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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

FOR PERIOD ENDING OCTOBER 31, 1952

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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

for Period Ending October 31, 1952

A. S. Householder, Chief
C. L. Perry, Editor

DATE ISSUED:

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MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

SUMMARY

Research participants S. D. Conte and S. L. Hull terminated on September 1; Conte returned to Wayne University and Hull took a position at The Citadel. W. A. Rutledge, who had been here as a temporary employee, returned September 19 to Alabama Polytechnic Institute. C. Perhacs terminated September 1 to begin graduate work at Carnegie Institute of Technology.

The name of the ORACLE (Oak Ridge Automatic Computer and Logical Engine) has been changed from ORAC to avoid confusion with the OARAC, which is being constructed for the Air Force. Individual memory units, fabricated at the Technitrol Corporation, Philadelphia, are being tested and assembled into the memory frame, and the control for the teletype input is being constructed. November 26 is the tentative date for assembly of the machine proper, and it is expected that by January 1, 1953, the input and output equipment will be attached and the machine will be ready to start on test problems. C. L. Perry is preparing a series of test routines, in collaboration with the engineers at Argonne.

Oak Ridge was well represented at the meeting of the Association for Computing Machinery, September 8 and 9, at the Computation Centre, University of Toronto. Papers on the logical design and on the engineering design of the ORACLE were given by C. L. Perry and by J. C. Chu, and a paper on computational techniques was given by A. S. Householder. Also attending the meeting were N. Edmonson, M. R. Arnette, and B. M. Drucker, from Oak Ridge, and E. W. Burdette and J. R. Klein, Oak Ridge engineers at Argonne.

The NEPA Linear Equation Solver is being installed in permanent quarters and is to be in operation early in November.

In September, J. H. Fishel and J. Moshman spent a few days in Philadelphia running some problems on the UNIVAC, with good success. The problems are described in this report. Other problems are being coded for the UNIVAC, but pressure from other projects for time on this machine makes uncertain how much time will be available for Oak Ridge problems.

At the meeting of the UNIVAC Program Council, October 2, proposals for the permanent site were examined and recommendations were made to the AEC. There was also some discussion of the exchange of information on the mathematical character of computations under way, and it was decided that brief reports would be given at subsequent meetings of the Council.

The course in Statistics for members of the Biology Division, given last year by A. W. Kimball, will be repeated this year. Classes meet once a week for one hour and are expected to start on or about November 15.

Papers presented during this quarter were:

C. L. Perry, *The Logical Design of the Oak Ridge Digital Computer*, September 8, Association for Computing Machinery, Toronto.

A. S. Householder, *Errors in Iterative Methods for Solving Systems of Linear Equations*, September 9, Association for Computing Machinery, Toronto.

W. C. Sangren, *Eigenfunction Expansions for Pairs of Linear First Order Differential Equations*, October 29, Mathematics Colloquium, University of Tennessee.

A. W. Kimball gave a seminar entitled *Estimation of Experimental*

Error, at a joint meeting of the Departments of Zoology and Botany of the University of Tennessee on October 14.

W. C. Sangren reviewed in the October 3 issue of *Science*, "Electronic Analog Computers" by G. A. Korn and T. M. Korn.

UNCLASSIFIED PROJECTS

CONSTRUCTION OF SUBROUTINES FOR THE ORACLE

Participating Members of Panel. Staff, M. R. Arnette, ORACLE Librarian.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1232, -1290, and -1360.

Background and Status. Work continues on the writing of standard subroutines for the ORACLE, since a large library of subroutines is essential for rapid coding of problems.

Subroutines for single-precision square-rooting, evaluation of sines and cosines, evaluation of a polynomial of arbitrary degree for an arbitrary number of values of the argument, and root reduction for polynomials are complete. Complete error analyses have been made for each of these subroutines. Also written for single-precision numbers are decimal-to-binary and binary-to-decimal conversion subroutines. These consider the decimal number N, where $-1 \leqslant N < +1$ and where N is punched on tape as the sign plus nine binary-coded decimal digits and is converted to a machine binary number of 40 bits (sign + 39 bits). Codes are written for fixed-point complex operations (addition, subtraction, multiplication, and division) and have been checked, but they have yet to be combined into a single subroutine. Also available are exponential and logarithm subroutines for single-precision numbers.

A subroutine for operations with double-precision numbers that includes complementation, summation, absolute value, multiplication, and division is coded and complete error analyses have been made. A square-rooting subroutine that uses these operations yields x accurate to 76 binary places if a is given to 78 binary digits ($x \doteq \sqrt{a}$).

Two floating-point subroutines have been coded but require a final checking. A third, which is only 96 words in length, including storage, is being checked. Complex operations, square root, and ln z, where z is a floating-point number, are written as subroutines that use floating-point arithmetic.

A subroutine for getting the inverse of a matrix, based on the discussion of von Neumann and Goldstine,[1] is coded. This subroutine uses floating-point arithmetic and occupies about 220 words in the memory. It has been checked and a few changes were made; therefore it must be rechecked. The flow diagram has been drawn for another subroutine that uses fixed-point arithmetic, but it is not yet coded.

A subroutine for getting the transpose of a matrix is coded and checked. This one transposes the matrix in the memory and then puts it on tape; the tapes are used only as extra storage. Another is being written that reads in and out from two tapes during the calculation and can therefore take care of a larger matrix.

Three subroutines have been written for getting the product of two matrices. Each of the three assumes $AB = C$, where A is stored by rows, B by columns, and C by rows, and each uses fixed-point operations, with the differences being in the methods of scaling. The

(1) J. von Neumann and H. H. Goldstine, *Am. Math. Soc. Bull.* **53**, 1021 (1947).

one that uses transient scaling, scales all terms when an overflow occurs. In semiprecision scaling, the sum is scaled after the sum is formed, and the scale factor is determined by the number of times an overflow occurs in the process; then each term is scaled by this amount. In precision scaling, the scale factor is determined as above, but the entire sum, not each term, is scaled. The code that uses transient scaling occupies about 45 words in the memory, but an excessive amount of scaling may be performed; the one that uses semiprecision scaling takes approximately 62 words; and the one that uses precision scaling occupies about 76 words but gives the optimum value when scaling is required.

Other subroutines, including assembly and binary-to-decimal and decimal-to-binary conversion subroutines for use with floating-point arithmetic, are being written.

TEST ROUTINES FOR THE ORACLE

Participating Member of Panel. C. L. Perry.

Background and Status. Several programs have been written for use in testing the ORACLE. Included in the routines are memory tests, adder tests, arithmetic register tests, a dispatch counter test, and tests of the control. In building the ORACLE, the engineers included several features for checking purposes, one of which is a switch that allows the computer to operate with the left half, with the right half, or with the entire memory. A test program for checking the memory has been written that will locate any defective memory unit.

