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PART I

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THE ELEMENTS OF NUCLEAR REACTOR THEORY

PART I

BY: SAMUEL GLASSTONE
MILTON C. EDLUND

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FOREWORD

This book is intended as an introduction to the subject of nuclear reactor theory for the use of physicists, engineers, and others who are being brought into the reactor program for the first time. Because of the varying backgrounds of the readers there is a considerable range in the scope and difficulty of the material. Some readers will consequently wish to omit certain chapters, and this can frequently be done without affecting the fundamental development of the subject.

The present form is a revision of the preliminary draft issued in 1950, that was developed from lectures given by M. C. Edlund at the Oak Ridge School of Reactor Technology at ORNL. The authors wish to take this opportunity to express their indebtedness to the many scientists whose cooperative efforts in connection with the Manhattan Project led to the development of the ideas discussed in this book. Mention may be made in particular of the contributions made by R. F. Christy, E. Fermi, F. L. Friedman, L. W. Nordheim, P. Morrison, G. Placzek, L. Szilard, E. Teller, A. M. Weinberg, J. A. Wheeler, and E. P. Wigner. Special thanks are also due A. M. Weinberg for his helpful advice and his valuable criticism of the preliminary draft.

A final manuscript is in preparation for publication by the Atomic Energy Commission.

Samuel Glasstone
Milton C. Edlund

CHAPTER I

NUCLEAR STRUCTURE AND STABILITY

Characteristics of Atomic Nuclei

Protons and Neutrons

1.1 The operation of nuclear reactors depends on various types of interaction of neutrons with atomic nuclei. In order to understand the nature and characteristics of these reactions, it is desirable to review briefly some of the fundamental aspects of nuclear structure and nuclear energy.*

1.2 An atom consists of a positively charged nucleus surrounded by a number of negatively charged electrons, so that the atom as a whole is electrically neutral. In the processes taking place in a reactor, leading to the release of atomic energy, it is only the atomic nuclei which are involved, and the electrons may be neglected. Chemical energy, such as is obtained by the combustion of coal and oil, results in a rearrangement of the atoms due to a redistribution of the electrons. Atomic energy, on the other hand, is a consequence of the redistribution of the particles within the atomic nuclei. For this reason, the term nuclear energy is frequently used as a more precise alternative to the historic name, atomic energy.

1.3 Atomic nuclei are built up of two kinds of primary particles, called protons and neutrons, respectively. Because they are the units of which the nuclei are composed, and for other reasons, protons and

*For a general elementary treatment of these and related topics, see S. Glasstone, "Sourcebook on Atomic Energy," D. Van Nostrand Co., Inc., 1950.

neutrons are often referred to by the general term nucleon. Both protons and neutrons can be obtained in the free state, that is, outside atomic nuclei, and their individual properties can thus be studied.

1.4 The proton carries a single unit positive charge, equal in magnitude to the electronic charge. This particle is, in fact, identical with the nucleus of a hydrogen atom, that is, a hydrogen atom without its single electron. Hence, the mass of a proton is taken as equal to the mass of a hydrogen atom minus the mass of an electron. Thus, expressed in atomic mass units, or amu,*

$$\begin{aligned}\text{Mass of hydrogen atom} &= 1.00813 \text{ amu.} \\ \text{Mass of proton} &= 1.00758 \text{ amu.}\end{aligned}$$

1.5 The neutron, which is of fundamental importance in connection with the release of nuclear energy, is electrically neutral and carries no charge.** Consequently, it does not suffer electrical repulsion when it approaches an atomic (positively charged) nucleus from outside, as does a charged particle, such as a proton. The mass of a neutron is somewhat greater than that of a proton, and even of a hydrogen atom, thus,

$$\text{Mass of neutron} = 1.00897 \text{ amu.}$$

Methods for the production of neutrons and a discussion of their interaction with atomic nuclei will be given later.

*The atomic mass unit is defined in terms of the mass of the atom of O_8^{16} the main isotope of oxygen. The weight of this atom is taken to be exactly 16.0000 atomic mass units. Masses expressed in atomic mass units are usually derived from mass spectrograph measurements. Conventional (chemical) atomic weights are obtained upon dividing the values in mass units by 1.00027.

**For an account of the discovery and properties of neutrons, see "Sourcebook on Atomic Energy," Chapters II and XI.

Atomic Number and Mass Number

1.6 For a given element, the number of protons present in the atomic nucleus, which is the same as the number of positive charges it carries, is called the atomic number of the element. It is usually represented by the symbol Z , and it is equal, apart from a few exceptions, to the number of the element in order of increasing atomic weight. Thus, the atomic number of hydrogen is 1, of helium 2, of lithium 3, and so on up to 92 for uranium, the element with the highest atomic weight existing in nature to any appreciable extent. A number of heavier elements, of which plutonium, atomic number 94, is important in connection with the release of nuclear energy, have been made artificially.

1.7 The total number of protons and neutrons in an atomic nucleus is called the mass number of the element, and is indicated by A . The number of protons is equal to Z , as stated above; hence, the number of neutrons in a given atomic nucleus is $A-Z$. Since both neutron and proton have masses which are close to unity on the atomic weight scale, it is evident that the mass number is the integer nearest to the atomic weight of the species under consideration.

Isotopes and Nuclides

1.8 It is the atomic number, that is, the number of protons, and not the atomic weight, which determines the chemical nature of an element. This is because the chemical properties depend on the electrons, and the number of the latter in an atom is equal to the atomic number. Consequently, atoms with nuclei containing the same number of protons, i.e., with the same atomic number, but with different numbers of neutrons, that is, with different mass numbers, are essentially identical chemically, although they frequently exhibit marked differences of nuclear stability.

Such species, having the same atomic number but different mass numbers, are called isotopes.

1.9 Most elements present in nature exist in two or more stable isotopic forms, which are virtually indistinguishable chemically, although their mass numbers and atomic weights are different. Altogether some 280 stable isotopes have been identified as occurring naturally, and in addition some 60 unstable species are found in nature. Another 700 or more unstable species have been obtained artificially by various nuclear reactions. In order to distinguish among the different isotopes of a given element, it is usual to indicate the mass number together with the name or symbol of the element. Thus, the isotope of uranium of mass number 238 may be represented as uranium - 238, U-238, or U²³⁸.

1.10 The element uranium, which is at present the most important for the release of nuclear energy, exists in nature in at least three isotopic forms, with mass numbers 234, 235, 238, respectively. The proportions in which the isotopes occur in natural uranium and the weights of the respective atoms in atomic mass units are given in Table 1.10. It is seen that uranium - 238 is by far the most abundant isotope, but all natural uranium contains a little over 0.7 per cent of uranium - 235. The proportion of uranium - 234 is so small that it is usually neglected in the study of nuclear reactors.

ISOTOPIC COMPOSITION OF NATURAL URANIUM			
<u>Mass Number</u>	<u>Per Cent</u>	<u>Nuclear Mass</u>	
234	0.006	234.11	amu
235	0.712	235.11	
238	99.282	238.12	

Table 1.10

1.11 While the majority of elements exist naturally as a mixture of isotopes, about 20 occur as single species only. For this and other reasons, it has been found desirable to introduce the term nuclide. It is used to describe an atomic species characterized by the composition of its nucleus, that is, by the numbers of protons and neutrons it contains. An isotope is consequently one of a group of two or more nuclides having the same number of protons, that is, the same atomic number, but different numbers of neutrons. An element, like fluorine, of which only one species exists in nature, is said to form a single stable nuclide.

Radioactivity

Radioactive Isotopes

1.12 It was stated above that a number of unstable isotopes (or unstable nuclides) are found in nature. Actually, the naturally occurring elements of highest atomic weight, such as polonium, thorium, radium and uranium, consist entirely of unstable nuclides or radioactive isotopes. These substances undergo spontaneous change, referred to as radioactive disintegration or radioactive decay, at definite rates. The decay is accompanied by the emission from the atomic nucleus of an electrically charged particle, either an alpha particle, which is a helium nucleus, or a beta particle, which is an electron. Frequently, the products of decay are themselves radioactive, expelling either an alpha or a beta particle. After a number of stages of disintegration, an atomic species with a stable nucleus is formed.

1.13 In many instances, when a nucleus suffers radioactive decay, the product (or daughter) nucleus is not in its lowest energy state or ground state. In other words, the product nucleus is in an excited

state, having energy in excess of the ground state. Within a very short time, perhaps 10^{-15} sec of its formation, the excited nucleus emits the excess (or excitation) energy in the form of radiation called gamma rays. These rays are similar in character to X-rays; they are highly penetrating, and have wave lengths in the range of 10^{-8} to 10^{-11} cm or less. The greater the excitation energy of the nucleus, the shorter the wave length of the gamma radiation.

1.14 While the elements of highest atomic number, from polonium (atomic number 84) onward, exist only in unstable, radioactive forms, thallium (81), lead (82) and bismuth (83) occur in nature largely as stable isotopes, and also to some extent as unstable isotopes. With a few exceptions, which are not important here, the elements below thallium, as found in nature, consist entirely of stable nuclides. However, in recent years there have been produced, by various nuclear reactions, unstable, i.e., radioactive, isotopes of all the known elements.

1.15 For reasons which will be apparent shortly (1.45 et seq), if a particular nuclide is to be stable, the ratio of neutrons to protons in its nucleus must lie within a certain limited range. This may be seen from Figure 1.15, in which the number of neutrons (ordinates) is plotted against the number of protons (abscissas) present in each of the known stable atomic nuclei. It is apparent that the points lie within a relatively narrow band, corresponding to a restricted stability range of neutron-to-proton ratios for any given mass number. As the mass number, i.e., the sum of the neutrons and protons, increase, the neutron-to-proton ratio for stability increases steadily from 1.00 to about 1.56. For each mass number there is a relatively small variation in the ratio within which stable nuclides occur.

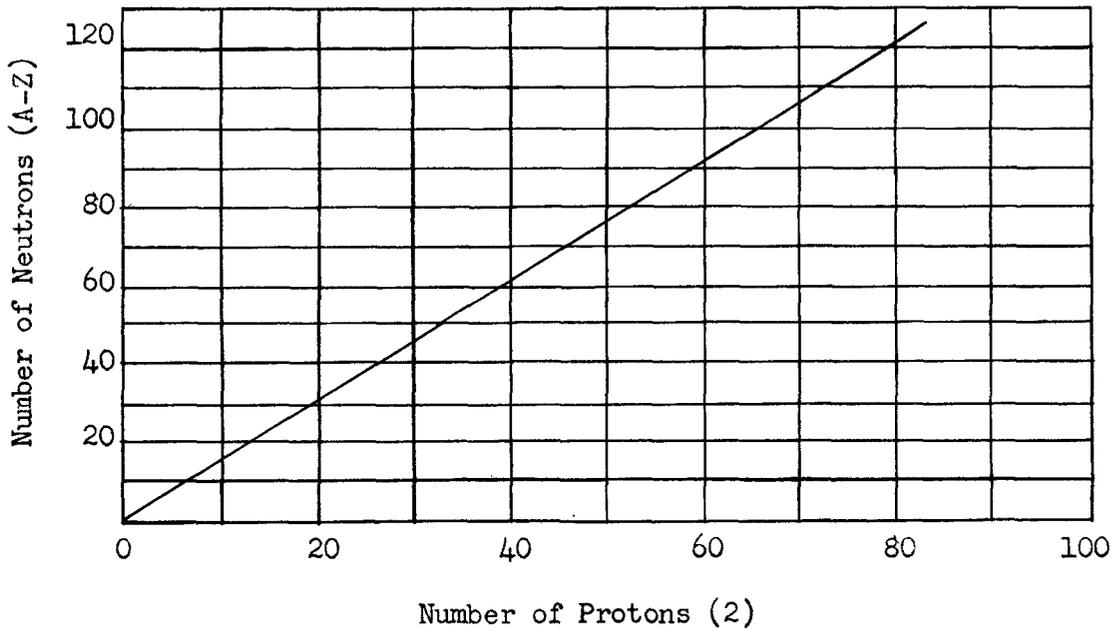
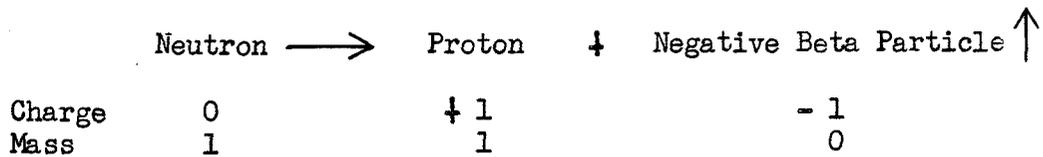


Figure 1.15

Radioactive Change

1.16 When the numbers of neutrons and protons in the nucleus of a given atomic species are such that the ratio lies outside the stability range for that mass number, the nuclide will be radioactive. The unstable nucleus will undergo spontaneous change in the direction of increased stability. Should the nucleus contain more neutrons or, what is the same thing, fewer protons, than are required for stability, a neutron will be spontaneously converted into a proton and at the same time a negative electron, i.e., a negative beta particle, will be expelled; thus,

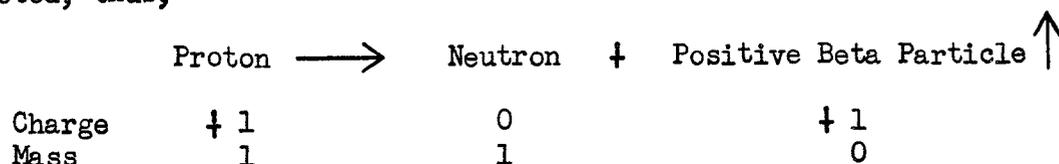


The charge and mass on the left-hand side, i.e., the neutron, are seen to balance those on the right-hand side, i.e., proton + negative beta particle. It may be mentioned that there are reasons for postulating

that another particle, called a neutrino, having essentially zero mass and no charge, is also formed, and carries off some of the energy liberated in the radioactive transformation.

1.17 The result of the change depicted above, is, effectively, to replace a neutron by a proton, so that the atomic number of the product (or daughter) element is one unit greater than that of the parent element, although its mass number is unchanged. In other words, the radioactive (negative beta) change leads to the formation of an isotopic form of another element with the same mass number as the parent element. In this new species the neutron-to-proton ratio will be less than in the nucleus of the parent, because the change of a neutron to a proton means a decrease in the number of neutrons and accompanying increase in the number of protons. Hence, in general, the daughter nucleus will tend to be more stable than the parent. It may not be entirely stable, however, and in this event, it will also be radioactive, expelling a negative beta particle, and forming an isotope of still another element. After one, two or more stages, in each of which a neutron is replaced by a proton and a negative beta particle is emitted, a stable species is formed.

1.18 A nuclide will also be unstable if the number of neutrons is too small, or the number of protons is too large, for the neutron-to-proton ratio to be within the stability range for the particular mass number. Now, however, a proton will be converted into a neutron, and at the same time a positive electron or positron, that is, a positive beta particle, will be ejected; thus,



The product nucleus will then have an atomic number one unit lower than its parent, although the mass number will be the same. As in the case considered above, the daughter may still be somewhat unstable, and then it will also be radioactive. In any event, after one or more stages of positive beta decay, a stable nucleus, having a neutron-to-proton ratio within the stability range, will be formed.

1.19 There are two other ways in which a nuclide with a ratio of neutrons to protons that is too low for stability can become more stable. One is by the emission of an alpha particle (α 1.12), and the other is by the nucleus capturing a negative electron from outside the atom, thus reversing the process described in β^- 1.16. In each case the change is associated with an increase in the neutron-to-proton ratio. Since neither of these modes of radioactive decay is important in connection with nuclear reactors, it is unnecessary to consider them in further detail.

Rate of Radioactive Decay

1.20 For a given radioactive species, every nucleus has a definite probability of decaying in unit time; this decay probability is characteristic of the particular species and has a constant value which cannot be changed in any known way. It is the same irrespective of the chemical or physical state of the element at all accessible temperatures and pressures. In a given specimen, the rate of decay at any instant is always directly proportional to the number of radioactive atoms of the isotope under consideration present at that instant. Thus, if N is the number of the particular radioactive atoms (or nuclei) present at any time t , the decay rate is given by

$$\frac{dN}{dt} = -\lambda N, \quad (1.20.1)$$

where λ is called the decay constant of the radioactive species. Upon

rearrangement of equation (1.20.1), an integration between any arbitrary zero time, when the number of radioactive nuclei of the specified kind present is N_0 , and a time t later, when the number of these nuclei remaining is N , it is readily found that

$$N = N_0 e^{-\lambda t} \quad (1.20.2)$$

It is evident that radioactive decay is an exponential process, the overall decay rate at any instant being determined by the decay constant λ and by the number of the particular nuclei present at that instant.

1.21 A convenient method for representing the rate of radioactive decay is by means of the half life of a particular nuclide. It is defined as the time required for the number of active nuclei (or the activity) to decay to half its initial value. This means that if in equation (1.20.2), N is set equal to $\frac{1}{2}N_0$, the corresponding time is the half life T ; thus,

$$e^{-\lambda T} = \frac{1}{2}$$

or

$$T = \frac{\ln 2}{\lambda} = \frac{0.6931}{\lambda}, \quad (1.21.1)$$

so that the half life is inversely proportional to the decay constant. The half lives of known radioactive species range from a small fraction of a second to billions of years.

1.22 For some purposes it is convenient to employ the reciprocal of the decay constant. The resulting quantity t_m is the mean life or average life of the radioactive species; thus,

$$t_m = \frac{1}{\lambda}. \quad (1.22.1)$$

It can be shown that the mean life is equal to the average life expectancy, before decay, of the nuclei present at any time.

Nuclear Forces

1.23 The remarkable fact about atomic nuclei is not that some show partial instability and are radioactive, but rather that they exhibit any stability at all. It might be thought, upon first consideration, that a system of closely packed (positively charged) protons, such as exists in an atomic nucleus, would fly apart because of the electrostatic repulsion of the charges. The stability of atomic nuclei is evidently related, at least partly, to the presence of neutrons, in addition to protons.

1.24 The existence of the stable nucleus of deuterium, the isotope of hydrogen of mass number 2, consisting of a neutron and a proton, shows that attractive neutron-proton forces must be involved. In addition, there is good evidence that at close range, as in an atomic nucleus, there are forces of attraction between protons themselves, as well as between neutrons. For example, the definite stability of the helium - 3 nucleus, containing one neutron and two protons, provides clear proof of the existence of proton-proton forces of attraction within the atomic nucleus. However, while it is accepted that attractive neutron-neutron, proton-proton and neutron-proton forces exist, little is known of the nature of such forces. The subject of nuclear stability cannot yet be satisfactorily treated from a theoretical standpoint, and so a semi-empirical approach, based on nuclear masses, will be used here.

Mass Defect and Binding Energy

1.25 If there were no energy changes, due to the operation of nuclear forces, the mass of a nucleus would be equal to the sum of the masses of its constituent Z protons and $A-Z$ neutrons (§1.7). The total mass of the atom as a whole would then be the sum of these quantities plus the mass of Z electrons. Since a proton and an electron make up a hydrogen atom.

it may be supposed that the mass of an atom of any nuclide of atomic number Z and mass number A should be equal to the mass of Z hydrogen atoms plus that of $A-Z$ neutrons, that is, to $Zm_H + (A-Z)m_n$, where m_H and m_n are the masses of a hydrogen atom and of a neutron, respectively.

1.26 Actual determinations of individual atomic masses, made by means of the mass spectrograph, are, however, always less than the values calculated in this manner. The difference between the calculated mass and the experimental mass M , called the true mass defect*, is represented by

$$\text{True mass defect} = Zm_H + (A-Z)m_n - M. \quad (1.26.1)$$

This mass defect represents the mass which would appear in the form of energy in the hypothetical process of assembling a particular atom from the requisite number of electrons, protons and neutrons. The same amount of energy would, of course, have to be supplied to the atom in order to break it up into its constituent particles. Hence, the energy equivalent of the true mass defect is taken as a measure of the binding energy of the particular atomic species.

1.27 In order to determine the energy equivalent of the mass defect, use is made of the Einstein mass-energy relationship

$$E = mc^2, \quad (1.27.1)$$

where E is the energy equivalent of the mass m , and c is the velocity of light. If m is in grams, and c in cm per sec, i.e., 3×10^{10} cm per sec, E will be in ergs. For the present purposes, it is more useful to express m in atomic mass units, where $1 \text{ amu} = 1.67 \times 10^{-24} \text{ gram}^{**}$; equation (1.27.1)

* The expression "true mass defect" is used here to distinguish it from a somewhat different quantity, incorrectly called the "mass defect," which appears in the nuclear science literature.

**This is one-sixteenth part of the actual mass in grams of an O^{16} atom.

then becomes

$$E(\text{ergs}) = m (\text{amu}) \times 1.49 \times 10^{-3} \quad (1.27.2)$$

1.28 In atomic studies it has become the practice to express energies in electron volt units. The electron volt, i.e., 1 ev, is the energy acquired by any charged particle carrying a unit (electronic) charge when it passes, without resistance, through a potential difference of 1 volt. From the known magnitude of the electronic charge, it is found that

$$1 \text{ ev} = 1.60 \times 10^{-12} \text{ erg.} \quad (1.28.1)$$

Consequently, equation (1.27.2) can be written as

$$E(\text{ev}) = m(\text{amu}) \times 9.31 \times 10^8.$$

Actually, the electron volt is too small a unit for many purposes, and the million electron volt unit, i.e., 10^6 ev, abbreviated to Mev, is used; hence,

$$E(\text{Mev}) = m(\text{amu}) \times 931. \quad (1.28.2)$$

In other words, a mass expressed in atomic mass units can be stated as its equivalent in Mev of energy upon multiplication by the factor 931.

1.29 Returning to equation (1.26.1) for the mass defect, it follows from the arguments presented in § 1.26, that the binding energy is given by

$$\text{Binding energy in Mev} = 931 \left[Zm_H + (A-Z)m_n - M \right], \quad (1.29.1)$$

where m_H is 1.00813 amu, m_n is 1.00897 amu, and M is the isotopic mass in amu. In this derivation the binding energy of the electrons to the nucleus has been neglected or, rather, it has been regarded as included in the Zm_H term. In any event, the electron binding energy is a very small fraction of the total. Consequently, equation (1.29.1) may be taken as giving a measure of the net binding energy of the constituent nucleons, i.e., protons and neutrons, in the nucleus of the atom under consideration.

1.30 By means of equation (1.29.1) the binding energies have been calculated for all nuclides whose isotopic weights are known with sufficient accuracy. If the result, in each case, is divided by the mass number, that is, by the total number of nucleons in the nucleus, there is obtained the mean binding energy per nucleon for the given species. The data secured in this manner have been plotted against the respective mass number in Figure 1.30. It will be seen that with the exception of He^4 , C^{12} and O^{16} , the values fall on, or in close proximity to, a single curve. The mean binding energy per nucleon in the elements of low mass number is low, but over a considerable range it is close to 8 Mev. The total binding energy is then approximately proportional to the mass number, that is, to the number of nucleons in the nucleus.

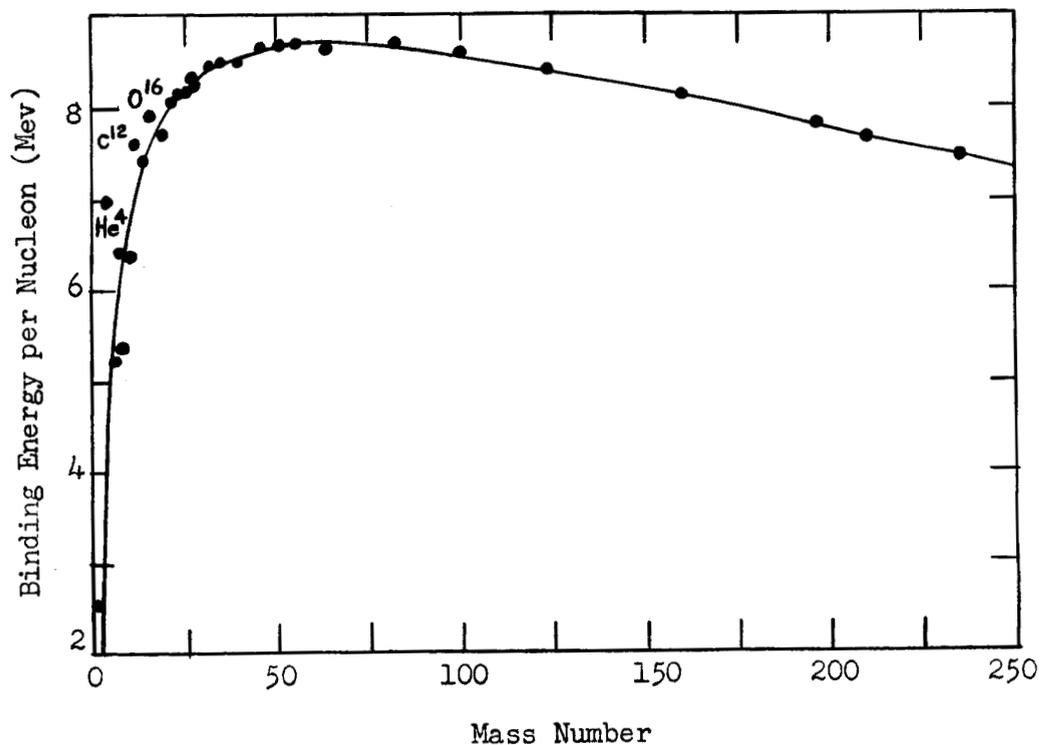


Figure 1.30

Liquid Drop Model of Nucleus

1.31 It has been found useful in certain respects to regard an atomic nucleus as somewhat similar in character to a drop of incompressible liquid. Just as the forces responsible for surface tension tend to maintain a liquid drop in a spherical form which resists distortion, it is believed that nuclear forces have a similar effect on the atomic nucleus. This point of view has been especially valuable in the consideration of nuclear fission.

1.32 In a liquid the forces between molecules are short-range forces; that is to say, they operate only between any given molecule and such others as are immediately adjacent to it. Hence, in a liquid, there is no appreciable interaction between more distant molecules. Similar considerations are apparently applicable to the forces operating between the nucleons in an atomic nucleus. This view is supported by the binding energy data given above. If nuclear forces had long-range character, so that each nucleon interacted with every other nucleon, the total binding energy would increase roughly as the square of the number of nucleons. Actually, the total binding energy is almost directly proportional to the number of nucleons, as stated in § 1.30.

1.33 Further evidence for the short-range character of nuclear forces is provided by determinations of nuclear radii. Three main methods have been used for this purpose. The first method, applicable to radioactive nuclids of high mass number which emit alpha particles, depends on the rates of decay and the energies of the alpha particles expelled. The second method is based on the existence of mirror nuclei up to about mass #41. Finally, there is a method for determining nuclear radii which can be used, at least in principle, for any species, irrespective of its stability or mass number. This involves measurement of the scattering cross section of fast neutrons.

1.34 The values obtained by different methods for a given nuclide are generally in good agreement with each other. Except for the elements of lowest mass number, the results can be expressed, with a fair degree of accuracy, by the formula

$$R = 1.5 \times 10^{-13} A^{1/3} \text{ cm,} \quad (1.34.1)$$

where R is the radius of a nucleus of mass number A . The fact that the nuclear radius is approximately proportional to the cube root of the mass number is of great importance. The volume of the nucleus is consequently directly proportional to the mass number, and hence to its actual mass. This means that all atomic nuclei, containing the same constituents, namely, neutrons and protons, have essentially the same density. The constancy of the nuclear density, irrespective of the number of nucleons, is just what is to be expected if the nucleus behaves like a liquid with short-range forces operating between the constituent particles.

Semi-empirical Calculation of Binding Energies

1.35 The liquid-drop model of the atomic nucleus can be used, in default of a complete theory of nuclear forces, to derive a semi-empirical expression for the binding energy. This is done by considering, in what is undoubtedly an oversimplified manner, the various factors which are believed to contribute to nuclear binding. The appropriate weighting constants are then derived from theoretical considerations where possible and from experimental data where the theory is still inadequate.

