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THE MOTION OF GAS BUBBLES IN THE HRE REACTOR CORE

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THE MOTION OF GAS BUBBLES IN THE HRE REACTOR CORE

by

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The University of Tennessee
Department of Chemical Engineering

Date Issued: JAN 4 1952

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THE MOTION OF GAS BUBBLES IN THE HRE REACTOR CORE

SUMMARY

A theoretical study of the motion of gas bubbles in a rotating liquid is presented to make comparison with experimental data from an investigation concerning the removal of gas bubbles from rotating liquids as described in ORNL-630. In particular; the following information is presented:

(1) The literature on the motion of gas bubbles in liquids is reviewed and a summary of extensive experimental studies at the University of Tennessee on this topic is presented. These results are presented in the form of quantitative expressions for the drag coefficients of gas bubbles in terms of the bubble size and physical properties of the system.

(2) An expression for the radial velocity of a gas bubble in a rotating liquid based on principles of mechanics is presented. This expression includes the effects of liquid velocity, drag coefficient of the gas bubble and the physical properties of the system. Computations of the maximum gas bubble residence time for the HRE mock-up studies were made based on this relation. The results of these computations are presented and are found to agree favorably with experimental data described in ORNL-630.

(3) A method for computing the steady state state gas hold-up in the HRE reactor core under power producing conditions is presented. The results of computations based on this method are presented. Such results indicate that 0.10 to 0.55 volume percent gas hold-up will be realized in the HRE reactor core. These values reflect the influence of gas bubble size over the range of 0.001 to 0.6 inch diameter and the influence of liquid velocities in the reactor core corresponding to a 2 inch and a 1 1/2 inch nominal diameter inlet pipe.

It is concluded that the relations and methods presented are general in scope, and could be applied to any size reactor core providing basic data on liquid velocity distribution, gas bubble size and physical properties are known.

INTRODUCTION

The removal of gas bubbles from the fuel solution by centripetal action resulting from rotary motion of the liquid within the spherical HRE reactor core was described in ORNL-630. As described in the report cited, it was found that a stable liquid flow pattern could be maintained with liquid being introduced on a chord of the sphere. Under such conditions the liquid moved in an approximately spiral path toward the center of the sphere with the formation of a cylindrical gas void aligned with

the vertical axis of the sphere. The liquid effluent was from an annular opening around the gas void at the top of the vertical axis.

It was also found upon introduction of gas bubbles into the liquid within the sphere that the gas bubbles could be separated from the liquid very effectively, the gas bubbles moving rapidly toward the central gas void in an almost plane, spiral path. Results were given of extensive experimental work involving the flow characteristics of air bubbles in water moving within a plastic sphere 1.5 feet in diameter. For a water flow rate of 100 GPM at approximately 25°C it was found that the gas bubbles moved with radial velocities of 1.5 to 3.0 feet per second. Under these conditions the radial and tangential components of the liquid velocity were found to be given by

$$V_t = 4.8\ R^{-0.77} \quad , \ 0.12 \leq R \leq 0.75 \text{ ft} \qquad (1)$$

$$V_t = 204\ R \quad , \ 0.01 \leq R \leq 0.12 \text{ ft}$$

$$V_r = \frac{0.0177}{R(0.5625-R^2)^{1/2}} \quad , \ 0.12 < R < 0.65 \qquad (2)$$

where,

V_t = tangential component of the liquid velocity at radius R relative to a fixed coordinate system, ft/sec

V_r = radial component of the liquid velocity at radius R relative to a fixed coordinate system, ft/sec

R = position radius measured from the vertical axis of the sphere, ft

The velocity distribution data were determined from pitot tube traverses at the equatorial plane of the sphere. The status of the experimental work was such that no information was available as to the effect of liquid properties on the observed bubble velocities or liquid velocity distribution. Further, the applicability of the velocity distribution as given by equations (1) and (2) to planes other than the equatorial was not known.

It is the purpose of this present work to examine the literature and summarize the data on the velocity of gas bubbles in liquids. Further, it is desired to consider the application of such data to these problems:

a. Development of relations to allow the quantitative prediction of the motion of gas bubbles in a rotating mass of liquid in terms of the physical properties of the system.

b. Computation of the mean bubble velocities in a sphere 1.5 feet in diameter for water at 25°C and at 250°C at a flow rate of 100 GPM showing the effect of bubble size on the velocities.

c. Computation of the anticipated total gas hold-up to be encountered in the HRE reactor core under power producing conditions; i.e. pressure-1000 psia, average temperature-250°C, power-1000 KW, fuel solution flow rate - 100 GPM.

The scope of this work is limited by the extent of existing knowledge on liquid velocity distribution in a sphere and the size of gas bubbles to be generated in the fuel solution. For the lack of other data

at the present time, the liquid velocity distribution data as expressed by equations (1) and (2) will be used for all points in the sphere. As further information on this topic becomes available the considerations can be revised.

Wilson[20] presented information showing that the size of gas bubbles to be generated under power producing conditions is conjectural, and it is possible that gas bubbles 6.6×10^{-5} ft to 2×10^{-2} ft radius may be released from the fuel solution. Again, for lack of specific information at the present time, this range of gas bubbles will be considered in the present work.

DATA ON GAS BUBBLE MOTION AND VELOCITIES

Theory

The problem of predicting the velocity of gas bubbles in a liquid is hydrodynamical in nature, and falls within the more general problem of the flow of fluids past bodies in a fluid stream, a question which has intrigued investigators in that field since the time of Newton. The earlier speculations were based on the hypothesis of an "ideal" fluid having zero viscosity. Reference may be made to texts on classical hydrodynamics such as Lamb[7], Milne-Thompson[10], Green[5], and Ramsey[13] for the results of such considerations. The conclusions are of little importance since for steady flow the "ideal" fluid offers no resistance to flow past a body immersed in the fluid stream. This is contrary to experience with all real fluids.

An analysis of the forces involved in the motion of a real fluid leads to the general differential equations of fluid motion known as the Navier-Stokes equations.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X_1 gc - \frac{gc}{\rho_1}\frac{\partial p}{\partial x} + \frac{\mu}{\rho_1}\nabla^2 u$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = X_2 gc - \frac{gc}{\rho_1}\frac{\partial p}{\partial y} + \frac{\mu}{\rho_1}\nabla^2 v \qquad (3)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = X_3 gc - \frac{gc}{\rho_1}\frac{\partial p}{\partial z} + \frac{\mu}{\rho_1}\nabla^2 w$$

where,

u,v,w = x,y,z components respectively of the fluid velocity, ft/sec

X_1,X_2,X_3 = x,y,z components respectively of any extraneous forces acting on a fluid element, pounds force/pound mass

p = pressure at a point (x,y,z), pounds force/(sq ft)

gc = conversion factor between force and mass

32.17 (pounds mass)(ft)/(sec)^2(pound force)

∇^2 = Laplacian operator

μ = liquid viscosity, pounds mass/(ft)(sec)

t = time, sec

ρ_1 = liquid density, pounds mass/ cu ft

This system of equations must be solved subject to the boundary conditions appropriate to the fluid motion situation involved. Attempts to obtain solutions by such an approach are usually hampered by unsurmountable

mathematical difficulties, and successful solutions have been achieved for only a few simple cases. In the main these simple cases have involved only unidirectional flow. However, for the case of symmetrical three dimensional fluid flow around a body immersed in the fluid stream, a solution may be obtained after certain simplifying approximations are made.

Stokes[16] considered the flow of a liquid past a solid sphere at very small velocities. For such small velocities the inertia terms $u\frac{\partial u}{\partial x}$, $v\frac{\partial u}{\partial y}$, ... become small compared with the frictional terms $\frac{\mu}{\rho_1}\nabla^2 u$, $\frac{\mu}{\rho_1}\nabla^2 v$, ... , and are neglected. With these approximations, Stokes solved system (3) for the boundary conditions of zero velocity at the surface of the sphere with the added assumptions of an infinite extent of liquid and gravity as the only extraneous force. The important result of the solution is that the total liquid resistance is given by

$$D = \frac{3\ \pi d \mu U}{gc} \tag{4}$$

where,

D = total liquid resisting force acting on the spherical solid, pounds force

d = diameter of the spherical particle, ft

U = velocity of the spherical particle relative to the liquid, ft/sec

This result can be applied to the rising of a gas bubble in a liquid under conditions for which equation (4) is valid. The equation of gravitational motion upon application of Newton's second law is

$$\frac{4/3\ \pi\ R_b^3\ (\rho_1 - \rho_2)g}{gc} - \frac{6\ \pi\ R_b\ \mu\ U}{gc} = \frac{4/3\ \pi\ R_b^3\ \rho_2}{gc}\ \frac{dU}{dt} \tag{5}$$

where

R_b = equivalent spherical radius of the bubble, ft

ρ_2 = density of the gas bubble, pounds mass/(ft)^3

When the bubble attains the steady state, or terminal velocity,

$\frac{dU}{dt} = 0$ and

$$U_\infty = \frac{2g(\rho_1 - \rho_2)\ R_b^2}{9\ \mu} \tag{6}$$

where

U_∞ = steady state velocity, ft/sec

This result is commonly known as Stokes' law. The restrictions imposed by the assumptions used in the development limit the application of this law to extremely small gas bubbles or very viscous liquids.

