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TRACK WIDTH OF A HEAVY CHARGED PARTICLE

Jacob Neufeld and Walter S. Snyder

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TRACK WIDTH OF A HEAVY CHARGED PARTICLE

Jacob Neufeld and Walter S. Snyder

The calculation of the energy dissipation of a heavy charged particle moving through tissue as a function of its velocity was presented in ORNL-1083.[1] In this report, the energy loss as a function of distance from the track of the particle is studied, and the energy loss density, as well as the cumulative distribution of these losses, is calculated. The curves presented serve to give a fairly detailed picture of the intensity of the disturbance due to the passage of the charged particle and may serve to determine an effective track thickness beyond which distance from the track the energy dissipation may be neglected. It will be seen from the curves that 99% of the energy is dissipated within a distance of about 5×10^{-7} cm of the track. This distance is of the same order of magnitude as the dimensions of certain viruses.

According to the method outlined in the appendix of ORNL-884,[2] each orbital electron in the stopping atom of the surrounding medium is to be considered as a harmonic oscillator. Then,

$$(1) \quad \left[-\frac{\Delta E}{\Delta z}\right]_r = \frac{2Z_1^2\omega_r^2 e^4}{mV^4}\left[K_0^2\left(\frac{\omega_r x}{V}\right) + K_1^2\left(\frac{\omega_r x}{V}\right)\right],$$

where $[-\Delta E/\Delta z]_r$ (in ergs/cm) is the energy loss occasioned by an electron having index r positioned at distance x from the track of the particle and having an "angular velocity" ω_r; Δz is the increment of the length of the path of the particle; and $Z_1 e$ and V are the charge and velocity of the particle.[3] The assumption is made that $\omega_r = I_r/\hbar$, where I_r is the energy in ergs required to remove the electron from the atom. (I_r is evaluated as in ref. 4.) K_0 and K_1 are modified Bessel functions of the second kind of orders 0 and 1, respectively.

For computation, the velocity has been taken in units of $e^2/\hbar$ by putting $w = \hbar V/e^2$. Thus Eq. 1 becomes

$$(2) \quad \left[-\frac{\Delta E}{\Delta z}\right]_r = \frac{h^2 I_r^2 Z_1^2}{2\pi^2 m e^4 w^4}\left[K_0^2\left(\frac{I_r x}{e^2 w}\right) + K_1^2\left(\frac{I_r x}{e^2 w}\right)\right].$$

Formula 2 is summed over the electrons contained in the atom, and formula 3 represents the "lateral distribution" with respect to x of the energy loss per atom due to resonance effects:

$$(3) \quad \left[-\frac{\Delta E}{\Delta z}\right]_{atom} = \sum_{r=1}^{Z_2}\left[-\frac{\Delta E}{\Delta z}\right]_r,$$

where Z_2 is the atomic number of the stopping atom.

The cumulative distribution of formula 3 is given by

$$(4) \quad C_{atom}(x) = \int_x^\infty \left[-\frac{\Delta E}{\Delta z}\right]_{atom} 2\pi x\,dx = \sum_{r=1}^{Z_2} \frac{h^2 I_r Z_1^2 x}{\pi m e^2 w^3} K_0\left(\frac{I_r x}{e^2 w}\right) K_1\left(\frac{I_r x}{e^2 w}\right)$$

and represents the energy dissipated in the medium at distances from the track exceeding x, with the medium normalized to one atom/cm^3. Formulas 2 and 4 apply for $x > R$, where R is

(1) W. S. Snyder and J. Neufeld, *On the Energy Dissipation of Moving Ions in Tissue*, ORNL-1083 (Nov. 14, 1951).

(2) J. Neufeld, *Ionization and Excitation Losses of Charged Particles of Intermediate Energies*, ORNL-884 (Dec. 14, 1950).

(3) *Ibid.*, p. 13, Eq. 18.

(4) *Ibid.*, p. 7, Eq. 5.

"the radius" of the outermost orbit of the atom.

The remaining contributions to the energy losses are due to "free collisions" and are assumed to occur in the immediate neighborhood of the track, that is, for $x < R$. This contribution can be expressed as

$$(5) \quad C_{\text{atom}}(0) - C_{\text{atom}}(R) = \frac{2\pi\, n(w)\, Z_1^2 \hbar^2}{m w^2} \ln \frac{2 m w^2 e^4}{\hbar^2 I_m},$$

where

$$I_m = \frac{1}{n(w)} \sum_{r=1}^{n(w)} \ln I_r$$

and $n(w)$ is the number of orbital electrons in the stopping atom that have velocities smaller than w.[5] The "radius" of the outermost orbit is given by $R = \hbar/mu_1$, where u_1 is the "orbital velocity" of the outermost electron in the atom.[6]

Formula 3 has been computed for protons ($Z_1 = 1$) of various velocities moving in hydrogen, carbon, nitrogen, and oxygen, and the results are plotted in Figs. 1, 2, and 3. A simple change of scale indicated on each of these graphs makes it possible to read them for values of w which differ by a factor of 10. These graphs are then combined to give the lateral distribution of the energy loss in tissue. The composition of the tissue used is given in Table 1, and the graph is shown in Fig. 4. Each of these graphs has physical significance only for values of x greater than the radius of the outermost electronic orbit, but to allow for change of scales, they have been, in some cases, continued beyond this point.

Formula 4 has been computed for w = 0.5, 1, 5, 10, 20, and 50. Transformations giving the cumulative energy loss for other values of w apply to formula 4 as to formula 3. However, no such simple transformation applies to formula 5. Figures 5 through 10 show the cumulative curve of energy loss for protons in hydrogen, carbon, nitrogen, oxygen, and tissue, and the curve includes the energy losses due to free collisions, as well as the resonance effects. Thus the ordinate at $x = 0$ represents the total energy loss per centimeter of track. The values used for the radius of the outermost orbit, as well as the energy losses due to "free collisions," have been tabulated in Table 2.

The dimensions of most viruses are of the order of 10^{-6} cm and for bacteria and cells they are generally much larger. It is apparent from Figs. 5 through 10 that in a distance of 5×10^{-7} cm from the track, 99% of the energy loss has occurred. This distance is of the order of the smallest dimensions of some of the viruses listed (see, for example, ref. 7).

