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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

FOR PERIOD ENDING JULY 31, 1952

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By: C. Goldberg 2.10.71

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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

for Period Ending July 31, 1952

A. S. Householder, Chief

C. L. Perry, Editor

DATE ISSUED

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ORNL-345	December, January, February, 1948-1949
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ORNL-1232	Period Ending January 31, 1952
ORNL-1290	Period Ending April 30, 1952

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TABLE OF CONTENTS

	PAGE
SUMMARY	1
UNCLASSIFIED PROJECTS	2
Test Problems for the ORAC Arithmetic Unit	2
Errors in Matrix Operations	2
Basic Studies in the Monte Carlo Method	3
Methods of Computation for Use with a High-Speed Automatic-Sequenced Computer	4
A Sturm-Liouville Problem	4
Approximate Steady-State Solution of the Equation of Continuity	5
Estimation of Volume of Lymph Space	6
X-Radiation Effects on Respiration Rates in Grasshopper Embryos	7
Temperature Effects in Irradiation Experiments with Yeast	7
Comparison of Beta-, Gamma-, and X-Radiation Effects on *Tradescantia*	7
LD_{50} Estimation for Burros and Swine Exposed to Gamma Radiation	8
Thermal-Neutron Penetration in Tissue	8
Numerical Evaluation of Integrals	9
Kinetics of HBr-$HBrO_3$ Reaction	9
Isotope Effect in Reaction Rate Constants	9
Experimental Determination of Diffusivity Coefficients	10
Dirac Wave Functions	11
Calculation of Racah Coefficients for the Angular Distribution in Nuclear Reactions	12
Fitting a Calibration Curve	12
Beta Decay (Field Factors)	14
Neutron Decay Theory	15
Calculation of Internal Conversion Coefficients with Screening	16
Determination of the Fast-Neutron Flux in the ORNL Graphite Reactor	16

[redacted]

[redacted]

[redacted]

Estimation of Alignment Ratios 16
Optimum Allocation of Counting Time 17
Efficiency of a Single Crystal 18
Calculation of Coincidence Probabilities for a Double-Crystal Detector 19
Reactor Inequalities 19
Heat Flow in Long, Rectangular Tubes 19
Kinetics of Boiling Reactors 20
Circulation in Boiling Reactors 20
Solving an Equation by Numerical Methods 20
Numerical Integration 20
Evaluation of a Function 20

SECRET PROJECTS 21

Depigmentation as a Biological Dosimeter - Operation Greenhouse . . . 21
Machine Computation of Solutions for the Neutron Transport Equation 21
Diffusion of Xenon-135 in ANP Fluoride Fuels 21

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MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

SUMMARY

S. D. Conte of Wayne University is spending the summer with the Mathematics Panel as a Research Participant. R. R. Coveyou, E. A. Forbes, and M. E. Tsagaris have transferred to the ANP Division. J. W. Tukey of Princeton University, a consultant in biometrics, spent July 18 at the Laboratory.

At the invitation of N. Metropolis, A. S. Householder and C. L. Perry spent three days at Los Alamos in June running a system of differential equations (cf., "Kinetics of HBr-$HBrO_3$ Reaction," this report) on the MANIAC. This machine has a Williams-tube memory and is very similar in construction and logical design to the ORAC. Repeated runs were made with various values of the parameters and different spacings with no machine failure and no detectable errors, except in a few instances in which the spacing proved too coarse.

The beta-decay problem [cf., "Beta Decay (Field Factors)," this report] is being run on the Whirlwind at MIT. C. L. Perry spent the month of July at MIT to carry through the problem, with the assistance of N. M. Dismuke the first week of July and H. B. Goertzel from July 24 to the end of the month.

Several problems are being planned for the UNIVAC, which the AEC is buying for a central AEC computing agency. An advisory Program Council, on which A. S. Householder is the Oak Ridge representative, makes recommendations as to allotment of time on the machine, and Oak Ridge has been promised approximately 20 hr per week beginning about August 15. Of this time, K-25 has need for roughly 200 hr before March 1, 1953. The Laboratory problems that might be put on the UNIVAC before the ORAC is in full operation are several systems of ordinary differential equations, two partial differential equations, and a Monte Carlo calculation of tissue damage by fast neutrons from an isotropic source. A similar computation for a parallel beam of neutrons was carried out with IBM equipment and has been reported in earlier quarterly reports. N. M. Dismuke, S. E. Ezzell, and J. H. Fishel spent June 16 and 17 at the Remington Rand plant to learn UNIVAC coding.

The arithmetic organ of the ORAC has been undergoing tests and "debugging" since early in June. Information can be put into the A, Q, and S registers by means of clip leads or by means of a test panel. It is possible to cause the machine to repeat the same operation, such as shift, multiplications, or division, continuously for as long as desired, and the normal clearing of A prior to a multiplication can be inhibited. It is possible to find numbers (cf., "Test Problems for the ORAC Arithmetic Unit," this report) for the three registers such that when either a division or a multiplication with inhibited clear is performed the same three numbers reappear in the same register. Hence it is possible to tell at a glance whether mistakes have been made in either multiplication or division regardless of how many such operations have been performed. This provides an incidental test of the simpler operations of shift, addition, and complementation. A series of such tests was run during the week of June 30 with no error on the part of the machine, except those induced by the conditions of the engineering tests.

Fabrication of the memory units and memory frame is in progress.

The following papers were published during the quarter.

A. W. Kimball, "Microbiological Assays with Nonparallel Response Curves" and "On Dependent Tests of Significance in the Analysis of Variance," *Colloquia in Mathematical and Applied Statistics*, Virginia Polytechnic Institute, Blacksburg, Virginia.

UNCLASSIFIED PROJECTS

TEST PROBLEMS FOR THE ORAC ARITHMETIC UNIT

Origin. ORAC engineering group.

Participating Members of Panel. A. S. Householder, W. A. Rutledge.

Background and Status. The ORAC has no memory, as yet. However, it is possible to introduce numbers into the A, Q, and S^U registers and orders into the S_L register and to read results in A and Q. For test purposes one wants the machine to perform a large number of computations at normal speed and to be able to tell at the end whether the results are correct. It turns out that there are numbers that if placed in the A, Q, and S^U registers reappear in the same registers after either a multiplication or a division. Hence, if the machine is made to multiply or divide repeatedly, the numbers will still be there when the machine stops if no mistakes have been made.

Let A, Q, and S represent positive integers $<2^{39}$. It can be shown that $a = 2^{-39}A$ in the A register, $q = 2^{-39}Q$ in the Q register, and $s = 2^{-39}S$ in the S^U register satisfy requirements provided

$$Q(2S - 1) = 2A(2^{39} - 1) .$$

Hence, $2S - 1$ must be some divisor of $2^{39} - 1$. An especially simple choice is $S = 2^{38}$, $A < 2^{38}$, but otherwise arbitrary, and $Q = 2A$.