The inability to solve the Navier-Stokes equations for systems where Stokes' law does not apply has led to the practice of employing an arbitrarily defined drag coefficient. The drag coefficient is defined by

$$D = \frac{C_D\ \rho_1\ U^2\ A}{2\ gc} \tag{7}$$

where

C_D = drag coefficient, dimensionless

A = cross section of particle perpendicular to the direction of flow, sq ft

The drag coefficient as defined is somewhat analogous to the Fanning friction factor used to correlate the data for the pressure drop accompanying

the flow of fluids through pipes. Using this analogy, dimensional analysis has been employed to gain some insight concerning the relation between C_D and the variables influencing it. Such treatment gives the result

$$C_D = \Psi\left(Re\right) \tag{8}$$

where

Ψ = arbitrary function, dimensionless

Re = Reynolds number, $\frac{2\, R_b\, U\, \rho_l}{\mu}$, dimensionless

The form of this unknown function, Ψ, can be computed for the case where Stokes' law applies by equating the two expressions for the total liquid drag resistance from equations (4) and (7) and assuming a spherical gas bubble. Thus,

$$\frac{6\, \pi\, R_b\, \mu\, U}{gc} = \frac{C_D\, \rho_l\, U^2\, \pi\, R_b^{\,2}}{2\, gc}$$

Solving for C_D,

$$C_D = \frac{12\, \mu}{R_b\, U\, \rho_l} = \frac{24}{Re} \tag{9}$$

For cases where Stokes' law does not apply the function, Ψ, must be determined from experimental data.

A Review of the Literature

A brief summary of the literature dealing with the drag coefficients for solid spheres moving in liquids is given here to serve as a basis for comparing drag coefficients of these with the corresponding values for gas bubbles. Lapple and Shepard[8] examined the data for solid spheres of 19 investigators. These data covered a variety of particle sizes and liquid

properties over a Reynolds number range of 10^{-1} to 3×10^{6}. The averaged data of all the investigators were correlated by the following relations:

$$C_D = \frac{24}{Re} \qquad Re \leqq 2 \tag{10}$$

$$C_D = \frac{18.5}{Re^{0.6}} \qquad 2 \leqq Re \leqq 500 \tag{11}$$

$$C_D = 0.44 \qquad 500 \leqq Re \leqq 2 \times 10^{5} \tag{12}$$

The result expressed by equation (10) is identical to Stokes law which has been shown to be synonymous with viscous flow. Equation (11) seems to represent the transition region between viscous and turbulent flow, while equation (12) applies to the turbulent flow condition.

Allen[1] was one of the first investigators to attempt verification of Stokes law for gas bubbles moving in liquids. Studies were made of the rate of rise of single air bubbles ($R_b < 0.001$ feet) in water and aniline, and the data indicated excellent agreement with Stokes law for $Re \leqq 1$. For $1 < Re < 200$, Allen found that the bubbles had a spherical shape, rose vertically in almost a straight path, and the velocity was very nearly proportional to the radius. Furthermore, when the data were expressed in terms of the dimensionless drag coefficient and Reynolds number they were found to agree favorably with the results for solid spheres. The conclusions from this work are that for aniline and water the results for solid spheres expressed by equations (10) and (11) are also valid for gas bubbles with the Reynolds number range modified to $Re < 1$ and $1 < Re < 200$ respectively.

Miyagi[11] studied the rate of rise of single air bubbles in water over a size range, $0.003 < R_b < 0.013$ feet, and found that the linear relationship between velocity and radius extended to $R_b < 0.005$ feet. Also, for $R_b > 0.005$ feet the velocity decreased up to $R_b = 0.009$ feet, and then tended to remain constant independent of radius. Empirical expressions were presented to fit the experimental data, but no general relations in terms of physical properties of the fluids were given by Miyagi. It was pointed out that in the size ranges where the linear relation between velocity and radius does not apply, the bubbles tend to flatten in the horizontal direction and that the path of ascent is spiraling in nature.

Bryn[2] studied the motion of air bubbles through aqueous solution of alcohol and glycerine over a wide range of sizes. The movements of the bubbles were classified into three distinct types:

1. "Small Bubbles," approximately spherical, rising in straight lines; for water at room temperature, $R_b < 0.005$ feet.
2. "Medium-sized bubbles," flattened horizontally, rising with rocking, pendulum-like spiral movements; for water at room temperature, $0.005 < R_b < 0.013$ feet.
3. "Large bubbles," greatly deformed, assuming a mushroom-like shape, rising relatively straight, very unstable tending to break easily into numerous smaller bubbles; for water at room temperature, $R_b > 0.013$ feet.

Bryn confirmed the definite maxima reported by Miyagi in the velocity-radius curve and was able to express the relation between velocity and bubble radius

quantitatively for the "large bubble" region by

$$U = \beta\sqrt{a} + \frac{\delta h_b}{\sqrt{a}} \tag{13}$$

where

β = arbitrary constant

δ = arbitrary constant

$a = \left[\frac{2\gamma g_c}{g\rho_1}\right]^{1/2}$ Laplace's capillarity constant, ft

γ = liquid surface tension, pounds force/ft

O'Brien and Gosline[12] surveyed the literature on the velocity of gas bubbles and conducted experiments in which the rise of air bubbles through water, mineral oil and livestock oil were studied in 1.18 inch, 2.24 inch and 6.00 inch glass tubes. They made a dimensional analysis of the variables that could affect the drag coefficient, and found that the drag coefficient should be some function of the Reynolds number, the Weber number and possibly other dimensionless groups. Having done this, they abandoned this analysis and did not determine the functional relationships between the dimensionless groups mentioned. Instead, their results for all experiments were plotted as curves showing the variation of the drag coefficient with Reynolds number. Their results indicate that for the three liquids studied the relation between C_D and Re for solids is valid for gas bubbles up to some critical value of the gas bubble radius peculiar to the liquid involved. Above this critical radius the drag coefficients are much larger than those for solid spheres at the same Reynolds number, and the C_D versus Re curve loses its generality. Classifications of three types of bubble movement

were described which were similar to those of Bryn. Limited experiments were conducted on the mean velocity of a stream of gas bubbles. No general correlations of these data were presented.

Robinson[14] made measurements of bubble velocities in lubricating oils for Re $\angle$ 1. The results confirmed those of previous investigators in that Stokes law is applicable to the movement of gas bubbles in this region.

Kaissling[6] made a dimensional study of the factors affecting the velocity of steam bubbles in vertical boiler tubes. It was claimed that the following dimensionless group functional relationship should apply

$$\frac{U \mu}{\gamma g_c} = f\left[\frac{\mu^4 g}{\rho_l \gamma^3 g_c^3}, \frac{\rho_l R_b^2 g}{\gamma g_c}\right] \tag{14}$$

where

f = arbitrary function.

Attempts to determine the functional relation were made without success using data given by Schmidt[15].

Wigner[19] made some speculations regarding the velocity of gas bubbles in liquids based on a dimensional analysis of the variables bubble velocity, liquid density and liquid viscosity. It was intended that such an analysis should give an expression for the terminal velocity of a bubble that was growing by transfer of material into it, hence the bubble radius was not considered in the analysis. Such an analysis led to

$$\frac{U_\infty \mu}{\gamma g_c} = f\left[\frac{\rho_l \gamma^3 g_c^3}{\mu^4 g}\right] \tag{15}$$

where

f = arbitrary function

U_∞ = steady state rising velocity, ft/sec

From data on air bubbles rising in water the arbitrary function was evaluated and expressed by

$$f = 2\left[\frac{\rho_1 \gamma^3 g c^3}{\mu^4 g}\right]^{-1/4} \tag{16}$$

Substituting equation (16) into equation (15) gives

$$U_\infty = 2\left[\frac{\gamma g\, gc}{\rho_1}\right]^{1/4} \tag{17}$$

No attempt was made to verify this deduction experimentally nor was an explanation of the mechanism that brings about this result put forth.

Levich[9] discussed the boundary layer theory with reference to liquid-gas interfaces and applied it to the computation of the total resisting force that acts upon a gas bubble rising in a liquid. The analysis of the boundary layer behavior applied to a spherical gas bubble led to the following expression for the steady state rising velocity,

$$U_\infty = \frac{R_b^2\, g\, (\rho_1 - \rho_2)}{9\,\mu} \tag{18}$$

This result was claimed to be valid over the range $1 < Re < 1500$. However, Levich pointed out that experimental verification of equation (18) would be difficult because of bubble deformation.

Gorodetskaya[4] conducted experiments on the rate of rise of single air bubbles in water and a number of normal alcohols with the express purpose of attempting verification of the conclusions of Levich. In the main the experimental bubble velocities were approximately 30 percent lower than those predicted by equation (18), but were somewhat higher than bubble velocities measured by other investigators under corresponding conditions. Gorodetskaya argued these discrepancies exist because the bubble velocities are very sensitive to the presence of the slightest trace of impurities in the liquids used, that all the liquids he used in the work were carefully purified whereas previous investigators had ignored this precaution. Data were presented to show that concentrations of surface-active substances as low as 10^{-6} molar are sufficient to affect the velocity of bubbles in liquids, these effects being attributed to changes in "interfacial viscosity."

Datta, Napier and Newitt[3] made measurements of the velocity of single air bubbles in water. Their data agree with those of Allen and Miyagi under corresponding conditions. In the range of gas bubble sizes described by Bryn as "large bubbles" ($R_b > 0.013$ feet), the velocities as measured by these investigators was almost independent of size.

Verschoor[18] studied the motion of a stream of air bubbles in a column of liquid. Measurements of mean bubble velocity were made for different rates of air passing through the column without measuring the bubble size. Attempts were made to correlate the data using the dimensionless groups suggested by Kaissling. No general relationships were presented.