Clearly, any distance which includes 90% or more of the energy losses will be of the same order as the dimensions of some of the smaller viruses, though for many cells, in particular red blood cells, the 99% distance is smaller by a factor of about 10^{-3}.

(5) *Ibid.*, p. 9.

(6) N. Bohr, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd.* **18**, 82, Eq. 3.3.2 (1948).

(7) F. C. Kelly and K. E. Hite, *Microbiology*, p. 102, Appleton-Century-Crofts, New York, 1949.

TABLE 1. COMPOSITION OF TISSUE

ELEMENT	AMOUNT OF ELEMENT IN TISSUE (wt %)	NO. OF ATOMS OF ELEMENT IN TISSUE
Hydrogen	10.11	6.045×10^{22}
Carbon	12.14	6.049×10^{21}
Nitrogen	4.02	1.766×10^{21}
Oxygen	73.64	2.752×10^{22}
Total	99.91	9.759×10^{22}

TABLE 2. VALUES USED FOR RADIUS OF OUTERMOST ELECTRON AND ENERGY LOSS DUE TO FREE COLLISIONS

ELEMENT	RADIUS OF OUTERMOST ELECTRON	ENERGY LOSS DUE TO FREE COLLISIONS (Mev·cm²/atom)					
		$w = 0.5$	$w = 1$	$w = 5$	$w = 10$	$w = 20$	$w = 50$
Hydrogen	0.531×10^{-8}	0	1.46×10^{-20}	1.20×10^{-21}	3.66×10^{-22}	1.08×10^{-22}	2.08×10^{-23}
Carbon	1.08×10^{-8}	0	1.84×10^{-20}	4.67×10^{-21}	2.10×10^{-21}	6.24×10^{-22}	1.21×10^{-22}
Nitrogen	1.12×10^{-8}	4.80×10^{-20}	1.86×10^{-20}	5.67×10^{-21}	2.38×10^{-21}	7.11×10^{-22}	1.38×10^{-22}
Oxygen	1.14×10^{-8}	4.88×10^{-20}	1.88×10^{-20}	5.70×10^{-21}	2.71×10^{-21}	8.09×10^{-22}	1.58×10^{-22}

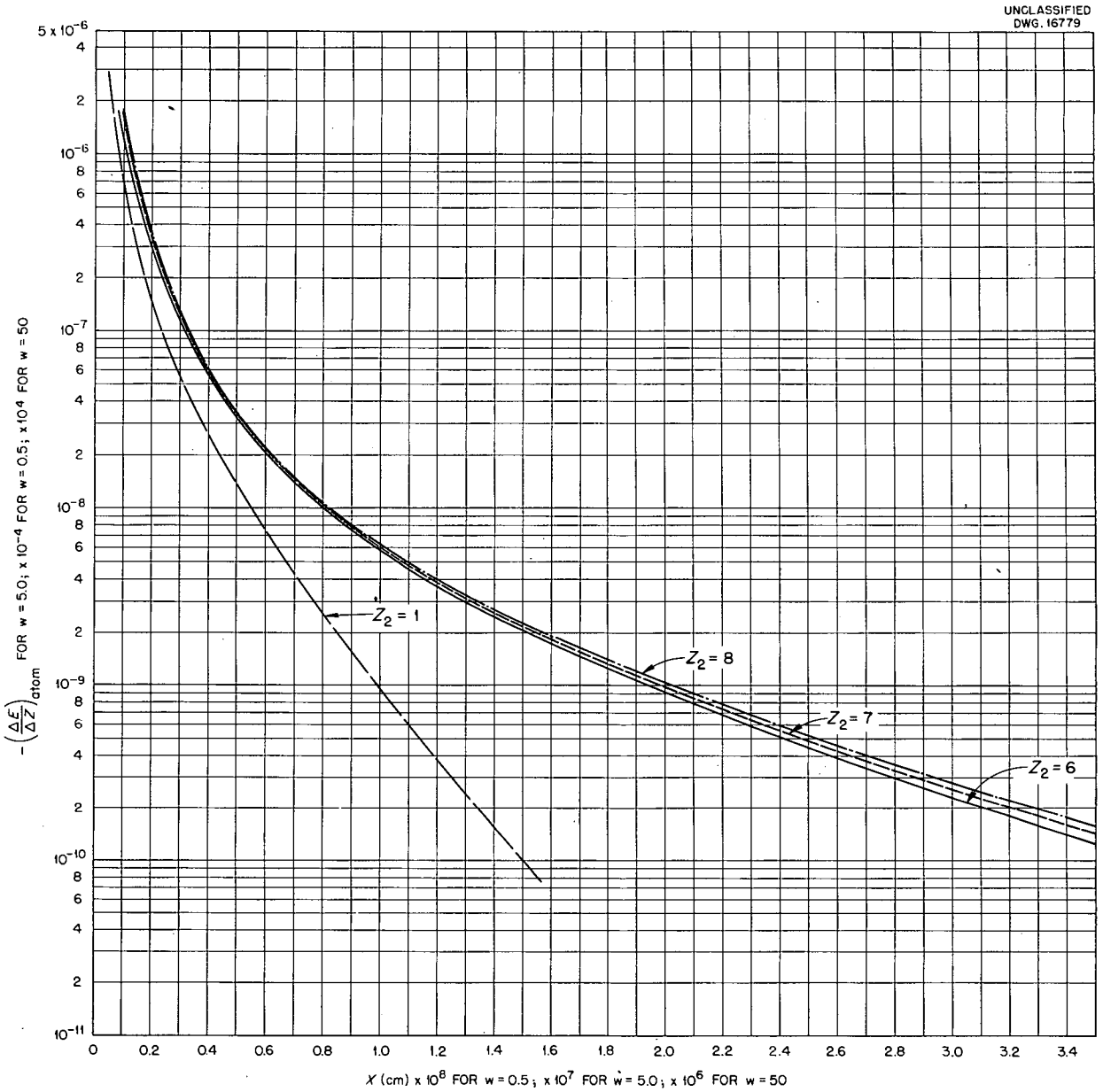

Fig. 1. Energy Loss Density $-(\Delta E/\Delta z)_{atom}$ Mev/atom for w = 0.5, 5, 50 (Proton Energy 0.00625, 0.625, 62.5 Mev).