METHODS OF COMPUTATION FOR USE WITH A HIGH-SPEED AUTOMATIC-SEQUENCED COMPUTER

Participating Member of Panel. W. Givens.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1232, -1290, and -1360.

Problem 3. Numerical Computation of Characteristic Values and Characteristic Vectors.

Status. In the course of preparing a report on a proposed method of computation, improved error bounds have been obtained.

GENERATION OF "RANDOM" DIGITS ON THE UNIVAC

Participating Members of Panel. J. Moshman, S. E. Ezzell.

Background and Status. With the definitions

$$\rho = 7^{13} ,$$

$$\rho_0 = 1 ,$$

$$\rho_{i+1} = \rho_i \cdot \rho \pmod{10^{11}} ,$$

there is obtained a sequence of 11 decimal digit numbers that are cyclic, with period $5 \cdot 10^8$. It was conjectured that the first six or seven digits would form a sequence of numbers that would fulfill requirements for randomness. The suggested criteria for randomness include correct proportional digital frequency, the number of multiplicities in successive numbers of the same digit in the same position, and the number of runs of varying length up and down.

The results of tests of the first 10,000 numbers in the sequence are being tabulated, and the probability of the deviations from expectation is being evaluated.

ESTIMATION OF THE MUTATION RATE FROM CLUSTERED SAMPLES

Origin. W. L. Russell, Biology Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. In the experiments, a population of k *Drosophila melanogaster* males is irradiated at a time when the total number of chromosomes in the spermatagonia of the ith male is N_i ($i = 1, \ldots, k$).

When these cells mature, each male is mated to several females, and out of N_i offspring examined, a_i are found to have mutations. If p is the probability of a mutation occurring in any one chromosome of the spermatagonia, the probability of a mutation occurring in the ith male is pN_i, since it is assumed that no more than one mutation can occur in one male. There is, in fact, a very small probability that more than one mutation will occur in the same male, but it is negligible compared to the probability of a single mutation. The N_i are unknown, and an estimate of p based on the observable a_i and n_i is desired.

Consider one male having N chromosomes. Let

B = occurrence of a mutation in the male,

$\overline{B}$ = occurrence of no mutation in the male,

A = occurrence of x mutations out of n offspring from the male.

Then

$$\text{Prob } [B] = pN \quad ,$$

$$\text{Prob } [\overline{B}] = 1 - pN \quad ,$$

$$\text{Prob } [A|B] = C_x^n \left(\frac{1}{N}\right)^x \left(1 - \frac{1}{N}\right)^{n-x} \quad ,$$

$$\text{Prob } [A|\overline{B}] = \begin{Bmatrix} 1 \text{ if } x = 0 \\ 0 \text{ if } x > 0 \end{Bmatrix} ,$$

and the usual formula for conditional probability leads to the relations

$$\text{Prob } [x = 0] = pN \left(1 - \frac{1}{N}\right)^n + (1 - pN) \ ,$$

$$\text{Prob } [x = a]$$

$$= pNC_a^n \left(\frac{1}{N}\right)^a \left(1 - \frac{1}{N}\right)^{n-a} \qquad (a > 0) \quad .$$

The expected values of a and a^2 are

$$E\ [a] = \sum_{a=0}^{n} a \cdot \text{Prob } [x = a]$$

$$= pN\left(\frac{n}{N}\right) = np \quad ,$$

$$E\ [a^2] = \sum_{a=0}^{n} a^2 \cdot \text{Prob } [x = a]$$

$$= pN\left[n\left(\frac{1}{N}\right)\left(1 - \frac{1}{N}\right) + \left(\frac{n}{N}\right)^2\right] \quad ,$$

from which the variance of a is found to be

$$V(a) = E\ [a^2] - E^2\ [a]$$

$$= np\left[1 + n\left(\frac{1}{N} - p\right) + \frac{1}{N}\right] \quad .$$

Like the case of the simple binomial, as estimate of p is given by a/n; but unlike the simple binomial, an estimate of $V(a)$ is not given by $n\ (a/n)\ (1 - a/n)$, since $V(a)$ is a function of the unknown N. It should be noted that $V(a)$ is larger than the simple binomial variance, since

$$np\left[1 + n\left(\frac{1}{N} - p\right) + \frac{1}{N}\right] > np\ (1 - p) \quad .$$

In the case of k males, it is clear that an unbiased estimate of p is given by

$$\hat{p} = \frac{1}{k}\sum \frac{a_i}{n_i} \quad ,$$

although no obvious method exists for calculating confidence intervals for p. If it were known exactly that h out of k males had mutated and if the N_i were known, the best estimate of p would be $h/\sum N_i$, and binomial or Poisson tables could be used to calculate a confidence interval for p. Alternatively, the same method might be used to calculate confidence limits for p based on the estimate $\hat{p}$, considering the quantity $\hat{p}$ to be a

mean of k observations. An estimate of $\sum N_i$ may be computed by equating the two estimates of p,

$$\hat{p} = \frac{h}{\sum N_i},$$

where h is the number of males producing mutated offspring. Then $h = \hat{p} \sum N_i$ can be referred to Poisson tables for limits on h, which are converted into limits on p by dividing by $\sum N_i$. As a rough check on the appropriateness of the method, the sample variance of the a_i/n_i should be compared with the mean. If these agree moderately well, the Poisson approximation should be satisfactory.

A more comprehensive treatment of the whole problem would be possible if something were known about the probability distribution of the N_i. On the basis of current genetic theory, it is unlikely that the N_i follow any of the well-known distributions, and any attempt to utilize one of them would be inadvisable.

TESTING OF SPERM BATCHES FROM IRRADIATED *DROSOPHILA MELANOGASTER*

Origin. W. K. Baker, Biology Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta, P. J. Brown.

Background and Status. Irradiated *Drosophila* males were mated to females at two different times after irradition. The two times are identified as sperm batches I and II. Four irradiation doses were used, and either two or three replications were performed at each dose. A control group was run with every treated group. The eggs laid by the females were examined and the proportions of eggs hatching were recorded for each female. A test was desired of the hypothesis that there is no difference between the hatchability of sperm batches I and II.

The first step was to examine the variability among the proportions of eggs hatching from the females inseminated by the same male. In almost every case the extraneous variation was a large percentage of the total variation, so the use of binomial weights was deemed unnecessary. The average proportions of eggs hatching were then subjected to an analysis of variance of the form illustrated in Table 1. The numbers of males in the various subclasses were not proportional, but it was possible to restore orthogonality to the data by dropping a few males from some subclasses. This was done by random selection, but since there were over 100,000 flies involved in the entire experiment, not much information was lost. The test of the hypothesis was provided by forming the F ratio of the mean square for between sperm batches (in controls or treated) to the mean square for males by sperm batch interaction.