Van Krevelen and Hoftijzer[17] made measurements of the velocity of a stream of air bubbles rising in water, cyclohexane, mineral oil and aqueous solutions of glycerine and saponine. When the results were expressed in terms of the drag coefficient and Reynolds number, deviations from the correlations for solid spheres were obtained similar to the results of O'Brien and Gosline. No general relationships in terms of the physical properties of the system were given.

Studies at the University of Tennessee

As a phase of a long range project at the University of Tennessee involving mass transfer in liquid-gas bubble systems, the motion of air bubbles in 22 different liquids was studied. The express purpose of the work was to discover general correlations of the velocity and the drag coefficients of gas bubbles in terms of the bubble size and physical properties of the system.

The experimental work consisted of determining the steady state velocity of single bubbles of known size rising in liquids of predetermined physical properties. The range of bubble sizes and physical properties used in the studies were such that a Reynolds number range of 0.5 to 2500 was covered.

Typical results of the steady state velocity of air bubbles rising in water and a ethyl acetate-cottonseed oil solution as a function of the bubble radius are given in Figure 1. Data for some of the other systems studied are shown in Figure 2 as a plot of the drag coefficient versus the Reynolds number. The data as given agree favorably with those of previous

investigators. As observed by O'Brien and Gosline the plot of the drag coefficient versus the Reynolds number is not a general correlation for the behavior of gas bubbles and other parameters are involved.

As may be noted in Figures 1 and 2, four distinct regions of behavior are exhibited by the motion of gas bubbles in a given liquid. The first two regions correspond to the motion of solid spheres in liquids. The third and fourth regions indicate the departure from similarity to the motion of solid spheres. Without discussing the empirical and theoretical methods used in the analysis of the experimental data, the drag data from all the experiments could be expressed by the relations given in Table I.

TABLE I

SUMMARY OF RELATIONS FOR THE DRAG COEFFICIENTS OF GAS BUBBLES MOVING IN LIQUIDS

Region	Drag Coefficient	Limits	
I	$C_D = 24\ Re^{-1}$	$Re \leqq 2.0$	(19)
II	$C_D = 18.7\ Re^{-0.68}$	$2.0 \leqq Re \leqq 4.02\ G_1^{-0.214}$	(20)
III	$C_D = 0.441\ G_2$	$16.32\ G_1^{0.144} \leqq G_2 \leqq 6.45$	(21)
IV	$C_D = 1.783\ G_2^{0.250}$	$6.45 \leqq G_2$	(22)

$$G_1 = \frac{g\,\mu^4}{\rho_1\,\gamma^3 g_c^3} \quad (23); \quad G_2 = \frac{g\,R_b^4\,U^4\,\rho_1^3}{\gamma^3\,g_c^3} \quad (24)$$

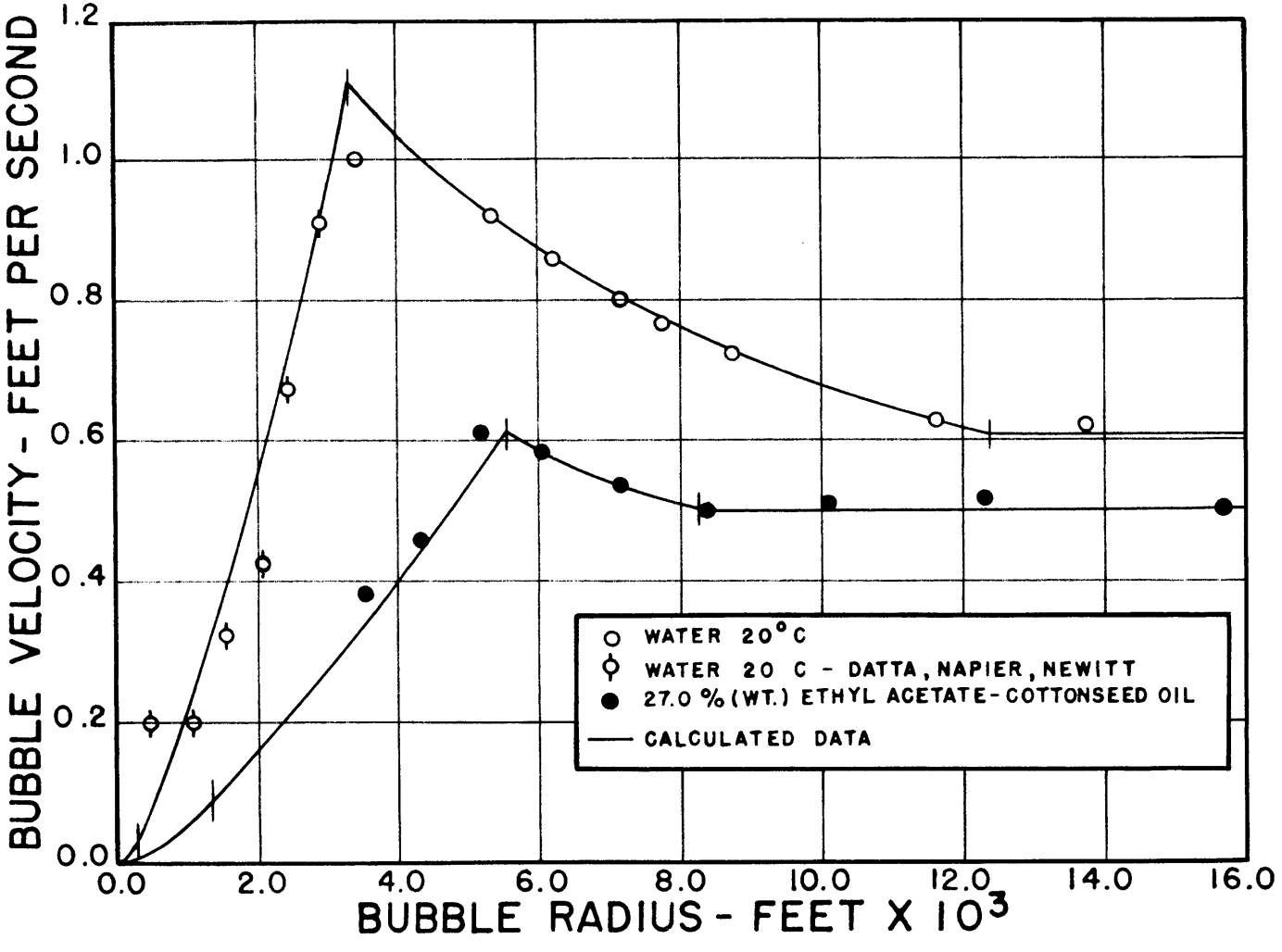


FIGURE I

TYPICAL RESULTS FOR BUBBLE VELOCITY AS A FUNCTION OF BUBBLE RADIUS

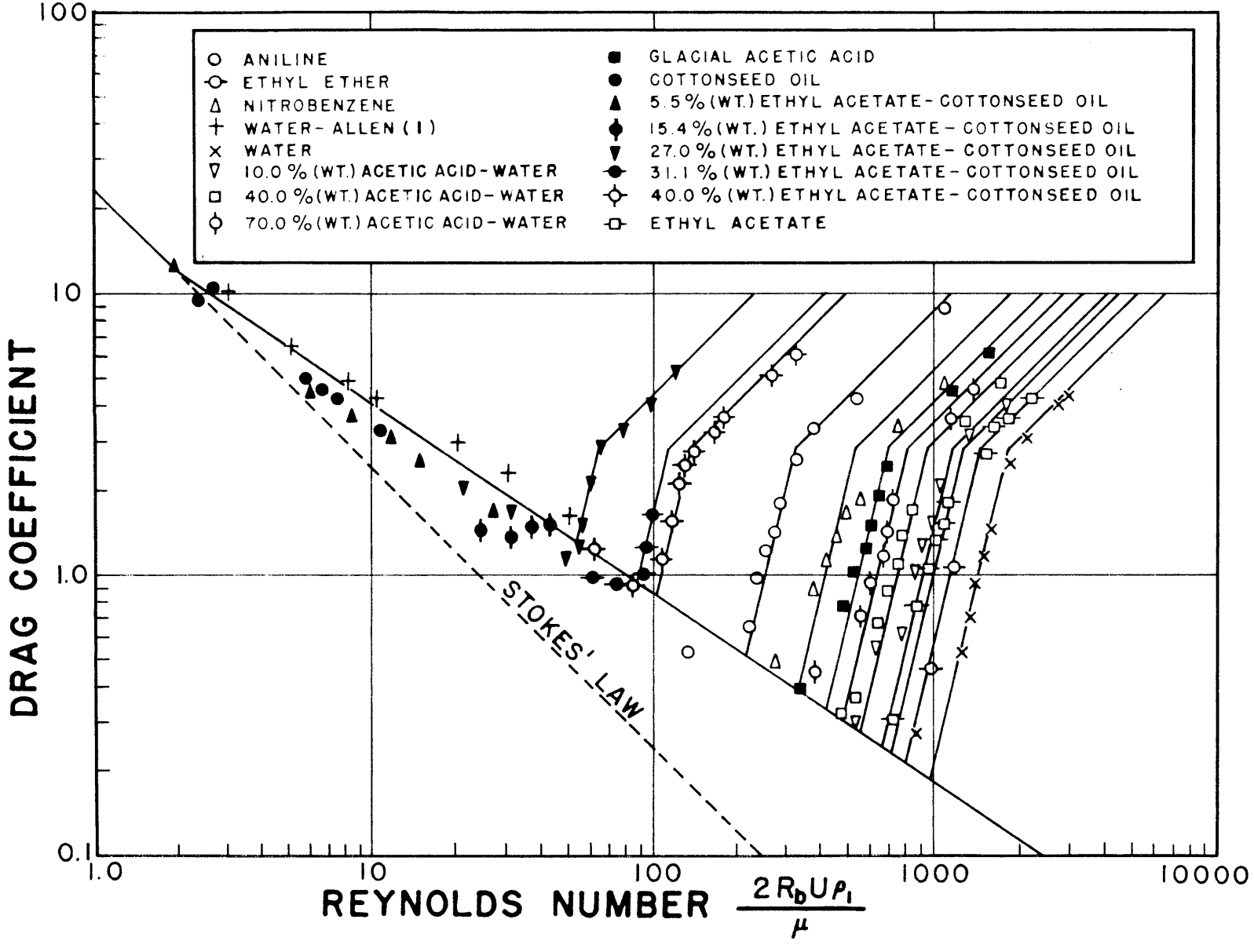


FIGURE 2

DRAG COEFFICIENTS FOR GAS BUBBLES

Recommended Relations

It is proposed to use the correlations expressed in Table I for the drag characteristics of gas bubbles moving in liquids in this present work. This proposal is based primarily on these factors:

a. No other general correlations for the drag coefficients of gas bubbles moving in liquids are known.

b. Even though the correlations are based on data from the motion of single gas bubbles, the corresponding data for masses of gas bubbles probably are not seriously different.

c. Intimate relation of the author with the work at the University of Tennessee.

