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R. K. Osborn
M. E. Rose

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1-3 γ-CORRELATION WITH MIXTURES AND POLARIZATION

R. K. Osborn and M. E. Rose

The purpose of this report is to exhibit explicitly the modifications induced in the γ-γ angular correlation function by mixed intermediate radiations; i.e., to compare the 1-3 correlation with mixed, unobserved intermediate radiation with the usual 1-2 correlation.

The processes under consideration herein consist of:

(a) A nuclear transition from state $(j_o\ m_o)$ to state $(j_1\ m_1)$ accompanied by the emission of a γ-ray described by propagation vector $\underline{k}_1$ and polarization vector $\underline{\epsilon}_1$;

(b) A nuclear transition $(j_1\ m_1) \longrightarrow (j_2\ m_2)$ accompanied by emission of a γ-ray $\underline{k}_2$, $\underline{\epsilon}_2$; and

(c) Nuclear transition $(j_2\ m_2) \longrightarrow (j_3\ m_3)$ accompanied by emission of γ-ray $\underline{k}_3$, $\underline{\epsilon}_3$.

The correlation function may be written in the form

$$W(\gamma_1\gamma_2\gamma_3) = \sum_{m_1 m'_1 m_2 m'_2} E_{m_2 m'_2}(\gamma_3) \quad S_{m_2 m'_2,\ m_1 m'_1}(\gamma_2)\ F_{m'_1 m_1}(\gamma_1),$$

where

$$F_{m'_1 m_1}(\gamma_1) = \sum_{m_o} (j_1 m'_1 \mid H(\gamma_1) \mid j_o m_o)(j_1 m_1 \mid H(\gamma_1) \mid j_o m_o)^*$$

$$S_{m_2 m'_2, m_1 m'_1}(\gamma_2) = (j_2\ m_2 \mid H(\gamma_2) \mid j_1 m_1)^*\ (j_2 m'_2 \mid H(\gamma_2) \mid j_1 m'_1)$$

and

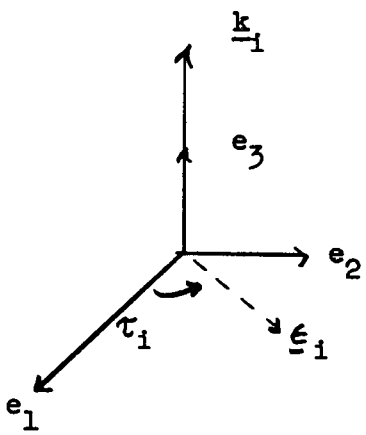
$$E_{m_2 m_2'}(\gamma_3) = \sum_{m_3} (j_3 m_3 \mid H(\gamma_3) \mid j_2 m_2)^* \, (j_3 m_3 \mid H(\gamma_3) \mid j_2 m_2')$$

are the density matrices for the transitions (a), (b) and (c) respectively, and

$$H(\gamma_i) = \underline{\epsilon}_i e^{i\underline{k}_i \cdot \underline{r}} \quad ;$$

$$\underline{\epsilon}_i^2 = 1,$$

$$\underline{\epsilon}_i \cdot \underline{k}_i = 0.$$



Choosing a set of base vectors $\{\underline{e}\}$ attached to the propagation vector $\underline{k}_i$ as shown in the figure, and defining a polarization angle as indicated; one may express

$$\underline{\epsilon}_i \, e^{i\underline{k}_i \cdot \underline{r}} = \frac{\pi}{\sqrt{2}} \sum_{L_i M_i P_i} e^{-iP_i \tau_i} \, (i)^{L_i} \sqrt{2L_i + 1} \; \mathcal{D}^{L_i}_{M_i P_i} (R_{e \to e'}) \left\{ \underline{A}^m (L_i M_i) + i P_i \, \underline{A}^e (L_i M_i) \right\} .^{(1)}$$

---

(1) L. C. Biedenharn, M. E. Rose, G. B. Arfken, ORNL 986; G. Goertzel, Phys. Rev. 70, 897 (1946).