For negative numbers, $q = -2^{-39}Q$, $a = -2^{-39}A$, $s = 2^{-39}S$, A, Q, and S must satisfy

$$Q(2S - 1) = 2A(2^{39} - 1) - (2^{40} - 1) .$$

Since $2^{39} - 1$ and $2^{40} - 1$ are relatively prime, one must take $A = Q$, a divisor of $2^{40} - 1$.

A number of combinations of both types have been computed and used in testing the machine.

ERRORS IN MATRIX OPERATIONS

Origin. Mathematics Panel.

Participating Member of Panel. A. S. Householder.

Background and Status. Although von Neumann and Goldstine have made an elaborate study of the accuracy of digital solutions of linear systems when solved by direct methods, no such consideration has been given to the results of iterative methods such as the Seidel method, which is used by the NEPA linear equation solver. That successive iterates agree to the last digit does not imply that the solution is correct to the last digit, although sometimes this inference seems to have been drawn. Since the problem of solving an integral equation or a boundary value problem reduces ultimately to the solution of an algebraic system, it is important to be able to assess the result of solution by whatever method might be employed.

If the equations are $Ax = y$, and A is positive definite, the NEPA machine starts with an approximate solution and improves it by a method that purports to reduce the quantity

$$x^T(Ax - 2y) = -x^T(y + r) ,$$

where $r = y - Ax$ for any x. Owing to roundoff, however, a step that replaces an approximation x_p by an approximation x_{p+1} could, in fact, increase the quantity in question.

For a residual $r_p = y - Ax_p$, the NEPA machine computes a digital vector $r_p^* = y - (Ax_p)^*$ in which each element of $(Ax_p)^*$ is obtained by summing the products and rounding off the sum. A step consists in adjusting an element of x_p so as to cause the corresponding element in r_p^* to vanish; a cycle consists of n steps adjusting each of the n elements. Since the machine carries four figures, the largest roundoff error is $\epsilon = 10^{-4}/2$. It turns out that if any element of r_p^* is equal to 10^{-4}, modifying the corresponding element of x_p so as to reduce this element to r_p^* could give an x_{p+1} that is worse than x_p; otherwise an improvement always results. In general, if x and x_2 are two approximate solutions, however obtained, then x_2 is certainly better than x_1 when

$$x_2^T(y + r_2^*) - x_1^T(y + r_1^*) > n^{1/2}\, \epsilon[N(x_1) + N(x_2)] ,$$

where $N(x)$ is the square root of the sum of the squares of the elements. If this inequality fails, the matter is uncertain.

Let λ be the smallest proper value of A, and let $Au = \lambda u$. If x satisfies $Ax = y$, then $x + u$ satisfies $A(x + u) = y + \lambda u$. Suppose u is normalized by making its numerically largest element unity. If λ is very small, say of the order of 10^{-5}, then the addition of the vector u to an approximate solution x_1 would not be detected by the machine since it would not affect the vector r_1^*. In this case, since no element of x can exceed unity, the result could be completely wrong since the element of x_1 in the same position as the unit element of u could be completely wrong. In general, the error in any one element of the approximation x_1 can be as great as $\epsilon\lambda^{-1}$ and can occur in that position in x_1 corresponding to the numerically largest element in u.

If the same method is programmed for a general-purpose digital computer, it is important that each product should not be rounded but only the sums, since otherwise the error can be n times as large.

BASIC STUDIES IN THE MONTE CARLO METHOD

Participating Member of Panel. G. E. Albert.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1232.

Background and Status. The basic studies in the Monte Carlo method are a continuing long-term research project initiated by the participant. The past status of the problem is given in the reference.

Following the last report on this project, investigations have proceeded in two directions. The first is a critical study of the possible utility of approximations to zero-variance Markoff chains in the estimation of the solutions of nonhomogeneous, Fredholm, integral equations. The second is a study of a method for the estimation of the solution of such an integral equation over its entire interval. The method amounts to

applying an expediently chosen interpolation formula to several point estimates of the solution. These investigations will be included in a comprehensive ORNL report that is being prepared on this project.

METHODS OF COMPUTATION FOR USE WITH A HIGH-SPEED AUTOMATIC-SEQUENCED COMPUTER

Participating Members of Panel. W. Givens, W. A. Rutledge.

Reference. *Mathematics Panel Quarterly Progress Reports,* ORNL-1232 and ORNL-1290.

Problem 1. Fields of Values of a Matrix.

Status. Results have now been published: "Fields of Values of a Matrix," *Proc. Am. Math. Soc.* **3**, 206-209 (1952).

Problem 3. Numerical Computation of Characteristic Values and Characteristic Vectors.

Status. The paper by V. B. Lidskii, "Concerning Characteristic Values of Sums and Products of Symmetric Matrices," *Doklady Akad. Nauk S.S.S.R.* **75**, 769-772 (1950), was translated and studied. A manuscript detailing the results obtained is being completed. There are several directions in which work can be continued.

Problem 4. Basic Matrix Operations.

Status. Preparation of coded sequences for the library of subroutines is being continued. Routines for computing the inverse of a matrix based on the Gauss elimination method have been prepared. Routines involving the use of the external memory of the ORAC are in preparation.

Problem 5. Coding Exercises for ORAC.

Background and Status. A set of exercises for use in training beginners in coding has been prepared. It is being reproduced and will be distributed to those concerned. The set consists of a series of exercises of graded difficulty designed to illustrate the need and use of various standard and special techniques in the preparation of coded sequences for the ORAC.

A STURM-LIOUVILLE PROBLEM

Origin. Mathematics Panel.

Participating Members of Panel. J. Z. Hearon, W. C. Sangren.

Status. Consider the diffusion problem

$$D \nabla^2 X^i + KX^i = \frac{\partial X^i}{\partial t}, \text{ in } V_0 \; ; \quad (1)$$

$$A \nabla^2 X^e = \frac{\partial X^e}{\partial t}, \text{ external to } V_0 \; ; \quad (2)$$

$$D \nabla X^i = A \nabla X^e$$
$$= \hat{n} H(X^i - X^e), \text{ on } S_0 \; ; \quad (3)$$

$$X^i \text{ bounded at origin and } X^i_j \geqslant 0, \text{ in } V, \text{ for all } t \; ; \quad (4)$$

$$X^e \to C, \text{ a prescribed constant, at } \infty \; ; \quad X^e_j \geqslant 0, \text{ external to } V, \text{ for all } t \; . \quad (5)$$

