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INTERACTION EFFECTS IN β-DECAY

M. E. Rose

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INTERACTION EFFECTS IN β-DECAY

M. E. Rose

The term interaction effects has been used by Sachs and Austern[1] in connection with the problem of γ-ray emission (or absorption) to describe the situation wherein the interaction energy between source and field depends explicitly on the nuclear forces. It has been shown[2] that if one treats the problem phenomenologically, starting with an essentially non-relativistic hamiltonian, then both electric and magnetic multipole transitions are subject to interaction effects.

The problem now arises as to whether the same situation occurs in the case of emission of β-particles. At first sight it may appear that the problem is artificial in this case. If one writes down a hamiltonian for nucleons coupled to an electron-neutrino field, a term $\mathcal{H}$ occurs representing the β-interaction. This term, which is actually the energy density corresponding to the β-coupling, appears as an explicit operator which apparently has nothing to do with the nuclear forces, although nuclear force operators may appear (phenomenologically) in other terms, in the zero order hamiltonian in fact. However, the starting point here is a relativistic hamiltonian and this manner of posing the problem obscures the difficulty. The assumption throughout is that a many-particle Dirac hamiltonian does describe the nucleus.

The difficulty lies in the fact that one is obliged to use relativistic nuclear wave functions. Now it is certainly true that a major difficulty impeding progress in nuclear physics is the lack of good wave functions. The only way to arrive at a representation of a nuclear wave function would appear to be the empirical approach and this practically necessitates non-relativistic wave functions. If such wave functions were available, or could be guessed, a reduction of the β-decay problem to the non-relativistic limit in the nucleon space would permit one to make somewhat more realistic estimates of nuclear matrix elements than it is presently possible to do. But it is just this process of non-relativistic reduction that will give rise to the interaction effects. The nuclear forces are contained in the wave functions and when one attempts to eliminate the subtleties in these wave functions, as represented by their small components, the nuclear forces appear explicitly in the β-interaction.

Even with sensible non-relativistic wave functions one must recognize that the small components of the relativistic wave function are needed, not only for the odd operators of the β-decay interaction but also for even operators if one wishes to account for such phenomena as ℓ-forbiddenness. The problem which presents itself is: given the non-relativistic zero order wave functions, how are the so-called small components to be obtained? This problem is answered, to a certain degree of approximation, in the course of our consideration of the interaction effect.

It should be mentioned that the possibility of interaction effects in β-decay has already received some attention in the literature. Thus, Peaslee[3] has considered the effect of pseudoscalar coupling as it affects the pseudoscalar interaction in β-decay. Our considerations are somewhat more general. The results we obtain should be equivalent, to a certain order of approximation, with those obtained by Chraplyvy[4] who used the Foldy-Wouthuysen canonical transformation to obtain the non-relativistic limit of the equations of motion. We use the alternative method of large components inasmuch as this seems to provide an opportunity to examine the small component-large component relationship. The results will be explicitly exhibited to first order in the reciprocal nucleon mass although higher order terms can be obtained readily enough. To this order, as will be seen, there are no interaction effects, but they do enter in the next order of the expansion. The question as to whether these interaction effects can be of importance (in the RaE spectrum, for example) is now being studied.

It is sufficient to consider a two-nucleon system since the generalization to an arbitrary number of nucleons will then be trivial. For the moment, spinor suffixes will appear as subscripts and particle labels (1 and 2) as arguments. Then for two nucleons, masses M(1) and M(2), the equation of motion will be taken as

$$i\dot{\psi} = \left\{ -\sum_j \alpha(j)\cdot p(j) + \beta(j)\, M(j) + V\omega(1)\,\omega(2) + U\Omega(1)\,\Omega(2) + \mathcal{H} \right\}\psi \tag{1}$$

Here the nuclear forces are represented by $V(r)\,\omega(1)\,\omega(2)$ and $U(r)\,\Omega(1)\,\Omega(2)$, where r is the distance between nucleons. The splitting of the nuclear interaction into two terms is made so as to distinguish between direct products of odd and of even operators. The fact that the force operators must be symmetrical in the two particles is taken advantage of. Then, in each space,

$$\omega = \begin{pmatrix} 0 & \omega_{12} \\ \omega_{21} & 0 \end{pmatrix} \qquad \Omega = \begin{pmatrix} \Omega_{11} & 0 \\ 0 & \Omega_{22} \end{pmatrix} \tag{2}$$

and each entry such as ω_{ij}, Ω_{ii} is a two by two matrix. Thus $\omega(1)\,\omega(2)$, for example is a 16 x 16 matrix. All single particle operators in (1) are 16 x 16 matrices composed of a direct product of a 4 x 4 Dirac operator in one space and a unit matrix in the other. Thus, for any operator $\Lambda = \lambda(1)\,\lambda'(2)$

$$(\Lambda\psi)_{\mu_1\mu_2} = \sum_{\nu_1\nu_2} \lambda_{\mu_1\nu_1}(1)\,\lambda'_{\mu_2\nu_2}(2)\,\psi_{\nu_1\nu_2} \tag{3}$$

and ψ is a 16 component spinor.

