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INTERNAL CONVERSION - X-RAY ANGULAR CORRELATIONS

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M. E. Rose

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INTERNAL CONVERSION - X-RAY ANGULAR CORRELATIONS

R. K. Osborn and M. E. Rose

Usually, the theory and application of angular correlations is concerned primarily with the observation of two or more radiations (γ-rays, internal conversion electrons, etc.) emitted in cascade by a given nucleus. Thus in order for an angular correlation to be unambiguous, it is in general necessary that the magnetic perturbations of the intermediate nuclear states be sufficiently weak in order that when the nucleus emits the second radiation it has not "forgotten" the state it was left in by the first radiation. In other words, the lifetime of the state should be short compared to the Larmor frequency of the intermediate nucleus in a particular magnetic substate subject to perturbing magnetic fields. Thus it is evident that, because of the difficulty of eliminating these perturbing magnetic fields in certain cases, the observation and interpretation of an angular correlation may not be feasible.

For this reason, it was felt to be worthwhile to investigate the possibility of an observable angular correlation between internal conversion electrons and the x-ray emitted subsequently by the electronic system as the atom returns to the ground state. One advantage, at least, of such a correlation, if it were in principle observationally feasible, would be the elimination of the difficulty referred to above.

The correlation function, neglecting scale factors, may be written as

$$W(\underline{x}\ \underline{e}) = \sum_{\mu_1 \mu_2 M M_1} \left| (\mathcal{H}_1 \mu_1 | H_x | \mathcal{H}_2 \mu_2)(I M \mathcal{H} \mu | H_e | I_1 M_1 \mathcal{H}_1 \mu_1) \right|^2 .$$

Electronic wave functions are here exhibited in the representation which diagonalizes the operator $\beta(\sigma \cdot L + 1)$ as well as J^2, and J_z; i.e.

$$J^2 \psi_{\varkappa\mu} = (|\varkappa| - \tfrac{1}{2})(|\varkappa| + \tfrac{1}{2}) \psi_{\varkappa\mu}$$

$$J_z \psi_{\varkappa\mu} = \mu \psi_{\varkappa\mu}$$

and $\beta(\sigma \cdot L + 1) \psi_{\varkappa\mu} = - \varkappa \psi_{\varkappa\mu}$.[1] The nuclear wave functions are in the usual angular momentum representation in which the operators I^2 and I_z, the total nuclear angular momentum and z-component respectively, are diagonal; i.e.,

$$I^2 \psi_{IM} = I(I + 1) \psi_{IM}$$

$$I_z \psi_{IM} = M \psi_{IM} .$$

The matrix element $(IM\varkappa\mu | H_e | I_1M_1\varkappa_1\mu_1)$ represents a nuclear and electronic transition from initial state $(I_1M_1\varkappa_1\mu_1)$ to the final state $(IM\varkappa\mu)$ induced by the interaction between the electrons and the nucleus. The matrix element $(\varkappa_1\mu_1 | H_x | \varkappa_2\mu_2)$ represents an electronic transition from initial state $(\varkappa_2\mu_2)$ to $(\varkappa_1\mu_1)$ accompanied by the emission of the observed x-ray. It is to be noted that there is no coherent sum involved in W as a consequence of the fact that the transitions are not coupled via the intermediate states, but rather through the fact that the initial state of the internally converted electron is also the final state of the electron emitting the x-ray.

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1. M. E. Rose, L. C. Biedenharn, and G. B. Arfken, Phys. Rev. 85, 5 (1952).

Thus it is not to be expected that the form of the correlation function for this process should bear any simple relation to that for angular correlation of internal conversion electrons and subsequent nuclear γ-rays.

If we define the density matrices

$$E_{\mu_1\mu_1} = \sum_{\mu_2} (\mathcal{H}_1\mu_1 | H_x | \mathcal{H}_2\mu_2)^* (\mathcal{H}_1\mu_1 | H_x | \mathcal{H}_2\mu_2)$$

$$F_{MM}(\mu_1) = \sum_{M_1} (IM\mathcal{H}\mu | H_e | I_1M_1\mathcal{H}_1\mu_1)^* (IM\mathcal{H}\mu | H_e | I_1M_1\mathcal{H}_1\mu_1)$$

it is clear that the correlation function assumes the form

$$W(\underline{x}\ \underline{e}) = \sum_{\mu_1 M} E_{\mu_1\mu_1}(\underline{x})\, F_{MM}(\underline{e},\mu_1).$$

In the light of the above remarks, this is to be contrasted with the correlation function for internal conversion electron-nuclear γ-ray,

$$W(\underline{\gamma}\ \underline{e}) = \sum_{mm'} E_{mm'}(\underline{\gamma})\, F_{m'm}(\underline{e}),$$

where $E_{mm'}(\underline{\gamma})$, the density matrix for the nuclear transition producing the observed γ-ray, is independent of the atomic state of the system.

The methods whereby the explicit angular dependence of W is extracted from the density matrices have been thoroughly discussed elsewhere.[2]

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2. L. C. Biedenharn and M. E. Rose, Theory of Angular Correlation of Nuclear Radiations, to be published in Rev. Mod. Phys.