GAS BUBBLE VELOCITY IN A ROTATING LIQUID

Development of Basic Equations

The accelerating motion of a gas bubble in the rotating liquid of the HRE core is approximated closely by a single equation for the radial component of the total motion, thus

$$\frac{4 \pi R_b^3 \rho_2}{3\ gc} \frac{d U_r}{dt} = \frac{4 \pi R_b^3 V_t^2 (\rho_1 - \rho_2)}{3\ gc\ R} - \frac{\pi C_D \rho_1 R_b^2 U^2}{2\ gc} \qquad (25)$$

where,

R = position radius measured from the vertical axis of the sphere, ft

t = time, sec

ρ_2 = gas density, pounds mass/cu ft

U_r = radial velocity of the bubble relative to a fixed coordinate system, ft/sec

Such an expression is based on negligible motion of the gas bubble with respect to the liquid in the tangential and vertical directions. Hence,

$$U_r = V_r + U \tag{26}$$

The combination of equation (26) and the relations for the drag coefficient as given by equations (18) through (24) would theoretically permit an analytical solution to equation (25). Such a procedure is extremely impractical and numerical integration is indicated. However, upon attempting such numerical integration, it is found that for a fixed value of bubble position radius, R, the acceleration to a terminal velocity is almost instantaneous. As a result, little accuracy is sacrificed by assuming

$$\left.\frac{d\,U_r}{dt}\right|_{R\,=\,\text{constant}} \equiv 0 \tag{27}$$

Incorporating this approximation into equation (25) leads to the simplification

$$C_D\,U^2 = \frac{8\,R_b\,V_t^{\,2}\,(\rho_1 - \rho_2)}{3\,R\,\rho_1} \tag{28}$$

Substituting the relations for the drag coefficient as given by equation (19) through (24) gives these expressions for the radial velocity,

$$U = \frac{N_G\,\rho_1\,R_b^{\,2}}{12\,\mu}\ ,\quad U \leqq \frac{\mu}{\rho_1\,R_b} \tag{29a}$$

$$U = 0.155\,N_G^{\,0.758}\left[\frac{\rho_1}{\mu}\right]^{0.515} R_b^{\,1.272}\ ,$$

$$\frac{\mu}{\rho_1\,R_b} \leqq U \leqq \frac{2.01\,\mu}{\rho_1\,R_b}\,G_1^{\,-0.214} \tag{29b}$$

[redacted]

$$U = 1.146 \left[\frac{N_G}{g}\right]^{0.167} \left[\frac{\gamma g c}{R_b \rho_1}\right]^{0.500},$$

$$\frac{2.01}{g^{0.250}} G_1^{0.036} \left[\frac{\gamma g c}{\rho_1}\right]^{0.750} \frac{1}{R_b} \leqq U \leqq \frac{1.594}{g^{0.250}} \left[\frac{\gamma g c}{\rho_1}\right]^{0.750} \frac{1}{R_b} \qquad (29c)$$

$$U = 0.825 \left[\frac{N_G^4}{g}\right]^{0.083} \left[\frac{\gamma g c}{\rho_1}\right]^{0.250},$$

$$\frac{1.505}{g^{0.250}} \left[\frac{\gamma g c}{\rho_1}\right]^{0.750} \frac{1}{R_b} \leqq U \qquad (29d)$$

where $N_G = \dfrac{8 V_t^2 (\rho_1 - \rho_2)}{3 R \rho_1}$

This system of equations permits the computation of the gas bubble velocity at a specified point in the rotating liquid in terms of the bubble size, physical properties of the system and the velocity of the liquid.

Computation of Maximum Gas Bubble Residence Time

Some of the experimental work on gas hold-up in the HRE core described in ORNL-630 involved the injection of measured volumes of gas bubbles into the mock-up spheres at the liquid inlet. From both motion picture studies and gas hold-up measurements, it was found that 0.25-0.40 seconds were required for gas bubbles to move from the liquid inlet to the central gas void (i.e. $R \cong 0.65$ ft to $R \cong 0.01$ ft). The sizes of bubbles involved in these studies were not known precisely but it was estimated that the sizes were from 0.10 - 0.30 inch in diameter.

In order to show the applicability of the previous equations for the gas bubble velocity in a rotating liquid, the maximum gas bubble residence time (the time required for a bubble to move from the liquid inlet to the

central gas void in the equatorial plane of the sphere) has been computed for a number of gas bubble sizes. Computations were made using physical properties of the fluids to correspond to the room temperature flow studies at approximately 70°F and to the actual operating conditions of 482°F and 1000 psia. The liquid properties were assumed to be those of water. The liquid velocities in the sphere were assumed to be those as computed from equations (1) and (2) which were the result of velocity traverses made in a sphere 1.5 ft inside diameter with a 2 inch nominal diameter inlet pipe. Computations were also made to correspond to the use of a 1 1/2 inch nominal diameter inlet pipe. In these computations it was assumed that the liquid velocities in the sphere would have the same type of distribution as given by equations (1) and (2) with only a change in the constant of equation (1) by the ratio of increase of inlet momentum.

Since the radial velocity of the gas bubbles varies with position in the sphere, it was necessary to compute the bubble velocity at a number of positions and then evaluate the residence time by integration over the distance traversed. The computational procedure was as follows for a particular gas bubble size, R_b:

a. For a specified value of the position radius, R, compute the radial and tangential components, V_r and V_t, of the liquid velocity using equations (1) and (2).

b. Using the value of V_t from computation (a) and the values of the physical properties evaluated at the operating temperature and pressure, compute the radial velocity of the gas bubble relative to the moving liquid, U, by the appropriate equation from system (29a-d).

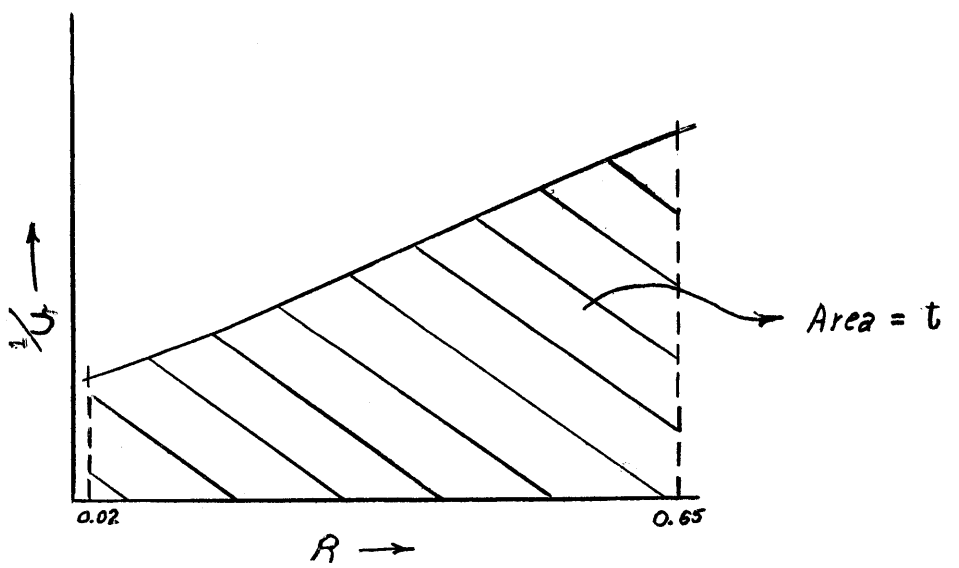
c. Compute the absolute radial velocity of the gas bubble, U_r, by equation (26) using the result of computation (b) and the value of V_r from computation (a)

d. Repeat the above computations for a number of values of the position radius in the range $0.02 \leqq R \leqq 0.65$ ft.

e. Compute the maximum gas bubble residence time, t, by evaluating the integral,

$$t = \int_{0.65}^{0.02} \frac{dR}{U_r} \tag{30}$$

this may be done graphically by a plot of this type



As an alternate procedure this integral may be evaluated numerically.