1.36 If the forces in the nucleus are like those in a liquid drop, each nucleon will, in the first place, be strongly attracted by those in its immediate vicinity, but will be unaffected by the others. This leads to an attractive contribution to the energy which is proportional to the number of nucleons in the nucleus. The attractive energy will thus

vary as the mass number A , and it can consequently be represented by

$$\text{Attractive energy} = a_1 A \quad (1.36.1)$$

where a_1 is a constant.

1.37 In stating that the attractive energy is proportional to the mass number, it is tacitly assumed that every nucleon has the same access to other nucleons. Actually, those at the surface of the nucleus will be less tightly bound than those in the interior so that the attractive energy, as given by equation (1.36.1), has been overestimated by an amount which depends on the surface area. The larger this area the greater will be the number of nucleons which are not completely surrounded by others. The amount whereby the attractive energy has been overestimated may thus be regarded as proportional to the surface area of the nucleus. It is frequently referred to as the surface tension effect, because it is due to a factor similar to that which causes surface tension in a liquid. Since, according to equation (1.34.1), the nuclear radius is proportional to $A^{1/3}$, the surface area varies as $A^{2/3}$, and hence

$$\text{Surface tension effect} = -a_2 A^{2/3}, \quad (1.37.1)$$

where a_2 is a constant.

1.38 In stable nuclei there is a tendency for groups of neutron-proton pairs to form. For example, the most stable species, such as He^4 , C^{12} and O^{16} (See Figure 1.30), are those consisting of equal numbers of neutrons and protons. Most nuclei, however, especially the heavier ones, have an excess of neutrons over protons. This excess is necessary in order that the attractive neutron-neutron and neutron-proton forces may compensate for the electrostatic repulsion between protons. At the same time, a degree of instability is introduced because the excess neutrons occupy a number of nuclear energy levels that contain no protons. The presence of more

neutrons than protons in the nucleus means that the estimate of the attractive energy given by equation (1.36.1) is too large. The appropriate correction can be made by a composition term, expressed by

$$\text{Composition term} = -a_3 \frac{(A-2Z)^2}{A}, \quad (1.38.1)$$

where a_3 is a constant and $A-2Z$ is the excess neutrons over protons in the nucleus.*

1.39 The sum of the three terms derived above probably represents essentially the net attractive energy in the nucleus. It is necessary now to examine the repulsive energy due to the mutual electrostatic repulsion of the protons. The potential energy of a uniformly charged sphere is proportional to Z^2/R , where Z is the number of unit charges, that is, the atomic number in the present case, and R is the radius of the sphere. As applied to the nuclear binding energy, the electrostatic repulsion can be represented by

$$\text{Repulsive energy} = -a_4 \frac{Z^2}{A^{1/3}}, \quad (1.39.1)$$

where the nuclear radius R has been replaced by $A^{1/3}$, to which it is proportional; as in the other expressions, a_4 is a constant.

1.40 Finally, consideration must be given to the influence of the odd and even character of the numbers of protons and neutrons. When these are both even, i.e., even-even type, the nucleus is exceptionally stable, and when they are both odd, i.e., odd-odd type, the system is particularly unstable. This may be attributed to the stabilizing effect of the pairing of nucleon spins, which is possible when there are even numbers of both protons and neutrons. Consequently, in an even-even nucleus there is an

*See E. Fermi, "Nuclear Physics," University of Chicago Press, 1950, Page 22.

additional positive contribution to the binding energy, whereas in an odd-odd nucleus, having a neutron and a proton with unpaired spins, there is a corresponding negative (or repulsive) effect. Purely empirical considerations, based on binding energies calculated from the isotopic mass by equation (1.29.1), shows that the spin effect contribution can be represented by

$$\text{Spin effect} = \pm \frac{a_5}{A^{3/4}}, \quad (1.40.1)$$

where the plus sign applies to even-even nuclei and the minus sign to odd-odd nuclei. For odd-even (or even-odd) nuclei the spin term is zero.

1.41 Upon combining the various contributions to the binding energy, as given in the preceding paragraphs, it follows that the total binding energy (B.E.) of a nucleus may be represented by

$$\text{B.E.} = a_1 A - a_2 A^{2/3} - a_3 \frac{(A-2Z)^2}{A} - a_4 \frac{Z^2}{A^{1/3}} \pm \frac{a_5}{A^{3/4}}, \quad (1.41.1)$$

where a_5 is zero for odd-even nuclei. Of the five constants in this equation, a_4 can be obtained from electrostatic theory, but the others must be derived empirically as follows.

1.42 Differentiation of equation (1.41.1) with respect to Z , with A constant, leads to

$$\frac{d(\text{B.E.})}{dZ} = 4a_3 \frac{A-2Z}{A} - 2a_4 \frac{Z}{A^{1/3}}$$

and, consequently, a maximum in the binding energy occurs when

$$4a_3 \frac{A-2Z}{A} = 2a_4 \frac{Z}{A^{1/3}}. \quad (1.42.1)$$

This equation should express the relationship between the mass number A and the atomic number Z of the most stable nuclei, as these will have the largest binding energy for each mass number. Since a_4 is known, as stated

above, a_3 can be determined by finding the value which, when inserted in equation (1.42.1), will best represent a plot of A against Z for the most abundant naturally occurring nuclides. Actually, no single constant is adequate for the whole range of mass numbers, and so a compromise has to be made in assessing the best value of a_4 to be used in equation (1.41.1).

1.43 With a_3 and a_4 known, the values of a_1 and a_2 can be determined from the known binding energies, calculated from the isotopic weights of any pair of odd-even nuclei, since a_5 is then zero. Finally, the value of a_5 is estimated from the binding energies of even-even nuclei, since only a very few stable odd-odd nuclei are known, and these are of low mass number.

1.44 Upon inserting the constants derived in the manner explained above, equation (1.41.1) for the binding energy, expressed in Mev, becomes

$$\text{B.E. (Mev)} = 14.0A - 13.0A^{2/3} - 19.3 \frac{(A-2Z)^2}{A} - 0.585 \frac{Z^2}{A^{1/3}} \pm \frac{33}{A^{3/4}} *$$

(1.44.1)

* \dagger for even-even
- for odd-odd

The relative effects of the various terms on the net binding energy can best be seen by using equation (1.44.1) to calculate the values for nuclides of low, medium and high mass numbers. The results for ${}_{20}\text{Ca}^{40}$, ${}_{47}\text{Ag}^{107}$ and ${}_{92}\text{U}^{238}$ are given in Table 1.44; the experimental values of the total binding energies obtained from the known isotopic weights are included for comparison. The agreement between the calculated and observed values is satisfactory, since the constants in equation (1.44.1) are given only to the three significant figures.

Calculation of Binding Energies			
	$^{40}_{20}\text{Ca}$	$^{107}_{47}\text{Ag}$	$^{238}_{92}\text{U}$
Attraction of nucleons	560	1498	3332
Surface effect	-152	-293	-501
Composition effect	0	-30.6	-236
Electrostatic repulsion	-68.4	-272	-799
Spin effect	3.2	0	0.5
Calculated binding energy	343	902	1796
Experimental binding energy	341	907	1785
B.E. per nucleon	8.5	8.4	7.5

Table 1.44

Nuclear Forces and Stability

1.45 The results derived above may be used to provide a qualitative interpretation of the fact that for any mass number there is a limited stability range for the neutron-to-proton ratio (≈ 1.15). As stated earlier, the actual value of this ratio increases from 1.00 for low mass numbers to about 1.5 to 1.6 for elements of high atomic weight. Since the neutron-neutron, proton-proton and neutron-proton attractive forces are approximately equal, a neutron-to-proton ratio close to unity is to be expected for stability; this is the case for nuclei of low mass number. However, as the atomic number increases, the electrostatic repulsion between protons begins to have an increasingly important effect. The electrostatic forces are long-range in character, and each proton repels, and is repelled by, all the other protons. Thus, as seen in $\S 1.39$, the repulsive force varies as $Z^2/A^{1/3}$, and so it increases rapidly with increasing atomic number.

1.46 In order to overcome the increasing repulsion of the protons and maintain stability in the heavier elements, the nuclei must contain an increased proportion of neutrons. The additional nucleon-nucleon attractive forces then partly compensate for the growing proton-proton repulsion. Consequently, the neutron-to-proton ratio in stable, heavier nuclei is greater than unity.

1.47 There is, however, a limit to the number of neutrons which can be present, for a stable system of given mass number because, as explained in § 1.38 in connection with the so-called composition term, an excess of neutrons over protons introduces some instability. This fact determines the upper stability limit for the neutron-to-proton ratio. The lower limit, on the other hand, arises because increasing the number of protons would lead to instability due to increased electrostatic repulsion. The fact that the stability range for the neutron-to-proton ratio is relatively small can thus be understood.

CHAPTER II
NUCLEAR REACTIONS

Rates of Nuclear Reactions

Comparison of Nuclear and Chemical Reactions

2.1 Under suitable laboratory conditions atomic nuclei can be made to react with other nuclei, especially those of the lightest elements, namely, hydrogen (protons), deuterium (deuterons) and helium (alpha particles). Atomic nuclei can also interact with neutrons, electrons and gamma radiation. However, at ordinary temperatures the rates of nuclear reactions, that is, the number of nuclei in a given volume reacting in a specified time, are very much less than for chemical reactions involving atoms or molecules. There are essentially two reasons for this marked difference in reaction rate between chemical processes and nuclear processes.

2.2 In the first place, the small size of the nucleus, whose diameter is of the order of 10^{-12} cm, as compared with 10^{-7} or 10^{-8} cm for the whole atom or molecule, means that nuclear collisions or encounters are much less frequent than atomic (or molecular) collisions. There are special circumstances to be referred to below (§ 2.9), in which a nucleus or nuclear particle of low mass and energy can behave as if it had a diameter approaching that of the whole atom. The rates of nuclear reactions are then greatly increased over the usual values.

2.3 The second factor responsible for the relatively low rate of interaction of one nucleus with another is the coulombic repulsion between them, arising from their positive electrical charges. The repulsion

energy is proportional to $Z_1 Z_2 e^2/R$, where Z_1 and Z_2 are the charges, i.e., the atomic numbers, of the interacting nuclei, and R is the distance between their centers. Since the two nuclei must approach each other to within distances of about 10^{-12} cm before they can interact, the repulsive energy which must be overcome is very large, especially for nuclei of high atomic number. Even for nuclei of low atomic number, e.g., hydrogen and helium, the coulombic energy is of the order of millions of electron volts.

2.4 For chemical reactions, on the other hand, the energies required to permit interaction of the electronic fields are rarely more than a few electron volts. At ordinary temperatures, there is an appreciable possibility that a pair of colliding atoms or molecules will possess this amount of kinetic energy. The reaction then takes place at an easily detectable rate. The probability that at ordinary temperatures two colliding nuclei will possess kinetic energy of a million electron volts is extremely small. Hence, not only is the number of encounters between atomic nuclei less than for atoms or molecules, under equivalent conditions, but the probability of interaction occurring upon collision is also considerably less. The rates of reactions between nuclei are thus very much less than for chemical reactions between atoms or molecules.

2.5 There are two ways in which nuclear reactions can be made to take place more readily. First, by increasing the temperature to several million degrees, the interacting nuclei will acquire sufficient kinetic energy to overcome their mutual electrostatic repulsion or coulomb barrier. Such nuclear processes, referred to as thermonuclear reactions, take place in the sun and stars; they represent the energy source of

these celestial bodies. In the laboratory, reactions involving atomic nuclei are studied by bombarding various materials with light nuclei, e.g., protons, deuterons or alpha particles, which have been accelerated until they have kinetic energies in the vicinity of a million electron volts or more. Cyclotrons and other devices are used for this purpose. Reactions of nuclei with highly accelerated electrons and with gamma rays and X-rays of high energy have also been achieved.

Interaction of Neutrons with Nuclei

2.6 While the foregoing nuclear processes are of great interest, they are not important in connection with the present discussion of nuclear reactors. As stated earlier, these devices involve interaction of atomic nuclei with neutrons, and such reactions differ in one important respect from those considered above. Since the neutron has no electric charge, it does not have to overcome any appreciable repulsive force in approaching an atomic nucleus. Consequently, even the so-called "slow" neutrons, having the same mean kinetic energy as ordinary gas molecules, e.g., about 0.03 ev at ordinary temperatures (§ 3.9), can readily interact with atomic nuclei.

2.7 The probability of interaction between a nucleus and a neutron is, in fact, generally greater for slow neutrons than for fast neutrons with energies of the order of several thousand or more electron volts. An explanation of this fact, in classical terms, might be that in an encounter with a nucleus, a slow-moving neutron spends, on the average, more time in the vicinity of the nucleus than does a fast-moving neutron. The chances of interaction would thus be expected to be larger in the former case. However, in quantum mechanics, the collision between a

neutron and a nucleus is regarded as the interaction of a neutron wave with the nucleus. As will be shown below, the effective wave length of the neutron is inversely proportional to its velocity. Hence, the wave length of a slow neutron is greater than that of a fast neutron, and the probability of interaction with a nucleus is increased correspondingly.

Neutron Wave Length

2.8 According to the wave theory of matter, all particles are associated with waves, called matter waves, or de Broglie waves, the wave length λ being given by

$$\lambda = \frac{h}{mv} , \quad (2.8.1)$$

where h is Planck's quantum theory constant, i.e., 6.62×10^{-27} erg sec, m is the mass of the particle and v is its velocity. Let E be the kinetic energy of the particle, then $E = \frac{1}{2} mv^2$, and equation (2.8.1) may be written as

$$\lambda = \frac{h}{\sqrt{2 mE}} . \quad (2.8.2)$$

If m is in grams, E in ergs and h is in erg sec, the wave length will be in centimeters. If m is expressed in amu units, which are 1.67×10^{-24} gram (§ 1.27), and E in electron volts, i.e., 1.60×10^{-12} erg, equation (2.8.2) becomes

$$\lambda = \frac{4.02 \times 10^{-9}}{\sqrt{2 mE}} \text{ cm} \quad (2.8.3)$$

2.9 For a neutron, which is the particle of special interest here, m is approximately unity in amu, and hence the expression for the neutron wave length becomes

$$\lambda = \frac{2.85 \times 10^{-9}}{\sqrt{E}} \text{ cm} \quad (2.9.1)$$

where E is the neutron energy in electron volts. For fast neutrons of energy about 1 Mev, the wave length is seen from equation (2.9.1) to be of the order of 10^{-12} cm, which is the same magnitude as the diameter of a nucleus. If the neutron energy is about 0.03 ev, however, λ is found to be about 1.7×10^{-8} cm. Thus, a slow neutron might have an effective diameter approaching that of the whole atom.* Even if the energy were 1000 ev, the neutron wave length (or effective diameter) would be of the order of 10^{-10} cm; this is still much larger than a nuclear diameter. The conditions under which slow neutrons thus behave as if they were almost as large as a whole atom, and hence have a relatively large probability of interacting with atomic nuclei, will be described in (§ 3.20).

The Compound Nucleus Model

Mechanism of Nuclear Reactions

2.10 Before considering various types of interaction of neutrons with atomic nuclei, a brief review will be given of some of the general features of nuclear reactions. In the first place, two broad classes of such reactions may be distinguished, depending on the energy of the particle, called the incident particle or projectile, which impinges on an atomic nucleus, called the target nucleus. In a nucleus the constituent nucleons are tightly bound, the degree of binding being measured by the average interaction energy per nucleon. As seen in §1.30, this interaction (or binding) energy is of the order of 8 Mev per nucleon for nuclei of medium or high mass number. If the kinetic energy of the incident particle is roughly equal to or greater than the average interaction energy between nucleons in the target nucleus, i.e., about 10 Mev

* It is because slow neutrons have equivalent wave lengths of the order of 10^{-8} cm that, like X-rays, they can be diffracted by crystals.

or more, the incident particle interacts with only a single nucleon or with a small number of nucleons. Since the operation of nuclear reactors depends on the interaction with matter of neutrons of energies considerably less than 10 Mev, this type of process need not be discussed further.

2.11 When the kinetic energy of the incident particle is less than the mean interaction energy per nucleon, the incident particle may be regarded as interacting with the nucleus as a whole. In these circumstances, the model of the compound nucleus, treated theoretically by Bohr and by Breit and Wigner (§ 2.38 et seq)*, is applicable. According to this model, a nuclear reaction is considered to occur in two stages. First, the incident particle is absorbed by the target nucleus to form a compound nucleus; then, after the lapse of a short time, the latter disintegrates expelling a particle (or a photon**) and leaving another nucleus, called the residual or recoil nucleus. The two stages may thus be written as follows:

(1) Formation of Compound Nucleus

Target nucleus + Incident particle \longrightarrow Compound nucleus

(2) Disintegration of Compound Nucleus

Compound nucleus \longrightarrow Recoil nucleus + Ejected particle.

It should be noted that the compound nucleus may well be a nucleus of a familiar atomic species, or it may be one which is unstable. In any event, the disintegration stage (2) described above is due to the fact

* N. Bohr, Nature, 137, 344, (1936); G. Breit and E. P. Wigner, Phys. Rev., 49, 519 (1936). The experimental evidence for the formation of a compound nucleus in nuclear reactions is largely due to W. D. Harkins (1935).

** A photon may be regarded as an "atom" or "particle" of radiation. according to quantum theory, the energy quantum carried by a photon is equal to $h\nu$, where h is Planck's constant (§ 2.8), and ν is the frequency of the radiation.

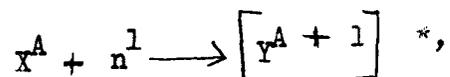
that the compound nucleus when formed in stage (1) is in an unstable, high-energy state (§ 2.16).

2.12 In order that the compound nucleus may be considered as a separate entity, its lifetime must be long compared to the time required for the incident particle to traverse a distance equal to the nuclear diameter. The time required for a slow neutron, having a speed of about 10^5 cm per sec, to cross a distance of about 10^{-12} cm, is of the order of 10^{-17} sec. Actually, after capture, an initially slow neutron will acquire additional kinetic energy, and hence move more rapidly. The time of transit is thus less than the 10^{-17} sec, just estimated. It will be seen shortly (§ 2.29) that the mean lifetime of the excited compound nucleus in many reactions with heavy nuclei is about 10^{-14} sec. Since this is less than the transit time by a factor of 1000, at least, the condition for the occurrence of the compound nucleus as a separate entity is satisfied.

Excitation Energy of Compound Nucleus

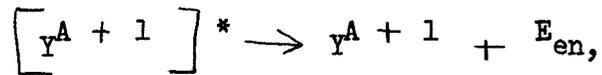
2.13 When a target nucleus captures an incident particle, the resulting compound nucleus is invariably in a higher energy, i.e., excited, state (§ 2.16). The excitation energy, i.e., energy above the ground state, is equal to the kinetic energy of the captured particle plus its binding energy in the compound nucleus. That this is so may be seen by considering the case of the capture of a neutron (n^1) of zero kinetic energy by a nucleus X of mass number A.

2.14 The formation of the compound nucleus Y, of mass number $A + 1$, may be represented as

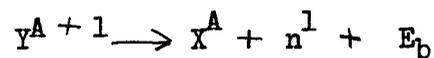


where the asterisk indicated an excited state of the Y^{A+1} nucleus.

Suppose now that the excited compound nucleus emits the excitation energy and thereby passes into its normal or ground state, thus



where E_{en} is the excitation energy, i.e., the difference in energy between the excited and ground states. Next, imagine a neutron of zero kinetic energy to be removed from the ground state of Y^{A+1} , leaving the ground state of the nucleus X^A . The energy which must be supplied is the binding energy E_b of the neutron in the compound nucleus; thus,



Upon combining the three stages, as a result of which the original condition is restored, it is seen that the excitation energy E_{ex} of the compound nucleus is equal to E_b , the binding energy of the neutron, which is about 5 to 8 Mev. It readily follows that if the neutron possessed kinetic energy, as it invariably does, the excitation energy is equal to the binding energy plus the kinetic energy of the neutron.

2.15 The situation may be represented diagrammatically by means of a potential energy curve, as in Figure 2.15, where the energy of the system of target nucleus and neutron of zero kinetic energy is plotted against the distance between these particles. At the extreme right, at B, the target nucleus and neutron are far apart and may be regarded as being quite independent. At the extreme left, at A, on the other hand, the target nucleus and the neutron may be regarded as being completely fused so as to form the ground state of the compound nucleus. The vertical distance between A and B represents the binding energy of the neutron in the compound nucleus. This amount of energy would have to be supplied in

a process starting from the ground state of the compound nucleus at A and ending with the separated target nucleus and neutron at B.

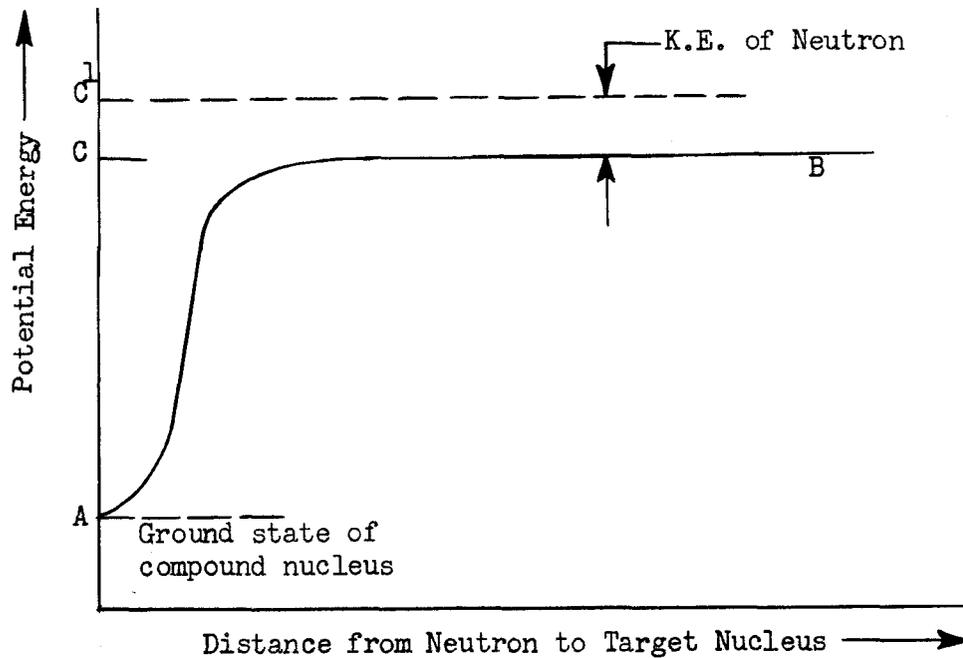


Figure 2.15

2.16 If a neutron of zero kinetic energy is gradually brought up to a target nucleus, there is no attraction or repulsion, and the energy of the system remains equal to that at B. Eventually, when the nucleus has completely absorbed the neutron, the energy state of the compound nucleus is at C. This nucleus is evidently in an excited state, the excitation energy being equal to AC, which is the same as the neutron binding energy. If the captured neutron has kinetic energy, the state of the compound nucleus is indicated by the line C', so that the excitation energy is given by AC'. It can be seen, in general, that when a compound nucleus is formed by neutron capture, the excitation energy, i.e., the (internal)

energy in excess of the ground state of the compound nucleus, is equal to the binding energy of the neutron plus its kinetic energy.

2.17 It may be pointed out that the foregoing conclusion is strictly true only when the target nucleus has infinite mass. In general, because of the conservation of momentum in a capture collision between a nucleus and a neutron, the compound nucleus will acquire some kinetic energy, even though the target nucleus was at rest. Consequently, only part of the energy of the neutron appears as internal, i.e., excitation, energy of the compound nucleus. For target nuclei of appreciable mass number, however, virtually, the whole of the kinetic energy of the neutron is transferred to the internal energy of the nucleus. Throughout the subsequent discussion, it will be assumed that this condition holds.

Statistical Distribution of Energy

2.18 Immediately after its formation, the excitation energy of the compound nucleus may be regarded as being concentrated on the captured particle. But, as a result of interactions within the nucleus, the additional energy is rapidly shared among the nucleons. This distribution occurs in a statistical manner. Thus, at a given instant, the excitation energy may be shared between two or more nucleons, while at a subsequent instant it may be shared by other nucleons or it may be concentrated on one of the nucleons. In the course of time, one particular nucleon, or combination of nucleons, in the compound nucleus may acquire sufficient energy to permit it to escape. This corresponds to the disintegration stage referred in § 2.11.

2.19 Because of the large number of ways that the excitation energy may be shared among the nucleons, the probability that a single nucleon

will acquire sufficient energy to permit escape from the compound nucleus during the time required for a captured particle to traverse the nucleus is usually small. Consequently, the average life of the compound nucleus will be long in comparison with the time required for a neutron to cross it.

2.20 An important consequence of the relatively long lifetime of the excited compound nucleus is that, for a given excitation energy, the manner in which the compound nucleus breaks up is independent of its mode of formation. If the lifetime of the compound nucleus is sufficiently long, the distribution of the excitation energy will depend only on the total excitation energy, the number of nucleons, and the energy levels of the compound nucleus. The mode of formation is thus "forgotten" by the compound nucleus as a result of the statistical manner in which the excitation energy is shared.

2.21 Because of its relatively long lifetime, there is a possibility that the excited compound nucleus will rid itself of its excess energy by the emission of radiation. Nuclear processes in which a particle is captured and the excess energy is emitted as radiation are called radiative capture reactions. The conditions for such reactions are especially favorable when a slow neutron is captured. In the majority of instances, although not always, the only process for which the compound nucleus has sufficient energy is the re-emission of a slow neutron. In other words, this stage would involve reversal of the capture process. However, before the redistribution of energy among the nucleons results in a particular neutron acquiring enough energy to escape from the compound nucleus, the excess (excitation) energy of the latter is emitted as gamma radiation,

i.e., radiation of 10^{-10} to 10^{-11} cm wave length.*

2.22 In certain reactions involving heavy atomic nuclei, the capture of a suitable particle results in the formation of an excited state of a compound nucleus so unstable that it splits up into two smaller nuclei. This is the process of fission, which is of fundamental importance for the operation of nuclear reactors. It will be referred to again in § 3.35 and discussed more fully in Chapter IV.

Nuclear Energy Levels

2.23 The existence of definite nuclear energy**levels or quantum states, is supported by various types of experimental evidence. In particular, mention may be made of the emission of gamma rays of definite energy (or wave length) in many radioactive processes. The nuclear energy levels are quite analogous to the familiar atomic or electronic energy levels which permit an interpretation of atomic spectra. However, since the forces acting between nucleons are not understood, it has not yet been possible to apply quantum mechanics to the study of nuclear energy levels with the same degree of success as has been done in connection with the problem of electronic energy levels. Nevertheless, certain aspects of the subject can be considered.

2.24 From various studies it has been concluded that the energy levels of atomic nuclei are relatively far apart for the low energy states, i.e., near the ground state, but become closer and closer as the internal energy of the nucleus increases. At very high energies, about 15 to 20 MEV or more, the energy levels are so close that they may be regarded as

* It can be shown from Planck's equation ($E = h\nu$)* that the wave length λ in cm is equal to $1.24 \times 10^{-10}/E$, where E is the energy, expressed in Mev, emitted by the nucleus in one step (cf. § 2.11, footnote).

** The energy under consideration is internal energy of the nucleus, and not kinetic energy of the nucleus as a whole.

virtually continuous.

2.25 For nuclei of mass number in the medium range, namely, from about 100 to 150, the separation (or spacing) of the levels near the ground state is in the vicinity of 0.1 Mev. However, when the energy is in the region of 8 Mev above the ground level, as in a compound nucleus formed by the capture of a slow neutron, the level spacing is only 1 to 10 ev. For light nuclei, the energy levels are somewhat further apart; the separations are of the order of 1 Mev near the ground state and roughly 10,000 ev when the internal energy is about 8 Mev above the ground level.