Here X^i and X^e are the column vectors of the internal concentrations X^i_j and

external concentrations X_j^e; D and A are the matrices of the internal and external diffusion coefficients, respectively; $\hat{n}$ is a unit (3-space) vector with direction of the outward normal to S_0, which is the surface enclosing the system of volume V_0. The surface S_0 is, physically, a membrane characterized by the matrix of permeability coefficients H. The operators ∇^2 and ∇ are the (3-space) vector operators div grad and grad, respectively. The problem is to show that with the following conditions on D, A, H, and the "source matrix" K, the X_j^i and X_j^e cannot be periodic functions of time. The simple laws of diffusion are assumed: D, A, and H are then diagonal. It can be shown that K is physically admissable if and only if $|K| = 0$; $k_{ij} \geqslant 0$, $i \neq j$; if $k_{rs} = 0$ then $k_{sr} = 0$; $k_{ij}k_{jm}k_{mi} = k_{im}k_{mj}k_{ji}$. In consequence of these properties a real matrix P can be found such that

$$PKP^{-1} = B = \tilde{B} , \qquad (6)$$

and B is real where $\tilde{B}$ is the transpose of B. Moreover, P is diagonal. If x^i and x^e denote the Laplace transform of X^i and X^e, respectively, and F and G are the initial values of X^i and X^e, then from Eqs. 1 and 2

$$\begin{aligned} D\,\nabla^2 x^i + Kx^i &= sx^i - F , \\ A\,\nabla^2 x^e &= sx^e - G . \end{aligned} \qquad (7)$$

The homogeneous problem associated with Eq. 7, with the transformed boundary given in Eqs. 3, 4, and 5, represents asystem of two-region Sturm-Liouville Systems. It is to be shown that the proper values, s, are real. Then by known properties of the inversion integral, X^i and X^e cannot be periodic vectors. A transformation of variables gives for the homogeneous equations

$$D\,\nabla^2 y + By = sy , \qquad (8)$$

$$A\,\nabla^2 \varphi = s\varphi . \qquad (9)$$

Similarly, Eq. 3 gives

$$D\,\nabla y = A\,\nabla \varphi = \hat{n}\,H(y - \varphi) . \qquad (10)$$

By Eq. 5, $\varphi \rightarrow$ constant, at ∞, and it is assumed that

$$\nabla \varphi \longrightarrow 0 \text{ at } \infty . \qquad (11)$$

Application of the divergence theorem to the difference between Eqs. 8 and 9 and their complex conjugates, the use of conditions 10 and 11, and the symmetry of B lead to $s = s^*$. It has also been shown for the spherically symmetric case that s is nonnegative. The problem of the inversion integral is being considered.

APPROXIMATE STEADY-STATE SOLUTION OF THE EQUATION OF CONTINUITY

Origin. Mathematics Panel.

Participating Member of Panel. J. Z. Hearon.

Status. An approximate method was presented previously[1] whereby the equation of continuity

$$D_i\,\nabla^2 C + Q = \frac{\partial C}{\partial t} , \text{ in } V ,$$

$$D_e\,\nabla^2 C = \frac{\partial C}{\partial t} , \text{ external to } V ,$$

is reduced to the ordinary differential equation

$$\frac{dC}{dt} = \frac{(C^0 - \overline{C})}{\lambda} + \overline{Q} . \qquad (1)$$

[1] N. Rashevsky, *Advances and Applications of Mathematical Biology*, U. of Chicago Press, 1940.

Here C^0 is the value of C at infinity; λ is a constant depending only on the dimensions of the system, the values of D_i and D_e, and a parameter associated with the properties of the boundary of V. $\overline{C}$ is the volume average,

$$\overline{C} = \int_V C \, dV ,$$

and $\overline{Q}$ is defined similarly.

It has been shown by means of the integral equation solution of the appropriate boundary value problem that for a spherically symmetric, steady-state system

$$\overline{C} = C^0 + \left(\epsilon + \frac{1}{\lambda}\right) \overline{Q} , \qquad (2)$$

provided the integral $\overline{Q}$ converges and is different from zero, and Q is bounded at the origin. In Eq. 2

$$\epsilon = \left(1 - \frac{\overline{r^2 Q}}{\overline{r^2}\,\overline{Q}}\right) \frac{r_0^2}{10\, D_i} \qquad (3)$$

where r_0 is the radius of the system. If Q does not depend on the spatial coordinates, Eqs. 1 and 2 agree. Otherwise, the "correction term" ϵ is different from zero. Values of ϵ have been computed for some special forms of Q:

$$Q(C) = -kC, \; k > 0 ,$$

and

$$Q(r) = Br \, e^{-\nu r} .$$

If $\overline{Q} = 0$, Eq. 2 is replaced by

$$\overline{C} = C^0 - \frac{\overline{r^2 Q}}{6\, D_i} , \qquad (4)$$

and it can be shown that if the physical problem is the diffusion, heat flow, or velocity-potential hydrodynamic problem, $\overline{Q} = 0$ if the flux vanishes at the surface of the system.

ESTIMATION OF VOLUME OF LYMPH SPACE

Origin. J. Furth, R. H. Storey, Biology Division.

Participating Members of Panel. J. Moshman, J. Z. Hearon, A. W. Kimball, G. J. Atta, K. P. Graw, N. D. Given.

Reference. *Mathematics Panel Quarterly Progress Reports,* ORNL-1091 and ORNL-1290.

Status. The equation

$$A_p V_p + A_l V_l + k_p V_p \int_0^t A_p \, dt + k_l V_l \int_0^t A_l \, dt = R_0 ,$$

where the notation follows that of ORNL-1290, was fitted to the experimental data from 4 dogs, and V_p, V_l, k_p, and k were estimated. The integral was evaluated by planimeter readings from the smoothed curves of plasma and lymph activity. The results were poor in that negative volumes and/or metabolism rates appeared.

Various hypotheses for the discrepancies are tenable, both in the model and in the experimental data.

In the model, there are three possible sources of error: (1) a three-compartment system is inadequate; (2) one metabolism rate per compartment is wrong; and (3) the volumes and metabolism rates are variable. Experimentally, errors may arise from (1) nonuniform mixing, (2) a nonuniform decomposition of the albumin, and (3) the lack of data between 6 and 24 hr, which makes the estimated activity

curves from which the integral was evaluated highly subjective.

X-RADIATION EFFECTS ON RESPIRATION RATES IN GRASSHOPPER EMBRYOS

Origin. S. R. Tipton, G. S. St. Amand, Biology Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. Respiration in grasshopper embryos, as measured by carbon dioxide production, increases linearly with time. The change in environment required for an irradiation experiment causes a shift in the location of the response but, apart from radiation effects, does not change the slope. Apparently the effect of x radiation is to decrease the respiration rate. To investigate this hypothesis, 12 irradiation experiments and 13 control experiments were performed. The control embryos were manipulated in the same manner as the treated embryos except for the actual irradiation. In each experiment the respiration rates before and after treatment were computed by the method of least squares.

The average differences in respiration rates before and after treatment were 0.00266 and 0.00507 for the control and treated embryos, respectively. Although these values differ in the direction expected, the standard error of the difference between them is 0.01041. Clearly, no definite conclusion can be drawn. With errors of this magnitude the number of experiments necessary to detect an irradiation effect would be prohibitive; accordingly, new methods of attacking the problem will be sought.