TABLE 1. FORM OF ANALYSIS OF VARIANCE

SOURCE OF VARIATION	DEGREES OF FREEDOM
Among replications	r
Between sperm batches in control males	1
Between sperm batches in treated males	1
Between treated and control males	1
Replication by treatment interaction	$3r$
Among males	m
Males by sperm-batch interaction	m

ESTIMATION OF ERROR IN CONCENTRATION RATIOS OBTAINED FROM AN EFFLUORADIOGRAM

Origin. W. S. Wilde, J. M. O'Brien, Biology Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. The outflux of potassium from cardiac muscle of the turtle has been measured by means of an effluoradiogram that was especially designed for this purpose.[2] The potassium-to-iodine ratio at time zero is given by

$$\frac{K_0}{I_0} = \frac{D\left[\frac{S_1}{t_1} - \frac{S_3}{t_3} - d\left(\frac{S_2}{t_2} - \frac{S_4}{t_4}\right)\right]}{d\left(\frac{S_2}{t_2} - \frac{S_4}{t_4}\right)},$$

where S_i is the number of counts recorded during the time interval t_i, and D and d are reciprocal decay constants. Subscripts 1 and 2 refer to potassium and iodine counts, respectively, and subscripts 3 and 4 are the corresponding background counts.

The ratio may be written in the form $D[(x/y) - 1]$, from which it is clear that

$$V\left(\frac{K_0}{I_0}\right) = D^2 V\left(\frac{x}{y}\right) .$$

By making use of the approximate formula for the variance of a ratio, the addition theorem for variances of linear functions of independent variates, and the fact that the square root of an observed Poisson variate is an estimate of its standard deviation, the approximate percentage error in K_0/I_0 is

$$\frac{1}{K_1}\left[\left(\frac{S_1}{t_1^2} + \frac{S_3}{t_3^2}\right) + d^2\left(1 + \frac{K_1}{dI_2}\right)^2\left(\frac{S_2}{t_2^2} + \frac{S_4}{t_4^2}\right)\right]^{1/2},$$

where $K_0 = DK_1$ and $I_0 = dI_2$. In this experiment K_1 and I_2 are directly observable, whereas K_0 and I_0 are not.

The data at present consist of seven runs of from 150 to 250 samples each. For each sample the estimate of K_0/I_0 and its standard error must be computed. The work is being done by the Numerical Analysis Laboratory at K-25, with the assistance of E. B. Carter. Since the problem is already coded for IBM, the additional runs anticipated in the near future can be handled with very little extra labor.

NOMOGRAM REPRESENTING $r = (1 - e^{-t})/(e^{\tau} - 1)$

Origin. W. Eister, Chemical Technology Division.

Participating Members of Panel. A. S. Householder, R. C. Weaver, H. B. Goertzel.

Status. The expression

$$r = \frac{1 - e^{-t}}{e^{\tau} - 1}$$

represents the total radiation at time t from material irradiated in equal batches for equal periods, one batch after the other, and allowed to decay for equal periods. The nomogram has been constructed.

ESTIMATION OF THE RATIO OF ISOTOPIC AND NORMAL RATE CONSTANTS IN A CARBON-14 EXPERIMENT

Origin. A. J. Weinberger, G. A. Ropp, Chemistry Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

[2] J. M. O'Brien and W. S. Wilde, *Science*, **116**, 193 (1952).

Background and Status. In a first-order reaction it is assumed that the amount of reactant remaining at time t is given by

$$y = \log Y = y_0 + k_y t \quad , \qquad (1)$$

where Y is the amount left at time t and y_0 is the logarithm of the amount present at $t = 0$. In one experiment involving both C^{12} and C^{14}, the amounts of these isotopes present were determined at various times after the start of the reaction. The errors in the time measurements were large compared with the analytical errors in determining C^{14} and C^{12}; therefore it was desirable, if possible, to eliminate the time variable from the calculations necessary for estimating the ratio of the rate constants. If Eq. 1 represents the equation for C^{14}, and

$$x = x_0 + k_x t \qquad (2)$$

represents the equation for C^{12}, elimination of t between Eqs. 1 and 2 yields

$$y = \left(y_0 - \frac{k_y x_0}{k_x}\right) + \frac{k_y}{k_x} x \quad . \qquad (3)$$

The slope of y plotted against x is thus an estimate of the desired ratio of rate constants.

Since both y and x are subject to error, the usual methods for linear regression fail. Several authors have considered the problem of fitting a straight line when both variables are subject to error. Bartlett,[3] utilizing some of the results of Wald,[4] gives a method for estimating the slope and for computing a confidence interval for the slope. The method was tried on the data from this experiment, and imaginary confidence limits were obtained. This result inspired a thorough investigation into the premises on which Bartlett's confidence interval method and many other methods in much wider use are based. Serious discrepancies have been uncovered and will be reported soon in the open literature.

In the meantime, some method of dealing with the problem at hand has to be found. As an approximation of a probably conservative nature, two sets of confidence intervals were computed, one from the regression of y on x and one from the regression of x on y. In each case the independent variate was assumed to be free of error. The extreme values from both sets were taken as the final confidence limits. As a simple method of taking account of the error in both variables, the procedure is satisfactory, although no real assurance can be given that the true confidence limits, if they exist, are narrower than those selected.

[3] M. S. Bartlett, *Biometrics* 5, 207-212 (1949).

[4] A. Wald, *Ann. Math. Stat.* 11, 284 (1940).

KINETICS OF HBr-$HBrO_3$ REACTION

Origin. O. E. Myers, Chemistry Division.

Participating Members of Panel. J. H. Fishel, C. P. Hubbard, R. C. Weaver.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1151, -1232, -1290, and -1360.

Background and Status. The numerical integration code used on the MANIAC for integrating the kinetic equations was transcribed into a code suitable for the UNIVAC. A numerical integration of the system, made by using a set of parameters that had been used on the MANIAC, was carried out on the UNIVAC for checking purposes, and the results compared favorably. The complete solution of this system for 16 sets of parameters required approximately 1.5 hr of UNIVAC time. The results of the integration have been sent to Myers in both tabulated and graphic form.

CONTROL CHART FOR STORES INVENTORIES

Origin. A. J. Cook, General Office Division.

Participating Member of Panel. J. Moshman.

Background and Status. Based on data from the 1951 and 1952 fiscal years, it was found that stationery stores consumption averaged $3.65 per month per Laboratory employee. The variation from month to month was in accordance with that expected from 95% control limits. Based on an average Laboratory population of 3,200 employees, the amount to be purchased each month in order to have a three-month supply at the beginning of each month could be expressed as

$$P = D + 25 - H ,$$

where P is the amount to be purchased; D is the expected demand per quarter; 25 is the upper control limit in excess of D; and H is the inventory on hand.

DAMAGE TO TISSUE FROM NEUTRON DISTRIBUTIONS WITH MAXIMUM ENERGIES RANGING FROM 1 TO 300 kev

Origin. W. S. Snyder, J. Neufield, Health Physics Division.