Lifetime and Level Width

2.26 To every excited quantum state of a nucleus there may be ascribed a mean lifetime τ ; this is the period of time, on the average, that a nucleus will remain in the given excited state before undergoing a change, e.g., emission of a particle or of radiation. Correspondingly, each quantum state has a level width Γ , which may be regarded as an indication of the indefiniteness involved in the determination of the energy of the particular state.* From the Heisenberg uncertainty principle the mean lifetime and the level width are related by

$$\tau \Gamma \approx \frac{h}{2\pi} \quad , \quad \equiv \quad \neq \quad (2.26.1)$$

where h is Planck's constant. The level width has the dimensions of energy and is usually expressed in electron volts. Making use of the conversion factor in 1.28, it follows that

$$\tau \Gamma \approx 0.7 \times 10^{-15} \quad (2.26.2)$$

where τ is in seconds and Γ in ev.

* This "indefiniteness" has nothing to do with the accuracy of the experimental methods. It is something fundamental to all measurements and may be considered as arising from the interaction of the measuring device with the system being measured.

2.27 Since a compound nucleus in a given excited state can frequently undergo change in several ways, e.g., emission of a neutron, proton, alpha particle or radiation, it is necessary to define a partial level width for each type of process. If τ_i is taken as the mean lifetime of the particular quantum state if the process i were the only possible way in which excitation energy could be lost, then the partial level width Γ_i for this process is given by

$$\tau_i \Gamma_i \approx \frac{h}{2\pi} \quad (2.27.1)$$

as in equation (2.26.1). The total level width for the given quantum state is then the sum of the partial level widths for all the possible processes which the compound nucleus in that state can undergo; thus,

$$\Gamma \approx \sum_i \Gamma_i \quad . \quad (2.27.2)$$

2.28 The level width has a simple and interesting physical significance: it is proportional to the probability that the compound nucleus, in the given energy state, will undergo change per unit time. As was seen in connection with radioactive disintegration (§ 1.22), the mean life of an unstable species is equal to the reciprocal of the decay constant, the latter representing the probability of decay per unit time. Since, by equation (2.26.1), the total level width is inversely proportional to the mean lifetime of the compound nucleus, the level width is related to the total probability that the latter will decay (or change) in unit time. Similarly, the partial level width Γ_i is a measure of the probability that the given excited quantum state will undergo the process i per unit time.

2.29 Level widths of about 0.1 ev have been observed for heavy nuclei which have captured neutrons of low energy, e.g., an electron volt or less. The mean lifetime of the compound nucleus, in such cases, is found from equation (2.26.2) to be about 7×10^{-15} sec. As stated earlier this is relatively long in comparison with the time required for a neutron to travel a distance equal to the nuclear diameter (2.12).

2.30 At high excitation energies, such as would result from the capture of a particle having a large kinetic energy, the level width is increased and the mean lifetime of the compound nucleus is correspondingly decreased. Thus, if Γ is 1000 ev, the mean lifetime would be 0.7×10^{-18} , which is of the same order as the time of transit of a nucleon. In these circumstances, the compound nucleus model, based on an exchange of energy among the nucleons, would break down. At the same time, the level width would exceed the spacing between the energy levels, especially for the nuclei of moderate mass (2.25). In other words, as far as experimental measurements were concerned, the energy levels would appear to overlap.

Resonance Absorption

Conditions for Resonance

2.31 In experimental studies of nuclear reactions, by bombarding different target elements with various projectiles, such as protons, neutrons, etc., it has been found that when the incident particles have certain specific energy values, there is a sharp increase in the reaction rate. In other words, for certain energy values the probability that the incident particle will be captured and a compound nucleus formed is exceptionally large. This phenomenon, which is very marked in connection

with nuclear reactions involving slow neutrons, is attributed to what is called resonance. For elements of moderate and high mass numbers, resonance absorption frequently occurs with neutrons of energy between roughly 1 ev and 10 ev. Uranium-238, for example, exhibits resonance absorption of neutrons with energies in the electron volt range.

2.32 It is generally accepted that a marked increase in the rate of the given nuclear reaction occurs when the energy of the incident particle is such that the resulting excited state of the compound nucleus is very close to one of the quantum states of the latter. This is what is meant by resonance absorption. The effect may be illustrated by considering Figure 2.32, the lines at the left indicating (schematically) the quantum levels of the compound nucleus. The line marked E_0 , at the right, represents the energy of the target nucleus plus that of a neutron with zero kinetic energy. Thus, the energy E_0 corresponds to that at the point C in Figure 2.15; it is above the ground state by an amount equal to the binding energy of the neutron in the particular compound nucleus (2.16).

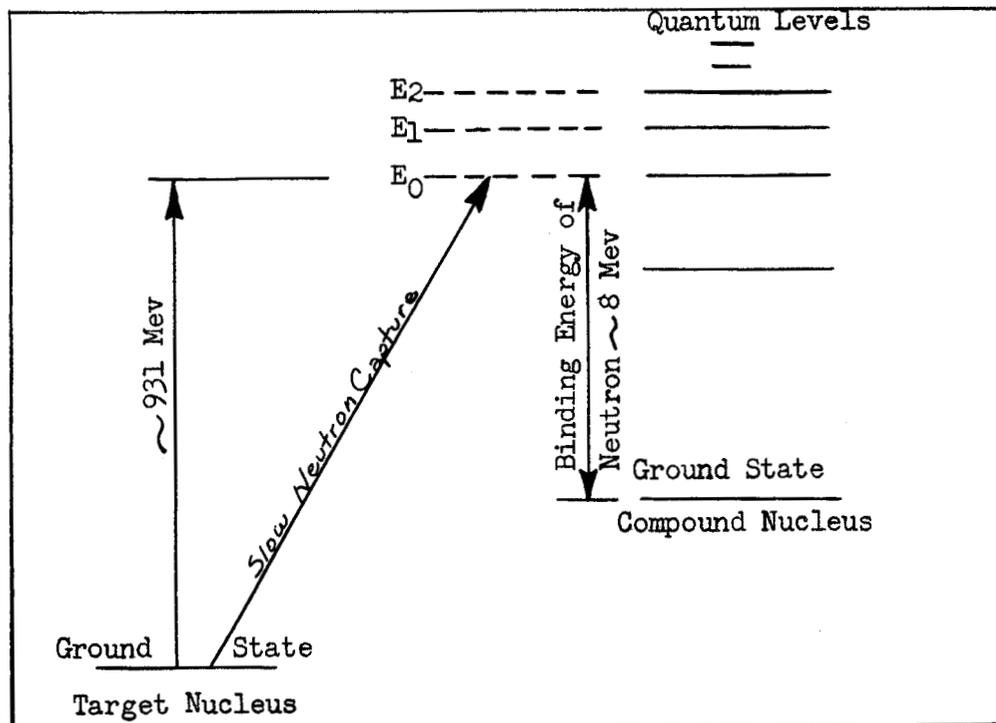


Figure 2.32

2.33 An examination of Figure 2.32 indicates that the energy E_0 does not correspond to any of the quantum levels of the compound nucleus. However, if the neutron has a certain amount of kinetic energy, sufficient to bring the energy of the system of target nucleus plus neutron up to E_1 , the energy of the compound nucleus will correspond to that of one of its quantum states. When the neutron has kinetic energy, $E_1 - E_0$, resonance absorption is said to occur. Similarly, there will be resonance absorption of neutrons with kinetic energy $E_2 - E_0$, so that the total energy E_2 of the system is equivalent to another quantum level of the compound nucleus, as seen in Figure 2.32.

2.34 According to quantum mechanics, the only states of a system which can be stable (or quasistable) are the definite quantum states. The probability of the formation of a compound nucleus in a given reaction will be greatest when its energy corresponds to that of one of its quantum states. Thus, when resonance absorption occurs the rate of the particular reaction will be markedly increased.

2.35 Experimentally, at least, as was seen in 2.26, the energies of the quantum level of a nucleus are not sharply defined, each level having a particular level width. More or less corresponding to this level width there is a spread of particle energies over which resonance absorption is observed. If the conditions are such, for example at high energies, that the level width is greater than the separation of the quantum levels, the spread will be so large that adjacent regions will overlap. In these circumstances, the concept of resonance absorption, like that of the compound nucleus (2.30), has no significance.

2.36 It was mentioned in 2.31 that the separation of the energy levels

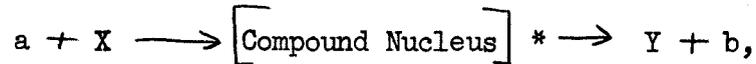
of a nucleus of moderate or high mass number in the 8-Mev region is about 1 ev to 10 ev. Hence, it is to be expected that resonance absorption will be observed for neutrons having certain kinetic energies of this order of magnitude. If the neutron has a large amount of energy e.g., about 1 Mev or more, the compound nucleus will have some 9 Mev of energy in excess of the ground state. In this region the level widths are frequently large enough, compared with the separations to make the resonance concept invalid.

2.37 For nuclei of low mass number the spacing of the energy levels in the 8-Mev region is larger than for species of moderate or high atomic weight. Hence, resonance absorption should occur only if the neutron has energy of the order of 10,000 ev., i.e., 0.01 Mev. Resonance effects of this kind have been observed, but the increased absorption is not very marked because, as will be seen later, there is a general tendency for the rate of absorption of neutrons to decrease as their energy increases.

The Breit-Wigner Formula

2.38 By applying the methods of wave mechanics to the compound nucleus concept, Breit and Wigner (2.11) derived an expression for the rates of nuclear reactions, including resonance absorption. The results are expressed in terms of a quantity called the nuclear cross section, represented by the symbol σ , which will be considered in some detail later, (3.38). For the present it is sufficient to state that the cross section is a measure of the probability of the occurrence of a particular nuclear reaction under prescribed conditions. It is a specific property of that reaction for incident particles of a given energy.

2.39 Consider a nuclear reaction represented, in general, by



where a is the incident particle, X the target nucleus, Y the residual nucleus and b the ejected particle. Let Γ_a and Γ_b be the level widths representing the probabilities of the emission of the particles a and b , respectively, by the compound nucleus in a specific quantum state (§ 2.28); the total level width Γ is then the sum of the partial widths, as stated in 2.27. If E is the total energy, both internal and kinetics, of the incident particle, and E_r is the energy value which would give exact resonance with the specified quantum level of the compound nucleus, then the Breit-Wigner formula for the variation of the nuclear cross section σ for the given reaction with the energy of the incident particle is represented (approximately) by

$$\sigma_{ad} \approx \frac{\lambda^2}{4\pi} \cdot \frac{\Gamma_a \Gamma_b}{(E - E_r)^2 + \frac{1}{4} \Gamma^2}, \quad (2.39.1)$$

where λ is the equivalent wave length of the incident particle as derived from its mass and velocity by the de Broglie equation (2.8.1).^{*} For simplicity, a factor allowing for the angular momenta of the nuclei and the spins of the particles has been omitted; it is generally of the order of unity.

2.40 The Breit-Wigner equation (2.39.1) is frequently referred to as the "one-level" formula. It applies to energies in the vicinity of any quantum level provided the latter is sufficiently widely separated from adjacent levels, so that the resonances do not interfere with one another. A one-level Breit-Wigner formula, with the appropriate values of Γ_a , Γ_b ,

^{*} In determining λ from the de Broglie equation, the mass m should be the "reduced mass" of the incident particle and the target nucleus. For nuclei of moderate or high mass number this is essentially the same as the mass of the incident particle.

Γ and E_r , will then apply to each resonance region or quantum state. When the resonances overlap, because of the small spacing of the quantum levels, a more complex equation must be used, but for present purposes the one-level formula is adequate.

Applications of Breit-Wigner Equation

2.41 In applying the Breit-Wigner equation to the resonance absorption of neutrons, a matter of particular interest in connection with nuclear reactors, the particle "a" is a neutron. Since this has no internal energy, E in equation (2.39.1) becomes merely the kinetic energy of the neutron. Further, by equation (2.28.2), the wave length λ is proportional to $1/\sqrt{E}$, where E is the kinetic energy. Hence, when the incident particle is a neutron,

$$\lambda^2 = k/E, \quad (2.41.1)$$

where k is a constant.

2.42 The level width Γ_b , representing the probability of the ejection of the particle "b", is believed to be independent of the energy of the incident particle, but Γ_a , which is a measure of the probability of the re-emission of the latter, can be shown to be proportional to its velocity. If "a" is a neutron, so that E is merely the kinetic energy, it follows, therefore, that

$$\Gamma_a = \Gamma_n = k' \sqrt{E}, \quad (2.42.1)$$

where k' is a constant. Upon inserting equations (2.41.1) and (2.42.1) into the Breit-Wigner equation (2.39.1), and combining the constants into A , it follows that

$$\sigma = \frac{A}{\sqrt{E}} \cdot \frac{\Gamma_b}{(E-E_r)^2 + \frac{1}{4} \Gamma^2} \quad (2.42.2)$$

This expression gives the variation of the cross section for neutron absorption with the energy E of the neutron, in the vicinity of a particular resonance widely separated from others. The quantities A , \int_b , E_r and \int are then constants.

2.43 An examination of equation (2.42.2) reveals a number of interesting qualitative features. When the neutron energy E is considerably less than the value E_r required for exact resonance, $(E-E_n)^2$ is large and almost constant. Equation (2.42.2) then reduces to the form

$$\sigma \approx \frac{B}{v} , \quad (2.43.1)$$

where B is a composite constant. Thus, at neutron energies that are small compared to the first resonance energy, the neutron absorption cross section will be inversely proportional to the neutron velocity energy. As indicated by the Breit-Wigner treatment, a low-energy range, in which the neutron absorption cross section is proportional to $1/v$, generally called the "1/v region", is often observed for slow neutrons (§ 3.68).

2.44 As the neutron energy E is increased and approaches E_r , it is evident from equation (2.42.2) that σ should increase rapidly, since $E-E_r$ becomes smaller. When E is equal to E_r the absorption cross section will be maximum. Subsequently, when E exceeds E_r , the cross section will decrease, at first sharply and then more slowly, with increasing neutron energy. This means that if σ is plotted as a function of E , there will be a fairly sharp peak, the maximum of which corresponds to the resonance energy E_n (Figure 2.44). Such peaks, called resonance peaks, are of frequent occurrence, as will be seen later. It will be noted from equation (2.42.2) that if \int_b does not vary greatly with energy, the maximum

value of the absorption cross section, which is given by

$$\sigma_{\text{max}} \approx \frac{A}{\sqrt{E_{\text{res}}}} \cdot \frac{b}{\frac{1}{4} \Gamma^2} \quad (2.44.1)$$

is approximately inversely proportional to the square root of the energy.

Thus, the cross section at the resonance absorption maximum will tend to be small when the resonance energy is large, and vice versa.

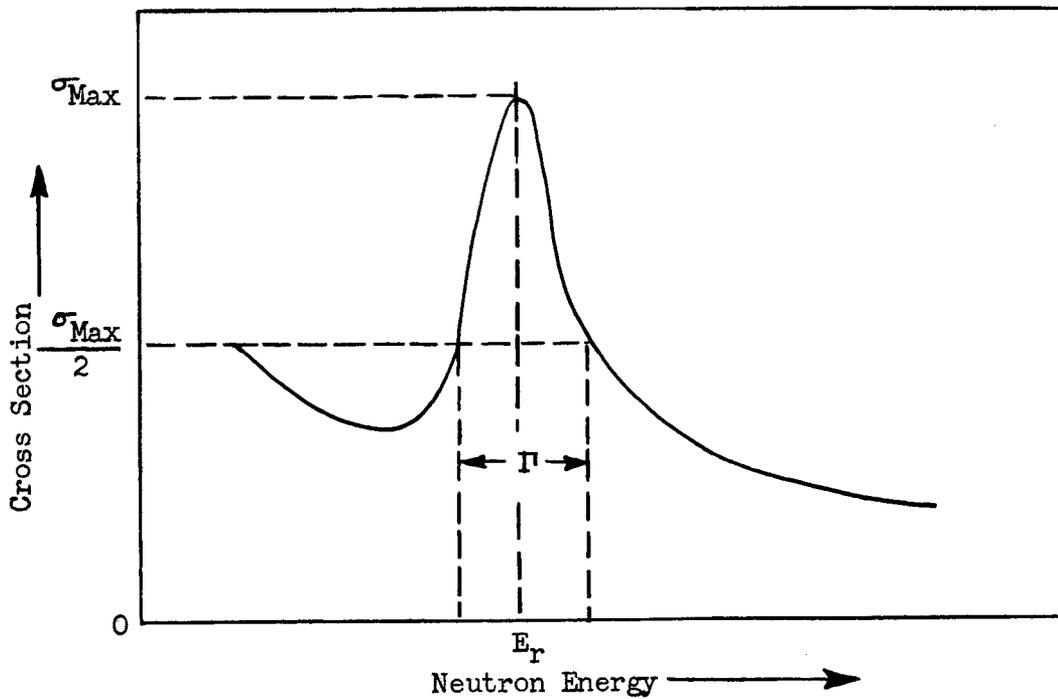


Figure 2.44

2.45 It can be readily shown from equation (2.42.2) and (2.44.1) that when the neutron absorption cross section σ is equal to $\frac{1}{2} \sigma_{\max}$, the width in ev of the resonance peak is equal to Γ , the total level width. For this reason, Γ is sometimes called the half width of the resonance peak.

2.46 Another special case of interest arises when the total width Γ of a particular level is large compared to $E - E_r$. Then $(E - E_r)^2$ in the denominator of equation (2.42.2) may be neglected in comparison with $\frac{1}{4} \Gamma^2$, and the expression for the neutron absorption cross section becomes

$$\sigma \approx \frac{A}{v} \cdot \frac{\Gamma b}{\frac{1}{4} \Gamma^2}, \quad (2.46.1)$$

so that the cross section is again inversely proportional to the neutron velocity. In other words, the $1/v$ law for neutron absorption will apply either (a) when the level width Γ is small and the neutron energy E is appreciably less E_r (§ 2.43), or (b) when the level width Γ is large compared to $E - E_r$. An example of the latter type of behavior will be given in § 3.77.

2.47 It should be noted that when Γ is large compared to E_r , there is no cross section maximum and no resonance peak in the plot of σ against E . The absorption cross section decreases steadily with increasing neutron energy, in accordance with equation (2.46.1). In these circumstances, it is not the practice to speak of resonance capture of neutrons.

Scattering of Neutrons

The Nature of Scattering

2.48 A possibility that has not been considered so far is that the

particle expelled in a nuclear reaction is identical with the incident particle captured. In the course of the redistribution of energy among the nucleons in the compound nucleus there is always a possibility that a particle of the same type as the incident particle will acquire enough energy to leave the nucleus. In general, the expelled particle will have less kinetic energy than the incident particle, some or all of its energy having been transferred to the target nucleus. A process in which the overall result is merely the transfer of energy from one particle (or nucleus) to another is called scattering. The scattering of neutrons by atomic nuclei plays a highly important part in the operation of nuclear reactors. Consequently, some general aspects of the subject will be considered here while a more extended treatment will be given in Chapter VI.

2.49 There are two types of scattering processes, namely, inelastic scattering and elastic scattering. In inelastic collision momentum is conserved, but kinetic energy is not, while in elastic collisions both momentum and kinetic energy are conserved.

Inelastic Scattering

2.50 When a neutron undergoes inelastic scattering, it is first captured by the target to form a compound nucleus; a neutron of lower kinetic energy is then expelled, leaving the target nucleus in an excited state. Thus, in inelastic scattering some (or all) of the kinetic energy of the neutron is converted into internal or excitation energy of the target nucleus. This energy is subsequently emitted in the form of gamma radiation, the target nucleus thereby returning to its ground state.

2.51 It was noted in § 2.25 that the spacing of nuclear energy levels near the ground state is about 0.1 Mev for nuclei of moderate or high mass number, but it is larger for nuclei of low mass number. Consequently, a neutron must possess at least 0.1 Mev of energy if it is to be involved in an inelastic collision process. If the scattering material has a low mass number, the required neutron energy is even higher. In nuclear reactors, the neutrons initially have high energies in the Mev range, and hence inelastic scattering then occurs to some extent. However, as will be seen later, the energy of the neutrons is soon reduced to values at which inelastic scattering is not possible.

Elastic Scattering

2.52 The situation in regard to elastic scattering is quite different. In this type of collision kinetic energy is conserved. Some (or all) of the kinetic energy of the neutron appears, after the collision, as kinetic energy of the initially stationary target nucleus. The process may be regarded as essentially a "billiard ball" type of collision, which may be treated by the laws of classical mechanics, based on the principles of the conservation of energy and momentum. In each collision with an essentially stationary nucleus, the neutron will transfer part of its kinetic energy to the nucleus; the amount of energy transferred will depend upon the angle through which the neutron is scattered. For a given scattering angle, the fraction of the neutron energy transferred will be greater the smaller the mass of the scattering nucleus(Chapter VI).

2.53 From the theoretical standpoint there are two aspects of elastic scattering of neutrons to be considered. First, there is resonance scattering, when the energy of the captured neutron is such that the excited compound nucleus formed is at or close to one of its quantum states.

And second, there is what is called potential scattering, which occurs for neutrons with energies on either side of the resonance level. In the language of wave mechanics, potential scattering is treated as being due to the interaction of the neutron wave with the potential at the nuclear surface. Effectively, the incident neutron then does not enter the target nucleus and there is no compound nucleus.

2.54 In resonance scattering of neutrons, a compound nucleus is formed which can be treated by the Breit-Wigner method and an expression derived for the corresponding cross section. As a rough indication of the behavior, it may be assumed that equation (2.39.1) is applicable, and since the particles a and b are both neutrons, this becomes

$$\sigma \approx \frac{\lambda^2}{4\pi} \cdot \frac{\Gamma_n^2}{(E-E_r)^2 + \frac{1}{4}\Gamma^2}, \quad (2.54.1)$$

where E is the kinetic energy of the incident neutrons. In this case, the level width Γ_n is essentially the same for the incident and outgoing neutrons, and each is proportional to \sqrt{E} , by equation (2.42.1); hence Γ_n^2 is proportional to E . Further, since λ^2 is inversely proportional to E , by equation (2.41.1), it is seen that $\lambda^2 \Gamma_n^2$ is approximately constant; consequently, equation (2.54.1) becomes

$$\sigma = \frac{A}{(E-E_r)^2 + \frac{1}{4}\Gamma^2},$$

where A is a constant. When E is equal to E_r , the resonance scattering cross section will be a maximum, but when $E \ll E_r$, i.e., the neutron energy is appreciably less than the resonance value, it is evident that the cross section will be independent of the neutron energy.

2.55 Except near resonance, the resonance scattering is usually much less than the potential scattering, and the cross section for the latter does not vary greatly with the neutron energy. While the resonance scattering at or near resonance does depend on the neutron energy, the variations are not great and the cross sections are of the same order of magnitude as for potential scattering. Consequently, it is generally accepted that the total elastic scattering cross section is independent of the kinetic energy of the incident neutrons. This is especially true for neutrons with energies less than about 0.1 Mev when scattered by nuclei of fairly low mass number, a situation of common occurrence in the study of nuclear reactors.

CHAPTER III

PRODUCTION AND REACTIONS OF NEUTRONS

Production of Neutrons

Alpha Particles and Light Nuclei

3.1 The neutron was first identified as a result of the interaction of alpha particles emitted by radioactive material, on the light elements beryllium boron and lithium. Subsequently it was found that other elements of low atomic number expel neutrons when bombarded by alpha particles. In the case of beryllium for example, the reactor may be represented by



where the subscript gives the atomic number, i.e., the nuclear charge, and the superscript the mass number in each case.* Since a neutron has a mass of unity but no charge, it is represented by the symbol ${}_0\text{n}^1$. The sum of the atomic numbers, i.e., the total number of protons, must be the same on each side of the equation. Similarly the mass numbers, i.e., the total number of nucleons, must balance.

3.2 A combination of an alpha emitter, such as radium or polonium, and a light element, such as beryllium or boron, makes a very simple, compact and useful source of neutrons for laboratory purposes. A mixture of 5 grams of beryllium and 1 gram of radium, for example, emits about 10 to 15 million neutrons per sec. Because of its long radioactive half life,

* The reaction may be written in the abbreviated form $\text{Be}^9(\alpha, \text{n})\text{C}^{12}$. In this method of representation, the first symbol, i.e., Be^9 , is the target nucleus; then, in the parentheses, are the incident particle (α) and the ejected particle (n); and, finally, the symbol of the residual nucleus, i.e., C^{12} , is given. Reactions are frequently described in terms of the particles involved; thus, an (n, α) reaction is one in which a neutron is captured and gamma radiation emitted. The symbol p is used to indicate a proton, i.e., a hydrogen nucleus.

about 1600 years, a radium-beryllium neutron source is virtually constant and permanent. Its chief disadvantages are the high cost of the radium and the strong gamma radiation which it emits. Polonium is frequently employed, with beryllium, in place of radium; the cost and gamma radiation are greatly reduced, but so also is the life of the neutron source.

3.3 The neutrons produced by the action of alpha particles on beryllium have moderately high energies, the minimum being about 5 Mev, and the maximum extending up to 12 Mev or more, depending on the energy of the incident particles. The sources described above are thus polyenergetic, the energy spectrum covering the range from about 5 Mev to 12 Mev. There may be, in addition, neutrons of energy lying outside this range which are produced by reactions due to gamma rays from the alpha-particle emitter, as will be seen in the next paragraph.

Photoneutron Sources

3.4 If monoenergetic neutrons, that is, neutrons of (approximately) the same energy, are required, use may be made of certain photoneuclear reactions. These are reactions in which gamma rays interact with a nucleus, transferring energy to the latter; the excited nucleus so formed may then eject a particle, such as a neutron. Photoneutron sources, as they are called, involve reactions represented by the symbol (γ, n) , since the gamma ray photon (γ 2.11) is the incident particle and a neutron is expelled.

3.5 Two (γ, n) processes which are possible with gamma radiation from available radioactive substances are the following:



and



In the former, the target element is beryllium and in the latter it is deuterium, the heavier (stable) isotope of hydrogen, having mass number 2. These reactions are of special interest in connection with the operation of nuclear reactors, as will be indicated later (§ 4.80).

3.6 The minimum threshold energy of the gamma rays necessary to bring about these (γ, n) reactions is 1.6 Mev for the beryllium reaction and 2.21 Mev for the deuterium reaction. Thus, gamma rays with energies less than these amounts will be unable to produce neutrons by the respective reactions. Any energy of the gamma ray in excess of the threshold energy will then appear mainly as kinetic energy of the emitted neutron, some being carried off by the recoil nucleus. Since gamma rays from a given radioactive source usually have a definite energy, so also will the emitted neutrons; consequently, when the latter are produced by the photo-nuclear (γ, n) reaction, they are essentially monoenergetic.

3.7 Possible sources of gamma radiation for use with neutron sources are the naturally occurring elements radium and mesothorium. The artificially produced isotopes Na^{24} , Ga^{72} , Sb^{124} and La^{140} are considerably cheaper although they have much shorter half lives. The target materials are beryllium metal and heavy water, i.e., water enriched in the heavier isotope of hydrogen. The lowest energy of neutrons obtainable in this manner is 0.03 Mev from the $\text{Sb}^{124}\text{-Be}$ reaction, and the highest is 0.88 Mev from the Ra^{228} or Ac^{228} process.

3.8 A simple and convenient source of approximately monoenergetic neutrons is available from the Isotopes Division of the Atomic Energy Commission. It consists of a rod of antimony, containing the radioactive Sb^{124} isotope, surrounded by a beryllium metal cup. When newly prepared the system emits neutrons of roughly uniform energy, 0.03 Mev, at the

rate of about 8 million per sec. The Sb^{124} has a half life of 60 days, and when the activity has decayed to such an extent that the neutron source has become appreciably weakened, it can be regenerated by exposure of the antimony rod to neutrons in a nuclear reactor.

Use of Accelerators

3.9 In addition to the relatively compact neutron source described above, there are various methods for producing neutrons, especially of fairly uniform energy, by using incident particles which have been accelerated by means of a cyclotron or, better, by a Van de Graaff machine, since the latter more readily yields particles of uniform energy. The action of accelerated protons, i.e., hydrogen nuclei, on lithium, or of deuterons, i.e., deuterium nuclei, on targets of lithium, beryllium or deuterium (as "heavy" ice or "heavy" paraffin) results in the formation of neutron beams of fairly uniform energy. The actual energy depends on that of the incident particles used, as well as on the particular process employed.

