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SUPPLEMENTARY REMARKS ON ANGULAR CORRELATION

M. E. Rose

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SUPPLEMENTARY REMARKS ON ANGULAR CORRELATION

M. E. Rose

Introduction

The theory of angular correlation of nuclear radiations has been discussed rather completely in a recent review article.[1] However, there are a number of additional remarks which can be made and these are herewith presented in the hope that they may be useful in the interpretation of the experiments. The present discussion is confined to three questions: (a) direction-direction correlation in which charged particles (notably, conversion electrons) are emitted, (b) polarization-direction correlations in γ-γ cascades and (c) γ-γ cascades in which an intervening radiation is unobserved. In (a) and (b) especially, we are concerned with the question of interpretation of the experiments in which appreciable spin coupling or other attenuating influences may occur.

For a direction-direction correlation it has been demonstrated that the correlation function always takes the form

$$W = \sum_{\nu} A_{\nu} P_{\nu}(\cos \vartheta) \tag{1}$$

Here ϑ is the angle between the directions of the two radiations. If the cascade is the successive emission of two γ-rays, the coefficients A_{ν} are as defined in Eq's (69a and b) of reference 1. We continue to use this notation so that A_{ν} refers to successive γ-γ cascades only and for other radiations

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1. L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. July 1953 issue.

an explicit particle parameter is introduced. Thus, for a cascade with particles x and y emitted, the correlation is

$$W = \sum_{\nu} b_{\nu}(x)\, b_{\nu}(y)\, A_{\nu}\, P_{\nu}(\cos\vartheta). \qquad (2)$$

Departures from the ideal case envisaged in the theory may occur for a number of reasons:

(1) Spin-coupling, either magnetic or electric

(2) Geometry corrections due to finite solid angles of the detectors or finite source size

(3) Scattering of the observed particles

Omitting consideration of the finite source size, and restricting our attention to cases in which the detector geometry is axially symmetric for each detector, it is clear that all these distorting influences affect the correlation function only by introducing attenuation factors (Q_{ν} say). Thus, A_{ν} in (1) or (2) is replaced by $A_{\nu}\, Q_{\nu}$ where $0 < Q \leqslant 1$ ($Q_0 = 1$) and the attenuation factors can depend on only the following:

For spin-coupling Q_{ν} will depend on the intermediate nuclear angular momentum and corresponding electric and/or magnetic moments, the intermediate state lifetime, on ν of course, and otherwise only on properties of the environment (magnetic field and/or electric field gradient at the nucleus). For geometry corrections, besides ν, only the geometry parameters and the efficiency of the detectors will be involved. For scattering corrections Q will contain only the parameters of the scattering function, geometry of the detectors,[2] and ν.

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2. S. Frankel, Phys. Rev. 83, 673 (1951).

The attenuation due to geometry and, for charged particles, due to scattering are essentially trivial in that they can easily be taken into account. For the spin-coupling, this is not the case. However, in many cases the spin-coupling effects can be made unimportant by the use of dilute aqueous solutions and in a large number of cases the effect is small enough, in any case, because of a small intermediate state half-life so that no ambiguity is introduced so far as spin assignments to the pertinent nuclear states is concerned.

Of course, the use of liquid sources would introduce impossible scattering corrections for radiations other than γ-rays. For this reason, and also for cases in which one suspects that spin-coupling has not been sufficiently well eliminated, it is desirable to see what conclusions are independent of the precise values of A_γ.

Conversion electron-gamma correlation

We consider a cascade in which the conversion coefficient for the second transition, at least, has an intermediate value so that either the γ-ray or the electron can be observed. If the γ-e correlation is compared with the γ-γ correlation, then representing the two correlation functions as above, the ratio of coefficients of P_γ in the correlation function gives the particle parameter $b_\gamma(e)$. This is true, of course, by virtue of the fact that the first step of the two cascades is the same and, hence, the intermediate state is the same. Scattering corrections for the conversion electron would be necessary if the source is not thin but a liquid source is unnecessary as long

as measurable anisotropy exists in both cascades. The above implies the same geometry correction for the second γ-ray as for the conversion electron but, if this is not the case, the appropriate corrections are easily made. The b_γ (e) depend on Z, the transition energy, the shell from which the conversion electron is ejected and on the parity and multipolarity of the second transition. The last two items constitute the interesting information which such an experiment might provide.