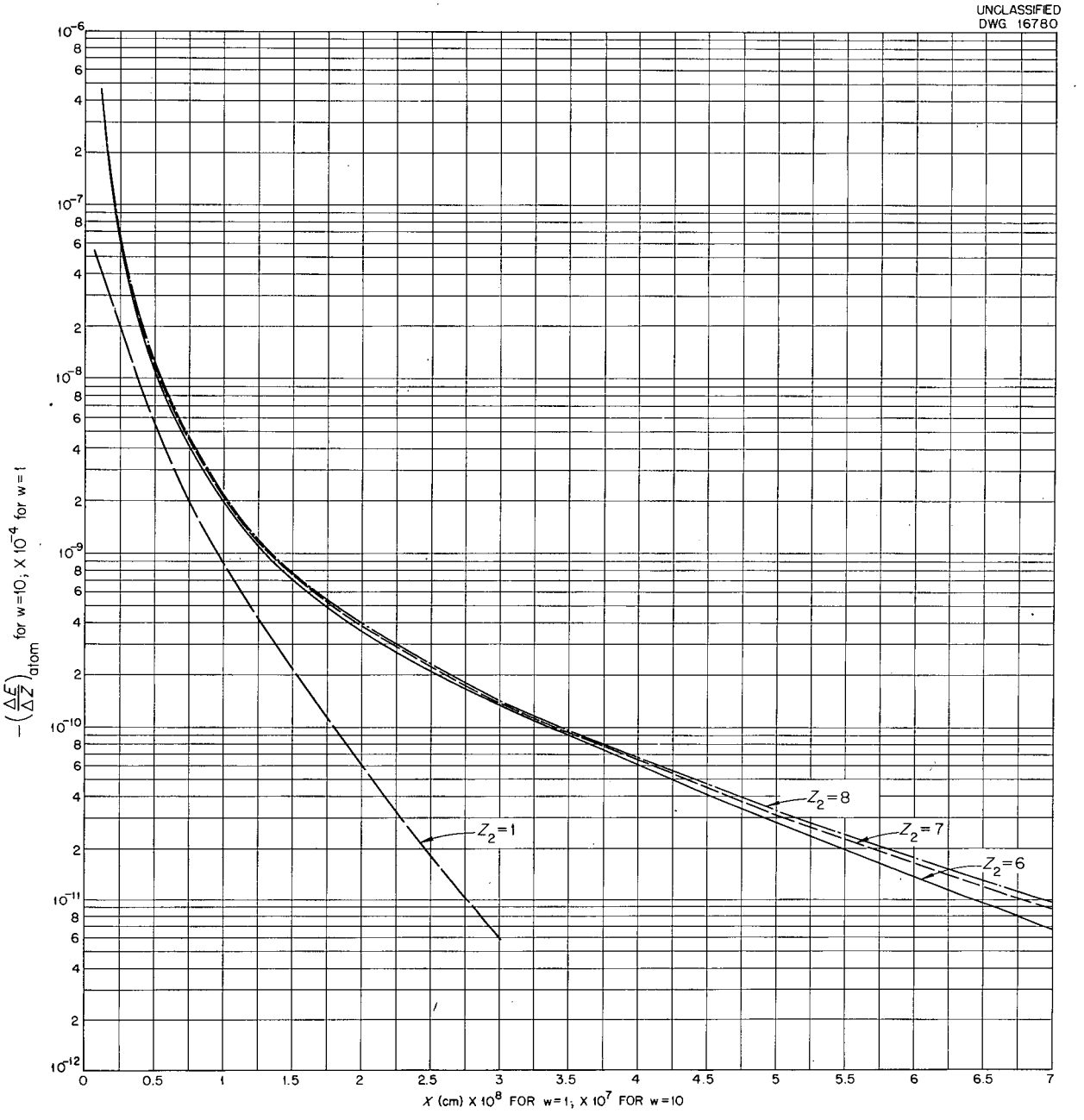

Fig. 2. Energy Loss Density, $-(\Delta E/\Delta z)_{atom}$ Mev/atom for w = 1, 10 (Proton Energy 0.025, 2.5 Mev).