Participating Members of Panel. P. J. Brown, R. C. Weaver, J. Moshman, N. M. Dismuke.

Background and Status. In order to determine the damage to human tissue from neutron irradiation, the following Monte Carlo problem is being programmed for the ORACLE. A parallelepiped of tissue is subjected to a monoenergetic source of neutrons with energy ranging from 1 to 300 kev. Cartesian coordinates were used, with the neutron source of energy E placed at the origin. The parallelepiped is placed so that its planes are given by

$$x = d , \qquad x = d + t ,$$
$$y = \pm w/2 ,$$
$$z = \pm h/2 ,$$

where t, w, and h are constants.

Neutrons are started in the x, z plane at an angle θ with the x axis and may have isotropic scattering collisions with hydrogen, oxygen, carbon, and nitrogen (and possibly with calcium and phosphorus), and absorptions with nitrogen. For each collision of each neutron, the average damage for that neutron energy is computed and recorded as a function of x, y, and z until the neutron leaves the body of tissue, is absorbed, or becomes degraded in energy to 1 ev. The parameters E_0, θ, and d are constants for a given problem.

The damaging effect of neutron distributions will be deduced by combining results of several problems. Also, the variation of the damage with distance from the source, d, can be studied. The published cross-section data[5] will be used. The size, shape, and composition of the parallelepiped were specified by the Health Physics Division. The flow chart for this problem has been drawn and detailed coding is in progress.

DESIGN OF A MULTIFACTOR EXPERIMENT IN A RADIOACTIVE WASTE DISPOSAL STUDY

Origin. C. P. Straub, Health Physics Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta, P. J. Brown.

Background and Status. As a continuation of previous work,[6] an experiment has been planned for investigating the simultaneous effects of 12 factors on the removal of radioactive waste from water. Originally it was thought that four levels of each factor would be needed, but, since this would require 4^{12} experimental units, the decision was made to consider only the extreme levels for each factor in order to uncover any possible interactions. The study of intermediate levels will be made later with smaller scale experiments. Even this reduced experiment would require $2^{12} = 4096$ experimental units,

(5) *Neutron Cross Sections*, compiled by AEC Neutron Cross Section Advisory Group, D. J. Hamilton, Chairman, AECU-2040 (May 15, 1952).

(6) *Math. Quar. Prog. Rep. Apr. 30, 1951*, ORNL-1029, p. 29.

which are clearly too many for a practical study.

Between 500 and 600 experimental units can be handled in one experiment. Accordingly it was decided to select two 1/16 replications of the basic 2^{12} design that would provide information on all the desired effects and several degrees of freedom for experimental error. The two 1/16 replications were constructed by finding the terms of like sign common to the *ABCDE*, *ABFGH*, *ABJKL*, and *ACFJM* interactions for one replication and the terms of like sign common to the *ABCDE*, *ABFGH*, *ABJKL*, and *ACFKM* interactions for the second replication. When this is done, it is found that exactly one half of the treatment combinations appearing in each replication are identical, and therefore 128 degrees of freedom for error will be available.

The problem of constructing the design reduces to finding the terms of like sign (either plus or minus) common to the four expressions

$$(a-1)(b-1)(c-1)(d-1)(e-1)(f+1)(g+1)(h+1)(j+1)(k+1)(l+1)(m+1) \quad ,$$
$$(a-1)(b-1)(c+1)(d+1)(e+1)(f-1)(g-1)(h-1)(j+1)(k+1)(l+1)(m+1) \quad ,$$
$$(a-1)(b-1)(c+1)(d+1)(e+1)(f+1)(g+1)(h+1)(j-1)(k-1)(l-1)(m+1) \quad ,$$
$$(a-1)(b+1)(c-1)(d+1)(e+1)(f-1)(g+1)(h+1)(j-1)(k+1)(l+1)(m-1) \quad ,$$

for the first replication, and a similar set for the second replication. The properties of each replication are: (1) the main effects are confounded only with four-factor and higher order interactions, (2) 31 of the two-factor interactions are confounded with four-factor and higher order interactions and 35 of them are confounded with three-factor and higher order interactions, (3) 8 of three-factor and 8 of the four-factor interactions are confounded only with five-factor and higher order interactions. The previous experiment indicated that three-factor and higher order interactions are negligible compared with the main effects and two-factor interactions; therefore the fractional design seems safe. Since some information will be obtained on three-factor and four-factor interactions, a check on this assumption will be available from the analysis.

As a check on the accuracy of the selection of the 1/16 replications, which admittedly are complex even to write down, arbitrary numbers were assigned to population means, and the expected values of each treatment combination were computed. An analysis of these means was performed and the results checked perfectly. Preparations for the execution of the actual experiment are now under way.

TABLE OF LOG *ft* VALUES

Origin. K. Way, National Bureau of Standards.

Participating Members of Panel. R. C. Weaver, N. M. Dismuke.

Reference. *Mathematics Panel Quarterly Progress Report for Period Ending January 31, 1951*, ORNL-979.

Background and Status. The original purpose of this computation was to prepare an up-to-date list of *ft* values for the group preparing punched cards of nuclear data. When the work was partly completed, it appeared that the table would be useful if distributed as a separate report.

Experimental data for the table of log *ft* values were furnished by the Nuclear Data Group of the National Bureau of Standards. Log *f* values were read from the graphs prepared by Feenberg and Trigg.[7]

[7] E. Feenberg and G. Trigg, *Rev. Mod. Phys.* 22, 399 (1950).

When the table had been completed and checks were being made, a manuscript by R. W. King, Physics Department, Washington University, St. Louis, Missouri, came to the attention of the group. The manuscript included most of the log ft values and the work was based primarily on data compiled by the Nuclear Data Group. King's work, of course, provided a very useful check. R. W. King and N. Dismuke met with K. Way in Washington on October 16, 17, and 18 to discuss differences in the two tables, and at that time it was decided to distribute a joint report, which will appear during the next quarter as an ORNL report.

CALCULATION OF RACAH COEFFICIENTS FOR THE ANGULAR DISTRIBUTION IN NUCLEAR REACTIONS

Origin. L. C. Biedenharn, M. E. Rose, Physics Division.

Participating Members of Panel. N. D. Given, K. P. Graw, S. L. Hull.

References. L. C. Biedenharn, *Tables of the Racah Coefficients*, ORNL-1098; *Physics Division Quarterly Progress Report for Period Ending March 20, 1951*, ORNL-1005; *Mathematics Panel Quarterly Progress Reports*, ORNL-1029, -1151, -1232, -1290, and -1360.