An alternative procedure (suggested by S. Frankel) would involve a comparison of γ-e correlations for both K and L electrons. The ratio of corresponding coefficients then gives b_γ (K)/b_γ (L). To date no relativistic calculations of b_γ (L) are available. In the non-relativistic range one should recognize that $b_\gamma(K)/b_\gamma(L_I) = 1$ for electric multipoles.

The possibility of obtaining useful information from a measurement of the correlation between the conversion electron and the subsequent X-ray seems not too promising since only L_{III} electrons give an anisotropic correlation function.[3,4]

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3. R. K. Osborn and M. E. Rose, ORNL 1499.

4. As an incidental remark, we note that a similar situation occurs in the correlation between the X-ray following capture and a succeeding γ-ray. Isotropy occurs unless L_{III} capture is involved. The latter is highly improbable, of course.

Polarization-direction correlation in the γ-γ cascade

This problem was first considered by Hamilton[5] and the experiments were first carried out by Metzger and Deutsch[6]. We wish to pursue a line of reasoning considered by Hamilton. The conclusions to be reached are that for pure multipoles and for certain, frequently occurring, types of cascades one can get all the necessary information without any knowledge of the A_ν other than their sign. The sign, of course, is not affected by attenuation.

As shown in reference 1, the correlation function when the first γ-ray is detected by the polarization-sensitive detector is

$$W_1(\vartheta\varphi) = \sum_\nu B_\nu \, \mathcal{P}_\nu(\vartheta, \varphi) \tag{3}$$

where $B_\nu = Q_\nu A_\nu$ and

$$\mathcal{P}_\nu(\vartheta\varphi) = P_\nu(\cos\vartheta) + (-)^{\sigma_1} \alpha_\nu(L_1) \cos 2\varphi \, P^2_\nu(\cos\vartheta). \tag{4}$$

Here φ is the angle between the electric field and the normal to the plane of the two γ-rays (it is the complement of the angle designated by the same symbol in reference 6). Also $\sigma = 0$ for electric, $\sigma = 1$ for magnetic multipoles and the parity dependence is contained in $(-)^{\sigma_1}$ where the subscript refers to the first radiation. The designations "first" and "second" are made on a basis of energy discrimination. In (4)

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5. D. R. Hamilton, Phys. Rev. 74, 782 (1948).

6. F. Metzger and M. Deutsch, Phys. Rev. 78, 551 (1950).

$$\alpha_\nu(L) = \frac{(\nu - 2)!}{(\nu + 2)!} \; \frac{2\,\nu(\nu + 1)\,L(L + 1)}{\nu(\nu + 1) - 2L(L + 1)} \qquad 2 \qquad (5)$$

$$\alpha_o(L) = 0$$

Following reference 6 we introduce the ratio of polarization intensities

$$p = \frac{J_\parallel}{J_\perp} = \frac{W(\vartheta, \pi/2)}{W(\vartheta, 0)} \qquad (6)$$

One measures $N_\parallel / N_\perp$, the ratio of triple coincidences for the detector of the Compton scattered photons in the γ-γ plane to triple coincidences for the perpendicular position. Throughout, no account of geometry corrections due to the finite extent of the Compton scatters are made. This will reduce the value of p - 1 presumably but only the sign of p - 1 is of interest. As shown in reference 6

$$\frac{N_\parallel}{N_\perp} = 1 - \frac{(p - 1)(R - 1)}{p\,R + 1} \qquad (7)$$

where $\bar{R}$ is the ratio of differential Compton cross-sections for $\varphi = 0$ to that for $\varphi = \pi/2$, $(R > 1)$.

In the measurements what one will usually observe is

$$W(\vartheta\,\varphi) = \eta_{12}\, W_1(\vartheta\,\varphi) + \eta_{21}\, W_2(\vartheta\,\varphi)$$

where $W_2(\vartheta\,\varphi)$ is the correlation function for the second photon going to the polarization-sensitive detector. The overall efficiencies for the W_1 and W_2 arrangements are η_{12} and η_{21} respectively. To obtain W_2 one need only replace L_1 and σ_1 in $\mathcal{B}_\nu$ by L_2 and σ_2.