TEMPERATURE EFFECTS IN IRRADIATION EXPERIMENTS WITH YEAST

Origin. S. Pomper, Biology Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. In some organisms resistance to radiation damage is increased at low temperatures. In an attempt to investigate the temperature effect in yeast, a number of experiments at effective doses were performed in which plate counts were made on control cultures and on x-irradiated cultures at three temperatures, 30, 18, and 12°C. In some experiments only two temperatures were used.

Randomized block analyses of variance based on the square roots of the plate counts were computed for each experiment. No effect of temperature on the control cultures could be demonstrated. Among the irradiated cultures, however, seven experiments showed increased survival at lower temperatures, six experiments showed decreased survival at lower temperatures, and in seven experiments no effect of temperature could be demonstrated. All tests of significance were made at the 5% level.

It appears that low-temperature protection is not a universal phenomenon and, in particular, that in yeast its effect cannot be predicted. Yeast cultures apparently lack homogeneity with respect to radiation-temperature interaction.

COMPARISON OF BETA-, GAMMA-, AND X-RADIATION EFFECTS ON *TRADESCANTIA*

Origin. J. Kirby-Smith, Biology Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. Chromosome and chromatid aberrations in *Tradescantia* have been classified as one-hit or two-hit aberrations depending on whether they increase linearly with dose or quadratically with dose. The present experiments were undertaken to evaluate the relative effects of beta, gamma, and x radiation on these aberration-dose relationships.

In each of several experiments, equations of the form $y = bx$ or $y = bx + cx^2$ were fitted to the number of aberrations per cell, y, and the dose, x, by the method of maximum likelihood. In general, x rays produce about twice as many aberrations as gamma rays at equivalent doses, whereas beta rays produce an effect just slightly greater than that of x rays.

LD_{50} ESTIMATION FOR BURROS AND SWINE EXPOSED TO GAMMA RADIATION

Origin. J. H. Rust, UT-AEC Agricultural Research Program.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. Following the construction of an exposure pen for whole-body irradiation of large animals,[2] two experiments with burros and one with swine were carried out for the primary purpose of estimating the dose that would kill 50% of the animals in 30 days. In the first experiment, eight groups of ten burros each were exposed to a range of doses from 450 to 1200 r. It was felt that this range would be certain to include the LD_{50}. With the results of the first experiment available, it was possible to reduce the range of doses for the second and third experiments and to use only five groups of ten animals each.

The LD_{50}'s and their 95% confidence intervals were estimated by the probit method.[3] The results are presented in Table 1.

THERMAL-NEUTRON PENETRATION IN TISSUE

Origin. W. S. Synder, Health Physics Division.

Participating Members of Panel. N. Edmonson, S. E. Ezzell, K. P. Graw, N. D. Given.

Background and Status. The problem is to calculate the collision density of thermal neutrons in a small animal considered as a small sphere, and the

[2] J. L. Wilding, C. S. Simons, and J. H. Rust, *Nucleonics* **10**, No. 5, 36-40 (May 1952).

[3] R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural, and Medical Research*, 3d ed., rev., Hafner Pub. Co., New York, 1949.

Table 1

ESTIMATED LD_{50}'s FOR BURROS AND SWINE EXPOSED TO GAMMA RADIATION

EXPERIMENT NUMBER	GAMMA SOURCE	ANIMALS	ESTIMATED LD_{50} (r)	95% CONFIDENCE INTERVAL FOR LD_{50} (r)
1	Ta^{182}	Burros	970	932 to 1009
2	Co^{60}	Burros	961	880 to 1116
3	Co^{60}	Swine	960	963 to 1042

results are to be compared with experimental data.

Mathematically the mode of calculation now being considered involves replacing an integral equation

$$f(x) = s(x) + \frac{\sigma_s}{2}\int_a^a K(x,y)\ f(y)\ dy \quad (1)$$

with

$$k(x,y) = E_1(\sigma\ |y - x|)$$

$$= \int_1^\infty t^{-1} \exp\ [-\sigma\ |y - x|\ t]\ dt\ , \quad (2)$$

by a system of simultaneous linear equations

$$f(x_i) = s(x_i) + \frac{\sigma_s \delta}{2} \sum_{j=1}^{2N} f(x_j)\ k_{ij}\ . \quad (3)$$

In system 3

$$k_{ij} = \delta^{-2} \int_{-a+(j-1)\delta}^{-a+j\delta} \int_{-a+(i-1)\delta}^{-a+i\delta} E_1(\sigma\ |y - x|)\ dx\ dy\ , \quad (4)$$

with

$$\delta = \frac{a}{N},$$

when N is a positive integer.

For the present, a = 1 and 2 cm with N = 100. Later, perhaps, larger values of a will be considered. Thus the solution of system 3 involves the inversion of a 200 by 200 matrix. Further work on this problem has been postponed until the NEPA computing machine is installed.

NUMERICAL EVALUATION OF INTEGRALS

Origin. R. B. Birkhoff, Health Physics Division.

Participating Members of Panel. N. Edmonson, S. E. Ezzell.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1290.

Background and Status. This problem was finished and the data were delivered.

KINETICS OF HBr-$HBrO_3$ REACTION

Origin. O. E. Myers, Chemistry Division.

Participating Members of Panel. C. L. Perry, A. S. Householder, W. C. Sangren, H. B. Goertzel, J. H. Fishel, C. P. Hubbard.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1290.

Background and Status. The numerical integrations code for integrating the kinetic equations was prepared for the MANIAC computer at Los Alamos. The numerical integration of the system was carried out and completed on the MANIAC for 18 sets of parameters. The coding of this problem is being completed for the UNIVAC in order to run off other sets of parameters.

ISOTOPE EFFECT IN REACTION RATE CONSTANTS

Origin. G. Ropp, Chemistry Division.

Participating Member of Panel. J. Z. Hearon.

Status. It is well known that the rate constants for normal and isotopic molecules are demonstrably different. For first-order reactions, the equation

$$\frac{k^*}{k} = \frac{\ln(1 - rf)}{\ln(1 - f)} \qquad (1)$$

is usually employed, where k^* and k are the first-order constants for the isotopic and normal reactions, respectively, f is the fractional completion of the reaction, and r is the ratio of the specific activity of the product at any time to that of the reactant at $t = 0$. The question arises as to the form of Eq. 1 when the reaction is not first order and the error committed when Eq. 1 is used and the reaction is in fact of order $n > 1$.

