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LABORATORY RECORDS 1954

MATHEMATICS PANEL
SEMIANNUAL PROGRESS REPORT
FOR PERIOD ENDING JUNE 30, 1953

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MATHEMATICS PANEL
SEMIANNUAL PROGRESS REPORT
For Period Ending June 30, 1953

A. S. Householder, Chief
W. C. Sangren, Editor

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MATHEMATICS PANEL SEMIANNUAL PROGRESS REPORT

INTRODUCTION

THE ORACLE

During the period covered by this report, the construction of the ORACLE was completed. The arithmetic unit and the computer power equipment had been finished prior to this period. Work on the memory unit continued throughout the period, and by June, the memory construction was completed. A temporary console has been built, and the teletype input-output equipment has been constructed and tested.

On June 2, the memory unit was wired to the arithmetic unit and the testing of the integrated computer began. The same day, the computer successfully executed a memory reading and writing test. In the following few days, several other test routines were successfully executed: an input-output unit test, read-around and other memory unit tests, and several arithmetic unit tests. All test routines run so far have indicated that the computer is functioning as expected.

During the next few weeks, testing of the ORACLE will continue. Some work remains to be done on the adjustment of the memory system and the selection of cathode-ray tubes. J. C. Chu and R. J. Klein are carrying out this program.[1] An electronic device to test read-around is planned for construction. Design work has already been started on a more permanent console. Design work continues on the magnetic-tape units; however, it is not expected that the magnetic-tape units will be ready for use for some time after the computer is installed and operating in Oak Ridge.

Plans for the installation of the computer in Oak Ridge are being executed on schedule so that the preliminary power wiring and air conditioning will be ready when the computer arrives. It is expected that the ORACLE will be installed and operating in Oak Ridge in the early fall.

LECTURES

Members of the Mathematics Panel presented the following lectures on the Traveling Lecture Program.

A. S. Householder, *Analysis of Error in Digital Computations,* University of Florida, Feb. 16, 1953; *Logical Structure of an Automatic Digital Computing Machine,* Georgia Institute of Technology, Feb. 18, 1953; *Analysis of Error in Digital Computations and Problem Preparation for Automatic Digital Computation,* University of Georgia, Feb. 19, 1953; *Logical Design on an Automatic Digital Computer,* University of Alabama, Feb. 23, 1953; *Errors in Iterative Solutions of Linear Algebraic Systems,* University of Illinois, Apr. 13, 1953.

A. W. Kimball, *Statistical Applications in Biological Research,* Oklahoma A. and M. College, Apr. 7, 1953.

W. C. Sangren, *General Purpose Automatic Digital Computing Machines,* Duke University, Mar. 10, 1953.

Another series of lectures on the ORACLE was given during the weeks of June 15 and June 22. These were given by A. S. Householder, J. Moshman, M. R. Arnette, and N. M. Dismuke.

At the request of the Analytical Chemistry Division, a series of ten lectures on the application of statistics in chemistry was presented, beginning on March 26, by A. W. Kimball. The topics covered include

---

[1] J. C. Chu, "The Oak Ridge Automatic Computer," p. 142; J. C. Chu and R. J. Klein, "Williams Tubes Selection Program," p. 110, *Proceedings of the Association for Computing Machinery* (Meeting at Toronto, Ont., Sept. 8-10, 1952), Sauls Lithograph Co., Washington, D. C.

quality control, factorial experiments, and curve fitting.

J. Z. Hearon presented a discussion *On the Concept of Master Reaction* at a seminar of the Biology Division on March 5, 1953.

W. C. Sangren discussed *General Purpose Automatic Digital Computing Machines* at a seminar of the Electromagnetic Division on March 13, 1953.

LIST OF PUBLICATIONS

A. S. Householder, "Errors in Iterative Solutions of Linear Systems," p. 30, *Proceedings of the Association for Computing Machinery*. (Meeting at Toronto, Ont., Sept. 8-10, 1952), Sauls Lithograph Co., Washington, D. C.

J. Z. Hearon, "Nonlinear Diffusion in Metabolic Systems," *Bull. Math. Biophys.* **15**, 15 (1953).

J. Z. Hearon, "Comments on the Approximate Solution of the Diffusion Equation: I," *Bull. Math. Biophys.* **15**, 23 (1953).

A. W. Kimball, "The Fitting of Multi-hit Survival Curves," *Biometrics*, June 1953.

C. L. Perry, "The Logical Design of the Oak Ridge Digital Computer," p. 23, *Proceedings of the Association for Computing Machinery* (Meeting at Toronto, Ont., Sept. 8-10, 1952), Sauls Lithograph Co., Washington, D.C.