Background and Status. A table of the R coefficients

$$R(J\ J_0;\ \nu L) \equiv \left[\frac{C^{L L \nu}_{1\ -1}}{C^{L L 0}_{1\ -1}}\right] \frac{W(L\ J\ L\ J;\ J_0\ \nu)}{W(L\ J\ L\ J;\ J_0\ 0)},$$

the Clebsch-Gordan coefficients $C^{L L \nu}_{1\ -1}$, and the Racah coefficients $W(L\ J\ L\ J;\ J_0\ \nu)$, as defined in ORNL-1098, has been completed for the range

$$\nu = 2(2)\ 8\ , \qquad L = 1(1)\ 4\ ,$$
$$J = 1(1)\ 5\ , \qquad J_0 = 1(1)\ 5\ ,$$

for integer spins, and for the range

$$\nu = 2(2)\ 8\ , \qquad L = 1(1)\ 4\ ,$$
$$J = \frac{3}{2}(1)\ \frac{13}{2}\ , \qquad J_0 = \frac{1}{2}(1)\ \frac{13}{2}\ ,$$

for half-integer spins. A less extensive table of the S coefficients

$$S(J\ J_0;\ \nu\ L) = C^{L\ L+1\ 0}_{1\ -1}\ W(L,\ J,\ L+1,\ J;\ J_0,\ \nu)$$

has also been completed.

CALCULATION OF THE EFFICIENCY OF ABSORPTION FOR A SINGLE CRYSTAL

Origin. P. R. Bell, F. K. McGowan, Physics Division.

Participating Members of Panel. N. Edmonson, N. M. Dismuke, J. H. Fishel, H. B. Goertzel.

Reference. *Mathematics Panel Quarterly Progress Report for Period Ending October 31, 1951*, ORNL-1151.

Background and Status. Computationally, the calculation of the efficiency of absorption for a single crystal consists of the evaluation of a definite integral of the form

$$T(E) = \frac{1}{2}\int_0^{\alpha_0} [1 - e^{-\tau(E)x(\alpha)}] \sin \alpha d\alpha.$$

This integral was evaluated by Simpson's rule. Since hand computations to three significant figures did not prove to be sufficiently accurate, the calculation has been coded for a UNIVAC computation to five significant figures. The flow diagram, code, and numerical data have been completed and checked and should be ready to send to the UNIVAC in Philadelphia on October 30, 1952.

CALCULATION OF COINCIDENCE PROBABILITIES FOR A DOUBLE-CRYSTAL DETECTOR

Origin. F. K. McGowan, Physics Division.

Participating Members of Panel. N. Edmonson, N. M. Dismuke, J. H. Fishel, H. B. Goertzel.

Reference. *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1952*, ORNL-1360.

Background and Status. This problem was coded for and computed on the UNIVAC. For checking purposes,

a desk computation was made for a typical computation. Agreement of this desk computation was obtained with the corresponding UNIVAC computation, and the reliability of the UNIVAC (a self-checking computer) was assumed for the other computations. The computed data have been sent to the Physics Division for further checking.

ANISOTROPY IN ANGULAR CORRELATION DATA

Origin. E. D. Klema, Physics Division.

Participating Members of Panel. C. L. Perry, J. Moshman, R. C. Weaver, K. P. Graw.

Background and Status. The angular correlation of gamma rays emitted in cascade from excited states in a given nucleus is measured by means of two scintillation counters. An estimate is desired of the parameters A, B, and C in the equation

$$y = A + B \cos^2 \theta + C \cos^4 \theta \ , \qquad (1)$$

where y is the coincidence counting rate between the two scintillation counters, one fixed and the other movable about the source, and θ is the angle between the lines joining the source and the fixed and movable detectors.

For various angles, the time, T_N, required for the observation of a given total number of coincidence counts, N, was observed. The random coincidence rate, R/T_R, was also determined for the experiment. Thus

$$y = \frac{N}{T_N} - \frac{R}{T_R} \ , \qquad (2)$$

with the variance of a given y being

$$s_y^2 = \frac{N}{T_N^2} + \frac{R}{T_R^2} \ . \qquad (3)$$

The parameters A, B, and C in Eq. 1 were estimated by using a classical least-squares procedure with weights $w_y = (s_y^2)^{-1}$.

It is possible to define

$$\psi = \frac{A + B + C}{A} \ ,$$

which is the ratio of the counting rate for $\theta = \pi$ to the counting rate for $\theta = \pi/2$, as the total anisotropy. By neglecting terms of higher order,

$$\mathrm{Var}\ (\psi) = \psi^2 \left[\frac{\mathrm{Var}\ (A) + \mathrm{Var}\ (B) + \mathrm{Var}\ (C) - 2\left\{\mathrm{Cov}\ (A,B) + \mathrm{Cov}\ (A,C) + \mathrm{Cov}\ (B,C)\right\}}{(A + B + C)^2} + \frac{\mathrm{Var}\ (A)}{A^2} - \frac{2\left\{\mathrm{Var}\ (A) + \mathrm{Cov}\ (A,B) + \mathrm{Cov}\ (A,C)\right\}}{A(A + B + C)} \right] .$$

A test of the hypothesis that the value of ψ is different for different chemical states of the source is desired. The estimation, based on 11 sample runs, is under way. Measurements were made at 10 angles in each run.

RESOLUTION OF RESONANCE CURVES

Origin. C. Moak, Physics Division.

Participating Members of Panel. C. P. Hubbard, W. C. Sangren, R. C. Weaver.

Background and Status. In the equation

$$f(x) = \int_a^b g(y)\ h(x - y)\ dy \ ,$$

the function $h(x)$ is given analytically and $f(x)$ experimentally. The problem consists of determining the $g(x)$ that satisfies the equation. It was assumed that $g(x)$ is a resonance type of curve. The integral was evaluated numerically and the results compared with the values of $f(x)$. By properly adjusting a parameter, fairly good results were obtained. The plotted results were given to Moak.

DIRAC WAVE FUNCTIONS

Origin. M. E. Rose, Physics Division.

Participating Members of Panel. S. D. Conte, W. C. Sangren.

Reference. *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1952*, ORNL-1360.

Background and Status. Two manuscripts have been prepared that present certain extensions of ordinary Sturm-Liouville theory to equations of the Dirac type of wave equations. It is believed that further extensions can be made and that then the original questions can be answered.

BETA DECAY (FIELD FACTORS)

Origin. M. E. Rose, P. R. Bell, Physics Division.

Participating Members of Panel. N. M. Dismuke, H. B. Goertzel, N. D. Given, C. L. Perry.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1029, -1232, -1290, and -1360.

Background and Status. The computations for $\pi pL/WGJ$, ..., $\pi pR/WGJ$ were completed at MIT during August. The remainder of the computation program to evaluate $\pi pL/W$, ..., $\pi pR/W$ is the multiplication of the numerical values obtained at MIT by the values of G and J, which were evaluated at Oak Ridge by using IBM computers. It is expected that this work will be completed in November.

FITTING A CALIBRATION CURVE

Origin. J. L. Meem, Physics Division.

Participating Members of Panel. J. Moshman, R. C. Weaver, J. H. Fishel.

