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1. NON-UNIQUE FORBIDDEN SPECTRA

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PHYSICS DIVISION

NUMERICAL ANALYSIS OF BETA SPECTRA:

I. NON-UNIQUE FORBIDDEN SPECTRA

M. E. Rose

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NUMERICAL ANALYSIS OF BETA SPECTRA: I. NON-UNIQUE FORBIDDEN SPECTRA

M. E. Rose

In the analysis of all beta spectra wherein the nuclear matrix elements enter only as a multiplicative factor, that is, allowed and spin change $\Delta j = n + 1$, parity change $\Delta \pi = (-)^n$ transitions, it is possible to construct a Kurie-Fermi plot. This is true when the interaction is a mixture as well as for pure interactions.[1] When the transitions does not conform to the above, a simple graphical representation of the data does not seem feasible. For $\Delta j = n$, $\Delta \pi = (-)^n$ one has the following transitions which are presumably second forbidden: Cl^{36}, Tc^{99}, Tl^{204} and possibly others. The first forbidden transitions seem to have the allowed shape, with the possible exception of RaE, and we therefore consider at least second forbidden transitions. The discussion given below is devoted to the question of analyzing such spectra.

When the experimental data are converted to the form of an observed correction factor by dividing the counting rate by the calculated spectrum for allowed transitions, one requires that the resulting data be fitted to a function of the form

$$G = p_1^2 \varphi_1(W) + p_2^2 \varphi_2(W) + p_1 p_2 \varphi_3(W) \tag{1}$$

where φ_1, φ_2 and φ_3 are given functions of the beta-particle energy W, and incidentally of Z and W_o, the endpoint energy. The latter is assumed known. The measurements are taken at discrete points W_i and we write $\varphi_\nu(W_i) = \varphi_\nu(i)$. The $\varphi_\nu(i)$ can be taken as accurately known and the error in $G_i = G(W_i)$ is

1. We are here assuming the validity of the Fierz conditions: no appreciable mixture of S and V and of T and A interactions.

statistical. While only one constant $S = p_2/p_1$ is involved so far as the shape of $G(W)$ is concerned, it is not desirable to divide out p_1^2, say, since this treats the data preferentially.

The $(\text{real})^2$ constants p_1, p_2 would be values of nuclear matrix elements if a pure interaction were assumed. Otherwise, they would be products of nuclear matrix elements and mixing constants. Alternatively, the matrix elements could be calculated on the basis of some model and then p_1, p_2 are mixing constants.[2] It is also possible that more than three terms are involved in the correction factor but attention is restricted to the simpler case. The extension to a more complex case will be evident.

For convenience we rewrite (1) in the form

$$G_i = a_1 \psi_1(i) + a_2 \psi_2(i) + a_3 \psi_3(i) \tag{2}$$

where $\psi_3 = \varphi_3/\sqrt{2}$, $\psi_1 = \varphi_1$, $\psi_2 = \varphi_2$, $a_1 = p_1^2$, $a_2 = p_2^2$, $a_3 = \sqrt{2}\ p_1 p_2$

and

$$g \equiv \frac{1}{2} a_3^2 - a_1 a_2 = 0 \tag{2a}$$

Then the fitting process, which is done by least squares, gives the normal equations

$$\sum_i \psi_n(i) \left[a_1 \psi_1(i) + a_2 \psi_2(i) + a_3 \psi_3(i) - G_i \right] - \lambda b_n = 0 \tag{3}$$

for $n = 1, 2, 3$ and

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 \\ a_1 \\ -a_3 \end{pmatrix} \tag{3a}$$

---

2. The phases can always be chosen so that the nuclear matrix elements are real. However, even without a special choice of phases the product $p_1 p_2$ is always real and p_1^2, p_2^2 are the square moduli of matrix elements. The fact that the mixing constants can be taken real has been demonstrated by L. C. Biedenharn and M. E. Rose, Phys. Rev. 83, 459 (1951).

Here λ is the Lagrange multiplier. The non-linearity of the problem is eliminated at this stage.

Introducing the symmetric matrix C whose lements are

$$C_{\ell m} = \sum_i \psi_\ell(i)\ \psi_m(i) \tag{4}$$

and the matrix M where

$$M = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{5}$$

which is also symmetric, one has

$$(C - \lambda M)\vec{a} = \vec{z} \tag{6}$$

where the vector $\vec{z}$ is given by

$$z_n = \sum_i G_i\ \psi_n(i) \tag{7}$$

The given results of observation fix $\vec{z}$, C. Then

$$\vec{a} = (C - \lambda M)^{-1}\ \vec{z} \tag{8}$$

where

$$(C - \lambda M)^{-1}_{\ell m} = \frac{K_{m\ell}}{|C - \lambda M|} \tag{9}$$

and $K_{\ell m}$ is the cofactor of $C_{\ell m} - \lambda M_{\ell m}$ in the determinant $|C - \lambda M|$. Note that $K_{\ell m} = K_{m\ell}$.

The mean square deviation in a_n is given by the diagonal elements of $(C - \lambda M)^{-1}$:

$$\overline{(a_n - \bar{a}_n)^2} = (C - \lambda M)^{-1}_{nn} = \frac{K_{nn}}{|C - \lambda M|} \tag{10}$$

The as yet undertermined Lagrange multiplier λ is given as a root of the equation resulting from (2a). This gives a quartic in λ (see Eq. 11 below) and the roots (which can be found numerically) are conditioned by the requirement that, when substituted back in (8), the resulting values of a_1 and a_2 are both positive.[3] This may, of course, give no solutions (in which case the original parity, spin change assumptions may be at fault) or more than one solution. Obviously λ must be real. It is clear, of course, that one is also interested in the value of a_3 since the sign of a_3 gives the sign of the ratio p_2/p_1.

For the determination of λ we have

$$\sum_{j,\ell=1}^{3} (K_{j1} K_{\ell 2} + K_{j2} K_{\ell 1} - K_{j3} K_{\ell 3}) z_j z_\ell = 0 \tag{11}$$

and

$$K_{11} = C_{22} C_{33} - C_{23}^2 + \lambda C_{22}$$

$$K_{22} = C_{11} C_{33} - C_{13}^2 + \lambda C_{11}$$

$$K_{33} = C_{11} C_{22} - (C_{12} - \lambda)^2$$

$$K_{12} = C_{13} C_{23} - (C_{12} - \lambda)(C_{33} + \lambda) = K_{21}$$

$$K_{23} = C_{13} (C_{12} - \lambda) - C_{11} C_{23} = K_{32}$$

---

3. This procedure is an alternative to the one described by A. M. Smith, Phil. Mag. Ser. 7, xliii, 915 (1952). A least squares fit without using Lagrange multipliers is also possible but gives simultaneous cubics in p_1 and p_2. Note that no difficulty should arise from a near singularity resulting from approximate linear dependence of the functions ψ_n. This would make $|C|$ small but not $|C - \lambda M|$.

$$K_{13} = C_{23}\,(C_{12} - \lambda) - C_{13}\,C_{22} = K_{31}$$

Also, for the a_ν (Eq's. (8) and (9)) one needs

$$|C - \lambda M| = |C| + \lambda \left[C_{11}\,C_{22} - C_{12}^2 + 2\,(C_{12}\,C_{33} - C_{23}\,C_{13})\right]$$

$$+ \lambda^2\,(2\,C_{12} - C_{33}) - \lambda^3.$$