EMPLOYEE CHANGES

E. L. Drabkin, N. Edmonson, Jr., and H. B. Goertzel terminated during this period. A. C. Downing, Jr., formerly of the University of Michigan, was added to the staff. C. L. Perry, Jr., formerly of the Mathematics Panel and now associated with the U. S. Naval Postgraduate School, Monterey, California, has been working with members of the Panel this summer in connection with the preparation for the arrival of the ORACLE this fall.

L. P. Burton from the University of California, Nathaniel Macon from the Alabama Polytechnic Institute, and W. H. Spragens from the University of Mississippi have joined the staff as research participants. Also, the following ORINS Fellows are participating in the work of the Panel: J. F. Elliott and R. D. Sheffield from the University of Tennessee and F. W. Stallard from the University of North Carolina.

UNCLASSIFIED PROJECTS

TEST ROUTINES FOR THE ORACLE

Participating Member of Panel. C. P. Hubbard.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1498.

Background and Status. At the time this report was written, the testing period for the Oak Ridge computer being built at the Argonne National Laboratory had begun. Prior to the beginning of the testing, many test routines had been programmed to determine whether or not the various units of the computer would function as planned. These routines included:

1. *Teletype, Input-Output Test.* The teletype, input-output test is for determining whether or not information is accurately read into and out of the computer.

2. *Memory Test.* The memory test is for determining whether or not the memory can accept and store zeros and ones at every address.

3. *Read-Around Tests.* Read-around is the phenomenon of unwanted altering of information in memory cells adjacent to the one cell being interrogated. The test is being made to locate and identify errors of this type.

4. *Arithmetic Unit Tests.* The arithmetic unit tests are designed to check the operation of all the various registers, drivers, and control circuits associated with the execution of the machine orders.

Now that the testing is actually in progress, these routines and the other routines being programmed are constantly being changed to meet the demands for a rigorous test program.

THE INPUT PROGRAM

Participating Members of Panel. M. R. Arnette, H. B. Goertzel, C. L. Perry.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1498.

Background. Four steps are required to put a problem on the ORACLE: the formulation of the mathematical model of the physical problem, the selection of a numerical method, the programming, and the coding. The first two steps are familiar to many scientists and engineers. The third step involves a step-by-step exposition of the numerical method to be used with the ORACLE. Three two-week courses have been given by the Mathematics Panel to acquaint Oak Ridge personnel with programming. Coding consists of clerical tasks, many of which can be performed by the ORACLE. An Input Program has been designed to perform many of these coding tasks.

A computing routine consists of

1. numbers,
2. commands,
3. groups of commands and numbers (for example, subroutines).

For numbers, the Input Program automatically converts from decimal representation to ORACLE binary representation, or it reads in previously converted numbers. The Input Program recognizes whether or not a number is in the decimal form by a code. The code prefix *D*/3 indicates to the ORACLE Input Program that the number is a decimal integer. The prefix *D*/4 indicates that the number is a decimal fraction, and the code *D*/0 indicates that the number is in ORACLE binary representation, that is, the ORACLE is to read-in without conversion. A modification of the Input Program also converts decimal floating-point numbers, that is, numbers in scientific notation.

For commands, the Input Program automatically inserts conditional stops (breakpoint stops) in orders, converts addresses from decimal to binary, and provides for arbitrary placement of the program in the memory; that is, the addresses can be relative or absolute. The modifications for a command are indicated by prefixes similar to those for orders. The

Input Program thus performs all changes of the *first* kind (changes which remain constant during the use of a subroutine in a program) in subroutines.

Three alternatives are possible with the Input Program:

1. conversion and input to the fast memory,
2. conversion and input to the magnetic-tape auxiliary memory,
3. conversion and storage on paper tape.

Status. The first Input Program, IP-1, has been completed and is being tested on the ORACLE. IP-2, the floating-point modification of the Input Program, is being completed. A more detailed description of the Input Programs can be found in the ORACLE Manual.

METHODS OF COMPUTATION FOR USE WITH A HIGH-SPEED AUTOMATIC COMPUTER

Participating Members of Panel. W. Givens, V. C. Klema.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1498.

Problem 3. Numerical Computation of Characteristic Values and Characteristic Vectors.