Employing this expansion for the $H(\gamma_i)$ and the Wigner-Eckart theorem one may immediately exhibit the density matrices as: (dropping purely multiplicative factors)

$$F_{m_1'm_1}(\gamma_1) = \sum_{L_1P_1L_1'P_1'\nu_1} F_\nu(L_1L_1'P_1P_1'j_oj_1)\,(j_1\|L_1\|j_o)^*\,(j_1\|L_1'\|j_o)$$

$$(-1)^{L_1+L_1'-m_1'-\nu_1}\, C^{j_1j_1\nu_1}_{m_1',-m_1}\, \mathcal{D}^{\nu_1}_{m_1-m_1',-\mu_1}(R_{e_1\to e'})$$

$$S_{m_2m_2';m_1m_1'}(\gamma_2) = \sum_{L_2} (j_2\|L_2\|j_1)^*\,(j_2\|L_2\|j_1)$$

$$C^{j_2L_2j_1}_{m_2,m_1-m_2}\; C^{j_2L_2j_1}_{m_2',m_1'-m_2'}\; \delta_{m_1-m_2,m_1'-m_2'}$$

and

$$E_{m_2m_2'}(\gamma_3) = \sum_{L_3P_3L_3'P_3'\nu_3} F_\nu(L_3L_3'P_3P_3'j_3j_2)\,(j_3\|L_3\|j_2)^*$$

$$(j_3\|L_3'\|j_2)\,(-1)^{m_2}\, C^{j_2j_2\nu_3}_{m_2,-m_2'}\, \mathcal{D}^{\nu_3}_{m_2'-m_2,-\mu_3}(R_{e_3\to e'});$$

where

$$F(LL'PP'jj') = (-1)^{P+j-j'}\sqrt{\frac{(2j'+1)(2L'+1)(2L+1)}{2}}$$

$$(i)^{L'-L}\, e^{i\tau\mu}\, (iP')^{\delta(L')}\, (-iP)^{\delta(L)}\, W(j'j'LL',\nu j)\; C^{LL'\nu}_{-PP'} \quad ,$$

the $(j_F \| L \| j_I)$ are the reduced matrix elements,

$$\mu_i = P_i - P_i', \quad (P_i = \pm 1)$$

C's and W's are Clebsch-Gordan and Racah coefficients respectively in a standard notation[2],

$$\mathcal{J}(L) = 1 \text{ for electric radiation}$$
$$= 0 \text{ for magnetic radiation,}$$

and $\mathcal{D}^{\nu}_{MM'}(R_{e_i \to e'})$ is the $(2\nu + 1)$ - dimension irreducible representation of the rotation that takes the $\underline{k}_i$ (e_i) coordinate system into the quantization system, $\{e\}$.

It is perhaps worth commenting that S takes the particularily simple form exhibited herein as a consequence of averaging over directions and polarization of γ_2 and because it represents nuclear transitions between energetically degenerate states of sharp (jm).

The magnetic quantum number sums in the expression for $W(\gamma_1\gamma_2\gamma_3)$ may now be performed, yielding

$$W(\gamma_1\gamma_2\gamma_3) = \sum_{L_3L_3'P_3P_3'} \sum_{L_1L_1'P_1P_1'} \sum_{\nu} \sum_{L_2} \Gamma_{\nu}(j_1j_2L_2)$$

$$F_{\nu}(L_3L_3'P_3P_3'j_3j_2)\,(j_3 \| L_3 \| j_2)^*\,(j_3 \| L_3' \| j_2)$$

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[2] L. C. Biedenharn and M. E. Rose, Theory of Angular Correlation of Nuclear Radiations, (to be published in Reviews of Mod. Phys.).

$$F_\nu\,(L_1L_1'P_1P_1'j_oj_1)\;(j_1\|L_1\|j_o)^*\;(j_1\|L_1'\|j_o)$$

$$(-1)^{L_1+L_1'-\mu_1}\;\mathcal{D}^{\nu}_{\mu_1,-\mu_3}\;(R_1^{-1}\,R_3);$$

where

$$\Gamma_\nu\,(j_1j_2L_2) = (j_2\|L_2\|j_1)^*\;(j_2\|L_2\|j_1)\;(-1)^{j_1+j_2-L_2}$$

$$W(j_2j_2j_1j_1,\,\nu\;L_2).$$

The sum over L_2 is an incoherent sum over the intermediate radiation mixture.

We wish to compare this with the correlation function, $W(\gamma_1\;0\;\gamma_3)$, for the case of no intermediate radiation. To specialize from the general case, we note that if there is no intermediate radiation,

$$\Gamma_\nu\,(j_1j_2L_2) \rightarrow (-1)^{j_1+j_2-L_2}\;W(j_2j_2j_1j_1;\nu L_2)\;\delta_{L_20}$$

$$= (-1)^{j_1+j_2-0+j_1+j_2-\nu}\;\frac{\delta_{j_1\,j_2}}{\sqrt{(2j_1+1)(2j_2+1)}}$$

$$= (-1)^{-\nu}\;\delta_{j_1j_2}$$ to within a normalizing factor. Hence

$$W(\gamma_1\;0\;\gamma_3) = \sum_{L_3L_3'P_3P_3'}\;\sum_{L_1L_1'P_1P_1'}\;\sum_{\nu}\;F_\nu\,(L_3L_3'P_3P_3'j_3j_2)$$

$$(j_3\|L_3\|j_2)^*\;(j_3\|L_3'\|j_2)\;F_\nu\,(L_1L_1'P_1P_1'j_oj_2)\;(j_2\|L_1\|j_o)^*$$

$$(j_2\|L_1'\|j_o)\;(-1)^{L_1+L_1'-\mu_1-\nu}\;\mathcal{D}^{\nu}_{\mu_1,-\mu_3}\;(R_1^{-1}\,R_3).$$