If the reaction is written as

$$nA + mB + \ldots \longrightarrow P + \ldots$$

and A is tagged, then the usual assumption is that

$$A^* + (n - 1)A + mB + \ldots \longrightarrow P^* + \ldots .$$

Accordingly, the rate equations are

$$\frac{dP^*}{dt} = k^* A^* A^{n-1} B^m \ldots , \qquad (2)$$

$$\frac{dP}{dt} = k A^n B^m \ldots . \qquad (3)$$

From Eqs. 2 and 3, if $P(0) = P^*(0) = 0$,

$$\frac{k^*}{k} = n \frac{\ln(1 - rf)}{\ln(1 - nf)} , \qquad (4)$$

where

$$f = \frac{P(t)}{A(0)}, \quad f^* = \frac{P^*(t)}{A^*(0)}, \quad r = \frac{f^*}{f} . \qquad (5)$$

From Eq. 5, r is the specific activity of the product P at t divided by the specific activity of the reactant A at $t = 0$, *provided* $P(t) >> P^*(t)$, $A(t) >> A^*(t)$. This condition is usually satisfied. If it is *not*, in Eq. 4

$$r = \frac{S_p(t)}{S_a(0)} \frac{1 - S_a(0)}{1 - S_p(t)} ,$$

where $S_a(t)$ and $S_p(t)$ are the specific activities of the reactant A and product P, respectively. For small t, when $f << 1$ *both* Eqs. 1 and 4 give

$$\frac{k^*}{k} \sim r .$$

Tables have been prepared giving the ratio of the actual and computed values of k^*/k as a function of n and f.

EXPERIMENTAL DETERMINATION OF DIFFUSIVITY COEFFICIENTS

Origin. J. R. Johnson, Metallurgy Division.

Participating Members of Panel. N. Edmonson, S. E. Ezzell.

Background and Status. The purpose of this computation is to supply data that will enable the thermal diffusivity, K, of a given material to be determined from an experiment described as follows: A cylinder of the material is placed inside a heater cylinder so that there is a space of uniform thickness h between the heater and the

specimen being tested. The temperature at the center of the test cylinder is determined by a thermocouple junction. The initial temperature T_0 of the test specimen is taken and the temperature T_F of the heater is known.

The quantity calculated is

$$\frac{T - T_0}{T_F - T_0} = 1 - \frac{8}{\pi} \sum_{p=0}^{\infty} \sum_{n=0}^{\infty} \frac{J_0(\beta_n r) \sin \left[\frac{(2p+1)}{L}\right] \pi z \; e^{-\{\beta_n^2 + (2p+1)^2\pi^2/L^2\} Kt}}{(2p+1)(\beta_n a) J_1(\beta_n a)} . \quad (1)$$

In this formula, $J_0(u)$ and $J_1(u)$ are the Bessel functions of the first kind of the zero and first orders, respectively. The numbers β_n are roots of the equation $J_0(\beta_n a) = 0$.

If the following substitutions

$$\beta_n a = \alpha_n , \quad \frac{r}{a} = s , \quad \frac{z}{L} = x , \quad \frac{a}{L} = \rho , \quad \frac{Kt}{a^2} = \tau$$

are made, formula 1 takes the dimensionless form

$$\frac{T - T_0}{T_F - T_0} = 1 - \frac{8}{\pi} \sum_{p=0}^{\infty} \sum_{n=0}^{\infty} \frac{J_0(\alpha_n s) \sin (2p+1) \pi x \; e^{-\{\alpha_n^2 + (2p+1)^2\pi^2\rho^2)\}\tau}}{(2p+1)(\alpha_n) J_1(\alpha_n)} . \quad (2)$$

The quantity $(T - T_0)/(T_F - T_0)$ is plotted against τ for $s = 0$, $x = 1/2$. From this curve and the experimental data, the values of the diffusivity coefficient, K, are determined.

DIRAC WAVE FUNCTIONS

Origin. M. E. Rose, Physics Division.

Participating Members of Panel. W. C. Sangren, S. D. Conte.

References. M. E. Rose and R. R. Newton, "Properties of Dirac Wave Functions in a Central Field," *Phys. Rev.* **82**, 470-477 (1951). E. C. Titchmarsh, *Eigenfunction Expansions Associated with Second-Order Differential Equations*, Oxford U. Press, New York, 1946.

Background and Status. The Dirac wave equations in a central field can be written in the following form,

$$F' = \alpha_{11} F + \alpha_{12} G ,$$
$$G' = \alpha_{21} F + \alpha_{22} G , \quad (1)$$

where

$$\alpha_{11} = -\alpha_{22} = \frac{k}{r} ,$$

$$\alpha_{12} = -W + 1 + V(r) ,$$

$$\alpha_{21} = W + 1 - V(r),$$

k is a nonzero integer, W is a parameter, and the prime denotes differentiation with respect to r. The basic mathematical problem is to investigate the influence of $V(r)$ and certain assumed boundary conditions upon the W spectrum and the eigenfunctions. Formal results have been obtained under the assumptions that $V(r)$ tends to a power (including zero) of r as r tends to zero and also to a power (possibly different) as r tends to infinity. One interesting result of the investigation is that the spectrum in the case

$$V = \frac{\beta}{r}, \quad \sqrt{\frac{3}{2}} \leqslant |\beta| \leqslant 1, \quad k = 1$$

is not uniquely determined by the customary boundary conditions.

The more general problem expressed by Eqs. 1, where

$$\alpha_{11} = \alpha_{22} = A(r) ,$$

$$\alpha_{12} = -W + V_1(r) ,$$

$$\alpha_{21} = W + V_2(r) ,$$

and appropriate boundary conditions are considered, is also being examined. It is believed that many of the ordinary Sturm-Liouville theorems, as well as Titchmarsh's results, can be extended to this problem.

CALCULATION OF RACAH COEFFICIENTS FOR THE ANGULAR DISTRIBUTION IN NUCLEAR REACTIONS

Origin. M. E. Rose, L. C. Biedenharn, Physics Division.

Participating Members of Panel. S. L. Hull, P. J. Brown, E. A. Forbes, R. C. Weaver.

References. L. C. Biedenharn, *Tables of the Racah Coefficients*, ORNL-1098; *Physics Division Quarterly Progress Report for Period Ending March 20, 1951*, ORNL-1005; *Mathematics Panel Quarterly Progress Reports*, ORNL-1029, -1151, -1232, and -1290.

Background and Status. The checking for errors in ORNL-1098 was completed. The new recursion relations developed by Biedenharn were used in the checking. A supplement entitled *Errata for ORNL-1098* was published.

Two groups of additional coefficients, $R(J\ J_0;\ \nu L)$ and $S(J\ J_0;\ \nu L)$, both functions of the Racah coefficients, were computed and checked. These are of interest in the problem of angular correlation of sucessive gamma radiation.

FITTING A CALIBRATION CURVE

Origin. J. L. Meem, Physics Division.

Participating Members of Panel. J. Moshman, G. J. Atta, K. P. Graw, C. Perhacs.

Background and Status. In the experiment disks of uranium coated with a thin aluminum foil are subjected to a radioactive source.

