to avoid spurious infinities. The amount of labor involved was very great due to the large number of digits which had to be used to obtain the desired accuracy.

The denominator of $\frac{d\epsilon}{d\theta^2}$ (Equation 5) can be expressed more simply as:

$(M + \mathcal{D}) \sqrt{1 - (\frac{e}{\mathcal{D}})^2 \sin^2 \theta} + e \cos \theta$ remembering that $\mathcal{D}$ is a function of e. There are two cases to consider:

(1) Using the negative sign the denominator is finite over the whole integration range of e. This case can be treated by usual methods.

(2) The denominator vanishes for the positive choice of sign. This case must be treated separately.

The infinity which arises in the second case is spurious. It occurs because for the positive choice of sign, the variable e is double valued for a given value of $\mathcal{D}$ near the upper limit, with the maximum e occurring exactly at the infinity given above. This fact can be obtained from the energy conservation equation as follows:

$$E + \mathcal{D} + \frac{p^2}{2m} = E_o \tag{12}$$

$$\frac{dE}{d\mathcal{D}} + 1 + \frac{p}{M} \frac{dp}{d\mathcal{D}} = 0$$

Setting $\frac{dE}{d\mathcal{D}} = 0$ and using the value for p derived from momentum conservation, we find that $(M + \mathcal{D}) \sqrt{1 - (\frac{e}{\mathcal{D}})^2 \sin^2 \theta} + e \cos \theta = 0$ which is exactly the condition for the infinity. To eliminate this infinity we introduced the single valued variable $\mathcal{D}$ in place of e for Case 2. The

vanishing denominator was cancelled by the $\frac{de}{d\delta}$ term used in changing the variable. Case 2 could then be integrated to the upper limit $\epsilon = \frac{1}{\sin \theta_2}$.

The integration then proceeded by standard methods. Since the details are of limited interest, we have omitted them from discussion. The specific case treated was at $\theta_2 = 120^\circ$. It was found that the contribution of the error term was less than $\frac{1}{2}$ of 1 per cent. The error term for the angles $140^\circ$, $160^\circ$, and $170^\circ$ was investigated somewhat less precisely and it was found that there was no reason to expect the contribution to exceed $\frac{1}{2}$ of 1 per cent. As previously stated, the error term vanishes at $90^\circ$ and $180^\circ$.

These results show that the neglect of the finite proton mass is well justified.