Reference. *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1952*, ORNL-1360.

Background and Status. Further experimentation at an activity greatly in excess of that used previously resulted in a curve with negligible variation. Theoretically the curve should be identical in shape to the calibration curve obtained at the lower activity level. The problem was resolved to one of determining the point on the lower curve that is known with greatest precision so that the known curve can be translated and tied to that point. It was shown that the optimum point was that corresponding to 30 minutes. As a check, it has been suggested that the known curve be translated to determine whether it does lie in the confidence belt constructed for the curve at the lower activity level.

DETERMINATION OF THE FAST-NEUTRON FLUX IN THE ORNL GRAPHITE REACTOR

Origin. D. K. Holmes, Physics Division.

Participating Members of Panel. J. Moshman, S. E. Ezzell, V. C. Carlock.

Reference. *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1952*, ORNL-1360.

Status. Actual histories are expected to be run on the UNIVAC in November and December 1952. Machine availability will determine the number of neutron histories that can be calculated and thus the precision that can be attained.

MINIMUM WEIGHT AND PRESSURE DROP STUDY FOR AN AIR HEAT EXCHANGER

Origin. W. S. Farmer, Reactor Experimental Engineering Division.

Participating Member of Panel. N. D. Given.

Background and Status. Algebraic formulas for the weight, length, and pressure change were evaluated for several combinations of the parameters to find the optimum dimensions for an air radiator made of stainless steel or nickel. The calculation has been completed.

CIRCULATION IN BOILING REACTORS

Origin. P. Zmola, R. V. Bailey, Reactor Experimental Engineering Division.

Participating Members of Panel. A. S. Householder, C. P. Hubbard, W. C. Sangren.

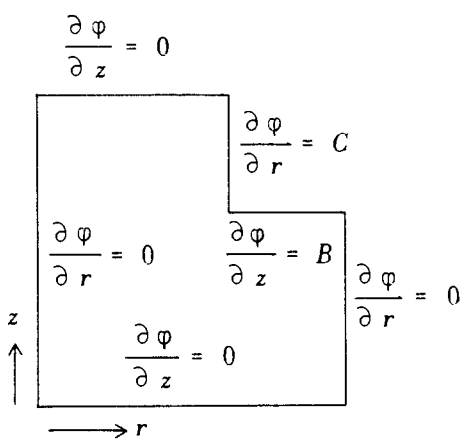
Reference. *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1952*, ORNL-1360.

Background and Status. The numerical method proposed for the solution of this problem has been modified to eliminate some difficulties relating to the boundary conditions. Since the method is applicable to other problems of this general type, one case will be considered here.

The equation

$$\frac{\partial^2\varphi}{\partial z^2} + \frac{4}{r}\frac{\partial\varphi}{\partial r} + 4\frac{\partial^2\varphi}{\partial r^2} = f(r,z) \quad ,$$

is given, with the boundary conditions shown below.



The region is divided into a mesh with n interior mesh points, and a four-point difference scheme is applied at each interior mesh point to obtain n equations of the form

$$\varphi_{i,j} = \frac{1}{10}\left[\left(4 + \frac{2h}{r_i}\right)\varphi_{i+1,j} + \left(4 - \frac{2h}{r_i}\right)\varphi_{i-1,j} + \varphi_{i,j+1} + \varphi_{i,j-1} - h^2 f_{i,j}\right] ,$$

where h is the interval length between points. Values for points on the boundary are given in terms of the adjacent interior points and the boundary conditions. The n^2 equations may now be written in matrix form:

$$M\Phi = \alpha + B\beta + C\gamma \ ;$$

Φ, α, β, and γ are column vectors. Φ_k is the kth element of $\varphi_{i,j}$, ordered systematically. M is the matrix of the coefficients of the $\varphi_{i,j}$. For problems of this type, a solution plus an arbitrary constant is also a solution. For this reason, the determinant of M vanishes, and solutions for the $\varphi_{i,j}$ exist only for the proper choice of the constants B and C. One of the $\varphi_{i,j}$ may be chosen arbitrarily, say $\varphi_{n,n} = b$. By incorporating the b's in the vector α and striking out the equation in $\varphi_{n,n}$, a system of $n - 1$ equations remains, which can be written

$$M_0\Phi_0 = \alpha_0 + B\beta_0 + C\gamma_0 \quad .$$

By solving the three systems,

$$M_0\Phi_{01} = \alpha_0 \ ,$$
$$M_0\Phi_{0B} = \beta_0 \ ,$$
$$M_0\Phi_{0C} = \gamma_0 \ ,$$

the desired solution can be constructed for

$$\Phi_0 = \Phi_{01} + B\Phi_{0B} + C\Phi_{0C} \ ,$$

where B and C must satisfy the relationship obtained from the discarded

equation in $\varphi_{n,p}$. Thus, given one of the constants, B or C, the other constant may be obtained that will ensure the existence of a nontrivial solution.

A more detailed description of this method has been forwarded to the originators of the problem in a memorandum, dated October 8, 1952. Preliminary steps have been undertaken in the coding of this problem for the ORACLE.

LINEAR STABILITY OF BOILING REACTORS

Origin. P. R. Kasten, Reactor Experimental Engineering Division.

Participating Members of Panel. A. S. Householder, V. C. Carlock, W. C. Sangren.

Background and Status. The stability under small perturbations of boiling reactors is being investigated by using linearized kinetic equations. The question of stability depends upon the roots ν of the characteristic equation associated with the linearized equations. Stability occurs when all roots of the characteristic equation have negative real parts. It can be shown that for any positive fixed values of the parameters λ, μ, a, ω^2, γ, the condition is always satisfied by taking ν sufficiently large. Since the roots of the equations are continuous functions of the coefficients, it follows that as ν decreases from $+\infty$, the condition first fails when some pair of the characteristic roots becomes pure imaginary. The condition for this is that a certain polynomial, $f(\nu;\ \lambda,\ \mu,\ a,\ \omega^2,\ \gamma)$, of the sixth degree in ν, shall vanish. It is therefore required to find the largest positive real root ν of $f = 0$, if such exists, or to determine that the equation $f = 0$ has no positive root, in which case the system is stable for all positive ν.

Further analysis shows that for fixed positive λ, μ, a, ω^2, a positive root always exists when γ is sufficiently large, that none exist when γ is small, and that as γ decreases from $+\infty$ the largest positive root decreases monotomically to some positive lower bound. Hence, if for any γ it is found that $f > 0$ for all positive ν, it is unnecessary to consider any smaller γ associated with the same values of λ, μ, a, ω^2. It should be possible therefore to eliminate certain combinations of the parameters a priori as the computation proceeds.