Slowing Down of Neutrons

Scattering and Moderation

3.10 All the sources described above yield neutrons of fairly high kinetic energy, usually in the Mev range. Such neutrons are referred to as fast neutrons. It was mentioned in § 2.52 that, as a result of elastic scattering collisions, neutrons can be deprived of some (or all) of their kinetic energy and are thereby slowed down. Slow neutrons, and particularly neutrons with energies in the vicinity of 0.025 ev at ordinary temperatures, called thermal neutrons (§ 3.13), are of importance in connection with nuclear reactors.

3.11 It will be seen in Chapter VI that, for a given scattering angle, the fractional decrease in the kinetic energy of a neutron in an elastic collision is greater the smaller the mass number of the scattering nucleus. This means that fast neutrons are slowed down most effectively by scattering in a medium containing nuclei of low mass number.

3.12 The slowing down of neutrons plays a significant part in most nuclear reactors, and the material used for the purpose is called a moderator. The process of slowing down neutrons as a result of scattering collisions is sometimes referred to as moderation. A good moderator is consequently a material which reduces the speed of fast neutrons in a small number of collisions; it will obviously consist of atoms of low mass number. Thus, ordinary water (H_2O), heavy water (D_2O), beryllium and carbon have been used as moderators in various reactors.*

3.13 After a number of scattering collisions, the velocity of a neutron is reduced to such an extent that it has approximately the same average kinetic energy as the atoms, or molecules, of the medium in which it is undergoing elastic scattering. As will be seen below, the energy depends on the temperature of the medium, and hence it is called thermal energy. Neutrons whose energies have been reduced to values in this region are designated thermal neutrons, and the process of reducing the energy of a neutron to the thermal region is known as thermalization. Neutrons with energies above thermal values are sometimes referred to as epithermal neutrons.

*A good moderator must not absorb neutrons to any great extent; consequently, the light elements lithium and boron, which absorb slow neutrons very strongly (λ 3.76), are not used as moderators.

Maxwell-Boltzmann Distribution

3.14 Thermal neutrons may strictly be defined as neutrons that are in thermal equilibrium with the atoms (or molecules) of the medium in which they are present. A particular thermal neutron undergoing collisions with the nuclei of the medium may gain or lose energy in any one collision. But, if a large number of neutrons diffusing in a non-absorbing medium are considered, there is no net energy change for all the neutrons. The kinetic energies of the neutrons will then be distributed statistically according to the Maxwell-Boltzmann distribution law, as derived from the kinetic theory of gases; thus,

$$\frac{dn}{n} = \frac{2 \pi}{(\pi kT)^{3/2}} e^{-E/kT} E^{1/2} dE, \quad (3.14.1)$$

where dn is the number of neutrons with energies in the range from E to $E + dE$, n is the total number of neutrons in the system, k is the Boltzmann constant, and T is the temperature on the Kelvin scale.

3.15 Since most media absorb neutrons to some extent, and the absorption cross sections, apart from resonance peaks, increase with decreasing energy, the Maxwell-Boltzmann distribution does not apply. The greater absorption of the slow neutrons results in an increase in the average energy above that to be expected from equation (3.14.1), the extent of the deviation increasing with the distance from the neutron source. This phenomenon is called hardening. However, provided the medium is not too strongly absorbing, the Maxwellian distribution may be assumed to be valid for thermal neutrons.

3.16 In conformity with a symbolism to be used later (§ 3.50), let $n(E)$ be the number of neutrons of energy E per unit energy interval*

* The volume of the system is usually taken as 1 cm^3 , but for present purposes the volume is immaterial.

Then $n(E) dE$ is the number of neutrons having energies in the range from E to $E + dE$; this is equivalent to dn in equation (3.14.1), and hence the latter may be written as

$$\frac{n(E)}{n} dE = \frac{2 \pi}{(\pi kT)^{3/2}} e^{-E/kT} E^{1/2} dE,$$

or

$$\frac{n(E)}{n} = \frac{2 \pi}{(\pi kT)^{3/2}} e^{-E/kT} E^{1/2}, \quad (3.16.1)$$

where the left-hand side represents the fraction of the neutrons having energies in the range from E to $E + dE$ per unit energy interval. The right-hand side of the equation can be evaluated for various E 's at a given temperature, and the plot obtained in this manner of $n(E)$ against the kinetic energy E of the neutrons is represented in Figure 3.16.

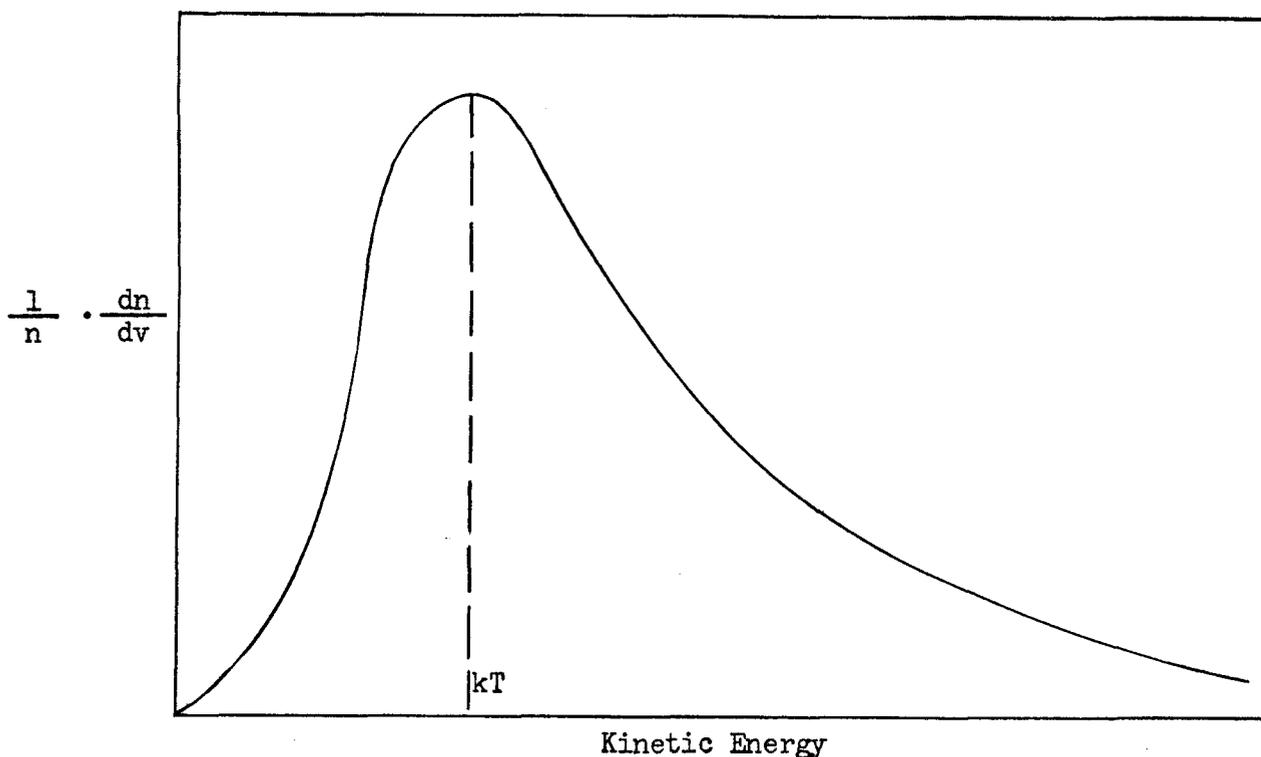


Figure 3.16

3.17 In the study of thermal neutrons, it has become the practice to state the energy as kT for the particular temperature T . Actually, this represents the kinetic energy corresponding to the most probable velocity for unit velocity.* The average kinetic energy of thermal neutrons, according to the Maxwell equation, is $(3/2) kT$.

3.18 Expressing energy, as usual, in electron volts, the Boltzmann constant k has the value 8.6×10^{-5} ev per degree; consequently, it is possible to write for thermal neutrons.

$$\text{"energy of thermal neutrons"} = 8.61 \times 10^{-5} T \text{ ev.} \quad (3.18.1)$$

A number of values for a series of temperatures are recorded in Table 3.19; at ordinary temperature, i.e., about 25°C or 298°K , the "energy of thermal neutrons" is approximately 0.025 ev.

3.19 The speed v of a neutron in cm per sec is related to its kinetic energy E by the equation

$$v = 13.8 \times 10^5 \sqrt{E} \text{ cm per sec,} \quad (3.19.1)$$

where E is expressed in electron volts. This result may be combined with equation (3.18.1) to give the most probable speed of thermal neutrons as a function of the temperature; thus,

$$\text{Most probable speed} = 1.28 \times 10^4 \sqrt{T} \text{ cm per sec.}$$

At room temperature, T is about 298°K and the most probable speed of thermal neutrons is then found to be 2.22×10^5 cm per sec. Values for other temperatures are given in Table 3.19.

* Since $E = (1/2) mv^2$, where v is the velocity, the Maxwell equation (3.16.1) can be written as

$$\frac{n(v)}{n} = 4 \pi \left(\frac{m}{2 \pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT}$$

if this is differentiated with respect to v and the result set equal to zero, the most probable velocity per unit velocity is equal to be $(2kT/m)^{1/2}$. The corresponding energy is then kT .

Energies and Speeds of Thermal Neutrons at Various Temperatures		
Temperature	Energy	Most Probable Velocity
300° K 27° C	0.026 ev	2.2 x 10 ⁵ cm/sec
400 127	0.034	2.6
600 327	0.052	3.1
800 527	0.069	3.6
1000 727	0.086	4.0

Table 3.19

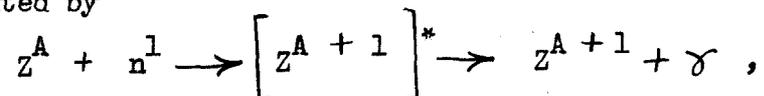
Reactions of Slow Neutrons

Types of Capture Reactions

3.20 Apart from scattering, slow neutrons undergo four types of capture reactions with atomic nuclei; these involve either (a) the emission of gamma radiation (n, γ); (b) the ejection of an alpha particle (n, α); (c) the ejection of a proton (n, p); or (d) fission (n, f). Of these, the radiative capture, i.e., (n, γ), process is the most common, for it occurs with a wide variety of nuclides from low to high mass numbers. The (n, α) and (n, p) reactions with slow neutrons are limited to a few elements of low mass number, while fission by slow neutrons is restricted to certain nuclei with high mass number.

Radiative Capture

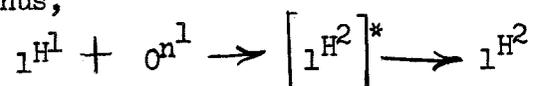
3.21 In radiative capture reactions, the target nucleus captures a slow neutron, and produces a compound nucleus in an excited state ($\} 2.14$). The excess energy is then emitted in the form of one or more gamma rays leaving the compound nucleus in its normal or ground state. The process may thus be represented by



where A is the mass number and Z is the atomic number of the target nucleus; the symbol $[Z^{A+1}]^*$ represents the compound nucleus in the excited state. The residual nucleus or product is Z^{A+1} , that is, a nucleus having the same atomic number as the target nucleus, but with a mass number one unit greater.

3.22 Since the capture of a neutron by a nucleus, followed by emission of gamma radiation, must be associated with an increase in the neutron-to-proton ratio, the product of an (n, γ) reaction is likely to be radioactive, especially if the ratio of neutrons to protons in the target nucleus is already near the upper limit of stability for the given atomic number. As seen above, the latter remains unchanged in the radiative capture of a neutron, while the number of neutrons increases by unity. If the product nucleus is unstable, it will almost always be a negative beta emitter, since this mode of decay means that the extra neutron is replaced by a proton (\S 1.16). In cases of this kind, the occurrence of the neutron capture reaction can be detected experimentally by the resulting radioactivity. The procedure is frequently used in various measurements with slow neutrons (\S 3.60, et seq.).

3.23 The simplest (n, γ) reaction occurs with hydrogen as the target nucleus; thus,



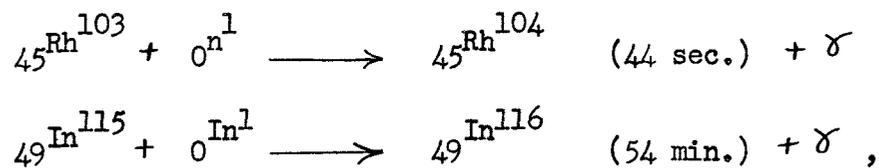
the product being deuterium. It will be seen that this process is exactly the reverse of that given in 3.5, for the action of gamma rays on deuterium as a source of neutrons. Since the minimum or threshold energy in that case is known to be 2.21 Mev, it follows that the energy of the gamma radiation resulting from the (n, γ) reaction with hydrogen

will have at least this value. The emission of such radiation, of relatively high energy and penetrating power, when neutrons pass through materials containing hydrogen, has been confirmed experimentally. Cognizance must be taken of this fact when such substances, e.g., concrete water, etc., are used in connection with nuclear reactors, either as a means for slowing down neutrons, as a coolant or as a shield to prevent the escape of neutrons.

3.24 When a slow neutron is absorbed in an (n, γ) reaction, the excitation energy of the compound nucleus above its ground state is approximately equal to the binding energy of the neutron in the compound nucleus, as was seen in § 2.14. Consequently, if the excited compound nucleus passes directly to the ground state by the emission of gamma radiation, the energy of the latter should be equal to the binding energy of the neutron. For example, in the case of the (n, γ) reaction with hydrogen referred to above, the binding energy of the neutron in the compound nucleus, i.e., in deuterium, is known to be about 2.21 Mev from the nuclear masses. This is in complete agreement with the measured energy of the radiation in the $H^1 (n, \gamma) H^2$ reaction, and with the threshold energy for the reverse process. With heavier target nuclei the slow neutron (n, γ) reaction leads to the formation of compound nuclei with higher excitation energies, and the so-called capture gamma rays may have energies of about 8 to 9 Mev.

3.25 The radiative capture of neutrons has been used extensively for the production of isotopes by exposing stable nuclides to the action of slow neutrons in a reactor. Over a hundred (n, γ) reactions leading to beta-emitting isotopes have been reported. Two of these, in which

rhodium-103 and indium-115, respectively, are the target nuclei, i.e.,

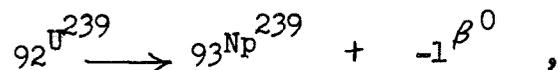


are of particular interest since they are used for the detection of neutrons of more or less specific energies, as will be described in 3.84. The half-life of the radioisotope formed as product is given in each case.

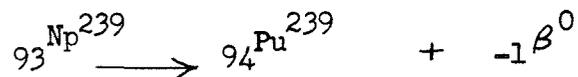
3.26 Perhaps the most notable of all (n, γ) reactions with slow neutrons is that undergone by uranium-238; thus,



The product, uranium-239, has a half life of 23 min., emitting negative beta particles (electrons), represented by ${}_{-1}\text{B}^0$ (charge -1, mass essentially zero); thus,



the daughter being an isotope of an element of atomic number 93, called neptunium (Np), which does not exist in nature to any detectable extent. Neptunium-239, with a half life of 2.3 days, is itself beta active, decaying by the process

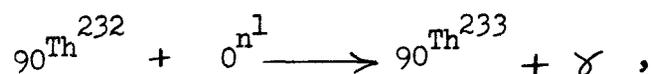


to form the isotope Pu^{239} of the element of atomic number 94, called plutonium (Pu).

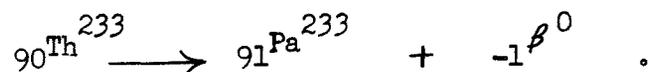
3.27 The element plutonium occurs naturally in the merest, almost

undetectable, traces.* Nevertheless, plutonium-239, which is in a sense an isotope of an artificial element, is being produced in appreciable quantities in nuclear reactors, as the result of the radiative capture of neutrons by uranium-238. The immediate product then undergoes two relatively rapid stages of beta decay, as stated above, forming the alpha emitter plutonium-239. The latter has a half life of 24,000 years, and is consequently relatively stable. It is an important substance from the standpoint of the release of nuclear energy, being used in atomic bombs.

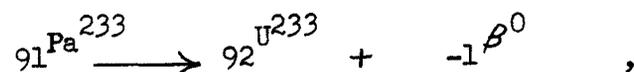
3.28 A series of processes similar to those just described is initiated by the (n, α) reaction with thorium-232; thus,



the isotope thorium-233 being formed. This is known to have a half life of 23 min, and the product resulting from negative beta decay is protactinium-233, i.e.,



The Pa^{233} is also a beta emitter, with a half life of 27.4 days, the decay process being



so that the daughter element is a new isotope of uranium which does not occur in nature, at least to any appreciable extent. It is radioactive, emitting alpha particles and having a half life of 1.63×10^5 years.

Thus, bombardment of thorium-232 by neutrons, and allowing time for the product to decay through two stages of beta activity, leads to the formation of the relatively stable uranium-233. This isotope, like the

* Such amounts of plutonium-239 as do exist in nature are believed to be found as a result of the capture of neutrons by uranium-238 and subsequent two-stage decay of the product, as described in § 3.26.

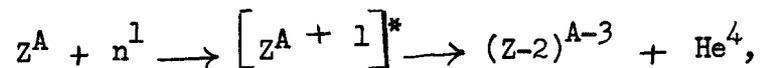
artificially produced plutonium-239 described in the preceding paragraph is of significance in connection with the problems of nuclear energy, i.e., "breeding".

Emission of Alpha Particles and Protons

3.29 Slow-neutron reactions accompanied by the emission of a charged particle, e.g., an alpha particle or a proton, are rare. The reason is that before a positively charged particle can be expelled from a nucleus it must acquire sufficient energy to overcome an electrostatic potential barrier, in addition to the energy needed for its detachment from the compound nucleus. Part of the requisite energy is provided by the addition of the neutron to the target nucleus; the remainder must be supplied by the kinetic energy of the neutron.

3.30 Since the kinetic energy of a slow neutron is very small, it is apparent that (n, α) and (n,p) reactions, with slow neutrons, can occur only when the electrostatic repulsion which the charged particle must overcome is small. This is the case for elements of low atomic number, and hence it is with a few species of this kind that (n, α) and (n,p) reactions with slow neutrons have been observed.

3.31 The (n, α) reactions may be written in the general form



where He^4 represents an alpha particle, i.e., a helium nucleus, with a mass number of 4 and an atomic number of 2. The recoil nucleus now has a mass number three units less and an atomic number two units less than the target nucleus. The absorption of slow neutrons by lithium-6 (Li^6), the rarer, naturally-occurring isotope of lithium, and by boron-10

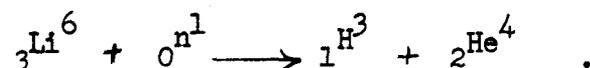
(B^{10}), the less common isotope of boron, lead to the emission of alpha particles. Both of these reactions have a special interest in the present connection.

3.32 The (n, α) reaction of boron-10 with slow neutrons may be represented by



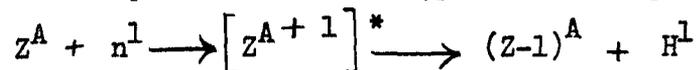
the recoil nucleus being the stable isotope lithium-7. This reaction is accompanied by the liberation of 2.5 Mev of energy, which is shared between the alpha particle and the nucleus. Both particles are therefore ejected, in opposite directions, with high velocities, so that they produce considerable ionization in their passage through a gas. It will be seen later that the (n, α) process with boron-10 is significant in several respects. It is utilized, for example, in an important method for the detection and counting of slow neutrons (§ 3.80), and also in the control of nuclear reactors (§ 4.73).

3.33 The other (n, α) reaction, which takes place readily with slow neutrons, is

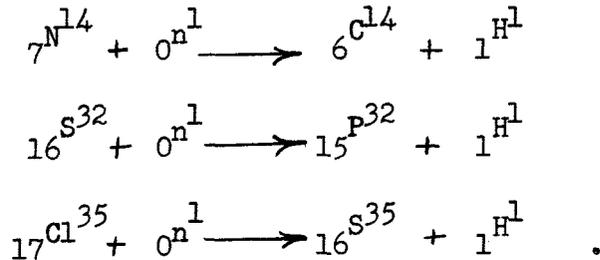


The residual (recoil) nucleus is here H^3 , a hydrogen isotope of mass number 3 called tritium. The isotope is radioactive, having a half life of about 12 years, and emitting a negative beta particle. Tritium has attracted attention because of its possible use in connection with the so-called "hydrogen bomb", as well as for other reasons.

3.34 The general representation of (n,p) reactions given by



so that the product has the same mass number as the target nucleus, but its atomic number is one unit less. A few isotopes of low atomic number, notable nitrogen-14, sulfur-32, and chlorine-35, undergo (n,p) reactions with slow neutrons; thus,



These processes can be carried out by exposing the respective elements to slow neutrons in a nuclear reactor. The products are all radioactive, emitting beta-particles; they have found many applications in investigations which make use of radioactive isotopes as tracers.

Fission

3.35 Another type of reaction caused by neutrons, which will be considered in more detail in Chapter IV, is nuclear fission. In the fission process the nucleus absorbs a neutron and the resulting compound nucleus is so unstable that it immediately breaks up into two more or less equal parts. Some nuclei, such as uranium-233, uranium-235, and plutonium-239, will readily undergo fission with slow neutrons, but others require fast neutrons. There are many different ways in which fission of a particular nucleus takes place, but in only a small proportion of the fissions does the nucleus break up in a symmetrical manner. This and other aspects of fission will be treated more fully below.

Reactions with Fast Neutrons

Capture and Fission Reactions

3.36 Reactions of fast neutrons with matter, other than scattering and fission, are not of major importance for the study of nuclear reactors; hence, they will be referred to only briefly here. Provided the energy is available, the expulsion of a charged particle from the excited compound nucleus is more probable than the emission of radiation. Thus, (n, α) and (n, p) reactions of nuclei with fast neutrons, with energies of 1 Mev or more, frequently occur more readily than the (n, γ) reaction. If neutrons of sufficiently high energy are used as projectiles, two or more nucleons may be expelled from the compound nucleus. For incident neutrons of energy of about 10 Mev, it is possible for two neutrons or a neutron and a proton to be emitted. Such reactions, which are not uncommon, are designated $(n, 2n)$ and (n, np) , respectively. If the neutron energy is still higher, processes such as $(n, 3n)$, $(n, 2np)$, etc., are possible.

3.37 Several nuclei which do not undergo fission by slow neutrons suffer this reaction as a result of the capture of fast neutrons. Thus, uranium-238 and thorium-232 require neutrons of about 1 Mev energy to cause fission at an appreciable rate. By the use of neutrons of very high energy, e.g., 100 Mev or more, the fission of a number of normally stable nuclei, such as bismuth, lead, thallium, mercury, gold, etc., has been achieved. Such fission, however, does not appear to have any immediate practical interest.

Neutron Cross Sections

Significance of Cross Section

3.38 The description of the interaction of neutrons with atomic nuclei can be made quantitative by introducing the concept of cross sections, defined generally in § 2.38. If a given material is exposed to the action of neutrons, the rate at which any particular nuclear reaction occurs depends on the number of neutrons, their velocity, and the number and type of nuclei in the given material. The cross section of a target nucleus for any given reaction is a property of the nucleus and of the energy of the incident neutron.

3.39 Suppose a uniform beam of I neutrons per cm^2 impinges perpendicularly, in a given time, on a layer one atom thick of target material containing N_a atoms per cm^2 , and let A be the number of individual nuclear processes, e.g., neutron absorptions, occurring per cm^2 , in that time. The nuclear cross section σ for a specific reaction is then defined as the average number of individual processes occurring per incident neutron in the beam per nucleus; thus,

$$\sigma \equiv \frac{A}{N_a I} \quad \text{cm}^2 \text{ per nucleus.} \quad (3.39.1)$$

Because nuclear cross sections are frequently in the range of 10^{-22} to 10^{-26} cm^2 per nucleus, it is the general practice to express them in terms of a unit of 10^{-24} cm^2 per nucleus, called a barn. Thus, a cross section of $1.8 \times 10^{-23} \text{ cm}^2$ per nucleus, would be written as 0.18 barn.

3.40 The significance of the cross section may be seen by rearranging equation (3.39.1); thus, consider the form

$$N_a \sigma = \frac{A}{I} \quad (3.40.1)$$

If every neutron falling on the target reacted, then I would be equal to the number of nuclei taking part in the reaction; hence the right hand side of equation (3.40.1) would represent the fraction of the incident neutrons which succeed in interacting with the target nucleus. Alternatively, $N_a \sigma$ may be regarded as the fraction of the surface which is capable of undergoing the given reaction; hence, of 1 cm^2 of surface, $N_a \sigma \text{ cm}^2$ is effective. Since the 1 cm^2 of surface contains N_a nuclei, the quantity $\sigma \text{ cm}^2$ is the effective area per single nucleus for the reaction. It is this interpretation of σ that leads to the use of the term cross section, although, as will shortly be apparent, it is related to the geometrical nuclear cross section only in certain special cases.

3.41 In the foregoing treatment, for purposes of defining the cross section, only the surface of the target material was considered. In order to determine cross sections experimentally, the attenuation of the neutron beam through a target of finite thickness is measured. For the present, the effect of scattering will be neglected. Consider a 1 cm^2 area inclosed by the dotted lines in Figure 3.41, of a slab of material $x \text{ cm}$ thick, and let I_0 be the number of incident neutrons striking this area from the left. If N is the number of target nuclei per cm^3 of material, then the number present in a thin layer dx , parallel to the surface, will be Ndx nuclei per cm^2 . This is equivalent to the quantity designated by the symbol N_a above. Hence, by equation (3.40.1), $Ndx \sigma$ is the fraction of the neutrons falling on this layer which react; this may be set equal to $-dI/I$, where $-dI$ is the decrease in the neutrons per cm^2 as a result of passing through the thickness dx of target material.

Consequently,

$$\frac{-dI}{I} = N \sigma dx \quad (3.41.1)$$

and integration over the thickness x of the material gives

$$I_x = I_0 e^{-N \sigma x} \quad , \quad (3.41.2)$$

where I_0 is the number of incident neutrons falling on a given area and I_x is the number which succeed in passing through x cm of the material over the same area. The experimental method for determining cross sections of nuclear reactions involving neutrons makes use of equation (3.41.2), as will be seen in § 3.57.

Macroscopic Cross Sections

3.42 The cross section σ for a particular process which applies to a single nucleus, is frequently called the microscopic cross section to distinguish it from $N \sigma$, called the macroscopic cross section, for the material for that process. Thus, representing the latter by Σ , the definition is

$$\Sigma \equiv N \sigma \text{ cm}^{-1}, \quad (3.42.1)$$

where N is the number of nuclei per cm^3 ; it is consequently the total cross section of the nuclei in 1 cm^3 of the material. It will be noted that the macroscopic cross section has the dimensions of a reciprocal length.