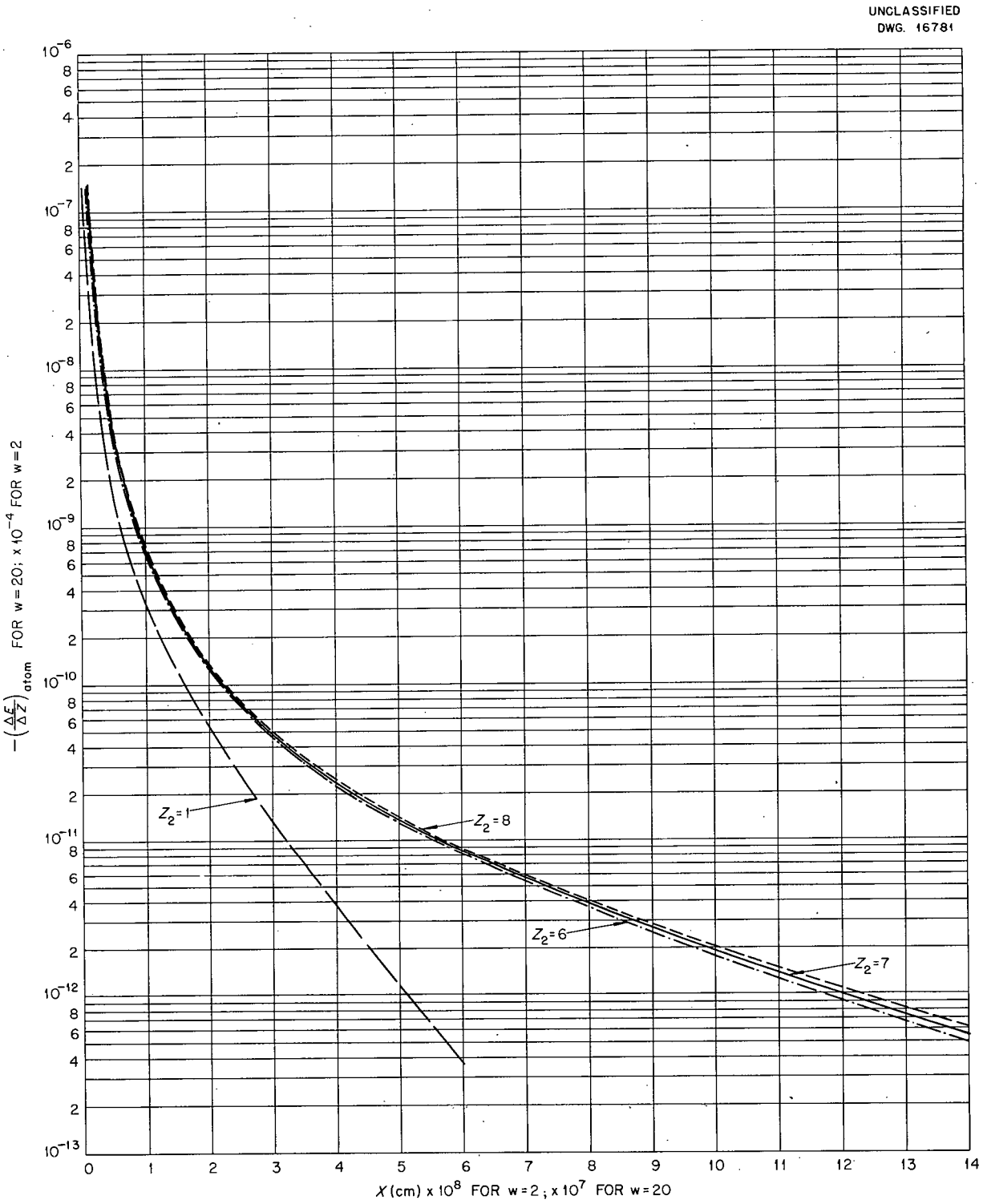

Fig. 3. Energy Loss Density $-(\Delta E/\Delta z)_{atom}$ Mev/atom for w = 2, 20 (Proton Energy 0.1, 10 Mev).

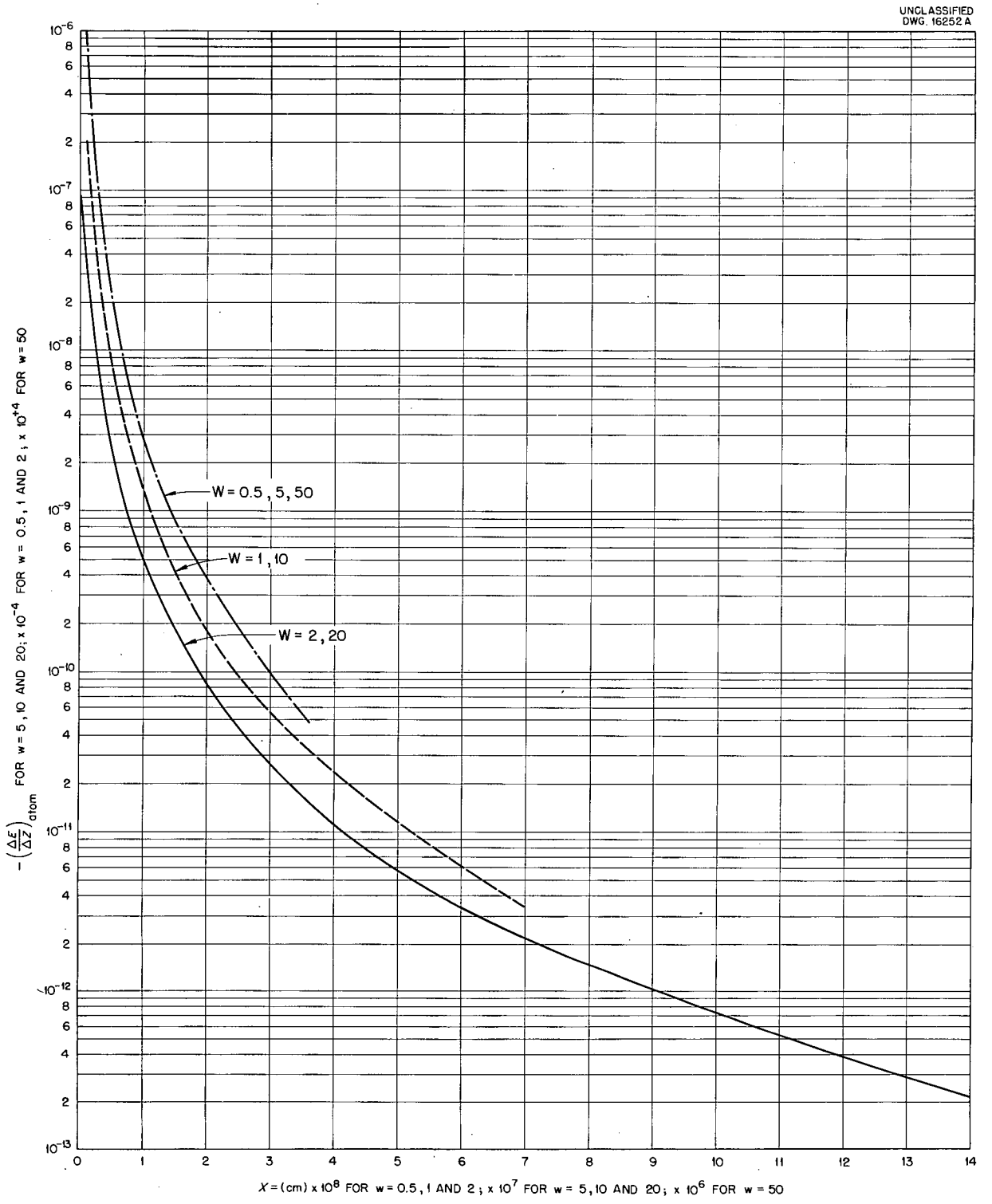

Fig. 4. Energy Loss Density in Tissue $-(\Delta E/\Delta z)_{atom}$ Mev/atom for w = 0.5, 1, 2, 5, 10, 20, and 50 (Proton Energy 0.00625, 0.025, 0.1, 0.625, 2.5, 10, 62.5 Mev).