The computations are being programmed for the ORACLE. Because of the complexity of the coefficients of ν in f, no feasible method has been found for getting a start, in the general case, with Newton's method. Consequently the plan is to compute the Sturm functions each time, find an upper bound to the positive roots, and proceed by successively subdividing the interval. A rough estimate indicates that 3 or 4 sec will suffice for setting up any one of the equations and solving it.

KINETICS OF BOILING REACTORS

Origin. P. R. Kasten, Reactor Experimental Engineering Division.

Participating Members of Panel. J. H. Fishel, C. P. Hubbard, W. C. Sangren, R. C. Weaver.

Reference. *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1952*, ORNL-1360.

Background and Status. The coding and programming necessary for obtaining the solution on the UNIVAC of the various systems of nonlinear differential equations was completed. The completed programs were taken to the UNIVAC and the solutions were obtained. After plotting the solutions, the results were turned over to Kasten. This problem was completed.

REACTOR INEQUALITIES AND EIGENVALUES FOR THE WAVE EQUATION

Origin. A. M. Weinberg, Research Director's Division.

Participating Members of Panel. H. B. Goertzel, C. P. Hubbard, W. C. Sangren.

Reference. *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1952*, ORNL-1360.

Background and Status. A memorandum was prepared and distributed concerning the preparation of a list or table of the fundamental eigenvalues for the wave equation $\nabla^2\varphi + B^2\varphi$. The objective of the memorandum was to obtain suggestions as to the value of such a table, the general make-up of the table, and the sources of information. Preliminary studies have been made of the programming necessary for computing the fundamental eigenvalues by using the NEPA machine and the ORACLE.

CORRELATION STUDY OF BODY SIZE IN FOUR SPECIES OF FARM ANIMALS

Origin. S. L. Hansard, UT-AEC Agricultural Research Program.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. Data on age, weight, and total blood volume in burros, cattle, sheep, and swine were collected for the purpose of determining whether age or weight is more highly correlated with total blood volume. This investigation is part of a larger study in which an attempt is being made to define an adequate measure of body size for use in animal husbandry research. In each case, the method derived by Hotelling[8] was used to test the hypothesis of equality of the correlations. In all four species it was found that the correlation between weight and blood volume is greater than the correlation between age and blood volume at the 1% level of significance.

Let x_i, y_i, and z_i ($i = 1, \ldots, N$) be the measurements of age, weight, and blood volume, respectively, on the N animals within a species, and let r_{xz}, r_{yz}, and r_{xy} be the product-moment correlations between the indicated variables. Then if

$$D = \begin{vmatrix} 1 & r_{xz} & r_{xy} \\ r_{xz} & 1 & r_{yz} \\ r_{xy} & r_{yz} & 1 \end{vmatrix},$$

the quantity

$$t = \frac{\sqrt{N-3}\,(r_{xz} - r_{yz})\sqrt{1 + r_{xy}}}{\sqrt{2D}}$$

is distributed approximately as "Student's" t with $N - 3$ degrees of freedom under the null hypothesis that $E(r_{xz}) = E(r_{yz})$. It should be noted that the usual test for the difference between two correlation coefficients based on Fisher's z transformation cannot be applied here, since r_{xz} and r_{yz} are not independent.

ANALYSIS OF ENDOGENOUS EXCRETION OF CALCIUM IN CATTLE

Origin. M. Visek, C. L. Comar, UT-AEC Agricultural Research Program.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. One of the perennial unknowns in animal nutrition is the effect of dietary factors on the amount of nutrient actually utilized by the animal. In an attempt to investigate this problem in cattle, two growing steers and two dairy cows were fed on normal calcium diets and high calcium diets, with a suitable period being allowed for adjustment after change in diet. Each day, for either 4 or 5 days, the endogenous calcium in the excretions of the animals was determined. The means are

(8) H. Hotelling, *Ann. Math. Stat.* **11**, 271 (1940).

shown in Table 2. The corresponding analysis of variance is shown in Table 3. There appears to be a difference between dairy cattle and growing steers, but none of the other factors seem to have an effect on endogenous calcium. The power of these tests is of course very poor because of the small number of animals. With such small numbers only very large effects can be demonstrated statistically.

TABLE 2. AVERAGE DAILY EXCRETION OF ENDOGENOUS CALCIUM

ANIMAL NO.	GROWING STEERS		MEANS	ANIMAL NO.	DAIRY COWS		MEANS
	Normal Ca*	High Ca*			Normal Ca*	High Ca*	
1	3.61	2.99	3.30	3	6.72	8.89	7.81
2	4.47	5.15	4.81	4	7.23	6.89	7.06
Means	4.04	4.07	4.06	Means	6.98	7.89	7.43

*In grams per day.

TABLE 3. ANALYSIS OF VARIANCE

SOURCE OF VARIATION	DEGREES OF FREEDOM	MEAN SQUARE	LEVEL OF SIGNIFICANCE (%)
Between types of animal	1	114.18	6.3
Between animals within same type	2	7.13	
Between normal and high calcium intake	1	2.23	> 50
Animal type by calcium intake interaction	1	1.98	> 50
Residual error	2	5.00	
Among daily measurements on same animal	30	1.36	

SECRET PROJECTS

DESIGN OF USAF SCHOOL OF AVIATION MEDICINE PRIMATE LABORATORY - ORNL PILOT TOLERANCE EXPERIMENT

Origin. ANP Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. In conjunction with the ANP shielding program, an experiment to provide information on pilot tolerance has been planned. The execution of the experiment is under the direction of the USAF School of Aviation Medicine Primate Laboratory, with assistance and advice from various ORNL personnel. Advice on the design of the experiment has been submitted jointly by members of the Department of Medical Statistics of the School of Aviation Medicine and a representative of the ORNL Mathematics Panel.

The experiment is factorial in nature. The four factors of major interest are rate of exposure, number of exposures, interval between exposures, and length of exposures. The total number of animals available for the experiment will be between 120 and 160. At first it was planned to run each factor at only two levels, which, with a control group, would have allowed for from 5 to 7 replications. It soon became evident, however, that for some types of biological measurements, linearity of effect could not be assumed and that no less than three levels of each factor would be required. For one replication of this experiment, $3^4 = 81$ animals are needed. If as many as 162 animals can be handled, two replications may be run and the analysis of the experiment will reduce to standard calculations, which can be performed on hand computers. If fewer than 162 animals are used, the analysis will require the inversion of a symmetric positive-definite matrix of order 81×81 and of symmetric and asymmetric positive-definite matrices of order somewhat less than 81×81. By the time the experiment is completed it is hoped that the ORACLE will be available for these computations if they become necessary.

The possibility of using fractional replication was investigated. Five or six replications of a one-third replicate of the basic 3^4 factorial could be handled with the number of animals expected to be available. If three- and four-factor interactions could be assumed negligible, this design would have some advantage over the previous one. A careful scrutiny of the types of measurement contemplated revealed that some of the psychological and possibly one or two of the biological measurements would actually be expected to yield significant three-factor interactions. Since the same design has to serve for all measurements, the fractional design had to be discarded.