(I) Overall parity change ($\eta_{12} = \eta_{21}$)

If the efficiencies are equal and one observes no polarization effect ($p = 1$) one can draw a conclusion as follows: For the above to hold

$$(-)^{\sigma_1} \alpha_\nu (L_1) + (-)^{\sigma_2} \alpha_\nu (L_2) = 0 \tag{8}$$

for all pertinent ν . The <u>only</u> solutions to (8) are

(a) $L_1 = L_2, \quad \sigma_1 \neq \sigma_2.$

These are E_L M_L or M_L E_L cascades.

(b) $L_1 = 1, L_2 = 2$ or $L_1 = 2, L_2 = 1$ and $\sigma_1 = \sigma_2$

In detail these are the cascades E_1 E_2, E_2 E_1, M_1 M_2, M_2 M_1. This case is referred to as the D-Q cascade.

In both these cases, (a) and (b), there is an overall parity change, as Metzger and Deutsch pointed out. Thus, $p = 1$ with $\eta_{12} = \eta_{21}$ means $\Delta\pi$ = yes, no or no, yes. The procedure for distinguishing between these two is discussed below. Note that it does not follow that an observation of a polarization effect with equal efficiencies would imply no overall parity change. This conclusion is safe only for the DQ cascade.

(2) No overall parity change ($\eta_{12} = \eta_{21}$)

We consider the DQ case, but not necessarily with $\sigma_1 = \sigma_2$, and $\vartheta = \pi/2$ in the polarization-direction measurement. Then we find for the correlation function W_1

$$p = \left(\frac{1 + a}{1 - a}\right)^{\Delta\pi_1} \tag{9}$$

where a is the anisotropy,

$$a = \frac{\overline{W}(\pi) - \overline{W}(\pi/2)}{\overline{W}(\pi/2)} \tag{10}$$

where $\overline{W}$ is the correlation function for the direction-direction correlation. In (9) the exponent refers to the parity change of the first transition and $\Delta\pi = 1$ means parity change "no", $\Delta\pi = -1$ means parity change "yes". Correspondingly, (9) applies to W_2 when $\Delta\pi$ refers to the second transition. Therefore it applies to the correlation measurement with both radiations entering the polarization sensitive detector if $\Delta\pi$ is the same for both transitions. We have the following possibilities

$\Delta\pi = 1$	$\Delta\pi = -1$
$E_2\ E_2$	$E_1\ E_1$
$M_1\ M_1$	$M_2\ M_2$
$E_2\ M_1$, $M_1\ E_2$	$M_2\ E_1$, $M_1\ E_2$

The possible results of both the direction-direction and polarization-direction correlation measurements are then listed in Table I.

Table I

p - 1	a	$\Delta\pi$	
+	+	no	no
-	-		
+	-	yes	yes
-	+		

Thus the sign of the product a(p - 1) distinguishes between the "no - no" and "yes - yes" cases.

(3) Overall parity change ($\eta_{12} \neq \eta_{21}$)

In order to identify the individual parity changes when there is an overall parity change it is necessary to consider different efficiencies. Then we consider cases (a) and (b) above.

(a) $L_1 = L_2 = L, \quad \sigma_1 \neq \sigma_2$

Then

$$p - 1 \approx \frac{2(\eta_{21} - \eta_{12})}{\eta_{21} + \eta_{12}} (-)^{\sigma_1} \frac{\sum_{\nu \geq 2} \alpha_\nu B_\nu P_\nu^2(\cos\vartheta)}{\bar{W}(\vartheta)} \tag{11}$$

An approximation based on the assumption $\eta_{21} - \eta_{12} \ll \eta_{12} + \eta_{21}$ has been made but even without this approximation the sign of p - 1, which is essential here, comes out the same as above.

The direction-direction correlation fixes the coefficients B_ν and hence the sign of the sum term appearing on the right hand side of (11). Then the measured sign of p - 1 and the known sign of $\eta_{21} - \eta_{12}$ gives $(-)^{\sigma_1}$ and hence the parity change of the first transition ($\Delta\pi = (-)^{L+\sigma}$). The parity change of the second transition is, of course, the opposite. That one is actually concerned with case (a) is to be determined on the basis of a previous measurement with $\eta_{12} = \eta_{21}$.

(b) $L_1, L_2 = 1, 2$ or $2, 1, \quad \sigma_1 = \sigma_2 = \sigma$.