Fission products of the uranium are embedded in the aluminum, which is then stripped off and placed in a counter. About a 30-min lag exists from the time the disks are removed from the source to the placement of the stripped foil in the counter. One-minute counts are taken at 15-sec intervals for about 75 minutes. The resulting calibration curves take the form

$$y = A\ e^{-g\tau} + B\ e^{-d\tau}\ .$$

Define

$$F \equiv (y - A\ e^{-g\tau} - B\ e^{-d\tau})w\ ,$$

where w is a weighting function, and let A_0, g_0, B_0, and d_0 be some approximate values, found graphically, for the parameters. F may then be expanded by Taylor's series about A_0, g_0, B_0, and d_0 to a first-order approximation to get

$$F \sim F^0 + F_A^0(A - A_0) + F_g^0(g - g_0) + F_B^0(B - B_0) + F_d^0(d - d_0)$$

$$\equiv F^0 + F_A^0\alpha + F_g^0\gamma + F_B^0\beta + F_d^0\delta\ ,$$

where $-\alpha = A - A_0$, $-\gamma = g - g_0$, etc., and the superscript denotes the numerical value at A_0, g_0, B_0, and d_0.

α, β, γ, and δ may be estimated by least squares by letting the square of F be a minimum. Differentiating with respect to each in turn, the following normal equations are obtained:

$$\alpha \sum F_A^{0^2} + \gamma \sum F_A^0 F_g^0 + \beta \sum F_A^0 F_B^0 + \delta \sum F_A^0 F_d^0 = \sum F_A^0 F^0$$

$$\alpha F_A^0 F_g^0 + \gamma \sum F_g^{0^2} + \beta \sum F_g^0 F_B^0 + \delta \sum F_g^0 F_d^0 = \sum F_g^0 F^0$$

$$\alpha F_A^0 F_B^0 + \gamma \sum F_g^0 F_B^0 + \beta \sum F_B^{0^2} + \delta \sum F_B^0 F_d^0 = \sum F_B^0 F^0$$

$$\alpha F_A^0 F_d^0 + \gamma \sum F_g^0 F_d^0 + \beta \sum F_B^0 F_d^0 + \delta \sum F_d^{0^2} = \sum F_d^0 F^0$$

which may be solved in the usual way for α, γ, β, and δ. Finally, the original parameters are estimated from relations such as $A = A_0 - \alpha$, etc.

Two sets of data have been analyzed and two more are being computed.

MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

BETA DECAY (FIELD FACTORS)

Origin. M. E. Rose, P. R. Bell, Physics Division.

Participating Members of Panel. C. L. Perry, N. M. Dismuke, H. B. Goertzel, K. P. Graw, N. D. Given, P. J. Brown, V. C. Carlock, R. C. Weaver.

References. *Mathematics Panel Quarterly Progress Report,* ORNL-1290.

Background and Status. The computations of

$$\frac{\pi p}{W}\frac{L}{GJ}, \quad \frac{\pi}{W}\frac{P}{GJ}, \quad \frac{\pi p}{W}\frac{M}{GJ}, \quad \frac{\pi p}{W}\frac{Q}{GJ}, \quad \frac{\pi p}{W}\frac{N}{GJ}, \quad \frac{\pi p}{W}\frac{R}{GJ},$$ [4]

where

$$G = \frac{2}{p}\left[\frac{(2p)^{\nu}\ \nu!}{(2\nu+2)!}\right]^2$$

and

$$J = \frac{p}{W}\frac{F_{\nu}}{F_0}$$

are being completed on the Whirlwind computer for $Z = 81, 83$ and $Z = 20(2)70$. The results of these computations will then be punched on IBM cards for multiplication by the factors GJ. The final values of

$$\frac{\pi p}{W}L, \quad \frac{\pi p}{W}P, \quad \frac{\pi p}{W}M, \quad \frac{\pi p}{W}Q, \quad \frac{\pi p}{W}N, \quad \frac{\pi p}{W}R$$

are expected to be obtained and reported in an ORNL report next quarter.

During the quarter Dismuke, Perry, and Goertzel worked with Charles Adams' applications group at the MIT Digital Computer Laboratory in writing the final code for the Whirlwind and in testing the code on the computer. The MIT applications group has done a great deal of developmental work in designing conversion, utility, and programmed arithmetic codes for automatic computers. Thus the programming of the field-factor computations has given the Mathematics Panel experience in using programs designed to simplify coding for automatic computers.

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[4] For the definitions of L, M, N, and F_{ν}, see E. Greuling, *Phys. Rev.* **61**, 568 (1942); for definitions of P, Q, and R, see D. L. Pursey, *Phil. Mag.* **42**, 1206 (1951).

NEUTRON DECAY THEORY

Origin. L. C. Biedenharn, H. Reynolds, A. H. Snell, Physics Division.

Participating Members of Panel. J. H. Fishel, C. Perhacs, V. C. Carlock, C. P. Hubbard.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1290.

Status. The integrals I_1 and I_2 given in the reference were the results of a simplifying assumption made on original integrands of greater complexity. An analysis of the exact integrand for I_1 was undertaken to determine the effect of this assumption. It was found that the effect was negligible, so the integration of I_1 and I_2 on IBM equipment was performed. In addition, the integration of the following equation was performed on IBM equipment:

$$G(\theta) = \int_0^{E_1} \sqrt{E(E+2)}\,(E+1)\,(E_0 - E)^2 \left[1 + n\,\frac{\sqrt{E(E+2)}}{E+1}\left\{\frac{\sqrt{E(E+2)}}{E_0 - E}\sin^2\theta + \cos\theta\sqrt{1 - \frac{E(E+2)}{(E_0-E)^2}\sin^2\theta}\right\}\right]\left[\frac{1 + \frac{E(E+2)\cos 2\theta}{(E_0-E)^2}}{\sqrt{1 - \frac{E(E+2)}{(E_0-E)^2}\sin^2\theta}} - 2\,\frac{\sqrt{E(E+2)}}{E_0 - E}\cos\theta\right] dE$$

$$E_1 = \frac{E_0^2}{2(E_0+1)} \qquad E_0 = 1.530$$

where

θ = 0, 30, 60, 90, 120, 130, 140, 150, 160, 170, 180 deg,

$n = 1, \frac{1}{3}, -\frac{1}{3}, -1$.

This project was completed.

CALCULATION OF INTERNAL CONVERSION COEFFICIENTS WITH SCREENING

Origin. M. E. Rose, Physics Division.

Participating Member of Panel. M. R. Arnette.[5]

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1290.

Status. Part I of the problem, calculation of the screening function, is completed. Part II, the calculation of the shell energy, has been recoded to use only the acoustic memory plus magnetic tapes and wire. The code was checked previously in using both acoustic and electrostatic memories, so this second check should be a simple matter. After the check, Part II can be run. Part III, the calculation of the coefficients, is coded but not checked. This part necessitates the use of the SEAC's complete memory, both internal and external, and cannot be thoroughly checked or run until the Williams-tube memory is working again. Since the trouble with this part of the memory has been localized, it is to be hoped that the complete memory will be again available in two to three weeks. The interpolation subroutine to be used in Part III has been checked in; and, for checking purposes, some new values are being computed by hand for Z of 25 by using the corrected starting formula.