The results of computations using this procedure are shown in Figure 3 and Figure 4 giving the maximum gas bubble residence time as a function of the gas bubble size. Figure 5 gives the radial bubble velocity for a bubble

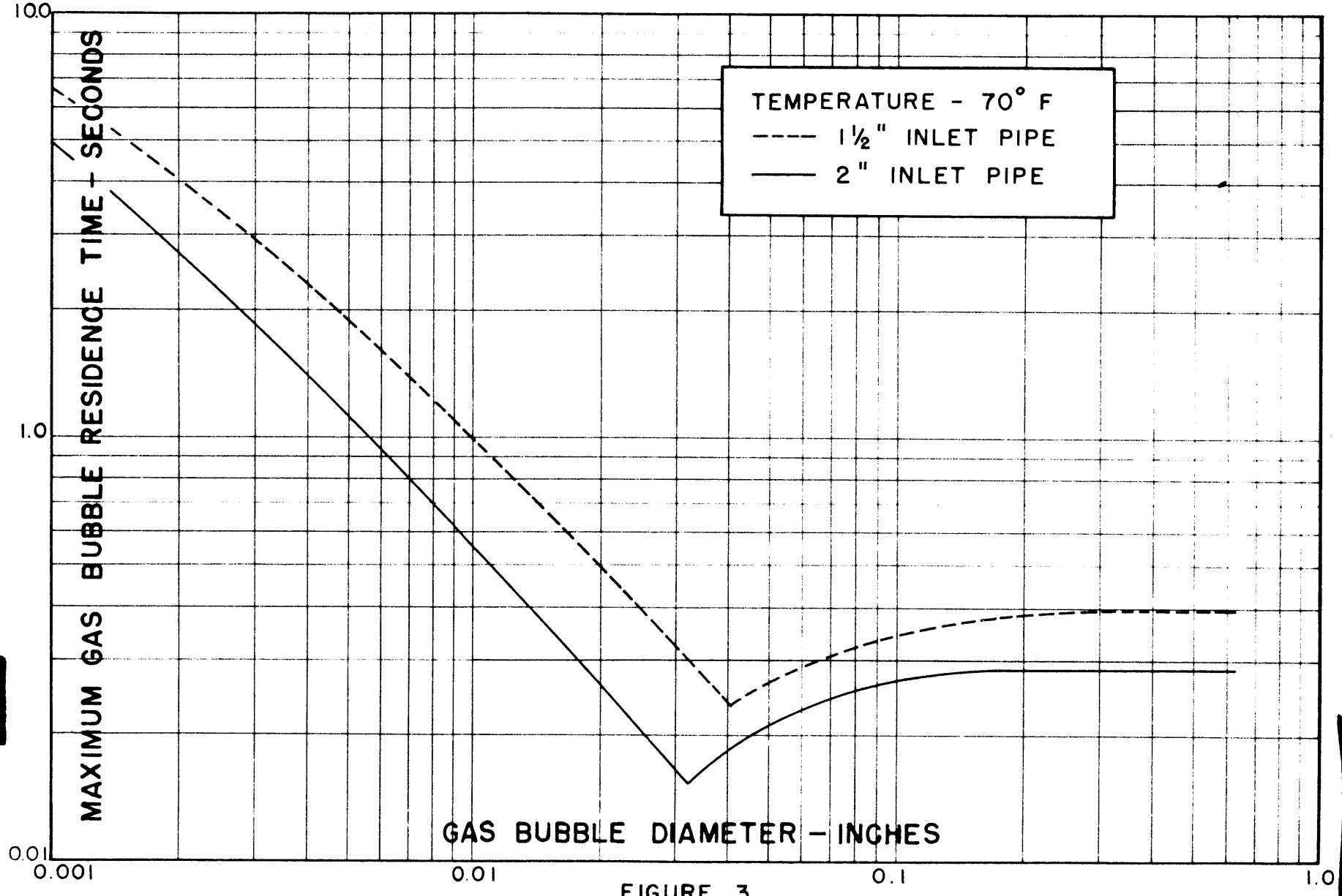


FIGURE 3

CALCULATED MAXIMUM GAS BUBBLE RESIDENCE TIME AT 70°F

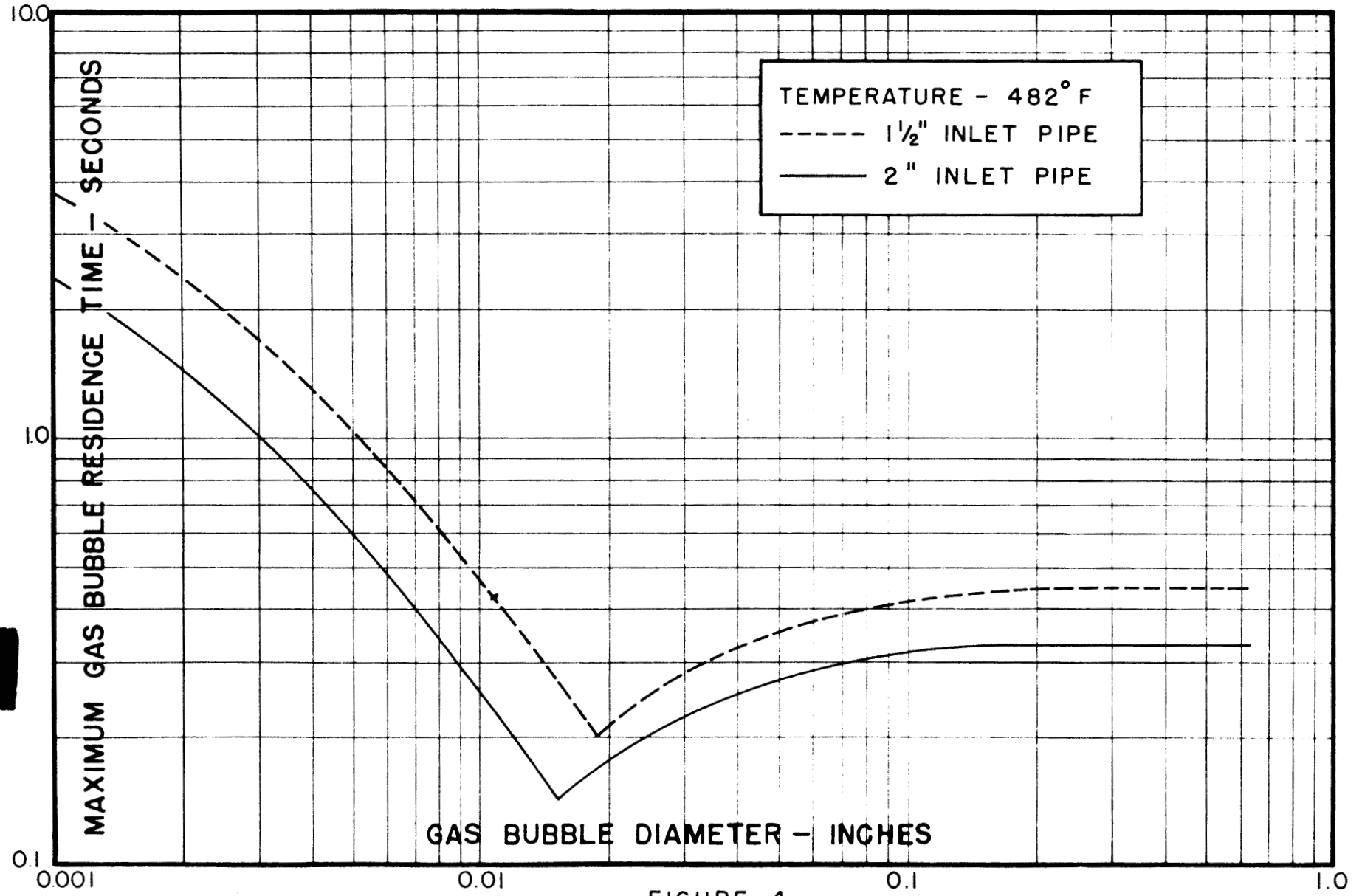


FIGURE 4

CALCULATED MAXIMUM GAS BUBBLE RESIDENCE TIME AT 482° F

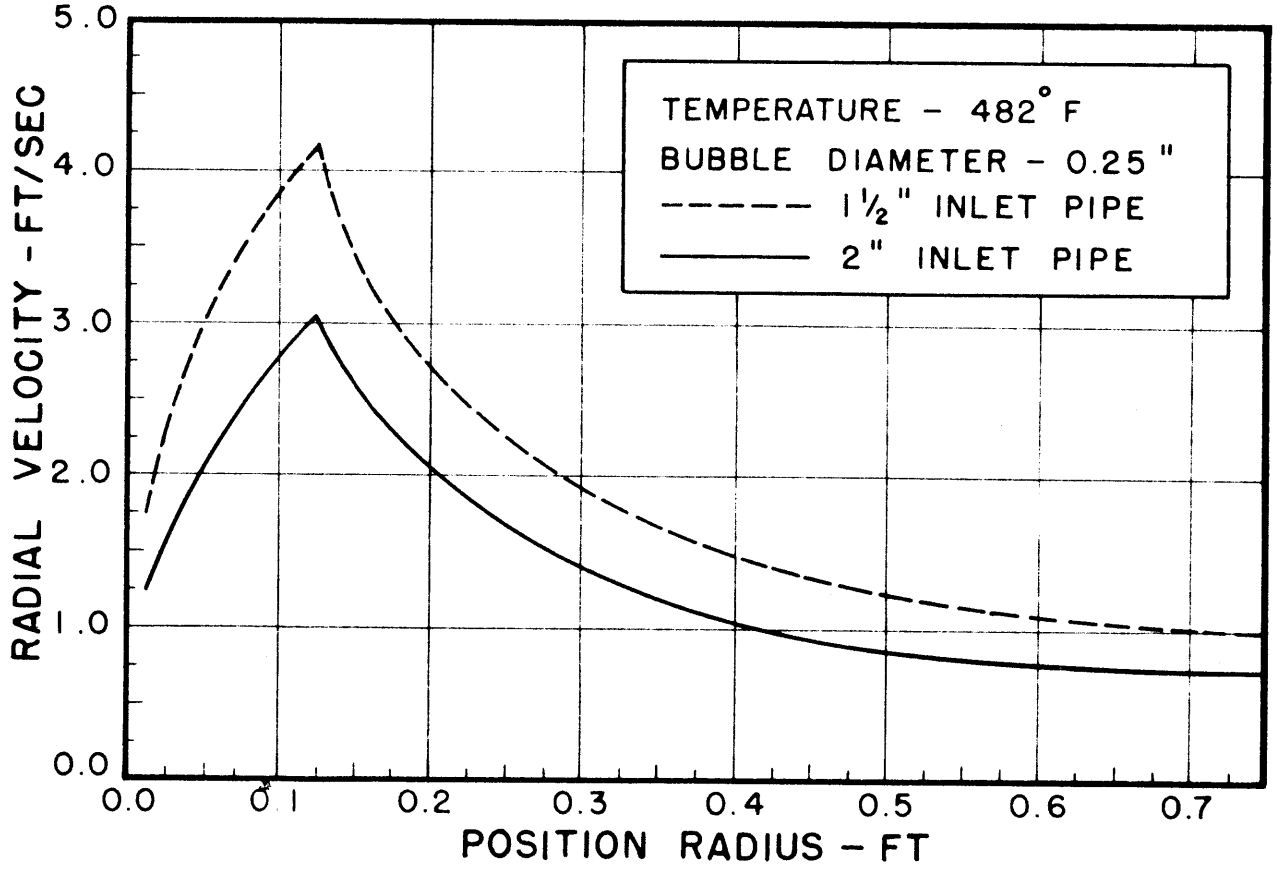


FIGURE 5

CALCULATED GAS BUBBLE RADIAL VELOCITIES IN THE HRE CORE

0.25 inch diameter as a function of position radius at 482°F to show the range of velocities predicted in the HRE core. It is gratifying to note that the maximum bubble residence times for bubbles 0.10 - 0.30 inch in diameter as calculated agree favorably with the experimental values given in ORNL-630.

TOTAL GAS HOLD-UP IN THE HRE REACTOR CORE

In the previous section computations of the maximum gas bubble residence time were made for the case where the gas was introduced at a single point in order to make a comparison with the HRE mock-up experiments. However, under actual operating conditions gas is to be generated throughout the liquid volume. Furthermore, the important information desired is the total gas hold-up volume within the sphere, not the time that a single bubble remains in the liquid mass. In this section the problem of predicting the extent of this total gas hold-up to be encountered in the HRE reactor core under power producing conditions will be considered.

For clarification the problem may be divided into these three steps:

(1) Considering any liquid volume increment, compute the number of gas bubbles contained in it; taking account of the number of gas bubbles generated in that volume of liquid, the number of gas bubbles moving into that volume from other parts of the liquid and the number of gas bubbles moving out of that volume into other parts of the liquid.

(2) Compute the gas hold-up within the entire liquid mass by summing the number of gas bubbles contained in all liquid volume increments within the sphere.

(3) Compute the volume of gas contained in the central gas void.

Again, an exact solution to the problem as outlined above will be difficult to obtain and certain simplifying assumptions will be made; namely:

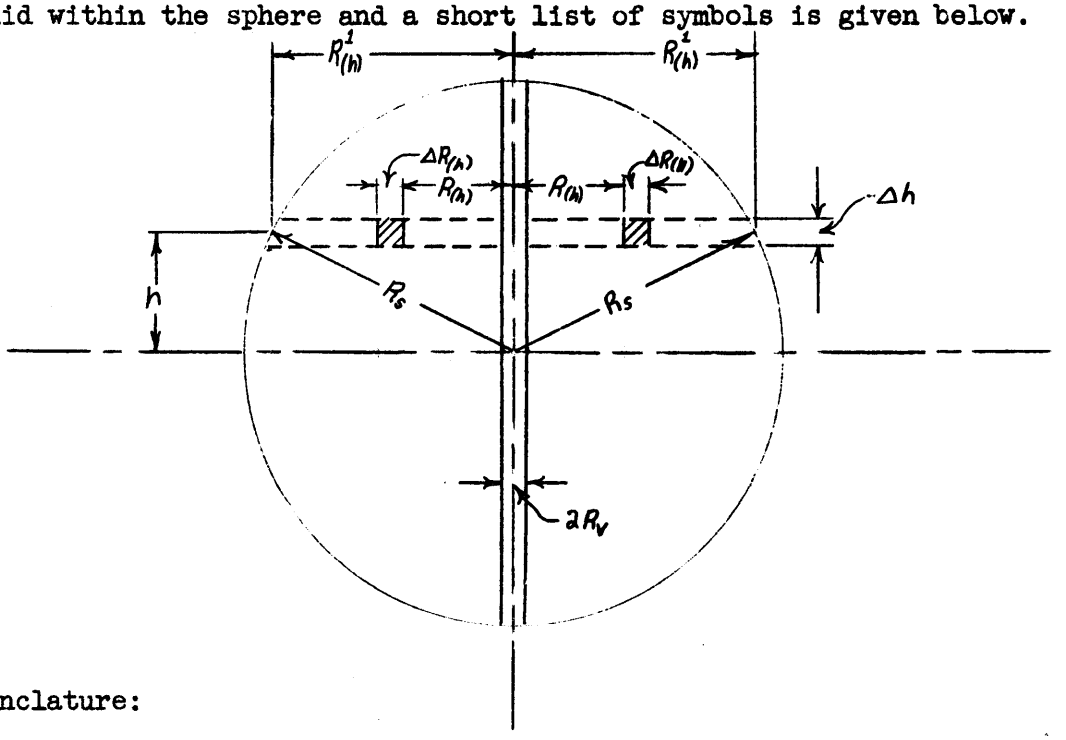
(a) gas generation rate per unit volume of liquid to be constant throughout the liquid mass and is to be computed from the total power generated (1000 KW) using 50 - 100 e.v. per water molecule decomposed

(b) gas bubbles move in horizontal planes from the point of generation to the central gas void

(c) for a given horizontal plane, the gas bubbles move at a constant, mean radial velocity which is to be evaluated in terms of liquid velocities, physical properties of the fluids and the bubble size by the methods presented in the previous section (such a mean value is used to expedite the computations, otherwise a laborious double graphical or numerical integration is required)