3.43 Replacing $N \sigma$ in equation (3.41.1) by Σ , in accordance with equation (3.42.1), it is seen that

$$-\frac{dI}{I} = \Sigma dx$$

or

$$\Sigma = -\frac{dI}{I} \cdot \frac{1}{dx} \quad .$$

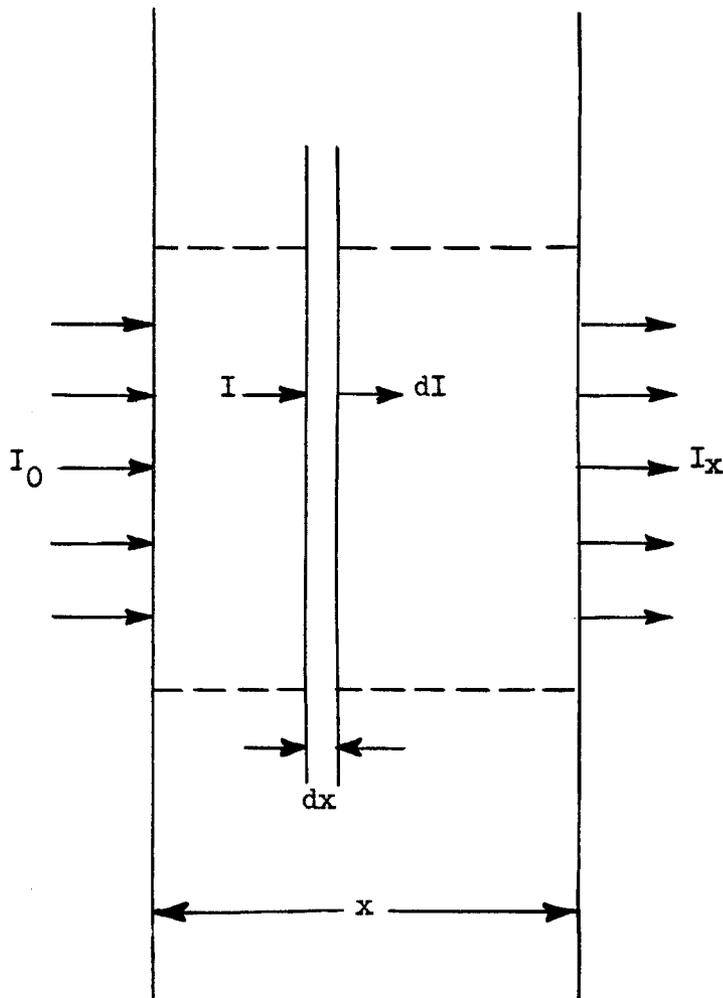


Figure 3.41

Since $-dI/I$ is the fraction of neutrons absorbed in the path dx , it is evident that the macroscopic cross section Σ is the probability that neutrons will be absorbed (or will interact) per unit path length of travel.

3.44 If ρ is the density of the absorbing material in grams per cm^3 ,

and A is its atomic weight, if an element, then ρ/A is the number of gram atoms per cm^3 . The number of atomic nuclei per cm^3 is then obtained upon multiplying by N_0 , the Avogadro number (6.02×10^{23}), which gives the number of individual atoms (or nuclei) per gram atom; thus,

$$N = \frac{\rho}{A} N_0 \quad (3.44.1)$$

and hence

$$\Sigma = \frac{\rho N_0}{A} \sigma \quad (3.44.2)$$

If the material under consideration contains several nuclear species, then the macroscopic cross section is given by

$$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + \dots + N_i \sigma_i + \dots \quad (3.44.3)$$

where, in general, N_i is the number of nuclei of the i th kind present in the material and σ_i is their microscopic cross section for the given process. For a compound, A in equation (3.44.1) must be replaced by the molecular weight M , and the result multiplied by the number ν_i of the absorbing atoms of the i th kind per molecule, to obtain N_i . The value of Σ is then given by equation (3.44.3); thus,

$$\Sigma = \frac{\rho N_0}{M} (\nu_1 \sigma_1 + \nu_2 \sigma_2 + \dots + \nu_i \sigma_i + \dots)$$

Mean Free Path and Relaxation Length

3.45 By introducing equation (3.42.1) into equation (3.41.2) it follows that

$$I_x = I_0 e^{-\Sigma x} \quad (3.45.1)$$

or

$$\frac{I_x}{I_0} = e^{-\Sigma x} \quad (3.45.2)$$

The quantity I_x/I_0 is the fraction of incident neutrons, which succeed in penetrating the thickness x of material without undergoing the reaction being considered. Hence $e^{-\sum x}$ may be regarded as the probability that a neutron will penetrate to a point x without being involved in the reaction. Since \sum is equal to the probability of neutron interaction per unit path length, as seen above, the probability that reaction will occur between x and $x + dx$ is given by $\sum dx$. Hence, the average distance λ a neutron will travel before being absorbed is given by

$$\lambda = \frac{\int_0^{\infty} x e^{-\sum x} \sum dx}{\int_0^{\infty} e^{-\sum x} \sum dx} = \frac{1}{\sum} \quad \text{cm,} \quad (3.45.3)$$

the integral in numerator and denominator being standard forms. This result is in agreement with the interpretation of \sum as the probability of capture per unit distance of travel.

3.46 The average distance λ calculated above is called the mean free path for the given nuclear reaction. It has, of course, the dimensions of length, since \sum is a reciprocal length. Replacing \sum in equation (3.45.2) by $1/\lambda$, the result is

$$\frac{I_x}{I_0} = e^{-x/\lambda} \quad (3.46.1)$$

If x is set equal to λ , then $I_x/I_0 = e^{-1}$, so that λ may also be regarded as the distance in which a fraction $1/e$ of the incident neutrons are absorbed.

3.47 When a neutron can take part in several different processes with a given target nucleus, there will be a different cross section and mean free path for each process. The equations derived above are quite

general and will apply to all the reactions in which neutrons are absorbed. It is then possible to define a total cross section for neutron absorption which is the sum of the individual cross sections. An equation of the form of (3.46.1) will give the total attenuation of neutrons due to absorption in a thickness x of medium through which they pass. In this case, the quantity λ , which is equal to the reciprocal of the total macroscopic absorption cross section, is sometimes called the relaxation length of the neutrons in the given medium. It is the distance in which the intensity of the neutron beam is reduced to a fraction $1/e$ of its initial value due to absorption of neutrons in the medium if there were no scattering.

Rates of Neutron Reactions

3.48 Suppose a neutron moves with a velocity v cm per sec, and λ cm is the mean free path for a given reaction; then, on the average, v/λ is the probability that a neutron will interact per second. If the neutron density, i.e., the number of neutrons per cm^3 , of the beam is n , then the number of neutron interactions is nv/λ cm^3 per sec. Since λ is equal to $1/\Sigma$, this number can be written Σnv , where Σ is the macroscopic cross section for a given process. In other words, it follows that

$$\begin{array}{l} \text{Number of neutrons involved} \\ \text{in a given process} = \Sigma nv \text{ per } \text{cm}^3 \text{ per sec, } \end{array} \quad (3.48.1)$$

a result of considerable importance. In the study of nuclear reactors, for example, it is frequently necessary to know the number of neutrons of velocity v absorbed per cm^3 per sec; this is equal to $\Sigma_a nv$, where Σ_a is the total macroscopic absorption cross section for these neutrons.

3.49 The product nv , expressed as neutrons per cm^2 per sec, is a significant quantity called the neutron flux, and represented by the symbol ϕ . It is the sum of the distances traveled by all the neutrons in one cubic centimeter in one second, and is therefore sometimes called the track length. Since the macroscopic cross section Σ is the probability of a particular neutron process occurring per unit track (or path) length, it follows that

$$\begin{aligned} \text{Number of neutrons involved} \\ \text{in a given process} \end{aligned} = \Sigma \phi \text{ per cm}^3 \text{ per sec.} \quad (3.49.1)$$

The same result can, of course, be obtained by substituting ϕ directly for nv in equation (3.48.1), but the foregoing derivation brings out the physical significance of the quantities involved. If Σ_a is the macroscopic absorption cross section for all processes, then $\Sigma_a \phi$ is the total number of neutrons absorbed by all nuclear processes per cm^3 per sec. This result will find frequent use in later sections.

Polyenergetic Neutron Systems

3.50 In the derivations given above, it has been assumed, for simplicity that all the neutrons have the same velocity, but this is not true in many nuclear reactor problems. As brought out in the preceding chapter, and considered further below, the cross section for a particular reaction varies with the energy, or speed, of the neutron. This introduces a complication for which due allowance must be made. If $n(E)$ is the number of neutrons of energy E per cm^3 per unit energy interval, then $n(E)dE$ is the number of neutrons in the energy range from E to $E + dE$. The total neutron flux ϕ , for neutrons of all energies (or velocities), is then given by

$$\phi = \int_0^{\infty} n(E)v dE \text{ per cm}^2 \text{ per sec,} \quad (3.50.1)$$

where the integration limits of zero and infinity are meant to be formal only, the implication being that integration is carried over the whole range of neutron energies. The velocity v corresponding to the kinetic energy E is defined by $v = \sqrt{2E/m}$, where m is the mass of the neutron.

3.51 An alternative form of equation (3.50.1) may be obtained by letting $\phi(E)$ represent the flux per unit energy of neutrons having energy E ; then $\phi(E)dE$ is the flux of neutrons in the energy range from E to $E + dE$. The total neutron flux is then given by

$$\phi = \int_0^{\infty} \phi(E)dE \text{ per cm}^2 \text{ per sec.} \quad (3.51.1)$$

Similarly, the corresponding form of equation (3.48.1) for a polyenergetic neutron system is

$$\begin{aligned} \text{Number of neutrons involved} &= \int_0^{\infty} \sum (E)n(E)v dE \quad (3.51.2) \\ \text{in a given process} & \end{aligned}$$

$$= \int_0^{\infty} \sum (E) \phi(E)dE \text{ per cm}^3 \text{ per sec,} \quad (3.51.3)$$

where $\sum(E)$ is the macroscopic cross section for the process for neutrons of energy E .

3.52 When the neutrons have a range of energies, an average macroscopic cross section $\bar{\sum}$ for a particular process may be defined so that, by analogy with equation (3.49.1).

$$\begin{aligned} \text{Number of neutrons involved} &= \bar{\sum} \phi \text{ per cm}^3 \text{ per sec,} \\ \text{in a given process} & \end{aligned}$$

where ϕ is the total neutron flux given by equation (3.50.1). It follows therefore, upon introducing equations (3.50.1) and (3.51.2), or equations (3.51.1) and (3.51.3), that

$$\bar{\sum} = \frac{\int_0^{\infty} \sum (E) n(E)v dE}{\int_0^{\infty} n(E)v dE}$$

$$= \frac{\int_0^{\infty} \sum (E) \phi (E) dE}{\phi (E) dE} \text{ cm}^{-1} \quad (3.52.1)$$

The corresponding average mean free path $\bar{\lambda}$ is expressed by

$$\bar{\lambda} = \frac{\int_0^{\infty} \lambda(E) \phi (E) dE}{\int_0^{\infty} \phi (E) dE} \text{ cm,} \quad (3.52.2)$$

where $\lambda (E)$, the mean free path for neutrons of energy E , is equal to $1/\sum (E)$ for the given process. It may be noted that, in general $\bar{\lambda}$ will not be equal to $1/\bar{\sum}$.

3.53 For thermal neutrons having a Maxwell-Boltzmann distribution $n(E)$ in the foregoing equations would be defined by equation (3.16.1), with n equal to the total number of neutrons per cm^3 . Because of absorption, however, the actual distribution of thermal neutrons is not strictly in accordance with the Maxwell-Boltzmann equation, as indicated in § 3.15. Nevertheless, for a weak absorber, the Maxwell-Boltzmann distribution would represent a very good approximation. The same would be true if the absorption cross section were essentially independent of the neutron energy. If the absorber obeys the "1/v" law, the absorption cross section may be expressed by $\sigma_a (E) = (a/E^{1/2}) + b$, and the average cross section is then

$$\bar{\sigma}_a = \frac{\int_0^{\infty} \sigma (E) n(E) v dE}{\int_0^{\infty} n(E) v dE} = \frac{a \int_0^{\infty} n(E) dE}{\int_0^{\infty} E^{1/2} n(E) dE} + b$$

The ratio of the integrals is seen to be the reciprocal of the average value of $E^{1/2}$; that is, it is equal to $1/\bar{E}^{1/2}$, and so

$$\bar{\sigma}_a = \frac{a}{\bar{E}^{1/2}} + b$$

The approximate average absorption cross section for polyenergetic neutrons in the case of a "1/v" absorber is thus the value at the velocity $\overline{1/v}$. For a thermal neutron distribution satisfying the Maxwell equation (3.16.1) it is readily found that

$$\overline{1/v} = \sqrt{\frac{4kT}{\pi}}$$

Consequently, the average absorption cross section for thermal neutrons having a Maxwellian distribution, if the absorber obeys the 1/v law, is precisely the value of the cross section at the energy equal to $4kT/\pi$.

Scattering Properties

Cross Sections and Mean Free Path

3.54 The results obtained above (§ 3.38, et seq.) are quite general being applicable both to the absorption of neutrons and to scattering, in which the incident neutron is not lost, as in absorption reactions, but is only deprived of some, or all, of its energy. The cross section

σ_s for scattering is defined by an expression similar to equation (3.39.1), where A is now the number of neutrons scattered out of the beam of I neutrons per cm^2 by the N_a nuclei. The macroscopic scattering cross section Σ_s is equal to $N \sigma_s$, where, as before, N is the number of nuclei per cm^3 of the scattering material.

3.55 In applying the arguments of § 3.41 to the scattering of neutrons, there is a change in the significance of the term I_x , it is now the number of neutrons which have escaped scattering, and not merely the number which succeed in passing through the material. Actually, the number of neutrons passing through in the x-direction is greater than I_x , since many neutrons will be scattered in this direction. However,

bearing in mind the correct interpretation of I_x , equations (3.41.2) and (3.45.2) apply to scattering. The quantity I_x/I_0 is the fraction of the neutrons which have escaped scattering, so that $e^{-\sum_s x}$ is the probability that a neutron will penetrate to a point x without being scattered, and $\sum_s dx$ is the probability that scattering will occur in the interval between x and $x + dx$. The scattering mean free path λ_s , which is the average distance a neutron travels before being involved in a scattering collision, can then be obtained by equation (3.45.3) as

$$\lambda_s = \frac{1}{\sum_s} \quad (3.55.1)$$

3.56 For a neutron of velocity v , the number of scattering collisions will be, on the average, v/λ_s per sec, and the total number of neutrons scattered per cm^3 per sec is nv/λ_s , or $nv \sum_s$, where n is the number of neutrons per cm^3 . As in § 3.48, it follows that the number of neutrons scattered per cm^3 per sec is $\sum_s \phi$, where ϕ is the neutron flux. The scattering cross sections do not vary so greatly with energy* as do absorption cross sections; nevertheless, when a beam of polyenergetic neutrons is being considered, the total number of scattering collisions per cm^3 per sec will be given by an expression analogous to equation (3.51.3). Average values of the macroscopic cross section and of the mean free path are given by equation (3.52.1) and (3.52.2).

Determination of Cross Sections

Transmission Method

* The potential scattering cross section σ_s for neutrons (§ 2.53) should theoretically be equal to $4\pi R^2$, where R is the "effective" radius of the scattering nucleus.

3.57 The most direct procedure for the measurement of cross sections, which gives the total of absorption and scattering cross sections, is the transmission method. The experimental arrangement consists of a neutron source S and a detector D, between which is placed a slab A of the material being investigated (Figure 3.57). By means of a suitable collimating shield, the neutron beam passing through to the detector is restricted to a relatively small solid angle. The purpose of the shield is to prevent, as far as possible, neutrons which have been scattered in the material from reaching the detector. This will be evident from a consideration of the dotted lines showing possible paths of scattered neutrons; if the aperture in the shield is small, the neutrons in general will not be scattered into the detector, as shown at a. However, if the aperture had been larger, there is a possibility that scattered neutrons would reach the detector, as indicated at b*

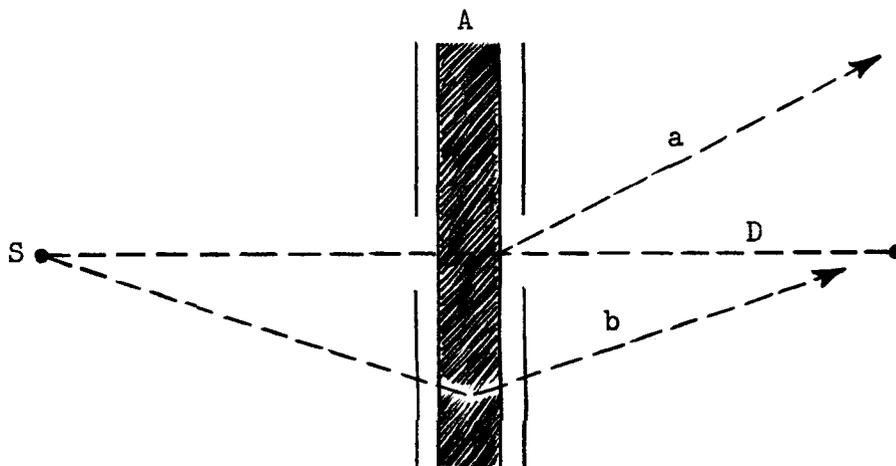


Figure 3.57

* It is assumed here that the slab is not too thick, so that the neutrons are not scattered more than once within the slab; otherwise, neutrons such as a might be scattered into the detector. It is on account of the scattering of neutrons that the exponential equation (3.41.2) for the attenuation of neutrons is not valid.

3.58 With the arrangement suggested above, the neutrons reaching D will be those which have escaped absorption and have not been scattered. If I_0 is the neutron intensity measured at D with the slab of material removed, and I_x is the value when the slab of thickness x is interposed between the source and the detector, then insertion of these results in equation (3.41.2) will permit the total (microscopic) cross section for absorption and scattering to be calculated. Alternatively, equation (3.45.1) may be used to give the total macroscopic cross section.

3.59 The scattering cross section can be determined by having the neutron detector in such a position that it can be reached by scattered neutrons only. This may be achieved by placing the detector at an angle approaching 90° from the incident beam; then only those neutrons which have been scattered through this angle will be counted. From the result obtained the total number of neutrons scattered through all angles by a known thickness of material can be calculated and hence the cross section can be evaluated. The difference between the total cross section obtained in § 3.58 and the scattering cross section gives the absorption cross section for all processes.* The method just described for obtaining scattering cross sections requires a strong neutron source, since the proportion of neutrons scattered is small, and only a fraction of these enter the detector placed at a selected angle.

Activation Method

3.60 If a particular neutron absorption reaction leads to the formation of a radioisotope (§ 3.22), the amount of which can be estimated from its radioactivity, it may be possible to determine the cross

* It should be noted that for neutrons of sufficiently high energy, where absorption is of the same order as, or smaller than, scattering, this method is inapplicable, since it involves the difference of two almost equal numbers containing experimental inaccuracies.

section for that reaction by what is known as the activation method. A thin foil of the material under investigation is exposed to the neutrons for a known time; it is then removed from the neutron flux and its activity is measured. If the target material is thin, the neutron density or flux may be regarded as constant throughout, and this simplifies the treatment of the results.

3.61 According to equation (3.49.1), the number of neutrons absorbed per cm^3 per sec, in a given process, is $\sum_a \phi$, where \sum_a is the macroscopic cross section for that process and ϕ is the neutron flux. If $V\text{cm}^3$ is the volume of the absorbing foil, the rate of absorption is $V \sum_a \phi$ neutrons per sec. Since each neutron absorbed results in the formation of the active species is $V \sum_a \phi$ nuclei per sec. However, the decay of the radioactive nuclide occurs to some extent while it is being produced. If λ is the radioactive decay constant (≈ 1.20), the net rate of increase of the active species at any instant is given by

$$\frac{dN}{dT} = V \sum_a \phi - \lambda N, \quad (3.61.1)$$

where N is the number of active nuclei present after T sec of exposure of the foil to the neutron flux ϕ .

3.62 The solution of this linear differential equation (3.61.1) noting that $N = 0$ when $T = 0$, is

$$N = \frac{V \sum_a \phi (1 - e^{-\lambda T})}{\lambda}. \quad (3.62.1)$$

The activity A of the foil measured by a counter is equal to N , which is the rate of emission of charged particles (or photons), so that

$$A = V \sum_a \phi (1 - e^{-\lambda T}). \quad (3.62.2)$$

If exposure to neutrons is continued for some time, so that T is large, and $e^{-\lambda T}$ is small compared with unity, equation (3.62.2) becomes

$$A_{\infty} = V \sum_a \phi . \quad (3.62.3)$$

The quantity designated A is called the saturation activity, and for a given neutron flux and foil it is directly proportional to the absorption cross section. It is the maximum or limiting activity the foil can acquire in the specified neutron flux.

3.63 After removal of the activated foil from the neutron flux, it continues to decay, and at any subsequent time t the activity is

$$\begin{aligned} A_t &= V \sum_a \phi (1 - e^{-\lambda T}) e^{-\lambda t} \\ &= V \sum_a \phi \left[e^{-\lambda T} - e^{-\lambda(T+t)} \right] . \end{aligned} \quad (3.63.1)$$

By determining the activity of the foil in a counter, after a period T of exposure to neutrons and a delay t before counting, and making allowances for decay during the process of counting, it is possible to evaluate the saturation activity $V \sum_a \phi$ from equation (3.63.1). If the neutron flux ϕ to which the foil is exposed and the volume of the foil are known, it is possible to determine the macroscopic cross section \sum_a for the process leading to the species whose activity was measured. The neutron flux may be determined directly by means of a suitable counter (§ 3.80). Alternatively, the procedure just described using a foil with a known absorption cross section, may be employed for the measurement of neutron flux. Once the latter, from a given source, is known, it may be utilized for the determination of absorption cross sections of other materials.

3.64 It should be noted that the cross section obtained by use of the

activation method not only refers to a particular process, but also to a specific isotopic constituent of the target material. For example, if silver is exposed to the action of slow neutrons, an active species, with a half life of 2.3 min, is formed. This has been identified as Ag^{108} formed by the (n, γ) reaction of slow neutrons with the stable isotope Ag^{107} . Hence, if the activity having a half life of 2.3 min is studied, the results will give the cross section for the (n, γ) reaction of the silver-107 isotope. Measurements made by the transmission procedure, described in 3.57 et seq., give an average value for both stable isotopes, of mass numbers 107 and 109.

Results of Cross Section Measurements

Variation of Cross Section with Neutron Energy

3.65 The problem of a complete determination of cross sections for neutron reactions is a very complex one; not only do the values vary from one isotope to another of the same element, and change with the nature of the reaction, they are also markedly dependent on the velocity or energy, of the incident neutrons. While the energy dependence of neutron absorption cross sections have been determined in many cases, the data have been obtained with the naturally occurring form of the target material, often consisting of two or more isotopes. In certain instances, where one isotope has a much larger absorption cross section than another, the former has been identified and its contribution estimated. The fact that the absorption cross sections are generally the total for all possible processes is not serious, because for nearly all elements, other than those of low atomic weight, the (n, γ) reaction takes place almost exclusively for neutrons of energy less than

several Mev. A few exceptional cases (§§ 3.32 - 3.34) are known, and then appropriate allowance can be made.

3.66 In order to study the effect of neutron energy on the cross section, it is necessary to have monoenergetic neutron sources. A few of these were described earlier, but they mostly refer to neutrons with energies of several thousand ev, at least. In the low energy range, neutrons of specific energies can be obtained from polyenergetic beams by the use of devices called velocity selectors. With their aid neutrons of any desired velocity, in the range from a fraction of an ev to about 1000 ev, can be studied.

3.67 With the exception of hydrogen in the unbound state, for which the value is as high as $20 \times 10^{-24} \text{ cm}^2$, i.e., 20 barns* the scattering cross sections of nearly all elements lie in the range from about 1 to 10 barns for neutrons of low energy. With increasing energy, the cross sections decrease somewhat, and for high-energy neutrons they approach the geometrical cross section, πR^2 , where R is the radius of the nucleus. The value of R is given, with a fair degree of accuracy, especially for the elements of higher atomic number, where A is the mass number, by the expression $1.5 A^{1/3} \times 10^{-13} \text{ cm}$. Hence, for an element of mass number 125, for example, the limiting value of the scattering cross section is about 2 barns.

3.68 For many nuclides, especially those of mass number exceeding 100, an examination of the variation of the absorption (or total) cross sections with the energy of the neutrons reveals the existence of three regions, in agreement with the general conclusions reached from the

* In the bound state, as in solid paraffin, the scattering cross section increases to 80 barns for neutrons of very low energy.

Breit-Wigner formula (§ 2.39). There is, first, a low energy region where the cross section decreases steadily with increasing neutron energy. The absorption cross section σ_a in this slow neutron region is inversely proportional to the square root of the neutron energy, as indicated by equation (2.43.1). Since the energy is kinetic in nature, σ_a is inversely proportional to the neutron velocity; this is the $1/v$ region referred to in § 2.43.

The Resonance Region

3.69 Following the $1/v$ region for slow neutrons, the elements under consideration exhibit a resonance region (§ 2.39). This is characterized by the occurrence of peaks where the absorption cross section rises fairly sharply to high values for certain neutron energies, and then falls again. Some elements, cadmium and rhodium, for example, have only one resonance peak in the electron volt region, while others, such as indium, silver, iridium and gold, have two or more peaks. The cross sections of cadmium and indium, as functions of the neutron energy, are shown in Figure 3.69. The scales are logarithmic in both directions, and so the $1/v$ region, at the left side of the figure, appears as a straight line. The resonance peak for cadmium occurs at 0.18 ev and the absorption cross section is then about 7200 barns. The main peak for indium is for neutrons of 1.44 ev energy, with a cross section exceeding 20,000 barns. There are also two lower resonance peaks at somewhat higher energies, about 4 and 10 ev.

3.70 Since the resonance peaks are found in regions of relatively low neutron energies and with elements of higher mass number, the reaction taking place must be of the (n, γ) type. As pointed out earlier, for

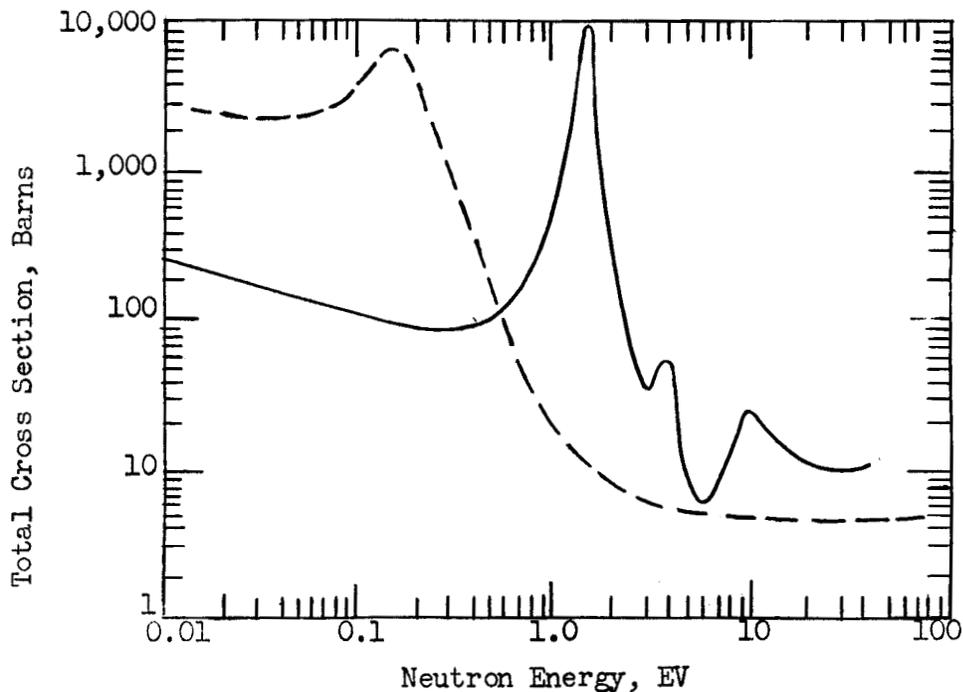


Figure 3.69

other processes, such elements would require neutrons having energies in the Mev range. The behavior of certain nuclides of low mass number undergoing (n, α) reactions with slow neutrons will be referred to below (§ 3.76).

3.71 The resonance peaks for (n, γ) reactions at low energies are usually sharp and narrow. This is in agreement with the Breit-Wigner treatment of resonance absorption. The partial level width Γ_{γ} for gamma-ray emission is apparently small, of the order of 0.1 ev, and since other reactions of the compound nucleus are not very probable for slow neutrons, the total level width Γ will also be small. It was shown in § 2.45, that the so-called half-width of the resonance peak

should be equal to the total level width. The resonance peaks should thus be relatively narrow, as they are in Figure 3.69.

3.72 Another point of interest is the high values of the resonance absorption cross sections. These are frequently of the order of 10,000 barns, i.e., 10^{-20} cm², compared with an actual ("geometrical") nuclear cross section of about 2 barns, i.e., 2×10^{-24} cm². Since the cross section may be thought of as the effective area of the nucleus as far as a given reaction is concerned (§ 3.40), it is apparent that the effective area can be much larger than the actual area.