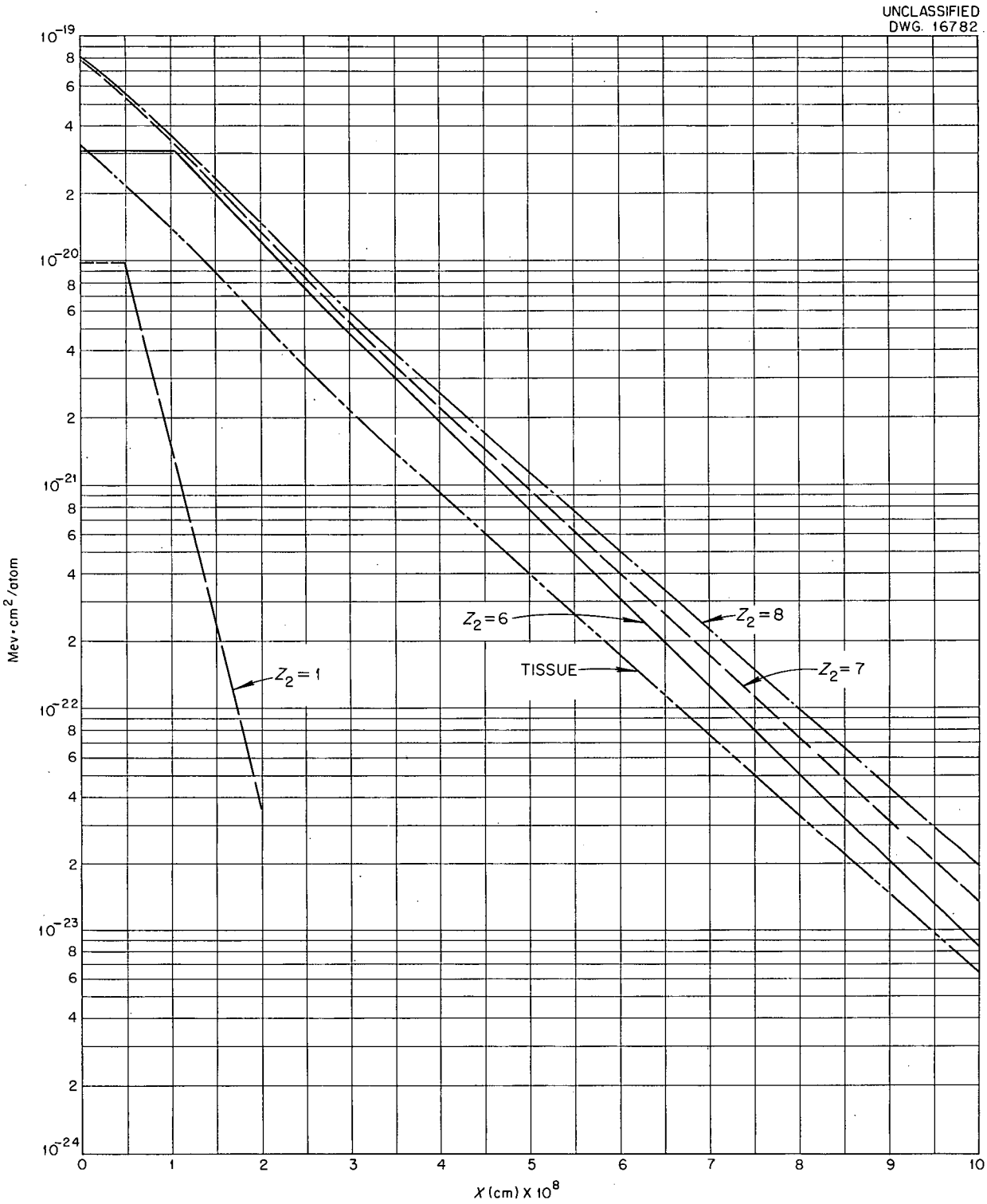

Fig. 5. Cumulative Energy Loss, w = 0.5 (Proton Energy 0.00625 Mev).