DEPIGMENTATION AS A BIOLOGICAL DOSIMETER – OPERATION GREENHOUSE

Origin. A. C. Upton, Biology Division.

Participating Members of Panel. J. Moshman, P. J. Brown, K. P. Graw.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1290 and -1360.

Background and Status. An alternate method of analysis to that described in the references is under investigation. A linear relation was assumed between dose and degree of depigmentation at the six anatomical sites, and, successively, all combinations of 1, 2, 3, etc. independent variables were examined to determine at which point the residual variance about the regression equation was not significantly reduced. For the specific data used in this analysis, the variance was reduced from 43,910 to 6,271 by using variable 1. The best combination of two variables further reduced the variance to 5938, and the best combination of three variables reduced the variance to 5812. Since 5938 was not significantly lower than 6271 on the basis of an F test with 229 and 228 degrees of freedom, the best linear function was that in which only variable 1 was used. The regression equation was

$$d = 275.74 + 109.47\, p_1 ,$$

where d is the dose received and p_1 is the degree of depigmentation at site 1. The value of p_1, however, was constant for the higher and lower doses. By omitting those doses that had the same effect on the loss of coloring and by repeating the procedure, the equation

$$d = 318.32 + 84.09\, p_1$$

was obtained, which is valid for $287 \leqslant d \leqslant 711$; the dose is expressed in roentgens.

It is planned to compare the above results with those of similar analyses applied to data for different periods following exposure.

CALCULATION OF ACTIVITY OF FISSION-PRODUCT GAS

Origin. T. H. J. Burnett, Health Physics Division.

Participating Member of Panel. E. N. Lawson.

Background and Status. Calculations have been made of the activity and concentration of the fission-product gas from the Teapot Reactor for use in studies of waste gas disposal.

POWER REMOVAL FROM BOILING REACTORS

Origin. R. V. Bailey, Reactor Experimental Engineering Division.

Participating Member of Panel. E. N. Lawson.

Background and Status. Calculations were made to obtain performance parameters for use in the study of power removal from boiling reactors.

STUDY OF FUEL ENRICHMENT REQUIRED FOR LARGE-SCALE, HOMOGENEOUS, BOILING REACTORS

Origin. P. C. Zmola, Reactor Experimental Engineering Division.

Participating Member of Panel. E. N. Lawson.

Background and Status. The calculated results are presented in the *Homogeneous Reactor Project Quarterly Progress Report for Period Ending October 1, 1952*, ORNL-1424.

CODING OF REACTOR CALCULATIONS FOR THE ORACLE

Origin. R. A. Charpie, Research Director's Division.

Participating Members of Panel. N. Edmonson, J. H. Fishel.

Reference. D. K. Holmes, *The Multigroup Method as Used by the ANP Physics Group*, ANP-58 (Feb. 15, 1951).

Background and Status. Since it is desired to have a reactor calculation ready as soon as the ORACLE is available for computations, coding is in progress on the procedure outlined in the reference. At this stage of the coding, no apparent difficulties have been encountered as far as memory capacity is concerned.

OPTIMUM SHIELDING FOR A CYLINDRICAL REACTOR

Origin. C. Clifford, Physics Division.

Participating Members of Panel. R. C. Weaver, C. L. Perry.

Background and Status. Preliminary calculations are being performed to determine the dimensions of an optimum, homogeneous, cylindrical, neutron shield for a cylindrical reactor. The optimization was with respect to shield weight, subject to the condition that the neutron dose at a point on the axis of the cylinder (external to the shield) has a fixed value. The empirical formula

$$D = N_x e^{-x/l_x} + N_y e^{-y/l_y} + N_z e^{-z/l_z} ,$$

where x, y, and z are the dimensions of the shield and N_x, N_y, N_z, l_x, l_y, l_z, and the dose D are known constants for the geometry and dose considered, was used as the relation between dose and shield dimensions.

DIFFUSIONS OF XENON-135 IN ANP FLUORIDE FUELS

Origin. M. T. Robinson, Solid State Division, ANP Project.

Participating Member of Panel. N. Edmonson.

Reference. *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1952*, ORNL-1360.

Background and Status. This problem has been formulated as the solution of the boundary value problem. If the concentration of the Xe^{135} is denoted by N_2, the following equations are obtained:

$$\frac{\partial N_2}{\partial t} = D_L \frac{\partial^2 N_2}{\partial X^2} - \lambda_2 N_2 + k\, e^{-\lambda_1 t} , \quad (1)$$

for the liquid phase;

$$\frac{\partial N_2}{\partial t} = D_V \frac{\partial^2 N_2}{\partial X^2} - \lambda_2 N_2 \quad , \qquad (2)$$

for the vapor phase;

$$\left.\frac{\partial N_2}{\partial X}\right|_{X=0} = 0 \quad ; \qquad (3)$$

$$-D_L \left.\frac{\partial N_2}{\partial X}\right|_{X=l} = -D_V \left.\frac{\partial N_2}{\partial X}\right|_{X=l} \quad , \qquad (4)$$

where D_L and D_V are, respectively, the diffusion coefficients for the liquid phase and the vapor phase, and l is the X coordinate of the phase interface;

$$\lim_{X\to\infty} N_L = 0 \quad ; \qquad (5)$$

$$N_2(X,0) = K \quad , \qquad (6)$$

where K is a constant for the liquid phase;

$$N_2(X,0) = 0 \quad , \qquad (7)$$

for the vapor phase;

$$\int_0^l N_2 \, dX + \int_l^\infty N_2 \, dX = f(t) \quad . \qquad (8)$$

A solution by the method of the Laplace transform is being attempted. The inversion will probably be complicated.

NEUTRON FLUX IN THE LITR

Origin. J. Trice, Solid State Division.

Participating Members of Panel. R. C. Weaver, C. L. Perry.

Background and Status. Threshold detectors may be used to investigate the distribution of neutron flux, $\Phi(E)$, as a function of E in the range from E = 0.5 Mev to E = 10 Mev. If it is assumed that the cross section of the detector is a simple step function, that is, $\sigma(E) = 0$ for $E < E_0$ and $\sigma(E) = \sigma_s$ (a constant) for $E > E_0$, a simple numerical procedure can be used to determine the E_0 and the σ_s from the experimentally obtained activities of the detector. The calculations for one experiment have been completed.

DETERMINATION OF LCAOMO ENERGY LEVELS IN CORONENE

Origin. M. T. Robinson, Solid State Division.

Participating Member of Panel. R. C. Weaver.

Background and Status. The energy levels of coronene may be represented by the characteristic values of a matrix of order 24. Group theory methods (based on the molecular symmetries of coronene) can be used to reduce the problem to finding the characteristic values of two determinantal equations of degree 8 and some of lower degree. These equations have been found and their roots evaluated to four significant figures.