3.73 An interpretation of this result can be found in the wave theory of matter. Thus, according to equation (2.9.1), a neutron with energy of 1 ev has a wave length of 2.9×10^{-9} cm. The neutron can consequently be regarded as a wave which can engulf many nuclei if the required energy conditions, namely, those for resonance, are fulfilled. The effective area of the nucleus for absorption of a neutron may then well approach a value of $(10^{-9})^2$, i.e., 10^{-18} cm².

3.74 The same general conclusion may be reached from the Breit-Wigner equation (2.39.1) by setting E equal to E_δ ; the corresponding value of the cross section, i.e., the maximum of the resonance peak, is then

$$\sigma_{\max} \approx \frac{\lambda^2}{\pi} \cdot \frac{\Gamma_a \Gamma_b}{\Gamma^2} \quad (3.74.1)$$

If $\frac{\Gamma_a \Gamma_b}{\Gamma^2}$ is taken to be about 0.1, then since λ is 2.9×10^{-9} cm for a neutron of 1 ev energy, the value of σ_{\max} might be roughly 10^{-19} cm², i.e., about 10^5 barns.

Fast-Neutron Region

3.75 Beyond the resonance region the nuclear cross sections decrease steadily with increasing neutron energy. This represents what may be called the fast-neutron region. The cross sections are usually low, being less than 10 barns in most cases and becoming even smaller at energies in the Mev range. For a neutron of 1 Mev energy the equivalent wave length is 2.9×10^{-12} cm, by equation (2.9.1), and hence the absorption cross sections are likely to be of the same order as the scattering cross sections.

Large Level Widths

3.76 A type of variation of cross section with neutron energy different from that described above has been observed with a few nuclei of low mass number for reactions in which a charged particle is expelled. Examples of this behavior are to be found in the (n, α) reactions with B^{10} and Li^6 , referred to earlier. The dependence on neutron energy of the cross section for the (n, α) reaction with boron, mainly due to the boron-10 isotope, is shown in Figure 3.76. The logarithmic plot of σ against E is essentially linear from about 0.01 ev to 0.1 Mev, although the data in the figure go only to 1000 ev. This means that the $1/v$ law holds at energies well beyond those found for (n, α) reactions, in fact beyond the usual resonance region.

3.77 The interpretation of this result may also be found in the Breit-Wigner formula. When the excited compound nucleus has sufficient excess energy to make emission of a charged particle possible, the probability that the process will occur is large. In other words, the

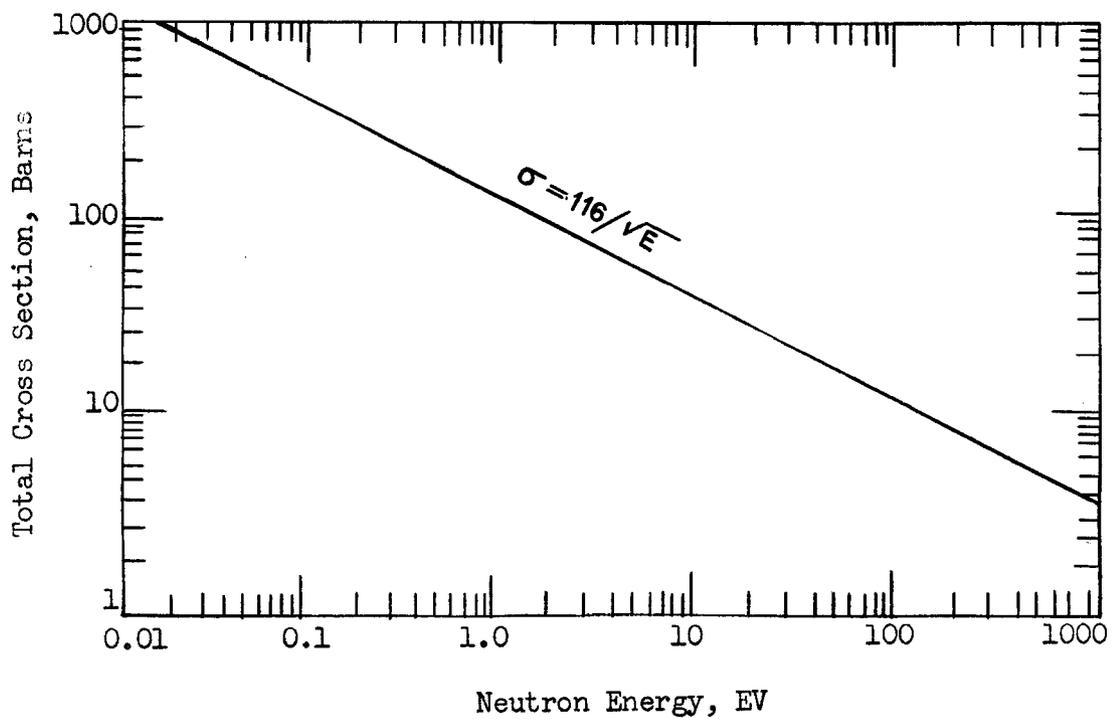


Figure 3.76

partial level width Γ_α is large and, consequently, so also will be Γ , the total width. In these circumstances, the quantity $E-E_r$ in the Breit-Wigner equation may be neglected in comparison with $1/4\Gamma^2$ over a considerable energy range. The cross section will then vary inversely as the square root of the neutron energy, or inversely

as its velocity, as shown in } 2.46. The extension of the $1/v$ region over such a large range of energies in the (n, α) reaction with boron¹⁰ (and Lithium⁶) may thus be explained by the large level width.

Elements of Low Mass Number

3.78 It is important to point out that not all elements exhibit the types of behavior described above. Most nuclides of low mass number, as well as several of high mass number, do not exhibit resonance absorption, at least not to any appreciable extent. The total neutron cross sections, including both absorption and scattering, are small, of the order of a few barns, over the whole energy range, from thermal values to several Mev.* Were it not for this fact the construction of nuclear reactors would be a virtual impossibility.

Detection and Counting of Neutrons

Secondary Ionization Counters

3.79 Neutrons produce very little direct ionization in their passage through a gas, and so they cannot be detected directly in such instruments as a Geiger counter or a cloud chamber. The operation of these and similar devices depends on the presence of ions produced by the entry of a particle and so they will not respond to neutrons directly. Nevertheless, instruments of this kind can be adapted to detect and count neutrons by utilizing certain secondary effects of these neutral particles

* The most notable exceptions are, of course, B¹⁰ and Li⁶.

3.80 The most common method for counting slow neutrons makes use of the (n, α) reaction with B^{10} , for which the cross section is large. As seen in 3.32, a Li^7 nucleus and an alpha particle are formed; both have relatively high energies and they produce considerable ionization in their paths. In order to take advantage of this process, a counter containing boron or a compound of boron is employed. A proportional counter, for example, may contain boron trifluoride as part of the filling gas, or the walls may be lined with a thin coating of elemental boron or of a solid compound, such as boron carbide. Since it is really the B^{10} isotope which is effective in the (n, α) reaction, better results are obtained if the boron compound used contains a larger than normal proportion of this isotope.

3.81 In a counter tube of proper design each neutron entering will produce sufficient secondary ionization to permit it to be counted without difficulty. If the area of the surface covered with boron is known, the slow neutron flux can be determined.

3.82 Another type of neutron detector and counter makes use of the fission reaction (3.35). Slow neutrons cause fission of the Uranium-235 nucleus, and the nuclear fragments which result have considerable ionizing effect. A simple device for observing slow neutrons thus consists of an ionization chamber of which one electrode is coated with uranium oxide, preferably somewhat enriched in the uranium-235 isotope. Again, each neutron entering the chamber and striking the coated electrode can be counted.

3.83 When fast neutrons are being studied, it is the general practice to make use of the ionization in the track of a light nucleus, e.g., a

proton, recoiling after being struck by a high-energy neutron. For this purpose a proportional counter may be filled with hydrogen; or, better, argon or one of the heavier inert gases is used as the filling gas and a thin sheet of a hydrogenous material, such as paraffin, is placed at one end of the chamber. Fast neutrons striking the paraffin cause protons to be ejected with relatively high energy; the latter produce ionization in their paths through the counter and so can be detected. Instruments of this type have been developed for determining the energy of the fast neutrons as well as counting them.

Activation Detectors

3.84 As indicated in § 3.63, the activation method can be applied to the determination of neutron flux. This procedure is often very convenient because thin foils, which cause little disturbance of the neutron density, can be placed in regions not accessible to counters. Further, the use of cadmium in combination with indium provides a means of distinguishing between neutrons of different energies. An examination of Figure 3.69 shows that indium, with a resonance peak at 1.44 ev neutron energy, has a high absorption cross section, 100 barns or more, for neutrons of energy less than about 2 ev. On the other hand, cadmium has a resonance peak at 0.18 ev, and the cross sections are high for energies less than about 0.5 ev. When indium is exposed to neutrons it becomes radioactive, but cadmium does not.

3.85 The foregoing facts are utilized in the following manner. First, an indium foil is exposed to neutrons for a known time and the activity measured in the usual manner; this is then used to calculate the saturation activity (§ 3.63), and from the known average absorption cross

section, the flux of neutrons with energy less than about 2 ev can be evaluated. The indium foil is then completely surrounded by cadmium foil and once more exposed to the neutrons. Because of the strong absorption by cadmium of neutrons with energies below about 0.5 ev, essentially the only neutrons reaching the indium will be those with energies in excess of this amount. Consequently, the indium foil will now be sensitive to neutrons in the range of 0.5 to 2 ev, approximately, and their flux can be determined from the saturation activity of the foil. The difference between the results obtained without a cadmium shield and with cadmium gives the flux of neutrons of energy less than about 0.5 ev.

A Few Thermal Neutron Cross Sections (.025 ev)

1

Element or Compound	Total σ (barns)	Absorption σ_a (barns)	Scattering σ_s (barns)
H*		0.32	
D ₂ O	15.3	0.00092	15.3
He	1.56	0.008	1.55
Be	6.9	0.009	6.9
B	722	718	3.8
C	4.8	0.0045	4.8
N	12.7	1.5	11.2
O	4.2	< 0.0009	4.2
F	4.0	< 0.001	4.0
Na	4.5	0.46	4.0
Al	1.6	0.22	1.35
S	1.6	0.47	1.1
Ca	4.4	0.4	4.0
Fe	13.5	2.5	11.0
Ni	22	4.5	17.5
Zr	8.4	0.4	8.0
Cd	3500	3500	6.5
In	193	191	2.2
Ph	8.5	0.2	8.3
Bi	9	< 0.01	9

1: Nuclear Data: NBS Circular 499 United States Department of Commerce National Bureau of Standards

* The scattering cross section of hydrogen increases rapidly at very low neutron velocities (Figure A). At low neutron energies equation (2.54.1) for the elastic scattering cross section becomes

$$\sigma(n,n) = \frac{\lambda^2}{4\pi} \frac{\Gamma_n^2}{E_r^2 + \Gamma^2}$$

The neutron wave length, λ is proportional to $\frac{1}{\mu v}$, where v is the velocity of the neutron and μ is the reduced mass of the system. It can be shown that Γ_n is proportional to $\mu^2 v$ so that if $\Gamma^2 \ll E_r^2$, the scattering cross section for very low neutron energies should be proportional to μ^2 .

When hydrogen is bound in a molecule, such as a hydrocarbon, the effective mass of the hydrogen is very large if the incident neutron energy is small compared to the binding energy. If the neutron energy is large compared to the binding energy, the hydrogen acts as if it were a free atom. Thus, the reduced mass is 1/2 if the neutron energy is large and is about 1 if it is small compared to the binding energy of the protons. Therefore, it is expected that the scattering cross section for bound hydrogen atoms should be about four times the cross section for scattering with free protons.

This result is verified by the experimental data for scattering by hydrogen in paraffin. The scattering cross section increases from about 20 barns at 10 ev to 80 barns for very slow neutrons.

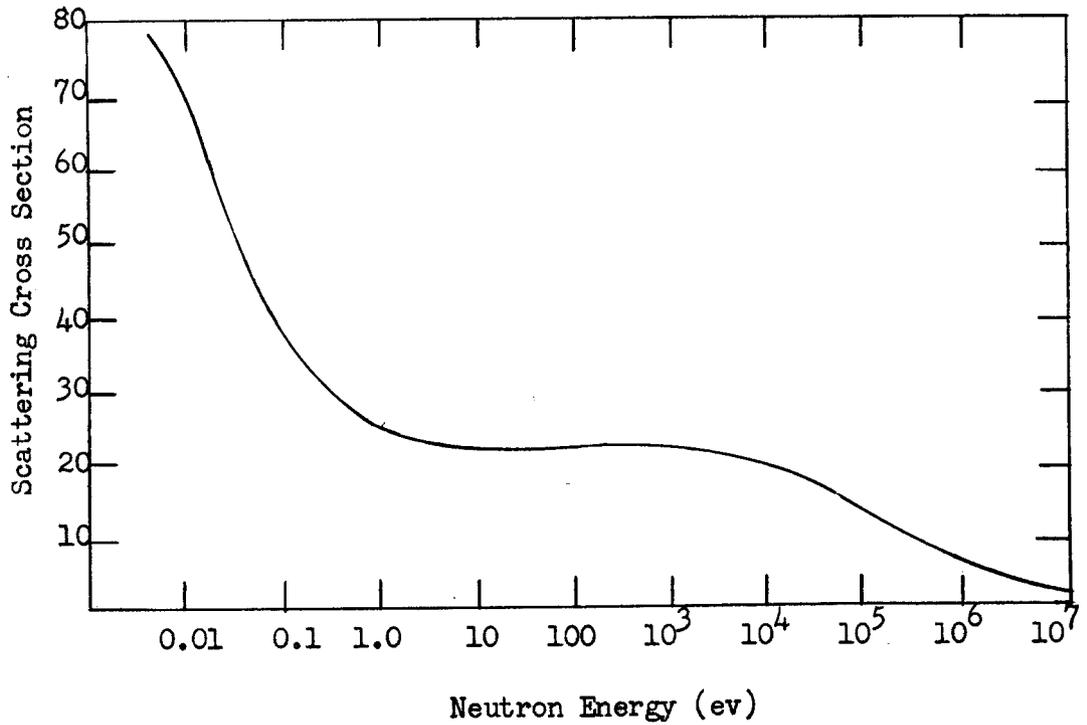


Figure A

CHAPTER IV
THE FISSION PROCESS

Characteristics of the Fission Reaction

Introduction

4.1 Although many nuclear reactions were known prior to 1939, they were all of the type in which a relatively light particle or a gamma ray photon were expelled, so that the atomic and mass numbers of the product nucleus were not very different from those of the target. In that year, however, there was discovered the process of fission, mentioned in § 2.22, whereby a uranium nucleus, after capture of a neutron, splits into two parts which differ considerably from the target element. From the standpoint of the utilization of atomic energy, the importance of the fission reaction lies in two facts. First, the process is associated with the liberation of relatively large amounts of energy and, second, the reaction initiated by neutrons is also accompanied by the liberation of neutrons. Consequently, it is possible under suitable conditions, which will be discussed in this book, for the process to be self-sustaining, and for energy to be generated continuously.

4.2 The theoretical interpretation of fission and its application in nuclear reactors will be considered later; but first the essential phenomena will be described. Uranium-235, which is present to the extent of 0.712 per cent in natural uranium undergoes fission with thermal neutrons, as well as with those of higher energy. The same is true for the artificially produced isotopes plutonium-239 and uranium-233 (§ 3.35). On the other hand, uranium-238 and thorium-232, which are the most abundant

isotopes of these elements as found in nature, require fast neutrons, of at least 1 Mev energy, to induce fission to an appreciable extent. It is of interest to note that natural uranium undergoes fission spontaneously, at a definite rate. Thus, in a gram of ordinary uranium, about 24 nuclei suffer spontaneous fission per hour, on the average.

4.3 In addition to uranium and thorium, other common elements of high, and even of moderate, atomic number exhibit fission, but only by the use of incident particles of very high energy. Fission processes of this kind, while of considerable general interest, do not appear to have any application in the field of nuclear energy, since these high energy particles are not produced by the reaction, and hence the process cannot be self-sustaining.

Emission of Neutrons

4.4 When a nucleus of high atomic number undergoes fission, splitting into two more or less equal parts, called fission fragments, the neutron-to-proton ratios of these fragments must lie on the dotted line in Figure 1.15. This is a straight line joining the origin to the points representing nuclides of high atomic number, such as those which are capable of undergoing fission. Because the fission fragment nuclei are roughly similar in size, it follows that their neutron-to-proton ratios must lie somewhere near the middle of the dotted line. Nuclei of this kind obviously contain too many neutrons for them to be stable; they can approach stability, however, either by ejecting one or more neutrons, or by conversion of a neutron into a proton with the simultaneous emission of a negative beta particle.

4.5 On account of the foregoing considerations, the possibility that

neutrons might be expelled in the fission of uranium was soon realized and verified experimentally. It has been found that in the fission of uranium-235 by slow neutrons, an average of 2.5 ± 0.1 neutrons are emitted for each nucleus suffering fission, i.e., for each neutron absorbed in a fission reaction. The number is not an integer because, as will be seen below, the uranium nucleus splits in many different ways, and although the number of neutrons expelled in any individual act of fission must obviously be zero or integral, the average may well not be an exact whole number.

4.6 The neutrons emitted as a result of the fission process can be divided into two categories, namely, prompt neutrons and delayed neutrons. The prompt neutrons, which are over 99 per cent of the total fission neutrons, are released within an extremely short interval of time, about 10^{-14} sec, of the fission process. The evidence indicates that they are not released directly from the compound nucleus which results when a uranium-235 nucleus, for example, captures a slow neutron. It appears that the compound nucleus first breaks up into two nuclear fragments, each of which has too many neutrons for stability, as well as the excess (excitation) energy, at least 6 Mev or so, required for the expulsion of a neutron. The excited, unstable nucleus consequently expels one or more neutrons within a very short time after its formation. The instantaneous gamma rays accompanying fission are apparently emitted at the same time.

4.7 The energies of the prompt neutrons cover a considerable range, probably from over 10 Mev down to thermal values; the majority, however, have energies of about 1 or 2 Mev. The energy distribution, represented

schematically in Figure 4.7, is referred to as the fission neutron or, in brief, as the fission spectrum. In the center of mass system of the fission fragment and the prompt neutron, the fission spectrum would probably be approximated by a Maxwell-Boltzmann distribution (\S 3.14), but in the laboratory system this is disturbed by the motion of the fission fragments, and by the fact that the neutron emission probability is a function of its energy.

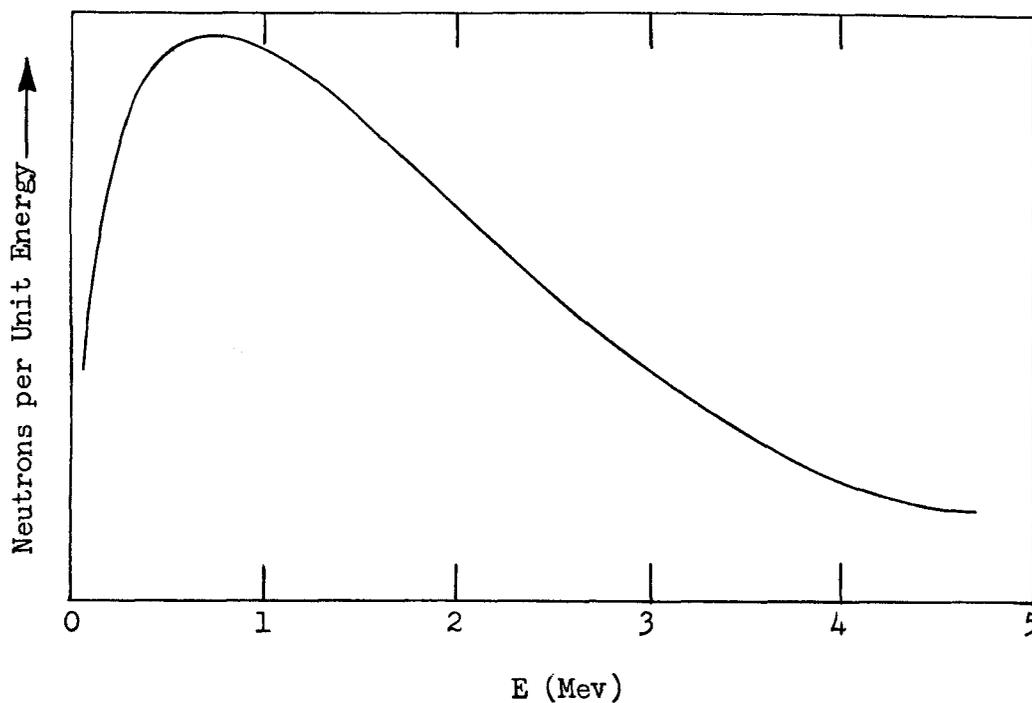


Figure 4.7

4.8 Whereas the expulsion of the prompt neutrons ceases within a very short time, the delayed neutrons are emitted, with gradually decreasing intensity, over a period of minutes. The delayed neutrons accompanying fission, fall into five, and possibly more, groups. The rate of decay of the neutron intensity in each group is exponential in nature, as it is for radioactive change in general. By observing the decay of the

delayed neutrons after fission has ceased, it has been found possible to associate a specific half life with each group. The characteristic properties of the five definitely established groups of delayed neutrons are given in Table 4.8; these include the half life T_i , the mean life t_i , i.e., $0.693 T_i$, the decay constant λ_i , i.e., $1/t_i$, the fraction β_i which the group constitutes of the total (prompt and delayed) fission neutrons, and the neutron energy, in each case. The last two columns apply only to the slow-neutron fission of uranium-235; the total fraction of delayed neutrons is about 0.0075 or roughly 0.75 per cent of the neutrons produced in fission. The same five groups of delayed neutrons are also formed when plutonium-239 suffers fission, but the proportions and energies are different from those recorded here.

Properties of Delayed Neutrons in Slow Neutron Fission				
Half Life T_i	Mean Life t_i	Decay Constant λ_i	Fraction β_i	Energy
0.43 sec.	0.62 sec.	1.61 sec^{-1}	0.00084	0.42 Mev
1.52	2.19	0.456	0.0024	0.62
4.51	6.50	0.151	0.0021	0.43
22.0	31.7	0.0315	0.0017	0.56
55.6	80.2	0.0124	0.00026	0.25

Table 4.8

4.9 The properties of the delayed neutrons have an important bearing on the time dependent behavior of nuclear reactors, and hence an explanation on their origin is of interest. By making rapid chemical separations of the fission fragments and their radioactive decay products, it has been found that neutrons decaying with the 55.6-sec. half life follow the chemistry of bromine, while those of 22.5-sec. half life follow that of iodine. It does not appear probable that the neutrons are expelled directly from the nuclei of isotopes of bromine or iodine, for if sufficient energy,

roughly 6 to 8 Mev, were available to permit the removal of a neutron, the process would be virtually instantaneous, with a half life of the order of 10^{-14} sec. The conclusion to be drawn, therefore, is that the emission of delayed neutrons occurs in an indirect manner from the bromine or iodine isotope, as follows.

4.10 One of the products of fission is a bromine isotope of high mass number, probably Br^{87} ; the nucleus contains too many neutrons for stability and is consequently a negative beta emitter. The half life of the Br^{87} is 55.6 sec., which is the same as the half life of one of the groups of delayed neutrons, and its decay product is Kr^{87} . The latter can evidently be formed in a highly excited state, with sufficient energy to permit it immediately to eject a neutron and form a stable Kr^{86} nucleus (Figure 4.10). Any excess energy available, after removal of the neutron appears as kinetic energy of the latter (see last column of Table 4.8). The observed rate of emission of neutrons is thus determined by the rate of formation of the neutron emitter Kr^{87} , and this is dependent on the decay of the precursor Br^{87} . Like all radioactive species, the latter decays in an exponential manner, the half life in this case being 55.6 sec, and hence the neutron emission falls off at the same rate.

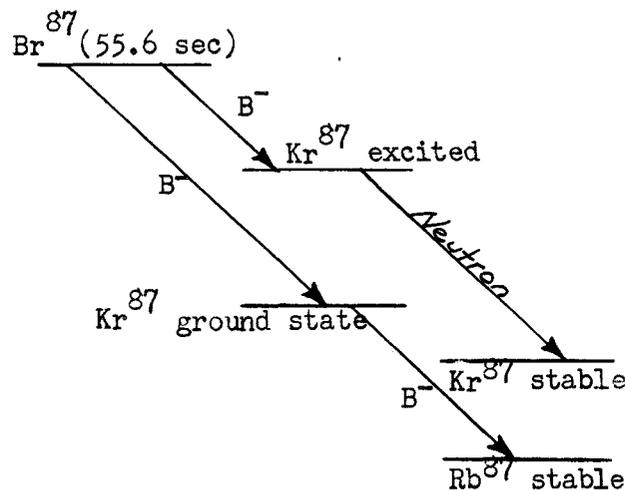
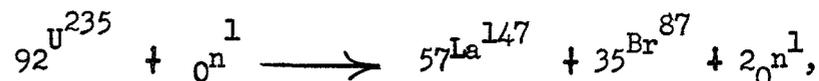


Figure 4.10

4.11 The precursor of the group of delayed neutrons with a half life of 22.5 sec is apparently I^{137} . It is known to have a half life of this magnitude and when it decays, by the emission of a negative beta particle, the product is Xe^{137} . The latter can presumably be in a state of high internal energy, so that it instantaneously expels a neutron to form stable Xe^{136} . Here again the emission of the neutron is delayed because the rate of formation of the Xe^{137} from which they originate depends on the rate of decay of the precursor I^{137} . The other three groups of delayed neutrons are undoubtedly produced in an analogous manner, although the precursors in these cases have not yet been definitely identified.

The Fission Product

4.12 The discovery of fission was made as a result of the detection of elements of moderate atomic weight, such as barium and lanthanum, when slow neutrons interact with uranium. Since lanthanum has an atomic number of 57, while that of uranium is 92, the other fission fragment* must presumably be bromine, atomic number 35. The fission process in this case might then be represented by the equation



using plausible mass numbers and assuming two neutrons to be emitted per fission. It will be noted that the two fragments, while considerably lighter than the uranium nucleus, have mass numbers which are appreciably different from each other. That the uranium nucleus tends to break up in an unsymmetrical manner was first indicated by the fact that the fission fragments

*The term fission fragments or primary fission products is used here to refer to the nuclei formed directly in fission or after emission of the prompt neutrons. The expression fission products, without qualification, is intended to include the fission fragments and the products of their radioactive decay.

were observed to fall into two groups with different ionizing powers and presumably different energies. The ratio of the kinetic energies was found to be about 1.45, and consequently, if momentum is conserved, the masses of the respective fragments must be inversely related to this ratio.

4.13 A more detailed investigation of slow-neutron fission has shown that uranium-235 splits up in more than 30 different ways, for more than 60 primary products have been identified. The range of mass numbers is from 72, probably an isotope of zinc (atomic number 30), to 158, possibly, an isotope of samarium (atomic number 62). In Figure 4.13, the mass numbers of the products are plotted against the corresponding fission yields, the fission yield being defined as the proportion (or percentage) of the total nuclear fissions yielding products of a given mass number.* Since the observed fission yields range from 10^{-5} to over 6 per cent, they are plotted on a logarithmic scale. It should be noted that as two nuclei result from each each act of fission, the total fission yield for all mass numbers adds up to 200 per cent. Incidentally, the reason why mass numbers, rather than atomic numbers, are considered is because the fission fragments are probably all radioactive, decaying by the loss of a negative beta particle. The atomic numbers, consequently, change with time, but the mass numbers are unaffected by the beta decay.

4.14 An examination of Figure 4.13 shows that, in accordance with the conclusions drawn from the energies of the fission fragments, the masses of nearly all the products fall into two broad groups, a "light" group, with mass numbers from 85 to 104, and a "heavy" group, with mass numbers from

*Similar curves, but with the maxima and minima displaced somewhat from Figure 4.13, have been obtained for the slow-neutron fission of plutonium-239 and uranium-233.