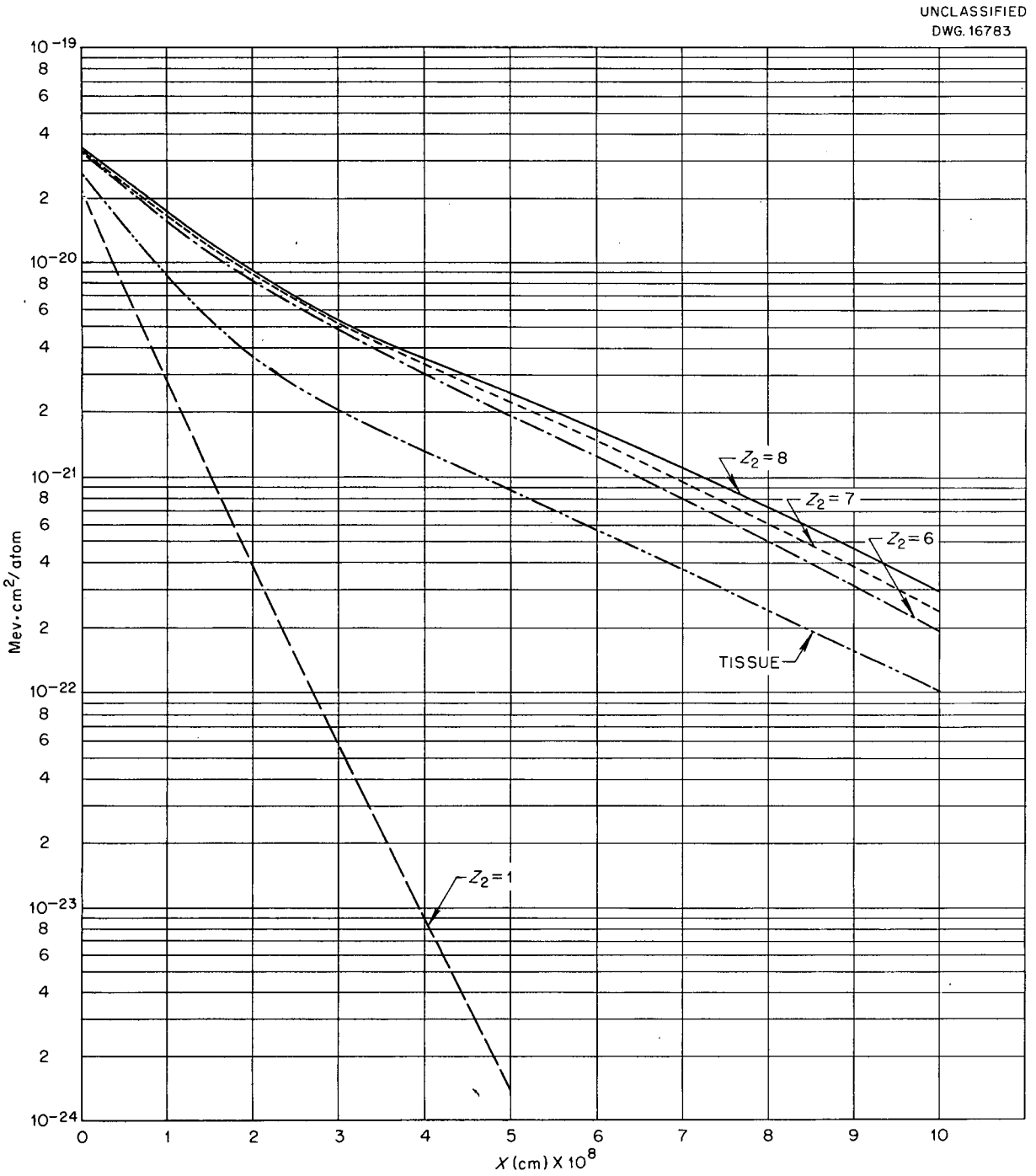

Fig. 6. Cumulative Energy Loss, w = 1.0 (Proton Energy 0.025 Mev).

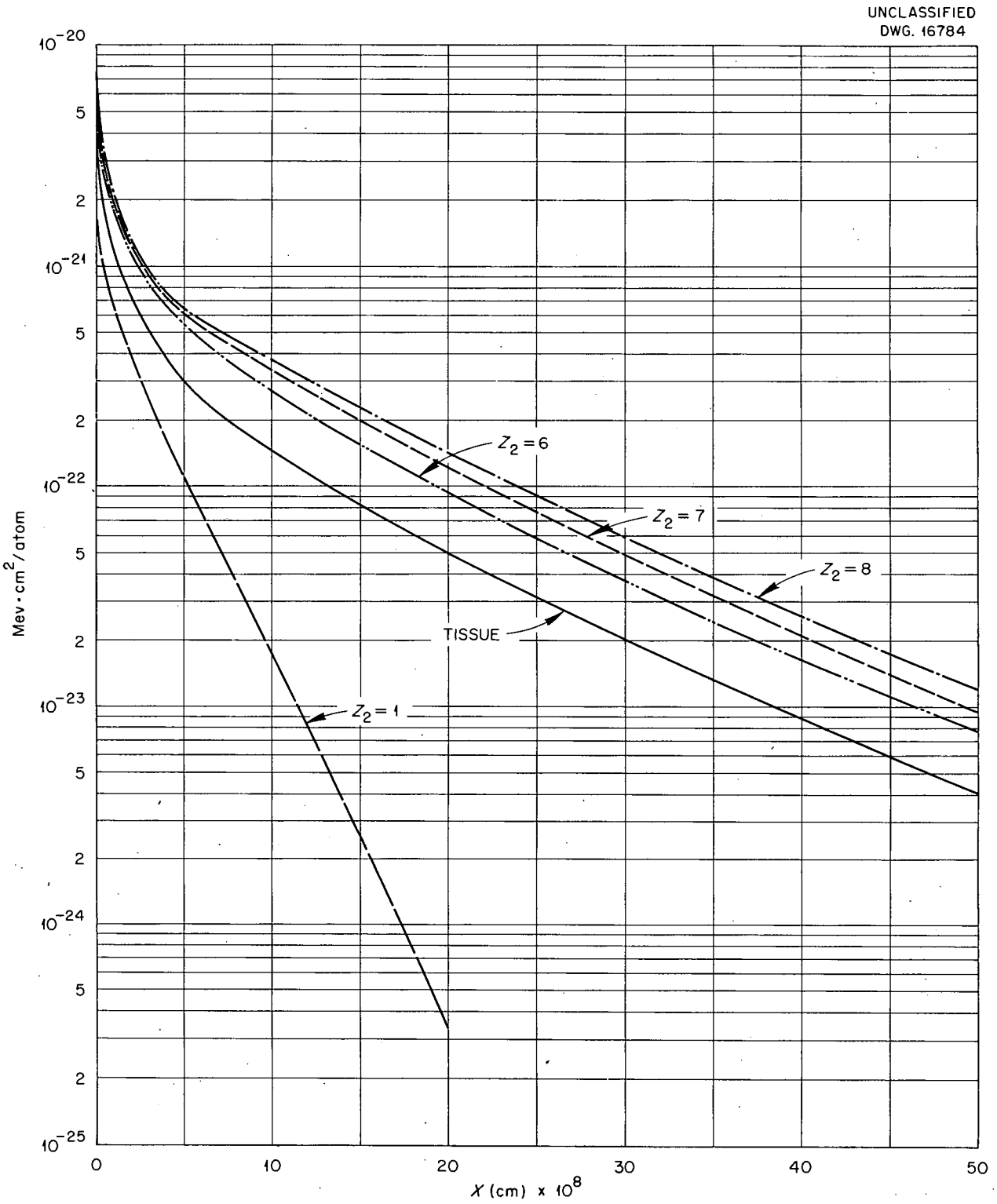

Fig. 7. Cumulative Energy Loss, w = 5.0 (Proton Energy 0.625 Mev).