(d) the gas bubbles are of a constant size and are small enough to permit the use of the calculus to describe the events.

Based on the assumptions, the problem as outlined may now be considered. A schematic diagram of a representative volume increment of liquid within the sphere and a short list of symbols is given below.



Nomenclature:

h = height of a horizontal plane above the equatorial plane of the sphere, ft

Δh = height of a horizontal layer of liquid, ft

n = number of gas bubbles per unit volume of liquid, ft^{-3}

R_s = radius of the sphere, ft

$R_{(h)}$ = position radius at a point in a horizontal layer of liquid at a height, h, ft

$\Delta R_{(h)}$ = increment of position radius, $R_{(h)}$, ft

$R^1_{(h)}$ = radius of a circle cut by the chordal plane at a height, h, ft

Rv = radius of central gas void, ft

t = time, sec

$U_{(h)}$ = mean radial velocity of all gas bubbles moving in a layer of liquid at a height, h, ft/sec

V_b = volume of a gas bubble, cu ft

V_G = volume of gas generated per unit time per unit volume of liquid, sec^{-1}

Now considering the annular element of liquid volume as formed by the increment of position radius, $\Delta R_{(h)}$, at a point $R_{(h)}$ with a height, Δh, it may be shown that the rate of change of the number of bubbles is given by

$$\frac{\partial n}{\partial t} = - U_{(h)} \frac{\partial n}{\partial R} - \frac{U_{(h)}\, n}{R} + \frac{V_G}{V_b} \tag{31}$$

Since for steady state conditions, $\frac{\partial n}{\partial t} = 0$, then n becomes a function of $R_{(h)}$ only and equation (31) is reduced to the ordinary differential equation

$$\frac{dn}{dR_{(h)}} + \frac{n}{R_{(h)}} = \frac{V_G}{U_{(h)} V_b} \tag{32}$$

this equation is integrated to

$$nR_{(h)} = \frac{V_G}{2U_{(h)} V_b} R^2_{(h)} + C \tag{33}$$

where C is a constant of integration. This constant may be evaluated from the boundary condition

$$n \longrightarrow 0, \quad R_{(h)} \longrightarrow R^1_{(h)} \tag{34}$$

Substituting this condition in equation (33), the gas bubble concentration distribution for steady state conditions is given by

$$n = \frac{V_G}{2U_{(h)}V_b}\left[R_{(h)} - \frac{R^1_{(h)}{}^2}{R_{(h)}}\right] \tag{35}$$

Equation (35) gives the gas bubble concentration as a function of $R_{(h)}$ within the horizontal layer of height, Δh, and then the number of gas bubbles in the annular element of liquid volume becomes

$$\Delta N = 2\pi\,\Delta h\;\; R_{(h)} n\;\Delta R_{(h)} \tag{36}$$

where ΔN represents the number of gas bubbles contained in this volume element. Now assuming the use of the calculus and passing to the limit as $\Delta R_{(h)} \rightarrow 0$ and $\Delta h \rightarrow 0$,

$$dN = 2\pi\, n\, R_{(h)}\; d\,R_{(h)}\; dh \tag{37}$$

Now integrating over the entire liquid volume, the total number of gas bubbles contained in the sphere becomes

$$\Sigma N = 4\pi \int_0^{R_s} \int_{Rv}^{R^1_{(h)}} n\, R_{(h)}\; dR_{(h)}\; dh \tag{38}$$