A direction-direction correlation would presumably fix the L's and a polarization-direction correlation with $\eta_{12} = \eta_{21}$ resulting in p = 1 would mean that case (b) applies. Then

$$p - 1 \approx \frac{2(\eta_{21} - \eta_{12})}{\eta_{21} + \eta_{12}} (-)^{\sigma} \frac{\alpha_2(L_1)\, B_2\, P_2^2(\cos\vartheta)}{\overline{W}(\vartheta)} \tag{12}$$

Note that $P_2^2 = 3 \sin^2 \vartheta > 0$, $\alpha_2(1) = 1/2$, $\alpha_2(2) = -1/2$.

The measurement of the anisotropy gives B_2:

$$B_2 = \frac{2a}{3 + a} \tag{13}$$

From the known sign of $\eta_{21} - \eta_{12}$ and the measured sign of $p - 1$ one obtains $(-)^{\sigma}$ when L_1 is known. If $(-)^{\sigma} = 1$ the cascade is $E_2\, E_1$ or $E_1\, E_2$. For $(-)^{\sigma} = -1$ the cascade is $M_2\, M_1$ or $M_1\, M_2$.

In greater detail assume $B_2 > 0$. Then Table II list the possibilities. We assume $\eta_{21} - \eta_{12} > 0$ so that the sign of $p - 1$ is the same as the sign of $(-)^{\sigma}\, \alpha_2(L_1)$.

Table II

$p - 1$	$(-)^{\sigma}\, \alpha_2(L_1)$	Possibilities	$\Delta \pi_1$	$\Delta \pi_2$
+	+	$E_1\, E_2$, $M_2\, M_1$	yes	no
-	-	$E_2\, E_1$, $M_1\, M_2$	no	yes

The case $B_2 > 0$ applies to the cascades $j \leftrightarrow j \leftrightarrow j + 2$. The cases $j \pm 1 \leftrightarrow j \leftrightarrow j \pm 2$ (signs incoherent) corresponds to $B_2 < 0$. In this case Table II is changed only by interchanging the entries + and - under $p - 1$.

Thus, one can identify the parity change occurring in each transition in all D-Q cases and also in case (a), see under (1). In the case of mixed multipoles no simple relations exist. However, the fact that the attenuation factors are the same as for the direction-direction correlation implies that parity determinations can be made in these cases as well even though the attenuation cannot be predicted on theoretical grounds.

Correlation with intervening radiation unobserved.

The correlation function for γ-γ cascades with pure multipoles was given in reference 1, Eq. (141). For mixtures one can easily generalize this result.

Consider that only the unobserved radiation is mixed. We designate the correlation function for pure multipoles by $W(L_1)$ where L_1 is the multipolarity index of the unobserved radiation. For the case in which this is a mixture of multipoles with multipolarity indices L_1 and L_1' the correlation function is simply

$$W = \beta_1^2 W(L_1) + \beta_1'^2 W(L_1') \tag{14}$$

where $\beta_1^2/\beta_1'^2$ is the intensity ratio of the 2^{L_1} to $2^{L_1'}$ pole. There is no interference term simply because the mixed radiation is not observed.

When one of the observed radiations is mixed the result is also easily generalized. We find, for example, when the first radiation is a 2^{L_o} and $2^{L_o'}$ mixture, that

$$W = \beta_o^2 W(L_o\, L_o;\, L_1,\, L_2) + \beta_o'^2 W(L_o'\, L_o';\, L_1,\, L_2)$$

$$+ 2\, \beta_o\, \beta_o'\, W(L_o\, L_o';\, L_1\, L_2)$$

$\beta_o^2/\beta_o'^2$ is again the intensity ratio and $W(L_o\ L_o,\ L_1\ L_2)$ is given in Eq. (141). To get $W(L_o'\ L_o';\ L_1\ L_2)$ replace L_o by L_o'. Finally $W(L_o\ L_o';\ L_1\ L_2)$ is obtained by making the same replacement of $F_\nu(L_o\ j_o\ j_1)$ by $(-)^{j_1-j_o-1}\sqrt{(2j_1+1)(2L_o+1)(2L_o'+1)}$ $G_\nu(L_o\ L_o'\ \ j_o\ j_1)$ as is done for the consecutive cascade. In other words, the effect of an intervening non-observed radiation is to introduce a factor which is the same for mixed or pure transitions preceding it.

The case in which the third radiation is mixed is obvious and needs no further comment.