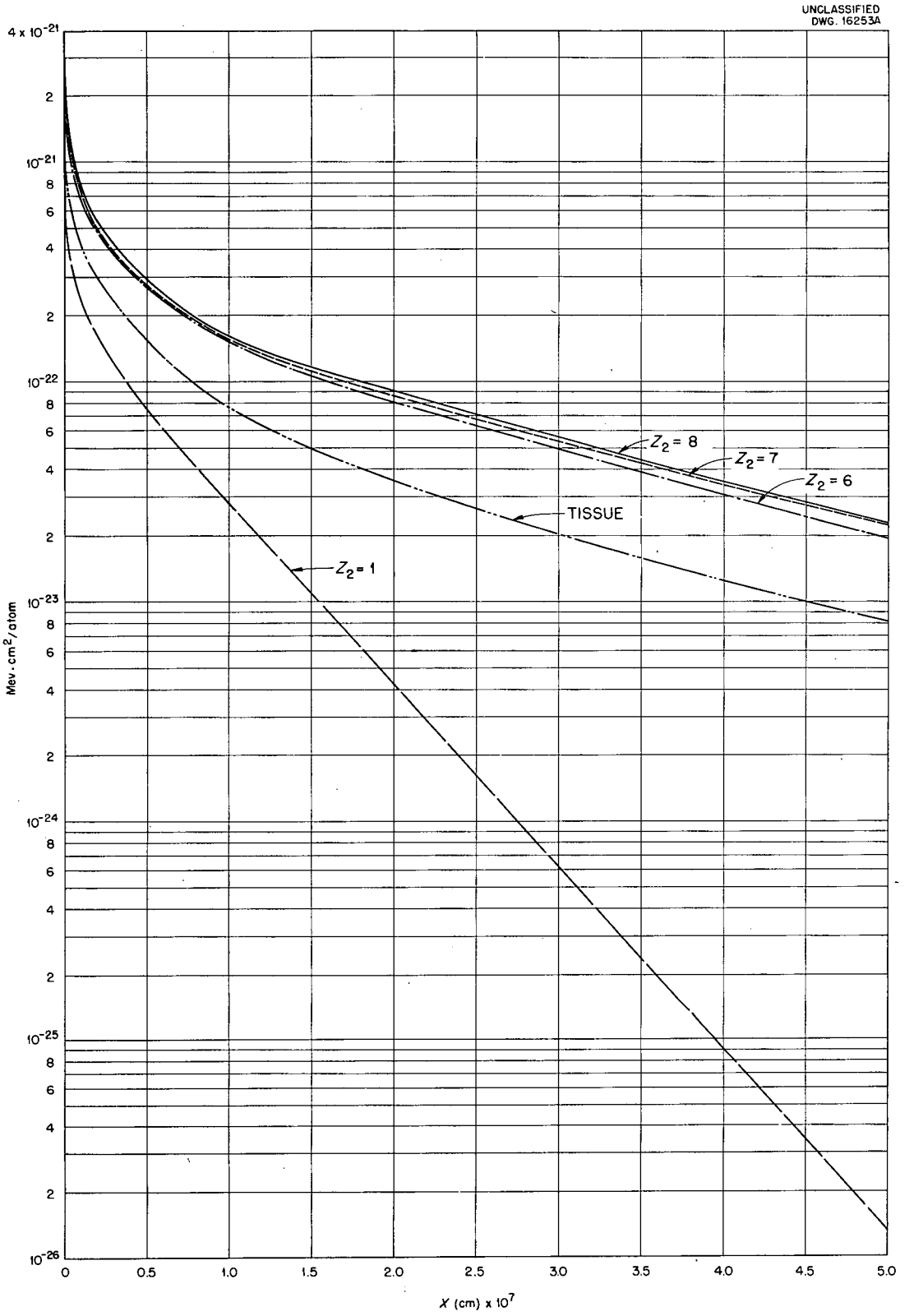

Fig. 8. Cumulative Energy Loss, w = 10 (Proton Energy 2.5 Mev).