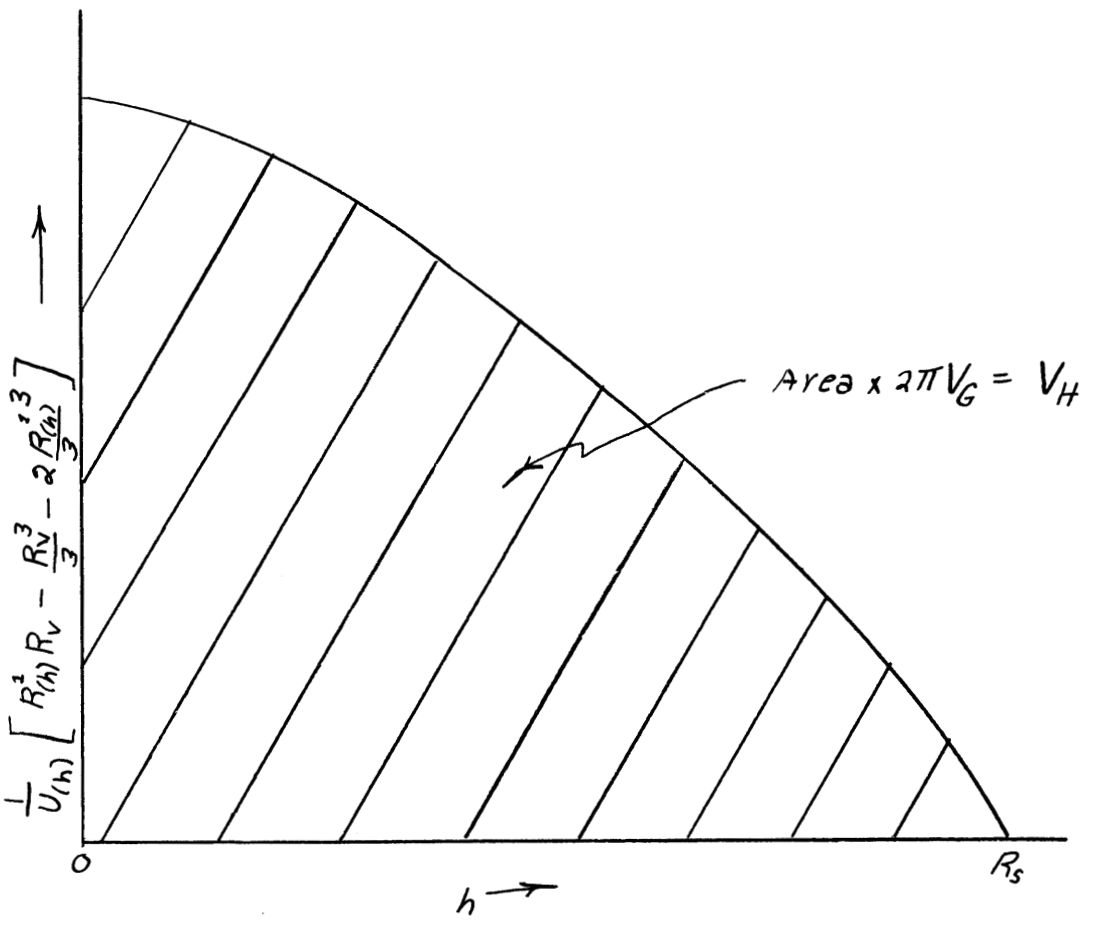
where ΣN represents this total number of gas bubbles. Letting V_H represent the volume of gas bubbles contained in the sphere and substituting the value of n from equation (35)

$$V_H = 2\pi\, V_G \int_0^{R_s} \int_{Rv}^{R^1_{(h)}} \frac{1}{U(h)}\left[R^2_{(h)} - R^{1\,2}_{(h)}\right] dR_{(h)} dh \tag{39}$$

The integration over $R_{(h)}$ may be performed analytically since $U_{(h)}$ is a constant for this integration, thus

$$V_H = 2\pi\, V_G \int_0^{R_s} \frac{1}{U_{(h)}} \left[R^1_{(h)} Rv - \frac{Fv^3}{3} - \frac{2\, R^{1\,3}_{(h)}}{3} \right] dh \qquad (40)$$

This final integration is best performed graphically as the relation between $U_{(h)}$ and (h) is not simple. However for a given value of h, $U_{(h)}$ may be evaluated by the method given in the previous section. Then V_H may be evaluated graphically using a plot of the type shown below.



The total gas hold-up in the sphere may be computed by

$$\% \text{ Hold-Up} = \frac{3\left[V_H + 2\pi R_V^2 R_S\right]}{4\pi R_S^3} \times 100 \qquad (41)$$

Using these results, the per centage total gas hold-up has been computed for the HRE reactor core under power producing conditions for a range of bubble sizes. These results are shown graphically in Figure 6. No experimental data are available for comparison. These computations are based on a value of Rv = 0.02 ft (approximately 0.5 inch diameter gas void).

DISCUSSION OF THE RESULTS

A number of assumptions and approximations to the actual HRE conditions have been made in this work. It is desirable to evaluate the effects of such approximations and to indicate the type of data that are needed to eliminate the approximations.

The basic approximation with regard to the motion of gas bubbles in the HRE reactor core is in the description of the liquid velocity given by equations (1) and (2). As has been pointed out these relations result from limited experimental data, and are known to be contradictory to qualitative information obtained in flow pattern studies now in progress at Oak Ridge National Laboratory. As quantitative information is made available from these studies and from similar studies now in progress at other laboratories, the computations can be revised to account for the more complete information on the liquid flow within the spherical core. It is felt that these revisions will be such that the vertical motion of the gas bubbles is accounted for

in the form of another equation of gas bubble motion in this direction similar to equation (25). A revision of this type will add greatly to the computational requirements but will permit a definitely more accurate description of the gas bubble motion within the reactor core.

It was necessary to use drag coefficient and velocity data for single gas bubbles in this present work since quantitative information on the motion of a stream of gas bubbles is not known. However, the limited data that are available indicate that this is accurate to approximately 25 percent. A project on the determination of such data on gas bubble streams is in progress at the University of Tennessee, Department of Chemical Engineering. Some results from this project should be available by August, 1952 and revisions to this work can be made in light of the new data.

It is to be noted that the procedure of assuming a size of gas bubble has led to positive results regarding the bubble motion and gas hold-up in the reactor core. Since little is known regarding the size of gas bubbles to be encountered in the reactor core, it was necessary to make computations for a range of bubble sizes that included all the probable sizes. Such a range of gas bubble sizes was taken to be 0.001" to 0.6" equivalent spherical diameter.

The other approximations and assumptions were for the most part computational in nature, i.e. "short cuts" to expedite the numerical work. In all such cases these approximations did not affect the results by more than 5 percent.

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APPENDIX

A. Table of Nomenclature

B. Computation Procedure

APPENDIX A

TABLE OF NOMENCLATURE

A = cross section of a particle perpendicular to the direction of flow, sq. ft.

a = Laplace's capillarity constant, ft.

C_D = drag coefficient, dimensionless

D = total liquid resisting force acting on a particle, pounds force

d = diameter of a particle, ft.

f = arbitrary function, dimensionless

G_1 = dimensionless parameter, $\dfrac{g\,\mu^4}{\rho_1\,\gamma^3 gc^3}$

G_2 = dimensionless parameter, $\dfrac{g\,R_b^4\,U^4\,\rho_1^3}{\gamma^3\,g^3c}$

g = gravitational acceleration, ft/sec^2

gc = gravitational conversion factor, 32.17(pounds mass)(ft)/(pound force)(sec)^2

h = height of a horizontal plane above the equatorial plane of the spherical core, ft.

Δh = height of a horizontal layer of liquid, ft.

N_G = defined variable, $\dfrac{8\,V_t^2\,(\rho_1 - \rho_2)}{3\,R\,\rho_1}$, ft/sec^2

N = number of gas bubbles, dimensionless

n = number of gas bubbles per unit volume of liquid, (cu ft)$^{-1}$

P = total pressure, pounds force/sq ft

R = position radius measured from the vertical axis of the sphere, ft

R_b = equivalent spherical radius of a gas bubble, ft

R_S = radius of the spherical core, ft

$R_{(h)}$ = position radius at a point in a horizontal layer of liquid at a height, h, ft

$R^1_{(h)}$ = radius of a circle cut by the chordal plane at a height, h, ft

$\Delta R_{(h)}$ = increment of position radius, ft

Rv = radius of the central gas void, ft

Re = Reynolds number, $\frac{2R_b\ U\ \rho_l}{\mu}$, dimensionless

t = time, sec

U = velocity of a moving particle relative to the liquid, ft/sec

U_∞ = steady state velocity, ft/sec

$U_{(h)}$ = mean radial velocity of all gas bubbles moving in a layer of liquid at a height, h, ft/sec

U_r = radial velocity of the gas bubble relative to a fixed coordinate system, ft/sec

u,v,w = x,y,z components respectively of the fluid velocity, ft/sec

V_r = radial component of the liquid velocity, ft/sec

V_t = tangential component of the liquid velocity, ft/sec

V_b = gas bubble volume, cu ft

V_G = volume of gas generated per unit time per unit volume of liquid, sec^{-1}

V_H = total volume of gas bubbles contained in the spherical core, cu ft

X_1,X_2,X_3 = x,y,z components respectively of any extraneous forces acting on a fluid element, pounds force/pound mass

x,y,z = space coordinates, ft

β = arbitrary constant, dimensionless

δ = arbitrary constant, dimensionless

∇ = Laplacian operator, dimensionless

γ = liquid surface tension, pounds force/ft

μ = liquid viscosity, pounds mass/(ft)(sec)

ρ_1 = liquid density, pounds mass/(cu ft)

ρ_2 = gas density, pounds mass/(cu ft)

Ψ = arbitrary function, dimensionless

APPENDIX B

COMPUTATION PROCEDURE

1. Radial Velocity of A Gas Bubble in the HRE Core at 482°F (See Figure 5)

For these conditions:

$$\rho_1 = 51.0 \text{ pounds mass/(cu ft)}$$

$$\rho_2 = 1.8 \text{ pounds mass/(cu ft)}$$

$$\mu = 0.75 \times 10^{-4} \text{ pounds mass/(ft)(sec)}$$

$$\gamma = 2.33 \times 10^{-3} \text{ pounds force/(ft)}$$

For this particular computation, assume:

$$R_b = 0.25 \text{ in} \cong 0.0208 \text{ ft}$$

$$R = 0.48 \text{ ft}$$

Inlet Pipe Diameter = 2 in.