In (1) $\mathcal{H}$ is the β-coupling and the explicit form of this operator is

$$\mathcal{H} = \sum_j \begin{pmatrix} \mathcal{E}_{11}(j) & 0 \\ 0 & \mathcal{E}_{22}(j) \end{pmatrix} + \begin{pmatrix} 0 & \mathcal{O}_{12}(j) \\ \mathcal{O}_{21}(j) & 0 \end{pmatrix} = \sum_j \begin{pmatrix} \mathcal{H}_{11} & \mathcal{H}_{12} \\ \mathcal{H}_{21} & \mathcal{H}_{22} \end{pmatrix} \tag{4}$$

where

$$\mathcal{E}_{11} = g_S\,L(\beta) + g_V\,L(1) + g_T\,\sigma\cdot L(\beta\sigma) + g_A\,\sigma\cdot L(\sigma) \tag{4a}$$

$$\mathcal{E}_{22} = - g_S L(\beta) + g_V L(1) - g_T \sigma \cdot L(\beta\sigma) + g_A \sigma \cdot L(\sigma) \tag{4b}$$

$$\mathcal{O}_{12} = - g_V \sigma \cdot L(\alpha) + g_T \sigma \cdot L(\beta\alpha) + g_A L(\gamma_5) - g_P L(\beta\gamma_5) \tag{4c}$$

$$\mathcal{O}_{21} = - g_V \sigma \cdot L(\alpha) - g_T \sigma \cdot L(\beta\alpha) + g_A L(\gamma_5) + g_P L(\beta\gamma_5) \tag{4d}$$

Here L(x) is a <u>time</u> <u>dependent</u> lepton covariant. The time dependence is

$$\frac{\partial}{\partial t} L(x) = i\, W_o\, L(x)$$

where W_o is the maximum energy of the β-spectrum.

Of course every Dirac operator is a direct product of a Pauli σ or a 2 x 2 unit matrix with the Dirac matrices ρ_1 or ρ_3

$$\rho_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \rho_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Thus

$$\alpha = \rho_1 \sigma \quad , \qquad \beta = \rho_3 1$$

Therefore, one can just as well work with a four component ψ function with components ψ_{11}, ψ_{12}, ψ_{21}, ψ_{22} and each of these is a four component spinor. ψ_{11} may be called the small-small component, ψ_{12} the small-large, ψ_{21} the large-small and ψ_{22} the large-large component. It is correct to say that in the non-relativistic limit where $p/M(j) \ll 1$ and also $U/M \ll 1$, $V/M \ll 1$, the wave function reduces to ψ_{22}. But it is apparently not necessarily true that ψ_{11} is much smaller than ψ_{12} or ψ_{21}. The

statement that this is the case is presumably based on consideration of a wave equation like (1) with V = 0. As the results given below demonstrate, when $V \neq 0$, $\psi_{11} \sim \psi_{12} \sim \psi_{21} \ll \psi_{22}$. Unless $V \ll p$ and no reason for this is apparent, the nomenclature "small-small" appears to be misleading.

We shift the zero of energy by introducing

$$\psi = e^{-iMt}\chi$$

where M = M(1) + M(2) is the total mass. Then $i\dot{\chi}$, for stationary state, has the value $E\chi$ where E is the total energy exclusive of rest energy. Using the fact that $\alpha_{12} = \alpha_{21} = \sigma$ and $\beta_{11} = -\beta_{22} = 1$ we find that the equation of motion becomes

$$i\dot{\chi}_{22} = -\sigma(1)\cdot p(1)\chi_{12} - \sigma(2)\cdot p(2)\chi_{21} + \sum_{\nu_1} \mathcal{H}_{2\nu_1}(1)\,\chi_{\nu_1 2}$$

$$+ \sum_{\nu_2} \mathcal{H}_{2\nu_2}(2)\chi_{2\nu_2} + V\,\omega_{21}(1)\,\omega_{21}(2)\,\chi_{11} + U\,\Omega_{22}(1)\,\Omega_{22}(2)\,\chi_{22}. \tag{5}$$