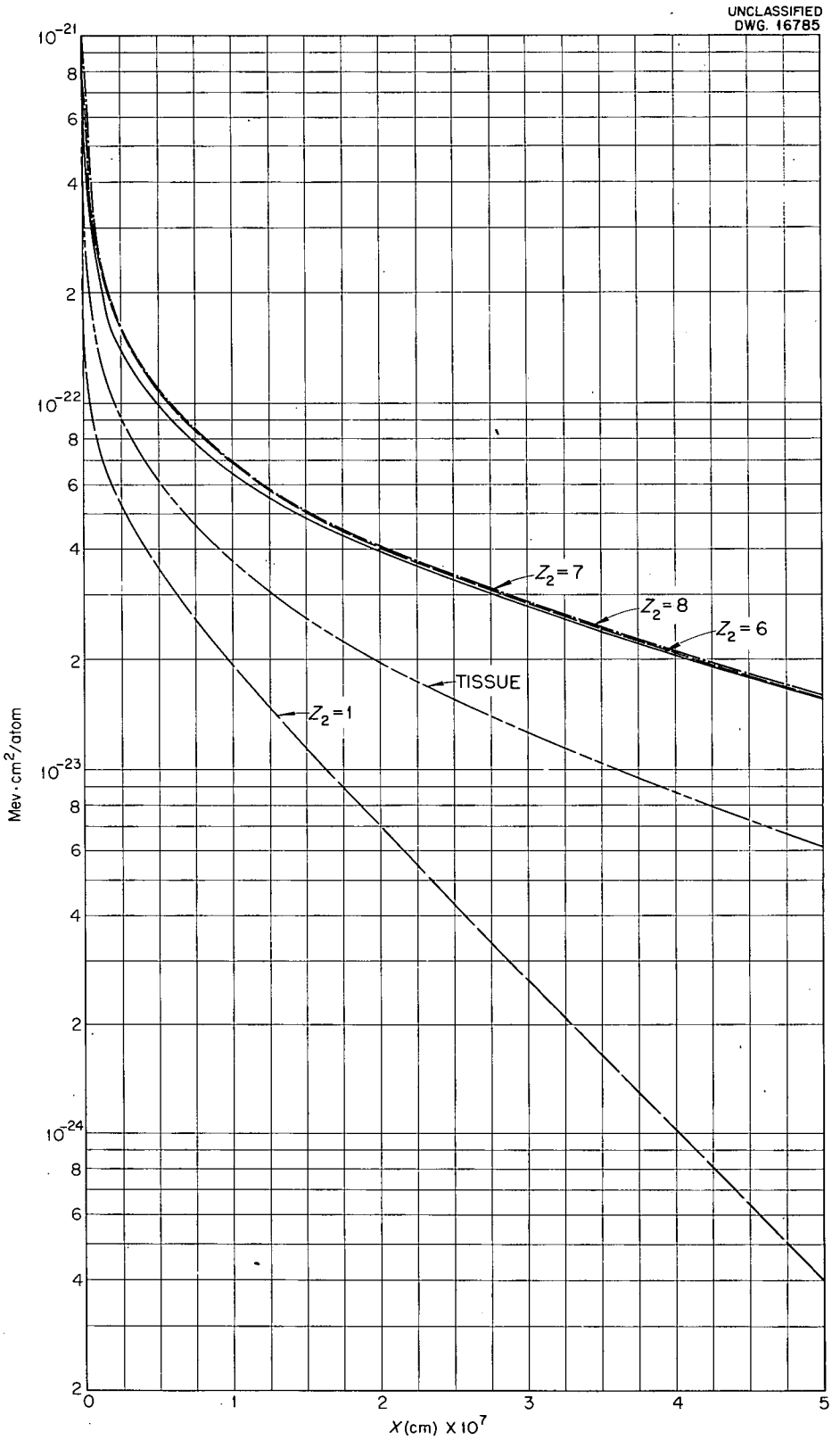

Fig. 9. Cumulative Energy Loss, w = 20 (Proton Energy 62.5 Mev).

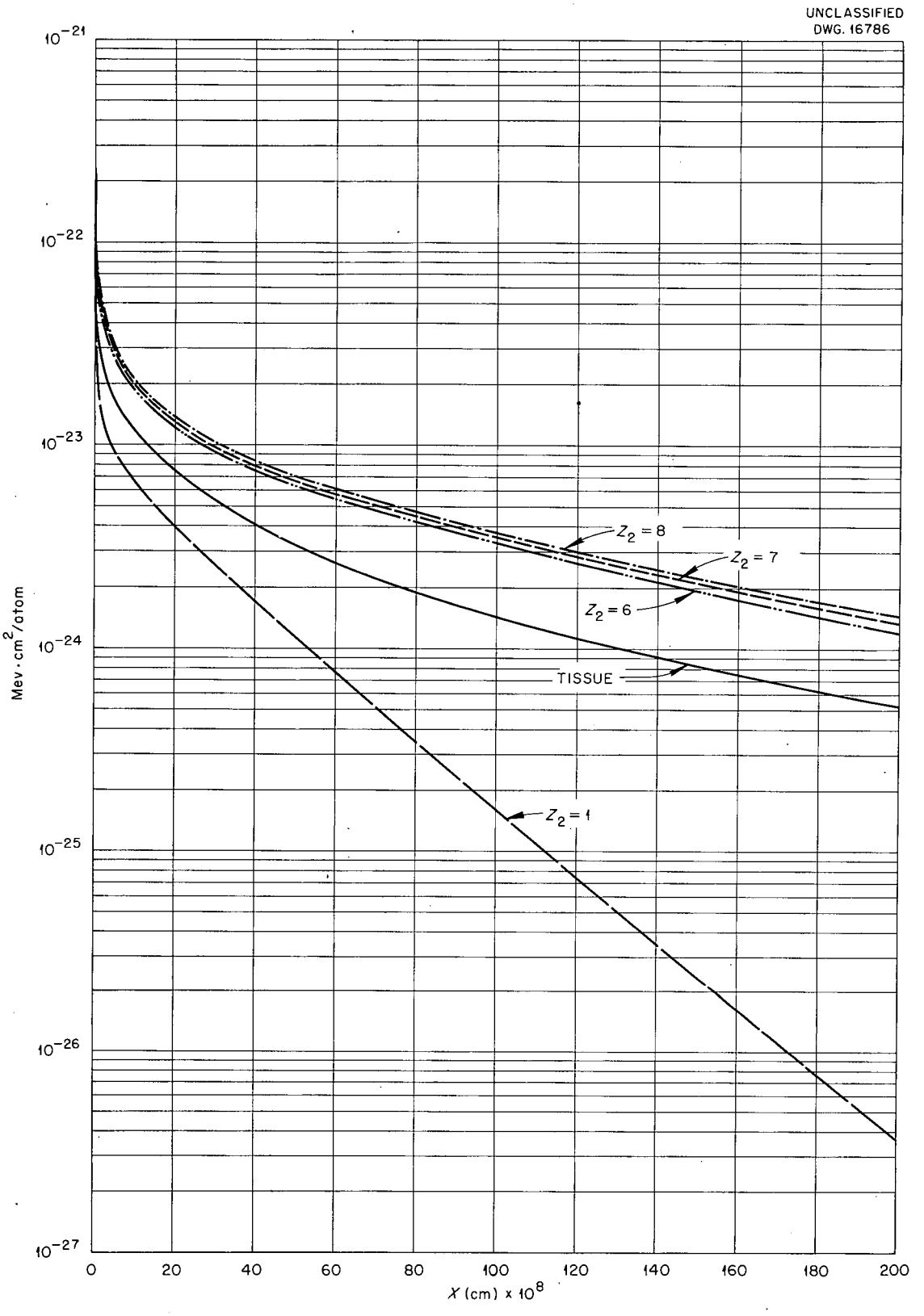

Fig. 10. Cumulative Energy Loss, w = 50 (Proton Energy 62.5 Mev).