Using equation (1) and (2),

$$V_t = 4.8\ (0.48)^{-0.77} = 8.43 \text{ ft/sec}$$

$$V_r = \frac{0.0177}{0.48(0.5625-0.48^2)^{1/2}} = 0.06 \text{ ft/sec}$$

From the definition of N_G,

$$N_G = \frac{8 \times 8.43^2(51.0-1.8)}{3 \times 0.48 \times 51.0} = 381.1 \text{ ft/(sec)}^2$$

By a trial and error procedure, it is found that equation (29d) applies to the computation of the bubble velocity under these conditions. Thus

$$U = 0.825 \left[\frac{381.1^4}{32.17}\right]^{0.083} \left[\frac{2.33 \times 10^{-3} \times 32.17}{51.0}\right]^{0.250} = 0.88 \text{ ft/sec}$$

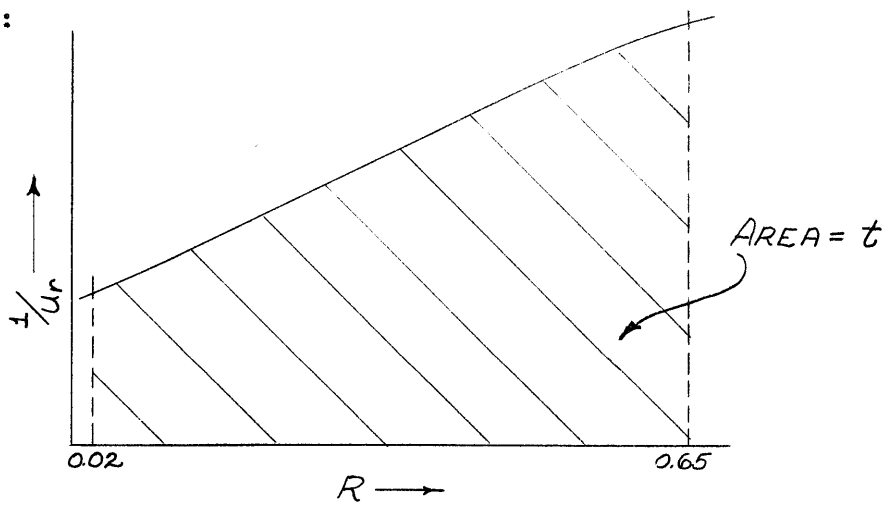
Then using equation (26),

$$U_r = 0.88 + 0.06 = 0.94 \text{ ft/sec}$$

These computations are then repeated for other values of the position radius, R. The results of such a series of calculations of this one size of gas bubble is shown in Figure 5.

2. Maximum Gas Bubble Residence Time (See Figure 4)

The maximum gas bubble residence time is given by equation (30) in the form of an integral to be evaluated. This integral may be evaluated graphically by using the results of computation (1) in a plot of the form shown below:



The area under this curve is equal to the maximum gas bubble residence time. The results of such a procedure are shown in Figure 4.

3. Total Gas Hold-Up in the HRE Core (See Figure 6)

From equation (40), the volume of gas contained in the core is given by

$$V_H = 2\pi V_G \int_0^{R_s} \frac{1}{U(h)} \left[R^1(h) Rv - \frac{R^3 v}{3} - \frac{2R^1(h)^3}{3} \right] dh \qquad (40)$$

[redacted]

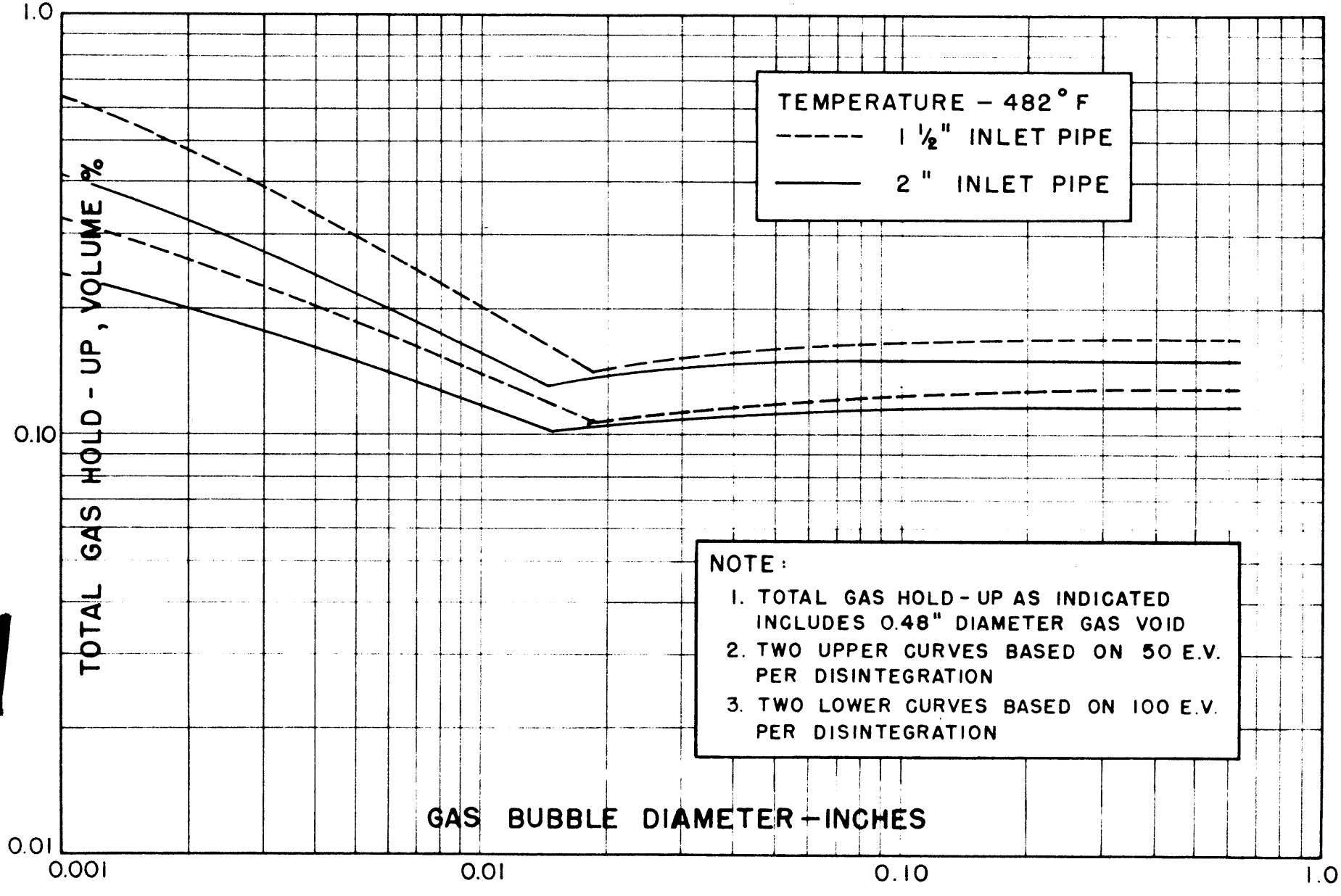


FIGURE 6

CALCULATED TOTAL GAS HOLD-UP IN THE HRE CORE

V_G can be computed from the total reactor power and the number of electron-volts required to decompose the water molecule. Using 1000 KW and 100 ev per water molecule decomposed -

$$V_G = 0.00306 \text{ cu ft/(sec)(cu ft of reactor volume)}$$

Further, $R^1_{(h)}$ can be calculated by the relation

$$R^1_{(h)} = (R_s^2 - h^2)^{1/2} \tag{42}$$

Using Rv = 0.02 ft and equation (42), the term in the bracket can be computed for a particular value of h.

For a particular gas bubble size the computations of the gas bubble radial velocity as a function of R can be used to compute $U_{(h)}$. Thus by definition

$$U_{(h)} = \frac{\int_{0.02}^{R=R^1_{(h)}} U_{(r)}\, dR}{R^1_{(h)} - 0.02} \tag{43}$$

By such a procedure, values of $U_{(h)}$ as a function of h can be evaluated and then the integral indicated in equation (40) can be calculated. Finally the volume percent gas hold-up can be calculated by equation (41).

This procedure was followed for a number of gas bubble sizes and the results are shown in Figure 6.