DETERMINATION OF THE FAST-NEUTRON FLUX IN THE ORNL GRAPHITE REACTOR

Origin. D. K. Holmes, Physics Division.

Participating Members of Panel. J. Moshman, E. B. Carter, K-25.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1290.

Status. IBM summaries are still being calculated. It is planned tentatively to use the IBM results as a pilot study in preparation for placing this problem on the UNIVAC.

ESTIMATION OF ALIGNMENT RATIOS

Origin. S. Bernstein, C. P. Stanford, Physics Division.

Participating Members of Panel. J. Moshman, C. L. Perry.

Background and Status. Given the functional relationship

$$\Delta_i = (\alpha A)X_i + BY_i ,$$

where Δ_i is the activity measured during time Y_i,

$$X_i = e^{-\lambda t_i} - e^{-\lambda t_i^*} ,$$

and

$$Y_i = t_i^* - t_i ,$$

it is desired to compare the ratio of the coefficients of X of sample to monitor for aligned, unaligned, and cross-aligned samples. Each experiment consists of noting the activity Δ_i during consecutive time periods, $t_i^* - t_i$, with a constant background, B. The ratio

$$R = \frac{(\alpha A)\ \text{sample}}{(\alpha A)\ \text{monitor}} ,$$

which is the statistic in question, is computed. According to the theory, $R \cong 1.09$ for unaligned samples, 1.13 for aligned samples, and 1.05 for cross-aligned samples. The experiment is designed to demonstrate the trichotomy and to estimate bounds for the ratio. A complicating factor is the

[5] Other participants: J. H. Wegstein, C. J. Swift, and G. E. Hawkins, Division 11.2, National Bureau of Standards.

heat loss effect, which tends to make the ratios drift towards 1.09 from either side.

A procedure was devised for estimating the (αA)'s and R. Since it is known in advance whether a specific sample was aligned, unaligned, or cross aligned, a nonparametric run was recommended to test whether the unaligned data differed significantly from either of the other two sets.

To estimate bounds for the ratio, it was assumed that the Gaussian distribution observed for the unaligned samples would be the same as the distributions about 1.05 and 1.13 if there were no heat loss. (The latter results in sharply skewed curves.) The sample standard deviation, s_ν, based on ν degrees of freedom, is estimated from the unaligned distribution. Let there exist n values of the ratio for the aligned series, say, which are ordered in magnitude $R_1 < R_2 < \ldots < R_n$. The last few values are considered "true" values, and the lower ones are contaminated by heat loss. Then there exists some number $K(n, \nu, \beta)$ such that

$$\text{Prob}\left\{\frac{R_n - \overline{R}}{s_\nu} \leqslant K(n, \nu, \beta)\right\} \geqslant \beta .$$

By simple transposition

$$\text{Prob}\ \{\overline{R} \geqslant R_n - K(n, \nu, \beta)\ s_\nu\} \geqslant \beta ,$$

which gives a lower bound for the mean aligned ratio with probability β. Values of $K(n, \nu, \beta)$ are calculable from the procedure of Nair.[6]

OPTIMUM ALLOCATION OF COUNTING TIME

Origin. S. Bernstein, C. P. Stanford, Physics Division.

Participating Members of Panel. J. Moshman, G. E. Albert.

Reference. "Estimation of Alignment Ratios," this report.

Background and Status. Let $(\alpha A)_s$ and $(\alpha A)_m$ be the true coefficients for sample and monitor, respectively, in the equation

$$D = (\alpha A)\ X + B\ Y .$$

The sample ratio may be expressed as

$$R = \frac{(\alpha A)_s + \epsilon_s}{(\alpha A)_m + \epsilon_m} , \qquad (1)$$

where the ϵ's are experimental fluctuations. Equation 1 may be written in the form

$$R = \frac{(\alpha A)_s}{(\alpha A)_m}\left[1 + \frac{\epsilon_s}{(\alpha A)_s}\right]\left[1 + \frac{\epsilon_m}{(\alpha A)_m}\right]^{-1} . \qquad (2)$$

Expanding Eq. 2 and neglecting terms of higher order

$$R - \frac{(\alpha A)_s}{(\alpha A)_m} = \frac{(\alpha A)_s}{(\alpha A)_m}\left\{\frac{\epsilon_s}{(\alpha A)_s} - \frac{\epsilon_m}{(\alpha A)_m}\right\} . \qquad (3)$$

Furthermore,

$$\text{Max}\left|R - \frac{(\alpha A)_s}{(\alpha A)_m}\right|$$

$$= \frac{(\alpha A)_s}{(\alpha A)_m}\left\{\frac{|\epsilon_s|}{(\alpha A)_s} + \frac{|\epsilon_m|}{(\alpha A)_m}\right\} . \qquad (4)$$

Let T_s and T_m be the time allotted to counting sample and monitor, and let

$$T_s + T_m = T \text{ (fixed)} . \qquad (5)$$

[6] K. R. Nair, *Biometrika* **35**, 118 (1948).

It may reasonably be assumed that the ϵ_i's are proportional to $\sqrt{(\alpha A)_i}/T_i$, so that

$$\epsilon_i = \frac{k\sqrt{(\alpha A)_i}}{T_i}, \qquad i = s, m, \tag{6}$$

where k is a proportionality constant. Substituting Eqs. 5 and 6 in Eq. 4,

$$\text{Max}\left|R - \frac{(\alpha A)_s}{(\alpha A)_m}\right| = \frac{k(\alpha A)_s}{(\alpha A)_m}\left\{\frac{1}{T_s(\alpha A)_s} + \frac{1}{(T - T_s)\ (\alpha A)_m}\right\} \equiv F . \tag{7}$$

To minimize F, one lets

$$\frac{dF}{dT_s} = \frac{k(\alpha A)_s}{(\alpha A)_m}\left[-\frac{1}{T_s^2(\alpha A)_s} + \frac{1}{(T - T_s)^2\ (\alpha A)_m}\right] = 0 \tag{8}$$

and finds

$$T_s = T_m \sqrt[4]{\frac{(\alpha A)_m}{(\alpha A)_s}} \equiv T_m r , \tag{9}$$

or

$$T_s = \frac{r}{1 + r} T , \qquad T_m = \frac{T}{1 + r} . \tag{10}$$

EFFICIENCY OF A SINGLE CRYSTAL

Origin. P. R. Bell, J. K. McGowan, Physics Division.

Participating Members of Panel. N. Edmonson, P. J. Brown, V. C. Carlock, H. B. Goertzel.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1151.

Background and Status. The problem under consideration is to calculate the integral given in the reference for further values of h and τ. The calculations have been finished and reported.

CALCULATION OF COINCIDENCE PROBABILITIES FOR A DOUBLE-CRYSTAL DETECTOR

Origin. J. K. McGowan, Physics Division.

Participating Members of Panel. N. Edmonson, N. M. Dismuke, H. B. Goertzel.