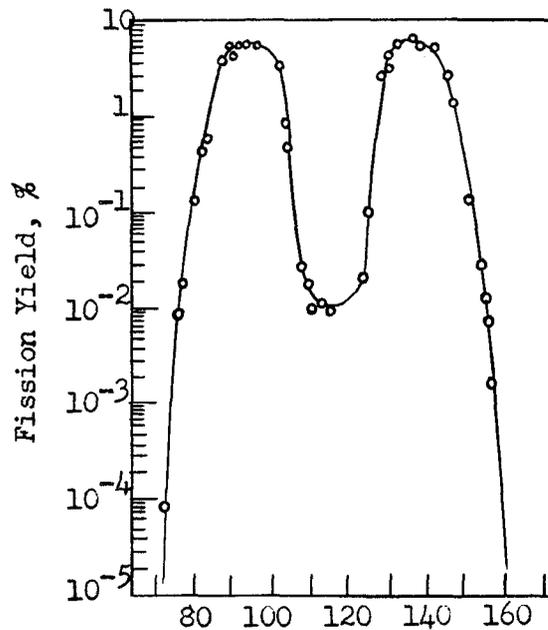
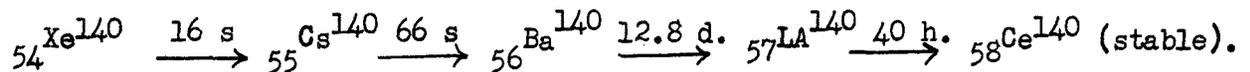


Figure 4.13

130 to 149. There are some products in the intermediate range, from 105 to 129, but altogether these represent no more than 3 per cent of the fissions. The most probable type of fission, comprising nearly 6.4 per cent of the total, gives products with mass numbers 95 and 139. It is apparent from these results that the thermal-neutron fission of uranium-235 is far from symmetrical. If the compound nucleus had split into two equal fragments, the mass of each would be 117 or 118; only 0.01 per cent of the nuclei undergoing fission break up in this manner.

4.15 One of the most important properties of the fission products is their radioactivity. As seen in § 4.4, the fragments formed, when a uranium nucleus splits up into two nuclei of somewhat similar mass, have neutron-to-proton ratios that are too high for stability. Even if a prompt neutron is expelled, the ratio of neutrons to protons will, in most cases, still be outside the stability range for the particular mass number.

Consequently, all or nearly all the fission fragments are radioactive, emitting negative beta particles. The immediate decay products are also frequently radioactive, and, while some decay chains are longer and some shorter, each fragment is followed, on the average, by three stages of decay before a stable species is formed. One of the important decay chains, because of its high yield (6.3 per cent) and also because it contains the barium and lanthanum which led to the recognition of the fission of uranium, is the following:



4.16 Since there are probably some 60 different radioactive nuclides produced in fission and each is, on the average, the precursor of two others, there are about 180 radioactive species present among the fission products after a short time. While it is possible theoretically to express the rate of decay of this complex mixture in terms of the fission yields and the radioactive decay constants, it is quite impractical. The rate of decay of the fission products is, therefore, represented by an empirical equation, which is probably accurate within a factor of two or less.* From about 10 sec after fission has taken place, the rates of the emission of beta particles and of gamma-ray photons per fission are as follows:

Rate of emission of gamma-ray photons

$$\approx 1.2 \times 10^{-6} t^{-1.2} \text{ per sec} \quad (4.16.1)$$

and

Rate of emission of beta particles

$$\approx 2.3 \times 10^{-6} t^{-1.2} \text{ per sec.} \quad (4.16.2)$$

where t is the time after fission in days. Taking the mean energy of the gamma rays as 0.7 Mev, and that of the beta particles as 0.4 Mev, the total

*Way and Wigner, Phys. Rev., 73, 1318 (1948)

rate of energy emission for both beta and gamma radiation is about $1.8 \times 10^{-6} t^{-1.2}$ Mev per sec per fission, with each type of radiation contributing an approximately equal amount. For practical purposes it is useful to express the result in terms of watts per gram of uranium-235 undergoing fission; this is found to be

$$\begin{aligned} & \text{Rate of dissipation of beta and gamma energy} \\ & = 1.1 \times 10^3 t^{-1.2} \text{ watts per gram,} \end{aligned} \tag{4.16.3}$$

at t days after fission. It will be seen below that about 5 per cent of the energy released in the fission of uranium-235 is initially present as latent beta and gamma energy of the fission products. The rate at which this is released is consequently of some significance during the steady-state operation of a nuclear reactor, and also after shutdown.

Energy of Fission

4.17 The fission process is remarkable for the magnitude of the energy released; this is about 200 Mev for each nucleus reacting, compared with a maximum of 20 Mev or so for other nuclear reactions.* The magnitude of the fission energy can be calculated in several ways, perhaps the most direct being to use the masses of the reacting species and of the fragments formed. The isotopic weight of uranium-235 is 235.124 and the mass of a neutron, on the same scale, is 1.00897, making a total of 236.133 amu for the total mass of the interacting particles. As stated in 4.14, the fission products obtained in greatest yield have mass numbers of 95 and 139, which add up to 234; it may be assumed, therefore that in this case two neutrons are liberated in the fission process. An examination of the masses of stable nuclides shows that the isotopic weights corresponding to the mass numbers 95 and 139 are 94.945 and 138.955, respectively. These,

* In chemical reactions, the energy released is never greater than a few electron volts for each atom or molecule reacting.

together with 2×1.00897 for the masses of the two fission neutrons, add up to a total mass, after fission, of 235.918. The difference between this and the mass of the interacting particles is converted into energy of the fission process; thus,

$$\begin{aligned}\text{Mass converted into energy} &= 236.133 - 235.918 \\ &= 0.215 \text{ amu}\end{aligned}$$

From equation (1.28.2) it is seen that 1 amu is equivalent to 931 Mev, consequently,

$$\begin{aligned}\text{Energy released per fission} &= 931 \times 0.215 \\ &= 198 \text{ Mev.}\end{aligned}$$

4.18 The energy calculation made above was for a particular mode of fission, and the actual energy will be the weighted mean for the 30 or more different ways in which the uranium-235 nucleus splits. However, as seen earlier, the great majority of the fissions yield products with mass numbers in a fairly limited region, and for all these the mass converted into energy, and hence the energy released, is approximately the same. It may be accepted, therefore, that 195 to 200 Mev of energy are released for every uranium-235 nucleus suffering fission.

4.19 The reason for the large magnitude of the fission energy will be apparent from an examination of Figure 1.30, which shows that the binding energy curve has a broad maximum. In the mass number range from about 80 to 150, which is that of the majority of the fission products, the binding energy per nucleon has an average value of 8.4 Mev. For higher mass numbers the value decreases and has fallen to 7.5 Mev per nucleon for uranium. This means that in the products formed by fission the binding energy is about 0.9 Mev per nucleon greater than in the latter original uranium nucleus. Since the latter contains some 230 nucleons, the total

difference in binding energy is about 200 Mev and this is the energy released in the fission process.

4.20 The large energy of fission is thus to be ascribed to the fact that in the fission products the nucleons are more firmly bound than they are in the nucleus which suffers fission. That is to say, more energy would be released in assembling the fission product nuclei from their constituent protons and neutrons than would be the case for the uranium nucleus. Consequently, when the latter breaks up into two parts, with mass numbers in the range from 80 to 150, there will be a liberation of energy. It can be seen from equation (1.29.1) that a smaller isotopic mass M would mean a higher binding energy. This is reflected in the total mass of the uranium nucleus and a neutron being greater than the total mass of the fission products, as considered in 4.17. The differences in mass and in the binding energy are thus equivalent; they are, in fact, both consequences of the same fundamental factor, namely, the forces acting between the nucleons in the different nuclei.

4.21 Consideration of the various energy terms in Table 1.44 shows that the decreased binding energy per nucleon for elements of high atomic number, which is related to the large value of the fission energy, is due mainly to the marked increase in the electrostatic repulsion of the protons. This, ultimately, is also the reason why fission has been observed to take place most readily with the elements of highest atomic number, as will be apparent from the discussion given below.

4.22 The energy data will shortly be expressed in terms of more practical quantities. In the meantime something must be said about the experimental determinations of the fission energy. From the extent of the ionization produced by the fission fragments it has been estimated that

this energy is 162 Mev. On the other hand, direct calorimetric measurement of the energy liberated as heat gave a value of 175 Mev. The explanation of the apparent disagreement between these figures and the difference from the calculated 195 Mev of energy is that the ionization measurement gives only the kinetic energy of the fission fragments. But the heat liberated includes also some other forms of energy which are associated with the fission process.

4.23 According to the latest estimates, the total amount of energy released by the fission products when they have decayed completely is about 21 Mev. Of this, about 5 Mev is beta energy, 5 Mev is gamma energy and the remainder is carried off by the neutrinos which accompany the beta emission (≈ 1.16). In addition, some 6 Mev of the fission energy are associated with the neutrons which are released, and 6 Mev appears in the form of the so-called instantaneous gamma radiation produced within an extremely short period. A complete energy balance, indicating the approximate distribution of the energy per fission is given in Table 4.23.

Distribution of Fission Energy	
Kinetic energy of fission fragments	162 Mev
Beta decay energy	5
Gamma decay energy	5
Neutrino energy	11
Energy of fission neutrons	6
Instantaneous gamma-ray energy	<u>6</u>
Total fission energy	195 Mev

Table 4.23

The kinetic energy of the fission products appears immediately as heat, and the neutron energy and instantaneous gamma-ray energy are degraded in a very short time. The beta and gamma energies of the fission products

however, are released gradually as these radioactive nuclides decay. Consequently, in the early stages of the operation of a nuclear reactor only about 174 Mev of energy are produced per fission, but this will increase gradually, and, when the fission products decay as fast as they are being formed, it will attain a maximum of 184 Mev, that is, 195 Mev less the energy carried off by the neutrinos.

4.24 Making use of the conversion units, $1 \text{ Mev} = 1.60 \times 10^{-6} \text{ erg} = 1.60 \times 10^{-13} \text{ watt sec}$, it is seen that the fission of a single uranium-235 nucleus is accompanied by the liberation of about $3.2 \times 10^{-11} \text{ watt sec}$. In other words, it requires 3.1×10^{10} fissions to release 1 watt sec of energy, so that fissions at the rate of 3.1×10^{10} per sec would yield 1 watt of power. Since 1 gram of uranium contains $6.02 \times 10^{23}/235 = 2.6 \times 10^{21}$ atoms, the energy produced by its complete fission would be $8.3 \times 10^{10} \text{ watt sec}$, which is 2.3×10^4 kilowatt hours or nearly 1 megawatt day. Thus, the fission of all the atoms in 1 gram of uranium-235, per day would yield 1 megawatt of power. Similar values apply to the fission of uranium-233 and plutonium-239.

Mechanism of Nuclear Fission

4.25 Whenever the mass of a nucleus exceeds that of the fragments into which it can be divided, the former will tend to be unstable with respect to the latter, since the process of subdivision would be accompanied by a loss of mass and a consequent liberation of energy. This condition certainly applies to all elements of mass number exceeding about 100, and hence for such elements spontaneous fission is theoretically possible. The reason why it is not observed is that the nucleus must acquire a certain critical energy or activation energy before it can break up. For species with mass numbers below about 210, this energy is so high that fission can occur only by bombardment with neutrons or other particles having energies exceeding 50 Mev.

4.26 Some understanding of the problem of critical energy may be obtained by considering the liquid-drop model of the atomic nucleus, referred to in Chapter I. Consider a drop of liquid to which a force is applied so that it is set into oscillation; the system passes through a series of stages, some of which are depicted in Figure 4.26. The drop is at first spherical as at A; it is then elongated into an ellipsoid as at B. If insufficient energy is available to overcome the force of surface tension, the drop will return to its original form, but if the deforming force is sufficiently large, the liquid acquires a shape similar to a dumbbell, as at C. Once it has reached this stage it is unlikely to return to the spherical form, but it will rather split into two droplets. These will, at first, be somewhat deformed, as at D, but finally they will become spherical, as shown at E.

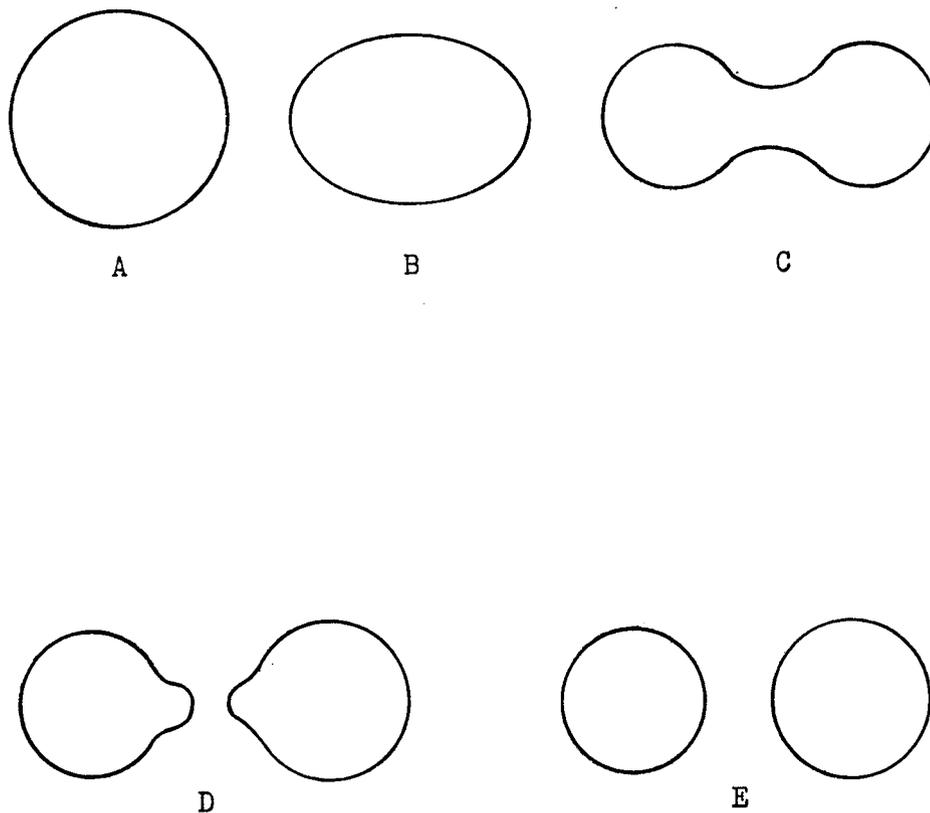


Figure 4.26

4.27 The situation in nuclear fission is believed to be analogous to that just considered. A target nucleus combines with a neutron to form a compound nucleus; the excitation energy of the latter is then equal to the binding energy of the neutron plus any kinetic energy the neutron may have had before capture (§ 2.16). As a result of this excess energy it is believed that the compound nucleus undergoes a series of oscillations, in the course of which it passes through a phase similar to Figure 4.26B. If the energy is insufficient to cause further deformation beyond B, the intranuclear forces will compel the nucleus to return to its original spherical form, and the excess energy will be removed by the expulsion of a particle from the excited compound nucleus.

4.28 However, if the nucleus has gained enough energy, as a result of absorbing the neutron, to permit it to form the dumbbell shape (Figure 4.26C), the restoration of the initial state A becomes improbable. This is because the electrostatic repulsion between the positive charges on the two ends of C can now overcome the relatively small portion of the nuclear binding force operating in the constricted region. Consequently, from C the system passes rapidly to D and then to E, representing fission into two separate nuclei which are propelled in opposite directions. The critical energy or activation energy requisite for fission to occur is thus the energy that must be added to the original nucleus in order to deform it to the state C, after which fission inevitably occurs, provided, of course, the mass requirements mentioned in § 4.25 are satisfied.

4.29 The critical fission energy may be considered, also with the aid of a potential energy curve, as in Figure 4.29. At the extreme right, at E, two fission fragments are supposed to be far apart, so that the

potential energy is virtually zero. As the fragments are brought closer together, there is an increase in potential energy due to the electrostatic repulsion of the positively charged nuclei. When the fragments reach the point C, where they are roughly in contact, the attractive forces become dominant and the potential energy decreases toward A. The latter point may be regarded as corresponding to the ground state of the compound nucleus, formed when the target nucleus captures a neutron (cf. §2.15); that is to say, it represents the energy of the compound nucleus without the excitation energy resulting from the neutron capture. (The letters A, C and E in Figure 4.29 correspond to the states of the liquid drop in Figure 4.26.) In order for fission to occur the system must pass from A to E, and it can only do so, in general, if the compound nucleus gains sufficient energy to raise it to the level of C. Thus, the energy difference between the states A and C represents the critical energy for fission.

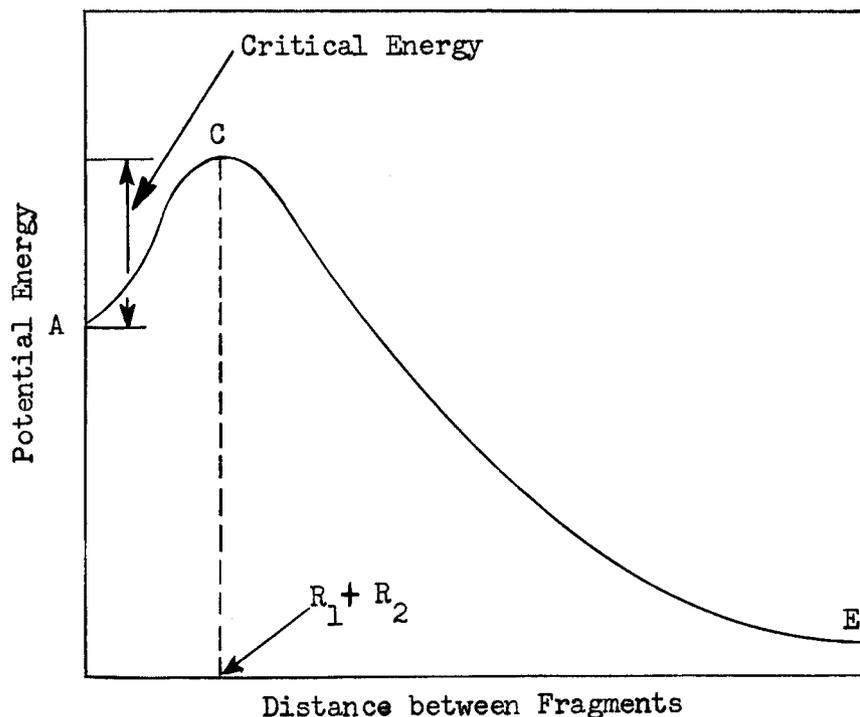


Figure 4.29

4.30 A qualitative idea of the magnitude of this critical energy may be obtained by considering the factors which determine the energies corresponding to the points A and C. The energy at A is the energy of the ground state of the compound nucleus with respect to the separated fragments at E; it is thus determined by the difference in mass of the target nucleus plus a neutron, on the one hand, and the sum of the masses of the two fission fragments, on the other hand. In view of the decrease of the binding energy, per nucleon with mass number, for nuclides of mass number exceeding about 80 (See Figure 1.30), this mass difference evidently increases with the mass number of the target nucleus. This conclusion is indicated by the curve E_A in Figure 4.30. The energy at C is determined, essentially, by the electrostatic repulsion of the fission fragments, and this is proportional to $Z_1 Z_2 / (A_1^{1/3} + A_2^{1/3})$, where Z_1, Z_2 and A_1, A_2 are the atomic numbers and mass numbers, respectively. Assuming for simplicity that $Z_1 = Z_2$ and $A_1 = A_2$, so that fission is symmetrical, the variation of E_C , i.e., the energy at C, with increasing mass number will be somewhat like that shown in Figure 4.30.

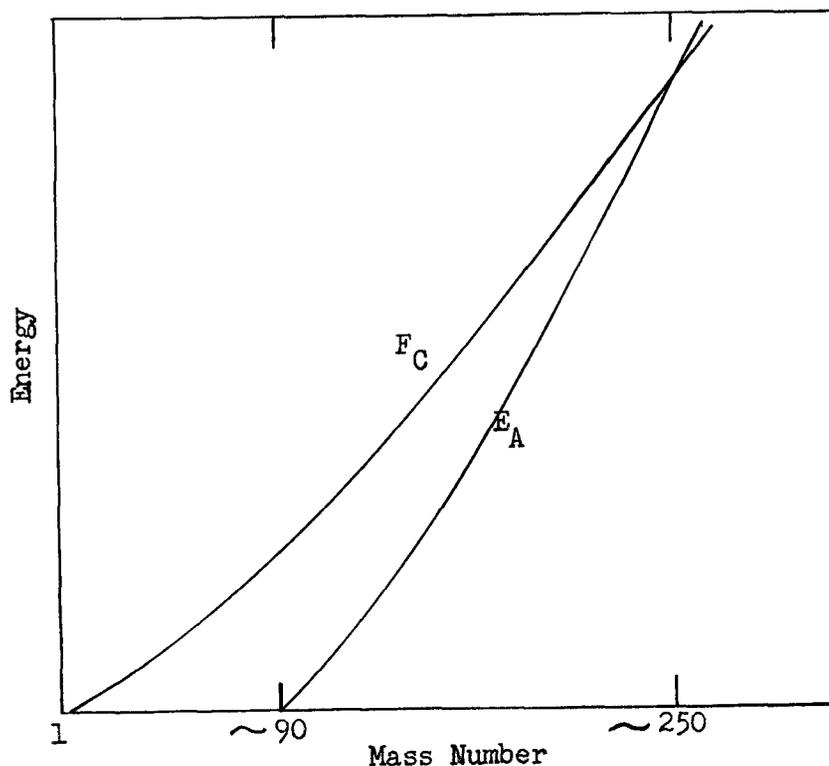


Figure 4.30

4.31 It is evident from the vertical distance between the curves for E_C and E_A that the critical energy for fission is at first very large, but it decreases with increasing mass number and atomic number. Thus, although fission by neutrons is theoretically possible for nuclei of mass numbers of about 80 or more, the critical energy required for the process to take place at an observable rate is greater than the largest neutron energies attainable at the present time. When the mass number is in the vicinity of 235 or so, the critical energy, as will be seen shortly, is down to about 6 Mev, so that fission by neutrons becomes an observable phenomenon. For nuclei of still higher mass number the E_A and E_C curves cross, so that E_A is above E_C . In these circumstances, that is, for mass numbers exceeding about 250, no critical energy is required for fission; in fact such nuclei are so unstable that if they could be produced in any way they would undergo spontaneous fission within an interval of about 10^{-20} sec.

4.32 It is of interest to note that the critical energy for fission decreases with increasing mass number because the curve for E_A in Figure 4.30 rises more rapidly than does that for E_C . This is due to the decrease in binding energy per nucleon at the higher mass numbers. The main factor contributing to this decrease is the increasing electrostatic repulsion energy of the protons. Consequently, the large magnitude of the mutual repulsion of the protons contributes to the decrease in the critical energy, thus making fission at an observable rate possible. As stated above, it also plays an important part in determining the large amount of energy liberated in fission.

4.33 Another quantitative aspect of the fission problem is provided by the liquid drop model discussed earlier. If it is assumed that the sphere does not change its volume when it is deformed into an ellipsoid (§ 4.26),

as is reasonable in view of the proportionality between the nuclear volume and mass ($\frac{2}{3} 1.34$), the change in energy will be due to only two of the five factors considered in Chapter I, namely, the surface tension effect and the electrostatic repulsion energy. The former will increase in magnitude when the nucleus is deformed, because of the increased surface area, but the latter will decrease, since the charges are separated to some extent. If ϵ is a parameter representing the degree of deformation, the energy required to produce this deformation is given by

$$\text{Deformation energy} = \epsilon^2 \left(5.2 A^{2/3} - 0.117 \frac{Z^2}{A^{1/3}} \right), \quad (4.33.1)$$

where the first term in the parentheses is the change in the surface tension effect and the second is that in the electrostatic repulsion energy*. When the deformation energy is zero, or negative, the spherical nucleus will deform, and consequently, undergo fission, spontaneously. The condition for spontaneous fission is, therefore,

$$0.117 \frac{Z^2}{A^{1/3}} > 5.2 A^{2/3}$$

or

$$\frac{Z^2}{A} > 44.5 \quad (4.33.2)$$

In 4.31 it was seen that a nucleus with mass number exceeding 250 would be expected to undergo instantaneous fission. If this result is combined with equation (4.33.2), it would appear that the maximum atomic number for stability against fission is about 105. These figures are, of course, approximate only.

4.34 The deformation energy considered above may be regarded as being equivalent to the critical energy for fission. Thus, equation (4.33.1) may be written as

$$\text{Critical energy} \propto A^{2/3} \left(5.2 - 0.117 \frac{Z^2}{A} \right).$$

* Bohr and Wheeler, Phys. Rev., 56, 426 (1939).

For a group of elements of high mass number $A^{2/3}$ is almost constant, and so the critical energy will decrease as Z^2/A increases. For plutonium-239 the value of Z^2/A is 37.0, for uranium-233 it is 36.4, for uranium-235 the value is 36.0, and for uranium-238 it is 35.5. The critical energy for fission should thus increase in this order. Calculations* based on the liquid drop model indicate that the critical energy is about 6.5 Mev for the compound nucleus formed as a result of neutron capture by uranium-235, and about 7.0 Mev for uranium-238.

4.35 The reference to spontaneous fission made above applies to cases in which none of the nuclei could exist for any appreciable time. It is important to point out, however, that there is always a certain probability that apparently stable or quasi-stable nuclei will undergo spontaneous fission. Even though the nucleus in its normal state does not have enough energy to permit it to pass through the critical deformation stage, the principles of wave mechanics require that there should be a definite, although small, probability for fission to take place.

Fast- and Slow-Neutron Fission

4.36 It will be recalled that the fission of uranium-235 can be brought about by slow (thermal) neutrons, with 0.025 ev energy, while uranium-238 requires neutrons of at least 1.1 Mev energy. The difference is partly accounted for by the smaller critical energy in the former case, but this is probably no more than about 0.6 Mev (} 4.34), and so there is still a discrepancy that requires explanation. Similarly, while thermal neutrons cause fission of uranium-233 and plutonium-239, fast neutrons are required for thorium-232, protactinium-231 and neptunium-237. The explanation of the difference, as will be shown below, lies in the fact that when fission

* Bohr and Wheeler, loc. cit.; see also, S. Frankel and N. Metropolis, Phys. Rev., 72, 914 (1947).

occurs a large proportion of the critical deformation energy is provided by the binding energy of the captured neutron, and this energy varies appreciably from one nucleus to another.

4.37 The excitation energy of the compound nucleus formed by the capture of a neutron of zero kinetic energy is equal to the binding energy of a neutron (§ 2.14). It is, consequently, the energy equivalent of the mass difference between the target nucleus plus a neutron, on the one hand, and the compound nucleus on the other hand. It can be readily shown from equation (1.29.1) that this is the same as the total binding energy of the compound nucleus minus the total binding energy of the target nucleus. These binding energies can be determined from equation (1.29.1) if the respective isotopic masses are known, or from equation (1.44.1) if they are not. Since the two masses are generally not known, it is preferable, for consistency, to use the latter equation to calculate the binding energies of both target and compound nuclei.

4.38 To determine the excitation energy of the compound nucleus in the case of uranium-235 as target nucleus, for example, the total binding energy is first calculated for this nucleus, for which A is 235 and Z is 92. The corresponding quantity is determined for the compound nucleus formed by the addition of a neutron, that is, with A equal to 236 and Z to 92. The difference between these two binding energies represents the excitation energy of the compound nucleus (U^{236}) formed when a neutron of zero kinetic energy is taken up by a uranium-235 nucleus. Using equation (1.44.1), the result is found to be

$$\text{B.E. } (U^{236}) - \text{B.E. } (U^{235}) = 6.8 \text{ Mev.}$$

Upon repeating the calculations for uranium-238, for which A is 238 and Z is 92, and the corresponding compound nucleus with A equal to 239 and

Z to 92, the excitation energy of the compound nucleus (U^{239}) is given by

$$\text{B.E. } (U^{239}) - \text{B.E. } (U^{238}) = 5.5 \text{ Mev.}$$

4.39 As mentioned in § 4.34, calculations indicate that the critical deformation energy of uranium-238 is about 7.0 Mev, but evidently only 5.5 Mev is acquired when the nucleus takes up a neutron of zero kinetic energy. It would appear, therefore, that the incident neutron would need to have at least $7.0 - 5.5 = 1.5$ Mev of kinetic energy to make fission of uranium-238 possible. Experiments show that the minimum neutron energy is about 1.1 Mev. The discrepancy between the observed (1.1 Mev) and the calculated (1.5 Mev) energies is, no doubt, due partly to the inexact nature of the calculations.