Choosing the quantization axis in the direction of the x-ray, we obtain for the density matrix for the electronic transition,

$$E_{\mu_1\mu_1}(\underline{x}) = -\sum_{\nu}\left[1+(-1)^{\nu}\right](-1)^{2j_2+j_1-\mu_1-L_x}$$

$$\times\ C^{j_1 j_1 \nu}_{-\mu_1,\mu_1}\ C^{L_x L_x \nu}_{1,-1}\ W(j_1 j_2 \nu\ L_x;\ L_x j_1)$$

$$\times\ (\mathcal{H}_1 \| L_x \| \mathcal{H}_2)^*(\mathcal{H}_1 \| L_x \| \mathcal{H}_2).$$

The C's and W are the Clebsch-Gordan and Racah vector addition coefficients, L_x is the angular momentum carried off by the x-ray; and the $(\| L_x \|)$ are the "reduced" matrix elements.

For purposes of simplification we consider here only pure magnetic 2^L pole transitions in the internal conversion process. For this case we obtain

$$F_{MM}(\mu_1) = \sum_{M_1 \mathcal{H}\mathcal{H}'\nu} (-1)^{\nu+\frac{1}{2}-M+M_1-\mu_1+\mathcal{H}+\mathcal{H}'}\ \Gamma(\mathcal{H}\mathcal{H}'\ell\ell'\ell_1 j_1 \nu)$$

$$\times\ C^{I_1 L_e I}_{M_1,M-M_1}\ C^{I_1 L_e I}_{M_1,M-M_1}\ C^{jj'\nu}_{M-M_1+\mu_1,M_1-M-\mu_1}\ C^{\ell\ell'\nu}_{oo}\ C^{L_e j_1 j'}_{M-M_1,\mu_1}$$

$$\times\ C^{L_e j_1 j}_{M-M_1,\mu_1}\ C^{L_e \ell_1 \ell'}_{oo}\ C^{L_e \ell_1 \ell}_{oo}\ W(\ell\ell' jj';\nu\tfrac{1}{2})$$

$$\times\ W(L_e \ell_1 j'\tfrac{1}{2};\ell' j_1)\ W(L_e \ell_1 j\tfrac{1}{2};\ell j_1)\ P_{\nu}(\theta),$$

where

$$\Gamma(\varkappa\varkappa'\ell\ell'\ell_1 j_1 \nu) = -i\, e^{i(\Delta_\varkappa - \Delta_{\varkappa'})}\, S(\varkappa)$$

$$\exp\left[\frac{\pi i}{2}\left\{\ell(-\varkappa') - \ell(\varkappa)\right\}\right]\left[(2\ell+1)(2\ell'+1)|\varkappa\varkappa'|/\pi(2\nu+1)\right]^{\frac{1}{2}}$$

$$\times\ (\varkappa + \varkappa_1)(\varkappa' + \varkappa_1)(R_3' + R_4')^*_{\varkappa'}\,(R_3' + R_4')_{\varkappa}$$

$$\times\ \frac{(2L_e+1)(2\ell_1+1)(2j_1+1)}{4\pi}$$

and θ is the angle between the conversion electron and the quantization axis. This expression for $F_{MM}(\mu_1)$ is obtainable directly from the work of Rose, Biedenharn, and Arfken[1] by combining their Eqs. (1), (25), and (26) and generalizing their Eq. (41) to arbitrary initial states. The notation employed herein is the same as in their paper.

Returning now to the correlation function under consideration here,

$$W(\underline{x}\ \underline{e}) = \sum_{\mu_1 M} E_{\mu_1\mu_1}\ F_{MM}(\mu_1)$$

and carrying out the sums over μ_1, M and M_1, we obtain finally:

$$W(\underline{x}\ \underline{e};\theta) = \sum_{\varkappa\varkappa'\nu} (-1)^{j_2+j_1+j_1+\frac{1}{2}+\varkappa+\varkappa'-L_e}\left[1 + (-1)^{\nu}\right]$$

$$\times\ \sqrt{(2j+1)(2j'+1)}\ (\varkappa_1 \| L_x \| \varkappa_2)^*\,(\varkappa_1 \| L_x \| \varkappa_2)\,\Gamma(\varkappa\varkappa'\ell\ell'\ell_1 j_1 \nu)$$

$$\times\ C^{L_x L_x \nu}_{1,-1}\ C^{\ell\ell'\nu}_{oo}\ C^{L_e \ell_1 \ell'}_{oo}\ C^{L_e \ell_1 \ell}_{oo}\ \Delta(I_1 L_e I)$$

$$\times\ W(j_1 j_1 L_x L_x;\nu j_2)\ W(j_1 j_1 jj';\nu L_e)\ W(\ell\ell' jj';\nu\tfrac{1}{2})$$

$$\times\ W(L_e \ell_1 j'\tfrac{1}{2};\ell' j_1)\ W(L_e \ell_1 j\tfrac{1}{2};\ell j_1)\ P_\nu(\theta),$$

where $\Delta(I_1\ L_e\ I) = 0$ unless I_1, L_e and I form a triangle, in which case it is unity.

It is to be noted that only those terms for which ν is even contribute to the correlation function, and hence for anisotropy to exist at all, $\nu \geqslant 2$. An examination of the Racah coefficients reveals that the permissible range of values for ν is from 0 to $2j_1$. Hence for anisotropy we conclude that

$$2j_1 \geqslant 2,$$

$$j_1 \geqslant 1,$$

and therefore $j_1 = 3/2$ is the lowest j-value for the initial state of the internal conversion electron which will make a contribution. Thus neither K-shell nor L_I and L_{II}-shell conversions will contribute to the anisotropy, though both will contribute to the isotropic background against which the presumably somewhat weaker L_{III}-shell-conversion contribution to the anisotropy is to be observed. It does not appear probable therefore that the internal conversion-x-ray angular correlation will prove to be a useful tool in the attempt to circumvent the difficulties associated with the perturbation of intermediate nuclear states.