and the remaining equations may be written as

$$\xi_{11}\chi_{11} + \xi_{12}\chi_{12} + \xi_{13}\chi_{21} = \eta_1\chi_{22} \tag{6}$$

$$\xi_{21}\chi_{11} + \xi_{22}\chi_{12} + \xi_{23}\chi_{21} = \eta_2\chi_{22} \tag{7}$$

$$\xi_{31}\chi_{11} + \xi_{32}\chi_{22} + \xi_{33}\chi_{21} = \eta_3\chi_{22} \tag{8}$$

where

$$\eta_1 = V\omega_{12}(1)\omega_{12}(2)$$

$$\eta_2 = -\sigma(1)\cdot p(1) + \mathcal{H}_{12}(1)$$

$$\eta_3 = -\sigma(2)\cdot p(2) + \mathcal{H}_{12}(2) \tag{8a}$$

$$\xi_{11} = 2M - U\Omega_{11}(1)\Omega_{11}(2) - \sum_i \mathcal{H}_{11}(i) + i\frac{\partial}{\partial t}$$

$$\xi_{22} = 2M(1) - U\Omega_{11}(1)\Omega_{22}(2) - \mathcal{H}_{11}(1) - \mathcal{H}_{22}(2) + i\frac{\partial}{\partial t}$$

$$\xi_{33} = 2M(2) - U\Omega_{22}(1)\Omega_{11}(2) - \mathcal{H}_{22}(1) - \mathcal{H}_{11}(2) + i\frac{\partial}{\partial t} \tag{8b}$$

$$\xi_{12} = -\eta_3, \quad \xi_{21} = \sigma(2)\cdot p(2) - \mathcal{H}_{21}(2)$$

$$\xi_{13} = -\eta_2, \quad \xi_{31} = \sigma(1)\cdot p(1) - \mathcal{H}_{21}(1) \tag{8c}$$

$$\xi_{23} = -V\omega_{12}(1)\omega_{21}(2), \quad \xi_{32} = -V\omega_{21}(1)\omega_{12}(2)$$

To lowest order the solutions of (8) are

$$\chi_{11} = \xi_{11}^{-1}\eta_1\chi_{22}$$

$$\chi_{12} = \xi_{22}^{-1}\eta_2\chi_{22} \tag{9a}$$

$$\chi_{21} = \xi_{33}^{-1}\eta_3\chi_{22}$$

since $\xi_{ii} \gg \xi_{ij}$ $(i \neq j)$ by virtue of the dominance of the rest mass terms.

In next order we obtain

$$\chi_{11} = \xi_{11}^{-1}(\eta_1 - \xi_{12}\xi_{22}^{-1}\eta_2 - \xi_{13}\xi_{33}^{-1}\eta_3)\chi_{22}$$
$$\chi_{12} = \xi_{22}^{-1}(\eta_2 - \xi_{23}\xi_{33}^{-1}\eta_3 - \xi_{21}\xi_{11}^{-1}\eta_1)\chi_{22} \qquad (9b)$$
$$\chi_{21} = \xi_{33}^{-1}(\eta_3 - \xi_{31}\xi_{11}^{-1}\eta_1 - \xi_{32}\xi_{22}^{-1}\eta_2)\chi_{22}$$

The equation of motion for χ_{22} is now obtained by substituting these results in (5). Then one finds

$$i\dot{\chi}_{22} = (H^{(o)} + H^{(1)} + H^{(2)} + \ldots)\chi_{22} \qquad (10)$$

where $H^{(n)}$ is defined as that part of the hamiltonian which explicitly contains $1/M^n$ (where, for classification purposes $1/M(1)$ and $1/M(2)$ are counted as $1/M$ terms). Then

$$H^{(o)} = U\Omega_{22}(1)\Omega_{22}(2) + \sum_j \mathcal{H}_{22}(j) \qquad (11)$$

To obtain higher terms we note that

$$(A + B)^{-1} = \sum_{n=0}^{\infty} (-)^n A^{-1} (BA^{-1})^n \qquad (12)$$

and in ξ_{ii} we identify A with the rest mass term. Then $H^{(1)}$ is obtained by taking the first term of (12), $(A + B)^{-1} \approx A^{-1}$, and using (9a) instead of (9b). In this way one finds

$$H^{(1)} = \sum_i \frac{p_i^2}{2M_i} - \sum_i \frac{1}{2M_i} \left[\sigma_i \cdot p_i, \mathcal{H}'(i)\right]_+$$

$$+ \frac{1}{2M} V\omega_{21}(1)\omega_{21}(2)\, V\omega_{12}(1)\omega_{12}(2) \qquad (13)$$

where for convenience the particle label on σ, p and M are written as subscripts. In (13) $\mathcal{H}'$ is the odd part of the β interaction and the operator involved is the anti-commutator. This gives exactly the same results for the β-interaction as is given by the Foldy-Wouthuysen transformation[5] to first order in 1/M.