4.40 Turning now to uranium-235, it will be seen that the conditions are quite different. The critical energy for fission has been estimated to be 6.5 Mev, but, as seen above, 6.8 Mev becomes available as the result of the capture of a neutron with zero kinetic energy. It is evident, therefore, that slow neutrons should be capable of causing fission of the uranium-235 nucleus, as indeed they are.

4.41 If a detailed examination is made of the calculations which lead to the appreciably different binding energies, given in 4.38, for the two isotopes of uranium, the cause of the discrepancy becomes apparent. It is due almost entirely to the effect of the odd-even or spin term in equation (1.44.1). Since the compound nucleus uranium-236 is of even-even character, this term makes a positive contribution of about 0.55 Mev to the binding energy; it is zero, however, in uranium-235, which is an odd-even nucleus. With the 238 isotope of uranium, the situation is

reversed; the compound nucleus uranium-239 has an odd-even character and the spin effect is zero, but uranium-238 is even-even and the spin effect term is again about 0.55 Mev positive. This effect alone is, consequently, responsible for a difference of $2 \times 0.55 = 1.1$ Mev in the excitation energies of the compound nuclei formed by uranium-235 and uranium-238, respectively, upon the addition of a neutron.

4.42 From the foregoing considerations it may be concluded, in general, that an odd-even nucleus, that is, one with an odd number of neutrons and an even number of protons, will produce a compound nucleus with a relatively large amount of excitation energy when it absorbs a slow neutron. Hence, provided Z^2/A for the nucleus is sufficiently high, such a neutron will be able to induce fission. Instances of this type, in addition to uranium-235, are uranium-233, plutonium-239 (${}_{94}\text{Pu}^{239}$), both of which are fissionable by slow neutrons. With even-even nuclei, like uranium-238 and thorium-232 (${}_{90}\text{Th}^{232}$), the excitation energy when a neutron of zero kinetic energy is captured is relatively smaller, and hence high energy neutrons are required to cause fission.

4.43 If a nucleus has an even number of neutrons and an odd number of protons, e.g., neptunium-237 (${}_{93}\text{Np}^{237}$), the compound nucleus (${}_{93}\text{Np}^{238}$), formed by the absorption of a neutron will be of the odd-odd type, with a negative spin contribution of about 0.55 Mev. In this event, the quantity $\text{B.E.}(\text{Np}^{238}) - \text{B.E.}(\text{Np}^{237})$, which is the excitation energy of the compound nucleus, will be similar to that for nuclei of the even-even type. For fission to be possible it would therefore be necessary to employ neutrons of high energy.

4.44 Finally, consideration may be given to the fission of an odd-odd nucleus; the compound nucleus formed by the capture of a neutron would

then be of the even-odd type, which has similar binding energy to the odd-even type. The spin term will be zero in the compound nucleus and negative in the target nucleus, so that the excitation energy will be relatively large. Thus, it is to be expected that odd-odd nuclei will undergo fission by slow neutrons. Since such species are relatively unstable in any event, the fact that they are fissionable by slow neutrons is of no practical value.

4.45 The results of the preceding paragraphs are summarized in Table 4.45; the signs of the spin terms for the compound nucleus and the target nucleus for each of the different cases are given. The relative magnitude of the excitation energy of the compound nucleus resulting from the capture of a neutron of zero kinetic energy, which is the difference between the total binding energies of the compound nucleus and the target nucleus, is indicated in the last column. In view of the natural instability of the odd-odd species, the only substances of interest for the release of nuclear energy by slow neutrons are consequently the nuclides with odd numbers of neutrons and even numbers of protons.

Spin Terms and Excitation Energies				
Target Nucleus		Spin Terms		Excitation Energy
Neutron	Proton	Compound Nucleus	Target Nucleus	
Odd	Even	+	0	large
Even	Even	0	+	small
Even	Odd	-	0	small
Odd	Odd	0	-	large

Table 4.45

The Fission Chain Reaction

The Multiplication Factor

4.46 With the information already given, it is now possible to consider the feasibility of applying the fission reaction to the practical utilization of nuclear energy. The essential condition is that a self-sustaining chain reaction should be maintained; in other words, once the fission process has been initiated in a few nuclei, it should be able to continue throughout the remainder of the material without external influence. Since at least two neutrons are released in each act of fission (§ 4.5), and these neutrons are capable of inducing the fission of other nuclei, and so on, it would appear that the requirements of a self-sustaining chain reaction can be met. However, account must be taken of the fact that the neutrons produced in the fission process can take part in other (nonfission) reactions. In addition to this competition for neutrons, there is the inevitable loss of neutrons from the system by leakage.

4.47 If a chain reaction is to be maintained, the minimum condition is that for each nucleus capturing a neutron and undergoing fission there shall be produced, on the average, at least one neutron which causes the fission of another nucleus. This condition can conveniently be expressed in terms of a multiplication factor or reproduction factor, defined as the ratio of the number of neutrons of any one generation to the number of corresponding neutrons of the immediately preceding generation. If the multiplication factor, represented by k , is exactly equal to, or slightly greater than, unity a chain reaction will be possible. But if k is less than unity, even by a very small amount, the chain cannot be maintained.

4.48 Suppose, for example, a particular generation starts with 100 neutrons, if the multiplication factor is unity, there will be 100 corresponding neutrons at the beginning of the second generation, 100 at the third and so on. Once it has started, the fission will continue at the same rate as at the commencement. For practical purposes, however, it is necessary that k be capable of exceeding unity, if power production is to be appreciable. As seen in § 4.24, fissions must take place at the rate of 3.1×10^{10} per sec to produce 1 watt of power. The simplest way in which a required power level can be attained is for the multiplication constant to exceed unity; the number of neutrons present, and hence, the fission rate will then increase until the desired rate is reached.*

4.49 The multiplication factor k is effectively the number of neutrons present at the end of a neutron generation for each neutron present at the beginning of that generation. Since one neutron is required to maintain the chain reaction, the number of neutrons will increase by $k-1$ in one generation. Thus, if there are n neutrons present initially, the rate of increase will be $n(k-1)$ per generation. If λ is the average lifetime of a neutron in the system under consideration, then it is possible to write

$$\frac{dn}{dt} = \frac{n(k-1)}{\lambda} = \frac{nk_{ex}}{\lambda}, \quad (4.49.1)$$

where k_{ex} is defined by

$$k_{ex} = k-1.$$

Upon integration of equation (4.49.1), it is seen that

$$n = n_0 e^{t(k_{ex}/\lambda)}, \quad (4.49.2)$$

*The number of nuclei undergoing fission per sec is $\phi \Sigma_f V$, where ϕ is the thermal neutron flux per cm^2 per sec, Σ_{fcm} is the macroscopic cross section for fission and V_{cc} is the volume of the reactor. Since 3.1×10^{10} fissions per sec are required to produce 1 watt, the power output will be $\phi \Sigma_f V / 3.1 \times 10^{10}$ watts.

where n_0 is the initial number of neutrons and n is the number after the lapse of time t . It is seen, therefore, that if the multiplication factor is greater than unity, the number of neutrons will increase exponentially with time.

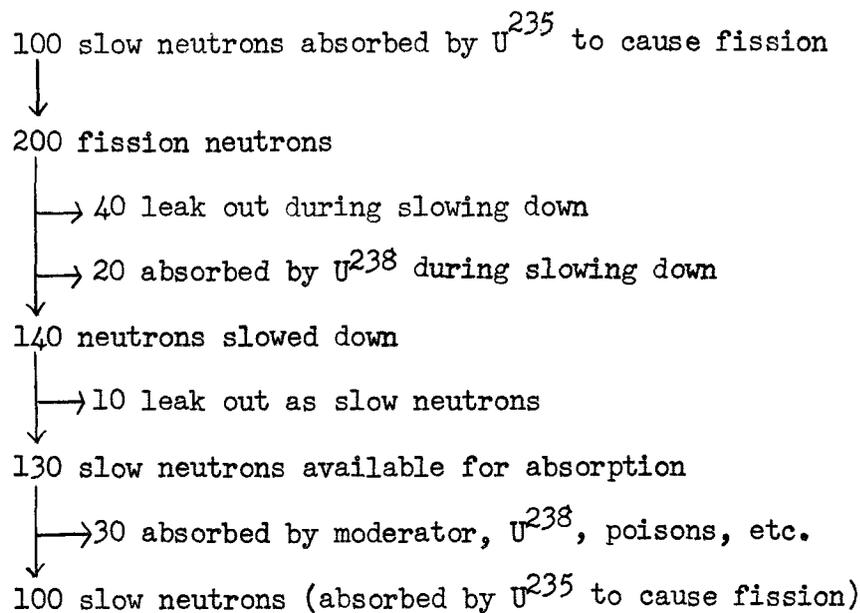
4.50 Suppose, in a particular case, k is 1.005, so that k_{ex} is 0.005. Taking the average neutron lifetime to have the reasonable value of 0.001 sec, it follows from equation (4.49.2) that after the lapse of a second the number of neutrons will have increased by a factor e^5 , i.e., roughly 150 fold. An increase in the number of neutrons will mean an increase in the fission rate and a consequent increase in the power output. When the desired output is attained, the multiplication factor should be reduced exactly to unity, either by introducing a nonfissionable neutron absorber or by allowing some neutrons to escape. The number of neutrons present in the system, the fission rate, and the power level will then remain constant.

4.51 If the multiplication factor is less than unity, the maintenance of a self-sustaining chain reaction becomes impossible. The rate of change in the number of neutrons is again represented by equation (4.49.2), but k_{ex} , equal to $k-1$, is now negative, and the neutron concentration decreases continuously in an exponential manner. As long as the multiplication factor is less than unity, by no matter how small an amount, the number of neutrons must inevitably decrease with time, and the maintenance of a self-sustaining reaction is not possible.

4.52 The value of the multiplication factor in any system consisting of fissionable material, e.g., uranium, and a moderator for slowing down the neutrons (§ 3.10) depends on the relative extents to which the neutrons

take part in four main processes. These are: (a) complete loss or escape of neutrons from the system, generally referred to as leakage; (b) nonfission capture, by uranium-235 and uranium-238, frequently designated resonance capture, since it is likely to occur mainly at resonance energies; (c) nonfission capture, sometimes called parasitic capture, by the moderator and by various extraneous substances ("Poisons") such as structural materials, coolant, fission products and impurities in the uranium and in the moderator; and, finally, (d) fission capture of slow neutrons by uranium-235, or of fast neutrons by both uranium-235 and uranium-238. In each of these four processes neutrons are removed from the system, but in the fourth process, i.e., in the fission reaction other neutrons are generated to replace them. Hence, if the number of neutrons produced in the latter process is just equal to (or exceeds) the total number lost by escape and by fission and nonfission capture, the multiplication factor will equal (or exceed) unity and a chain reaction should be possible.

4.53 An illustration of the type of neutron balance that might exist in a system for which the multiplication factor is exactly unity is depicted below. It is assumed that fission results only from the capture of slow neutrons, and it is supposed, for simplicity, that exactly two neutrons are produced, on the average, in each fission process. Since 100 slow neutrons are absorbed in fission processes at the beginning, and 100 are available for similar absorption at the end of the generation, the conditions for a self-sustaining chain are satisfied.



4.54 A nuclear reactor is a system usually consisting of a moderator and fissionable material (fuel), together with coolant and structure, in which a self-sustaining chain reaction can be maintained. In such a reactor fast neutrons are produced in the fission process; these neutrons may suffer scattering collisions, mainly elastic, as a result of which their energy is decreased; they may be absorbed by the various material present in the system; or they may escape. Depending on the relative amounts of the moderator, fuel and other substances, their geometrical arrangements, and the dimensions of the system, which largely determines the leakage, the main portion of the neutron captures leading to fission will take place in a certain energy range.

4.55 If most of the fissions result from the capture of thermal neutrons, the system is referred to as a thermal reactor. When most of the fission processes are due to the absorption of neutrons of higher energy, sometimes called epithermal neutrons or intermediate neutrons, the term intermediate reactor is used. An important type of intermediate reactor is one in which most of the fissions are induced by neutrons

with energies from thermal values up to about 1000 ev. Finally, if the main source of fissions is the capture of fast neutrons by the fuel, the system is a fast reactor.

4.56 Because of the relatively low cross sections for fission with neutrons of high energy, especially compared with the cross sections for nonfission reactions, it is impossible to maintain a nuclear chain reaction with fast neutrons in natural uranium, consisting of nearly 99.3 per cent of uranium-238 and 0.7 per cent of uranium-235 (Table 1.11). By using fuel material enriched in the latter isotope, or containing a sufficient proportion of plutonium-239, a chain reaction with fast neutrons can be achieved.* With natural uranium, a chain reaction is possible only if fission is mainly due to slow, i.e., thermal, neutrons; this is because the fission cross section of uranium-235 is sufficiently large for thermal neutrons to compensate for the nonfission absorption. In order to slow down the neutrons a moderator must be used, and this may be heavy water, beryllium (or beryllium oxide), carbon (graphite), or even ordinary water if an enriched fuel is used.

4.57 Because of their importance, and incidentally because they are most susceptible to theoretical treatment, some consideration will now be given to the conditions which determine the value of the multiplication factor for thermal reactors, especially those using natural uranium as fuel. In order to avoid, for the present, the problem of the loss of neutrons by leakage, it will be postulated that the multiplying system is infinite in extent. Suppose that at a given instant, representing the initiation of a generation, there are available n thermal neutrons which are captured in fuel. Let η be the average number of fast fission

*The atomic bomb is a reactor in which fast neutrons maintain an uncontrolled nuclear chain in uranium-235 or plutonium-239.

neutrons emitted as a result of the capture of one thermal neutron in fuel material, that is, in both uranium-235 and -238. Then, due to the absorption of the n thermal neutrons $n\eta$ fast neutrons will be produced. It should be noted that since the neutrons captured in fuel do not all necessarily lead to fission, the value of η differs, in general, from the average number (2.5 ± 0.1) of fast neutrons released per slow neutron fission (≈ 4.5). If the latter number is represented by ν , then

$$\eta = \nu \frac{\sum_f \sigma_f}{\sum_{\text{fuel}} \sigma} \quad (4.57.1)$$

where $\sum_f \sigma_f$ is the macroscopic cross section (≈ 3.42) for slow neutron fission, and $\sum_{\text{fuel}} \sigma$ is the total cross section for absorption of thermal neutrons, by fission and nonfission processes, in the fuel material.

4.58 Before the $n\eta$ fast neutrons have slowed down appreciably some will be captured by, and cause fission of, uranium-235 and -238 nuclei, mainly of the latter. Since more than one neutron is produced, on the average, in each fission, there will be an increase in the number of fast neutrons available. Allowance for this effect may be made by introducing the fast fission factor, denoted by ϵ and defined as the ratio of the total number of fast neutrons produced by fissions due to neutrons of all energies to the number resulting from thermal-neutron fissions. Consequently, as a result of the capture of n thermal neutrons in fuel, $n\eta\epsilon$ fast neutrons will be formed. For natural uranium fuel the value of ϵ has been found to be about 1.03, with either graphite or heavy water as the moderator.

4.59 As a result of collision, mainly elastic, with the moderator, the fast neutrons will ultimately be thermalized. However, during the

slowing down process some of the neutrons are captured in nonfission processes, so that not all of the $n \gamma \in$ fast neutrons reach thermal energies. The fraction of the fast (fission) neutrons which escape capture while being slowed down is called the resonance escape probability and is represented by p . Consequently, the number of neutrons which become thermalized is $n \gamma \in p$.

4.60 When the energy of the neutrons has been reduced to the thermal region, they will diffuse for some time, the energy distribution remaining essentially constant, until they are ultimately absorbed by fuel, by moderator or by such poisons as may be present. Of the thermal neutrons, therefore, a fraction f , called the thermal utilization, will be absorbed in fuel material; the value of f is represented by

$$f = \frac{\text{Thermal neutrons absorbed in fuel}}{\text{Total thermal neutrons absorbed}} \quad (4.60.1)$$

where the denominator is the total number of thermal neutrons absorbed by fuel, moderator and other materials present in the reactor. The number of thermal neutrons captured in fuel is consequently $n \gamma \in pf$.

4.61 Since, for the present purpose, the multiplication factor (ξ 4.2) may be defined as the ratio of the total number of thermal neutrons produced, on the average, in one generation divided by the number of thermal neutrons absorbed in that generation, on the average, in an infinite medium, it follows that

$$k = \frac{n \gamma \in pf}{n} = \gamma \in pf. \quad (4.61.1)$$

This result is sometimes referred to as the four factor formula. As seen above, the condition for a self-sustaining chain reaction in a system of infinite size is that the multiplication factor should be

unity; the criterion for a natural uranium system is, therefore, that $\eta \epsilon pf = 1$.

4.62 In the special case of a reactor in which the fuel material contains only uranium-235 and no uranium-238, both the fast fission factor ϵ and the resonance escape probability will be virtually unity. This is because fission by fast neutrons and absorption of neutrons in the resonance region occurs mainly in the heavier isotope. In these circumstances equation (4.61.1) would reduce to

$$k = \eta f.$$

4.63 Of the four factors involved in equation (4.61.1), η and ϵ are more or less fixed by the character of the fuel, but p and f can be varied to some extent. In order to insure the propagation of the nuclear fission chain, p and f should be as large as possible, although they are, of course, always less than unity. Unfortunately, such changes in the relative proportions of fuel and moderator as cause f to increase cause p to decrease. If the system contains a relatively small amount of moderator, thermal utilization will be increased [cf. equation (4.60.1)], but the larger proportion of uranium-238 means a decrease in the resonance escape probability. The reverse will be true if the proportion of moderator is large. In actual practice, therefore, it is necessary to find the composition and arrangement which gives the maximum value for the product pf , in order to maintain the chain reaction.

4.64 In the case of a system consisting of natural uranium and graphite an increase in the value of the product pf can be achieved by a heterogeneous lattice arrangement consisting of fairly large lumps of uranium

imbedded in a mass of graphite. For reasons which will be considered in Chapter IX, the resonance escape probability is greater than for a homogeneous mixture of uranium and graphite of the same composition. However, there is some decrease in the thermal utilization, but by a proper design of the lattice and proper choice of the proportion of fuel to moderator, an overall increase in the product pf is achieved as compared with that for a uniform mixture.

4.65 One way in which the multiplication factor of a reactor can be increased is by the use of enriched fuel material, containing a larger proportion than normal of the fissionable isotope, uranium-235. If the isotopic ratio of U^{235} to U^{238} is increased, η becomes larger and the resulting multiplication constant is increased. Another smaller effect is a decrease in p for a given ratio of U^{235} atoms to moderator atoms.

Leakage of Neutrons

4.66 It should be emphasized that equation (4.61.1) was derived for an infinite system, for which there is no leakage of neutrons; for this reason the factor k is sometimes written as k_{∞} , and is called the infinite multiplication factor. In a finite multiplying system, a factor k can be defined as above, by the right-hand side of equation (4.61.1). It will, however, not be the same as k_{∞} for an infinite medium, for the following reasons. In calculating the resonance escape probability and the fast fission factor, the neutron energy distribution is important, since both quantities are functions of the energy. When, as in a reactor of finite size, there is neutron leakage, the energy distribution will not be the same everywhere in the system, as is true for an infinite (homogeneous) system, but will depend on the position in the reactor.

Consequently, p and f , and hence the multiplication factor with no leakage, for the finite reactor will differ from the values for the infinite system. However, for most practical purposes, k (finite size but no leakage) can be considered equal to k_{∞} , as defined above.

4.67 For a reactor of finite size the condition that the multiplication factor should be unity is no longer adequate for a self-sustaining chain reaction. It is required, now, that for every thermal neutron absorbed in fuel there shall be produced, on the average, one thermal neutron in addition to the loss by leakage from the reactor. If P is the total nonleakage probability, that is, the probability that a neutron will not escape either during the slowing down process or while it diffuses as a thermal neutron, then the condition for a chain reaction to be maintained is

$$kP = 1, \quad (4.67.1)$$

where k is defined by equation (4.61.1). Only for the infinite system is the nonleakage probability unity, and then $k = 1$, satisfying the condition for the chain reaction. For a finite reactor, P is less than unity, and hence the multiplication factor must exceed unity if the nuclear chain reaction is to be maintained.

4.68 As indicated above, the value of k is determined by the composition of the system, that is, by the nature of the fuel and proportion of moderator, and also by the arrangement of the material. Hence, if these are specified, a chain reaction will be possible only if P is large enough to make kP equal to or greater than unity. In other words, if the chain reaction is to be self-sustaining for any given fuel-moderator system, the nonleakage probability must exceed a minimum value; thus,

the permissible leakage of neutrons relative to the number being produced must be less than a certain amount.

Critical Size of Reactor

4.69 The proportion of neutrons lost by escape from a finite reactor can be diminished by increasing the size of the system. The escape of neutrons occurs at the exterior, but absorption, leading to fission and neutron production, occurs throughout the whole of the interior of the reactor. The number of neutrons lost by escape thus depends on the external surface area, while the number formed is determined by the volume. To minimize the loss of neutrons, and thereby increase the non-leakage probability, it is necessary to decrease the ratio of area to volume; this can be done by increasing the size of the reactor. The critical size is that for which the nonleakage probability P is such that kP is just equal to unity. Since the area to volume ratio depends on the geometrical shape, the nonleakage probability will be determined by the shape of the reactor. For a given volume, a sphere has the smallest ratio of area to volume; hence, leakage from a spherical reactor will be less than for any other shape. The critical volume of such a reactor will consequently also be less. Leakage of neutrons can be diminished by surrounding the reactor with a suitable reflector, that is, a material generally itself a moderator, which returns a proportion of the escaped neutrons to the reactors.

4.70 Since the value of k depends on the fuel-moderator composition and structure, it is apparent that the maximum permissible neutron leakage for a self-sustaining chain reaction will also be determined by these factors. The critical size, even for a reactor of specified geometry

will thus not be a constant, but will vary with the nature and structure of the fuel-moderator system. For example, if k is increased, by the use of enriched fuel, then it will be permissible for the nonleakage probability P to decrease, and hence the neutron leakage to increase, and still satisfy the condition that kP is equal to or greater than unity. Consequently, the critical size of the enriched reactor will be less than that of a reactor of the same geometrical shape and structure using natural uranium as fuel.

4.71 The product kP is called the effective multiplication factor of a reactor of finite size. The critical condition is that the effective multiplication factor shall be exactly unity [cf equation (4.67.1)] The chain reaction will then be maintained at a constant rate of fission and power level; this is sometimes referred to as a steady state or stationary state of the reactor. Up to a point, a given reactor can have an indefinite number of such states corresponding to different fission rates and power levels. If the effective multiplication factor of a reactor exceeds unity, the system is said to be supercritical. In such a reactor the rate of fission and hence the neutron density (or flux) and the power level increase steadily. When the effective multiplication factor is less than unity, that is, in a subcritical reactor, the neutron density (or flux) and power level decrease steadily.

Reactor Control

4.72 For practical operation, a reactor must be constructed so that it is actually appreciably greater than the critical size. One reason is that having an effective multiplication factor exceeding unity provides the only feasible means of increasing the number of neutrons, and

hence the fission rate, up to the point where the required power level is attained (§ 4.48). Once this has been reached, it is necessary to decrease the effective multiplication factor to unity, and then the reactor will remain in a steady state, neutrons being produced just as fast as they are used up by leakage and capture.

4.73 The adjustment of the multiplication of neutrons in a thermal reactor is achieved by the insertion of control rods of cadmium or boron steel. Both cadmium and boron have large capture cross sections for slow neutrons (see §§ 3.69, 3.76); hence, by varying the position of the control rods the effective multiplication constant can be made to vary over a suitable range. In order to shut down the reactor, the control rods are inserted to an extent that permits them to absorb additional neutrons. The system now loses neutrons faster than they are formed by fission; the effective multiplication factor sinks below unity, and the chain reaction dies out.

Effect of Delayed Neutrons

4.74 In the calculation in § 4.49 of the rate of neutron increase in a reactor with a multiplication factor exceeding unity, the mean lifetime of a neutron was taken to be 0.001 sec. This is actually the average value of the time elapsing between the birth of a neutron and its ultimate capture in a thermal reactor; it gives the correct rate of neutron increase only if all the fission neutrons are released promptly, that is, essentially at the instant of fission. It was seen in § 4.8, however, that about 0.75 per cent of the fission neutrons are delayed, and this affects the calculation of the rate of neutron increase (or decrease).

4.75 The mean lives of the five groups of delayed neutrons range from about 0.6 sec to 80 sec (table 4.8). By weighting the values appropriately, according to the fraction in each group, the mean delay time, averaged over all the fission neutrons, is about 0.1 sec (7 4.8). The average time between the fission capture of a neutron in two successive generations is, consequently, about $0.1 + 0.001$ sec; the first term is the average time elapsing between fission and the complete release of the neutron, while the second is that between release and capture in a fission process. In other words, the effective lifetime of a neutron is roughly 0.1 sec.

4.76 Using the value 0.1 sec for l in equation (4.49.2), and taking k to be 1.005, as before, it is found that the number of neutrons actually increases by a factor of $e^{0.05}$, i.e., about 1.05 per sec, as compared with a factor of 150 per sec if all the neutrons were prompt.* Clearly, the effect of the delayed fission neutrons, when the multiplication factor exceeds unity, is to make the rate of neutron increase much slower than it would have been had all the neutrons been released promptly.

4.77 Suppose that, in general, β is the fraction of the fission neutrons which are delayed, so that $1-\beta$ represents the fraction of the prompt neutrons. Of the total number of fast neutrons produced for each thermal neutron absorbed in fuel $(1-\beta)\eta$ are emitted instantaneously, while $\beta\eta$ are delayed and expelled gradually over a period of time. Consequently, the multiplication factor may be considered as consisting of two parts; one, equal to $k(1-\beta)$, representing the prompt neutron multiplication factor, and the other, equal to $k\beta$, being

*The calculations given here are approximate only and are intended mainly to provide a general indication of the effect of delayed neutrons. The subject will be treated more completely in Chapter X.

due to the delayed neutrons. If, in the operation of a reactor, the quantity $k(1-\beta)$ is adjusted so as to be just less than, or equal to, unity, then the rate of increase in the number of neutrons from one generation to the next will be determined essentially by the delayed neutrons. Since β is actually 0.0075 (β 4.8), this condition can be realized by having the effective multiplication factor between unity and 1.0075. When this is the case, the neutron flux (or density) and power level of the reactor will increase relatively slowly and adequate control is possible.

4.78 When the effective multiplication factor is exactly 1.0075, the condition of a reactor is described as prompt critical, since the nuclear fission chain can be maintained by means of the prompt neutrons alone. If k exceeds this value, multiplication will occur due to the prompt neutrons, irrespective of those delayed, and the neutron density will increase rapidly right from the commencement. In this condition, a reactor is difficult to control and hence it should be avoided in practice.

4.79 Just as the delayed neutrons affect the rate of increase of neutrons when the effective multiplication factor exceeds unity, so they influence the decay in the neutron density when the reactor is made subcritical, that is, when it is being shut down. The delayed neutrons continue to be emitted for some time, and this maintains a fission rate that is considerably higher than would be the case if all the fission neutrons were prompt. The ultimate rate at which the neutron flux in a thermal reactor decreases after shutdown is determined essentially by the most delayed group of neutrons, i.e., those with a mean life of 80 sec. (see Table 4.8).

4.80 In reactors using heavy water as moderator, shutdown is further delayed by the neutrons formed as a result of the interaction of gamma radiation from the fission products with deuterium ($\sigma \approx 3.5$).

<u>Thermal Neutron Cross Sections for Uranium</u>			
	Fission	Radiative Capture	Scattering
U ²³⁵	545 b.	100 b.	8.2 b.
U ²³⁸	0	2.6	8.2
Natural	3.9	3.3	8.2