Background and Status. In the experiment, a homogeneous source of radiation is placed on the axis of two right-cylinder crystals. The apparatus is arranged so that coincident absorptions are recorded. The mathematical problem is to calculate the coincidence probability

$$I_l = \frac{1}{\Omega_1 \Omega_2} \int d\Omega_1 \, d\Omega_2 \, P_e(\cos\theta) \, (1 - e^{-\tau_1 x_1}) \, (1 - e^{-\tau x_2}) \; ;$$

in this integral, Ω_1 and Ω_2 are the solid angles subtended at the source S by the crystal ends, τ is the total absorption cross section of the crystal material, x_1 and x_2 are the total path lengths within the crystals, and θ is the angle between two arbitrary radius vectors from the source S to the crystals. It is clear that the integral measures the probability of a simultaneous absorption.

The analysis of this problem has been checked and the computation is being coded for the UNIVAC.

REACTOR INEQUALITIES

Origin. A. M. Weinberg, Research Director.

Participating Members of Panel. C. P. Hubbard, W. C. Sangren, S. D. Conte.

Reference. G. Pólya and G. Szegö, "Isoperimetric Inequalities in Mathematical Physics," *Annals of Mathematics Studies,* No. 27, Princeton University Press, 1951.

Background and Status. The reference discusses among other things the principal frequency of a vibrating membrane. Certain inequalities are derived for the value of the principal frequency that depend upon the geometric shape of the membrane. Since the critical equation for reactors is the three-dimensional analog of the membrane problem, in which the buckling is identified with frequency, the reference is being examined to see what relevant ideas and theorems can be extended to three dimensions.

HEAT FLOW IN LONG, RECTANGULAR TUBES

Origin. H. C. Claiborne, Reactor Experimental Engineering Division.

Participating Members of Panel. N. D. Given, S. D. Conte, W. C. Sangren.

Background and Status. The solutions of the boundary value problems for the case of constant wall temperature and the case of constant flux, which were obtained by Claiborne, were checked. By using the solutions, the Nusselt number was computed as a function of distance along the tube for the case of constant flux. Preliminary calculations have been completed for the computation of the Nusselt number in the constant-wall-temperature case.

MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

KINETICS OF BOILING REACTORS

Origin. P. R. Kasten, Reactor Experimental Engineering Division.

Participating Members of Panel. C. P. Hubbard, J. H. Fishel, W. C. Sangren.

Background and Status. Preliminary steps have been taken in preparation for solving systems of nonlinear differential equations on the UNIVAC.

CIRCULATION IN BOILING REACTORS

Origin. P. Zmola, R. B. Bailey, Reactor Experimental Engineering Division.

Participating Members of Panel. C. P. Hubbard, J. H. Fishel, H. B. Goertzel, S. L. Hull, W. C. Sangren.

Background and Status. Since the boundary value problem involved appears to be unsolvable by analytical methods, work has been started on the coding of the problem for the UNIVAC. Desk computations have been started for checking the feasibility of the numerical method to be used in solving the problem on the UNIVAC.

SOLVING AN EQUATION BY NUMERICAL METHODS

Origin. M. Tobias, Reactor Experimental Engineering Division.

Participating Members of Panel. H. B. Goertzel, C. Perhacs.

Background and Status. The equation to be solved is

$$1 = f(\gamma, Z) \equiv \frac{a\gamma}{\eta_1 [eZ^2 + b\gamma + c]} ,$$

where

$$\frac{Z}{\tan Z} \equiv 1 - Af[(b\gamma + c)g + h]\ 10^{-24} .$$

The parameters a, c, and e are functions of β, β/F^2, and p, which were tabulated by Tobias. The parameters η_1, A, f, and h are dependent upon temperature and were also tabulated by Tobias. The equation was to be solved for various parameter combinations. The solution was obtained by a method of successive approximations.

NUMERICAL INTEGRATION

Origin. C. G. Lawson, Reactor Experimental Engineering Division.

Participating Members of Panel. W. C. Sangren, C. Perhacs.

Background and Status. The integral to be evaluated is

$$\frac{y}{y_0} = 1 - e^{-a} \int_0^x e^{-t}\ I_0(2\sqrt{at})\ dt$$

for x = 10, 25, 50, 75, 100 and a = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0, 2.0, 3.0, 4.0, 5.0, 8.0, 10.0. This problem is being analyzed for possible IBM computation.

EVALUATION OF A FUNCTION

Origin. M. Tobias, Reactor Experimental Engineering Division.

Participating Members of Panel. J. H. Fishel, C. Perhacs.

Background and Status. The expression to be evaluated was

$$\gamma = \frac{3.196 + 0.001824\beta + A\dfrac{\beta}{F^2}}{Bp - 641.294} .$$

A and B are dependent upon temperature. B, β/F^2, and p were tabulated by Tobias. The expression was evaluated for 224 combinations of temperature and β. This problem was completed.

SECRET PROJECTS

DEPIGMENTATION AS A BIOLOGICAL DOSIMETER – OPERATION GREENHOUSE

Origin. A. C. Upton, Biology Division.

Participating Members of Panel. J. Moshman, K. P. Graw.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1290.

Background and Status. A discriminant function of the form

$$y = \sum_{i=1}^{6} \lambda_i P_i$$

was set up for the purpose of distinguishing between dose stations, if possible, on the basis of the amount of greying at each of six anatomical sites. The calculated values of λ and their standard errors are

SITE	λ	±S.E. (λ)
Back of head	85.83	9.20
Back of neck	36.53	10.26
Fore shoulders	-10.57	8.37
Abdomen	1.26	9.90
Upper back	-1.80	8.53
Lower back	12.80	8.11

The head and neck having values of $|\lambda| > 3$ S.E. (λ) are significant in distinguishing among dose groups. Further calculations are planned to determine the effectiveness of the discriminant function of p_1 and p_2 in distinguishing among dose stations.

MACHINE COMPUTATION OF SOLUTIONS FOR THE NEUTRON TRANSPORT EQUATION

Origin. W. K. Ergen, ANP Project; A. S. Householder, Mathematics Panel.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1290.

Participating Member of Panel. N. Edmonson.

Background and Status. This study is almost to the point that an actual, bare, spherical reactor can be coded for the ORAC.

DIFFUSION OF XENON-135 IN ANP FLUORIDE FUELS

Origin. M. T. Robinson, Solid State Division, ANP Project.

Participating Members of Panel. N. Edmonson, J. Z. Hearon.

Background and Status. The behavior of the Xe^{135} formed in the fissioning of the fused fluoride fuels proposed for the Aircraft Reactor is of the greatest importance with reference to the poisoning effect of Xe^{135} on the reactor operation. It is desired that the Mathematics Panel make a mathematical investigation of the diffusion of Xe^{135} out of the liquid fuel into the gaseous phase. This problem is being checked for errors in formulation. It is proposed to code the problem for ORAC.