Status. A report on the proposed method of computing the characteristic values of a real symmetric matrix has been completed and is being reproduced for distribution. The report contains a detailed description of the computation and bounds obtained for the maximum possible round-off error. Flow diagrams and codes have been prepared but have not yet been machine tested. They are designed to be applicable to matrices capable of being stored in the internal memory and hence of maximum order about 36. Variants of these codes that will use the external memory are needed if larger matrices are to be processed or if the characteristic vectors are also wanted. An outline of a straightforward method for obtaining the characteristic vectors is included in the above-mentioned report, but error bounds have not been obtained.

Problem 5. Roots of a Polynomial.

Status. Work on this project is continuing.

REACTOR INEQUALITIES AND EIGENVALUES FOR THE WAVE EQUATION

Origin. A. M. Weinberg, Research Director's Division.

Participating Members of Panel. H. Goertzel, W. C. Sangren.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1360, -1435, and -1498.

Background and Status. The fundamental eigenvalues for concentric spheres, three semi-infinite slabs, and segments of a circle were computed on the UNIVAC, and the results were published in a memorandum. The coding necessary for finding the fundamental eigenvalue for a pair of concentric squares has been completed for the ORACLE.

In a report to be issued presently, many of the results of G. Polyá and G. Szegö are shown to carry over from the plane to n-space. In particular, the fundamental eigenvalue is shown to decrease under Steiner symmetrization. It is also shown that for a very large class of regions, in n-space with a given volume, the sphere has the smallest fundamental eigenvalue. Simple proofs of the continuity of the fundamental eigenvalue as a function of the region and the basic inequality – if B' is a subregion of B then the fundamental eigenvalue of B' is greater than that of B – are given for a space of any dimension. An example is given which shows how fairly precise bounds on the fundamental eigenvalue may be obtained by using these results.

A memorandum has been prepared which deals with the question of the self-adjointness of wave equations associated with M-group and N-region problems.

RADIATION-INDUCED SEX DIFFERENCES IN *DROSOPHILA* MELANOGASTER

Origin. W. K. Baker, Biology Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. For some time, it has been known that x radiation alters the usual one-to-one sex ratio in favor of males among offspring from irradiated *Drosophila* males. The present experiment was designed to evaluate simultaneously the oxygen effect and the age of sperm as related to the same phenomenon.

Males were exposed to 4000 r of x radiation. Each male was then mated to several unirradiated females at two different times after irradiation to provide two different sperm batches representing the age of sperm effect. In each experiment, one-half the males were irradiated in an atmosphere of air and one-half in nitrogen. After testing for homogeneity of sex ratios among the offspring of the males in each treatment group, the offspring from all males in each group were pooled. The results are shown in Table 1.

The total numbers of offspring varied from 152 to 1232. The data seem amenable to treatment by analysis of variance, but even after the arcsin transformation is made, weights varying by a factor of almost 10 would be required. Even for a two-way classification, such a weighted analysis is difficult to interpret, and for a three-way classification, it would be even harder.

Accordingly, a chi-square analysis was substituted for the analysis of variance. For each experiment, each pair of sperm batches yielded a chi-square with one degree of freedom. For air, the chi-squares were 0.940, 2.648, 1.239, 0.027, and 0.050, respectively. Since the square root of a chi-square with one degree of freedom is a normal variate with zero mean and unit variance, the statistic,

$$t = \frac{0.970 + 1.627 + 1.113 + 0.165 - 0.223}{\sqrt{5}} = 1.63,$$

provided a test of the equality of sperm batches. Each square root is assigned a sign corresponding to the sign of the difference of the two percentages. The t for air corresponds to a significance level of 10%. Likewise, for nitrogen,

$$t = \frac{-2.057 - 2.319 - 1.583 + 0.440}{\sqrt{4}} = -2.760,$$

which corresponds to a significance level of 0.6%. On the basis of these tests, it appears that in nitrogen the

TABLE 1. PERCENTAGES OF MALES AMONG OFFSPRING

EXPERIMENT NUMBER	MALE OFFSPRING (%)			
	From Males Irradiated in Air		From Males Irradiated in Nitrogen	
	Sperm Batch 1	Sperm Batch 2	Sperm Batch 1	Sperm Batch 2
1	69.4	67.1		
2	65.7	61.6	57.9	67.9
3	61.8	57.6	57.9	65.3
4	66.1	65.6	60.0	64.0
5	61.9	62.8	59.6	58.4

second sperm batch yields a higher percentage of males than the first, while in air, the effect, if it exists, is in the opposite direction.

An alternative approach would have been to pool the chi-squares to obtain total chi-squares with 5 and 4 degrees of freedom, respectively. This method, however, ignores the direction of the differences and results, in general, in a less powerful test of the hypothesis than is provided by working with the normal variates.