We may now, without practical loss of generality, specialize to the case in which the first radiation is pure and unpolarized; i.e.,

$$W(L_1\gamma_2\gamma_3) = \left\langle W(\gamma_1\gamma_2\gamma_3)\right\rangle_{\tau_1} \delta_{L_1'L_1}\, \delta_{L_1L_1} \; .$$

Now

$$\left\langle F_\nu\,(L_1L_1'P_1P_1'j_oj_1)\right\rangle_{\tau_1} \delta_{L_1L_1}\, \delta_{L_1'L_1}$$

$$= F_\nu\,(L_1j_oj_1)\, \delta_{\mu_1 0}\, \delta_{L_1L_1}\, \delta_{L_1'L_1}$$

where

$$F_\nu\,(L_1j_oj_1) = (-1)^{j_o-j_1-1} \sqrt{2j_1+1}\;(2L_1+1)\, C^{L_1L_1\nu}_{1\;-1}\; W(j_1j_1L_1L_1;\nu j_o)$$

for ν even,

$= 0$ for ν odd.

Thus we now obtain:

$$W(L_1\gamma_2\gamma_3) = \sum_{L_3L_3'P_3P_3'} \sum_\nu \sum_{L_2} \Gamma_\nu(j_1j_2L_2)\, F_\nu\,(L_3L_3'P_3P_3'j_3j_2)$$

$$(j_3 \| L_3 \| j_2)^*\,(j_3 \| L_3' \| j_2)\, F_\nu\,(L_1j_oj_1)\; \mathcal{D}^\nu_{0,-\mu_3}\;(R_1^{-1}\,R_3),$$

and

$$W(L_1\;0\;\gamma_3) = \sum_{L_3L_3'P_3P_3'} \sum_\nu F_\nu(L_3L_3'P_3P_3'j_3j_2)$$

$$(j_3 \| L_3 \| j_2)^*\,(j_3 \| L_3' \| j_2)\, F_\nu\,(L_1j_oj_2)\; \mathcal{D}^\nu_{0,-\mu_3}\;(R_1^{-1}\,R_3).$$

We note finally that

$$\sum_{P_3 P_3'} F_\nu(L_3 L_3' P_3 P_3' j_3 j_2)\, \mathcal{D}^{\nu}_{0,P_3'-P_3}(R_1^{-1} R_3)$$

$$= F_\nu(L_3 L_3' j_3 j_2)\ \mathcal{P}_\nu(L_3 L_3' \theta \tau_3)$$

where

$$F_\nu(L_3 L_3' j_3 j_2) = (-1)^{j_3 - j_2 - 1} \sqrt{(2j_2+1)(2L_3+1)(2L_3'+1)}$$

$$W(j_2 j_2 L_3 L_3';\ \nu\ j_3)\ C^{L_3 L_3' \nu}_{1\ -1} \qquad \text{and}$$

$$\mathcal{P}_\nu(L_3 L_3' \theta \tau_3) = (-1)^{\delta(L_3')}\ g(L_3 L_3' 0)\ \ P_\nu(\theta)$$

$$+ \frac{C^{L_3 L_3' \nu}_{1\ 1}}{C^{L_3 L_3'}_{1\ -1}}\ g(L_3 L_3' \tau_3) \sqrt{\frac{(\nu-2)!}{(\nu+2)!}}\ \ P^2_\nu(\theta)$$

$$g(L_3 L_3' \tau_3) = \frac{(i)^{L_3' - L_3}}{2} \Big[(-1)^{L_3' - L_3} (-i)^{\delta(L_3')} (-i)^{\delta(L_3)} e^{2i\tau_3}$$

$$+ (i)^{\delta(L_3')} (i)^{\delta(L_3)} e^{-2i\tau_3} \Big] .$$

We thus write finally:

$$W(L_1 \gamma_2 \gamma_3) = \sum_{L_3 L_3' \nu} \sum_{L_2} \Gamma_\nu(j_1 j_2 L_2)\, F_\nu(L_1 j_0 j_1)$$