Eq. (13) shows that there are no first order (1/M) interaction effects on the β-coupling. The even part of the nuclear forces enters in $H^{(o)}$ and the odd part in $H^{(1)}$. The last term is given by Chraplyvy[4] as $\frac{1}{2M}\left[V\omega(1)\omega(2)\right]^2$. The form given in (13) is somewhat more general. For pseudoscalar coupling $\omega_{12} = -\omega_{21} = -1$ and the last term would be

$$\frac{1}{2} \sum_{i>j} \frac{1}{M_i + M_j} V_{ij}^2$$

for an arbitrary number of nucleons. More generally it is

$$\frac{1}{2} \sum_{i>j} \frac{1}{M_i + M_j} V(r_{ij})\omega_{21}(i)\omega_{21}(j)\, V(r_{ij})\omega_{12}(i)\omega_{12}(j) \qquad (14)$$

The term $H^{(2)}$ will contain interaction effects. It is obtained using (9b) in (5) with the second term of (12) used in ξ_{ii}^{-1} and the first term in the $\xi_{ii}^{-1}\,\xi_{jj}^{-1}$ products which appear in (9b). Here one uses

$$i\frac{\partial}{\partial t}\mathcal{H} = -W_o\mathcal{H} + \mathcal{H}\, i\frac{\partial}{\partial t} \cong -W_o\mathcal{H} + \mathcal{H}H_o^{(o)}$$

$$i\frac{\partial}{\partial t}\sigma\cdot p \cong \sigma\cdot p\, H^{(o)}$$

The interaction dependent term can be written most simply by introducing

$$A_1 = -\sigma_1 \cdot p_1 + \mathcal{H}'(1)$$

$$A_2 = -\sigma_2 \cdot p_2 + \mathcal{H}'(2)$$

$$\bar{\mathcal{E}} = U\Omega(1)\Omega(2) + \mathcal{H}''(1) + \mathcal{H}''(2)$$

$$\bar{\Theta} = V\omega(1)\omega(2)$$

where again

$$\mathcal{H}' = \begin{pmatrix} 0 & \sigma_{12} \\ \sigma_{21} & 0 \end{pmatrix}$$

and

$$\mathcal{H}'' = \begin{pmatrix} \mathcal{E}_{11} & 0 \\ 0 & \mathcal{E}_{22} \end{pmatrix}$$

Then the interaction dependent term is

$$\mathcal{H}_\beta^{(2)} = \frac{1}{8M_1M_2}\left\{ \left[\left[A_1, \bar{\Theta}\right]_+, A_2\right]_+ + \left[\left[A_2, \bar{\Theta}\right]_+, A_1\right]_+ \right\}$$

$$+ \frac{1}{8M_1^2}\left[\left[A_1, \bar{\mathcal{E}}\right], A_1\right] + \frac{1}{8M_2^2}\left[\left[A_2, \bar{\mathcal{E}}\right], A_2\right] \qquad (15)$$

and all terms quadratic in $\mathcal{H}'$, $\mathcal{H}''$ or bilinear in these two operators should be dropped for single β-emission. It is seen that both the even-even as well as odd-odd nuclear interactions are involved in this part of the β-coupling. It is to be emphasized that $H_\beta^{(2)}$ does not represent the entire contribution

to the β-coupling in second order. There are other terms as well which are interaction independent.

The importance of the interaction terms depends on an estimate of V and/or U. From (11) and (13) one might expect that since U and $V^2/2M$ represent essentially the nuclear force interaction in non-relativistic approximation, these should be of order $p^2/2M$ (the kinetic energy). Then $V \sim p$ and a typical term of (15) (arising from V) is of order $p\ V\ \mathcal{H}/M^2 \sim (p^2/M^2)\mathcal{H}$. A term arising from U is of order $U\ p\ \mathcal{H}/M^2 \sim (p^3/M^3)\mathcal{H}$. These terms should therefore be negligible. However, a closer examination of them may be necessary. The ordering of the terms in the hamiltonian, as indicated by (10), may not in every case represent a separation into contributions of distinctly different order of magnitude.

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