$$(j_3 \| L_3 \| j_2)^* (j_3 \| L_3' \| j_2)\, F_\nu(L_3 L_3' j_3 j_2)\ \ \mathcal{P}_\nu(L_3 L_3' \theta \tau_3)$$

and

$$W(L_1 0 \gamma_3) = \sum_{L_3 L_3' \nu} F_\nu (L_1 j_0 j_2) (j_3 \| L_3 \| j_2)^* (j_3 \| L_3' \| j_2)$$

$$F_\nu (L_3 L_3' j_3 j_2) P_\nu (L_3 L_3' \theta \tau_3).$$

We note that for an (M_{L_3}, E_{L_3+1}) mixture;

$$\mathcal{P}_\nu (L_3 L_3' \theta \tau_3) = P_\nu (\theta) + (-1)^{\delta (L_3')} \frac{C_{1\,1}^{L_3 L_3' \nu}}{C_{1\,-1}^{L_3 L_3' \nu}} \sqrt{\frac{(\nu - 2)!}{(\nu + 2)!}} \cos 2\tau_3 \, P_\nu^2(\theta);$$

whereas for an (M_{L_3+1}, E_{L_3}) mixture, $\mathcal{P}_\nu$ is the difference instead of the sum of these two terms.

We see that the only effect on the correlation function of the presence of an intermediate radiation is to introduce the factor $\Gamma_\nu (j_1 j_2 L_2)$ which is completely independent of the character of the initial and final transitions. In order to exhibit the consequences of this fact most explicity we note that

$$\langle W(L_1 0 \gamma_3) \rangle_{\theta \tau_3} = | (j_3 \| L_3 \| j_2) |^2 + | (j_3 \| L_3' \| j_2) |^2 ;$$

since

$$F_0(L_1 j_0 j_2) = F_0(L_3 L_3 j_3 j_2) = F_0(L_3' L_3' j_3 j_2) = 1$$

and

$$F_o(L_3L_3'j_3j_2) = 0 \text{ for } L_3 \neq L_3'.$$

Thus if we define the intensity ratio

$$= |(j_3 \| L_3 \| j_2)|^2 / |(j_3 \| L_3' \| j_2)|^2 = \delta^2$$

we may then exhibit the mixed correlation function with no intermediate radiation as

$$W(L_1 0 \gamma_3) = \omega_I + \delta^2 \omega_{II} + 2\delta\, \omega_{III} ,$$

where

$$\omega_I = \sum_\nu F_\nu (L_1 j_o j_2)\, F_\nu (L_3 L_3 j_3 j_2)\, \mathcal{P}_\nu (L_3 L_3 \theta \tau_3)$$

$$\omega_{II} = \sum_\nu F_\nu (L_1 j_o j_2)\, F_\nu (L_3' L_3' j_3 j_2)\, \mathcal{P}_\nu (L_3' L_3' \theta \tau_3)$$

and

$$\omega_{III} = \sum_\nu F_\nu (L_1 j_o j_2)\, F_\nu (L_3 L_3' j_3 j_2)\, \mathcal{P}_\nu (L_3 L_3' \theta \tau_3).$$

Now, in this notation, the correlation function for mixed final and intermediate radiations may be expressed as

$$W(L_1 \gamma_2 \gamma_3) = \sum_\nu \left[\Gamma_\nu (j_1 j_2 L_2) + \Gamma_\nu (j_1 j_2 L_2') \right] N(j_1 j_2)$$

$$\times \Big[F_\nu (L_1 j_o j_1)\, F_\nu (L_3 L_3 j_3 j_2)\, \mathcal{P}_\nu (L_3 L_3 \theta \tau_3)$$

$$+ \delta^2 F_\nu (L_1 j_o j_1)\, F_\nu (L_3' L_3' j_3 j_2)\, \mathcal{P}_\nu (L_3' L_3' \theta \tau_3)$$

$$+ 2\delta\, F_\nu (L_1 j_o j_1)\, F_\nu (L_3 L_3' j_3 j_2)\, \mathcal{P}_\nu (L_3 L_3' \theta \tau_3) \Big] ,$$

where $N(j_1 j_2)$ is a normalization factor introduced to make

$$N(j_1 j_2)\ \Gamma_o(j_1 j_2 L_2) = |\ (j_2 \| L_2 \| j_1)|^2 \qquad ;\ \text{i.e.,}$$

$$N(j_1 j_2) = \sqrt{(2j_1 + 1)(2j_2 + 1)}\ .$$

Thus we see that $N(j_1 j_2)\ \Gamma_o(j_1 j_2 L_2)$ is just the intensity of the L_2-multipole in the intermediate radiation.