TESTING FOR INDEPENDENCE IN GENETIC RECOMBINATION EXPERIMENTS WITH BACTERIOPHAGE T4

Origin. A. H. Doermann, Biology Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background. Recombination experiments with bacteriophage T4 using *r* and *tu* markers yield about 90% of the parental type and 10% recombinants. The results of a typical experiment are shown in the first line of Table 2. The desired test for independence refers only to the recombinants. In particular, the null hypothesis is that the events leading to the two types of single recombinants are independent. Since the greatest effort is expended in counting plaques and since 90% of the plaques yield no direct information concerning the independence hypothesis, the experiments were divided into two parts. In the first part, all classes were counted, but in the second part, only the recombinants were counted. In the experiment shown in Table 2, this procedure resulted in a reduction of about 60% in the number of plaques counted. The statistical problem was to combine both parts of the experiment in a single test of independence.

Status. The observed and expected frequencies corresponding to this type of experiment may be represented as in Table 3, where p_1 and p_2 are the probabilities of the events leading to recombinants r^+ and tu^+, respectively, and where N is the total number of plaques in the second part of the experiment. In this problem, N, p_1, and p_2 all appear as parameters to be estimated.

Under the null hypothesis, the probability of the sample is

$$\frac{M!}{a_1!a_2!a_3!a_4!}\,[(1-p_1)(1-p_2)]^{a_1}\,[p_1(1-p_2)]^{a_2}\,[p_2(1-p_1)]^{a_3}\,[p_1p_2]^{a_4}$$

$$\times\frac{N!}{[N-a_5-a_6-a_7]!\;a_5!a_6!a_7!}\,[(1-p_1)(1-p_2)]^{N-a_5-a_6-a_7}\,[p_1(1-p_2)]^{a_5}$$

$$\times\,[p_2(1-p_1)]^{a_6}\,[p_1p_2]^{a_7}\,.$$

TABLE 2. PLAQUE COUNTS

	PARENTALS	r^+	tu^+	r^+tu^+	TOTALS
All classes counted	7801	230	524	18	8573
Only recombinants counted		400	1073	27	

TABLE 3. OBSERVED AND EXPECTED FREQUENCIES

		PARENTALS	r^+	tu^+	r^+tu^+	TOTALS
All classes counted	Observed	a_1	a_2	a_3	a_4	M
	Expected	$M(1-p_1)(1-p_2)$	$Mp_1(1-p_2)$	$Mp_2(1-p_1)$	Mp_1p_2	M
Only recombinants counted	Observed		a_5	a_6	a_7	
	Expected		$Np_1(1-p_2)$	$Np_2(1-p_1)$	Np_1p_2	

For large N and particularly where ratios of factorials are involved, Stirling's approximation for $N!$ is quite accurate. When this substitution is made, the maximum likelihood equations are found to be

$$\frac{a_2 + a_4 + a_5 + a_7}{p_1} - \frac{a_1 + a_3 + N - a_5 - a_7}{1 - p_1} = 0 ,$$

$$\frac{a_3 + a_4 + a_6 + a_7}{p_2} - \frac{a_1 + a_2 + N - a_6 - a_7}{1 - p_2} = 0 ,$$

$$\log (1 - p_1) + \log (1 - p_2) + \frac{N + \frac{1}{2}}{N} + \log N - \frac{N + \frac{1}{2} - a_5 - a_6 - a_7}{N - a_5 - a_6 - a_7}$$

$$- \log (N - a_5 - a_6 - a_7) = 0 .$$

If the term

$$\frac{N + \frac{1}{2}}{N} - \frac{N + \frac{1}{2} - a_5 - a_6 - a_7}{N - a_5 - a_6 - a_7}$$

is ignored as negligible, the three equations reduce to

$$\hat{p}_1 = \frac{a_2 + a_4 + a_5 + a_7}{M + \hat{N}} , \qquad \hat{p}_2 = \frac{a_3 + a_4 + a_6 + a_7}{M + \hat{N}} ,$$

$$[2(M - a_1) + a_7 - a_2 - a_3]N^2 - [(a_1 - a_7)(a_1 + a_2 + a_3 - a_5 - a_6 - a_7)$$

$$+ (a_3 - a_5)(a_2 - a_6) + 2M(a_5 + a_6 + a_7) - M^2]N - [M^2(a_5 + a_6 + a_7)] = 0 ,$$

and $\hat{N}$ is given by the positive root of the third equation.

With the use of these estimates, χ^2 may be computed in the usual manner. Since three parameters have been estimated and since there are two linear restrictions, that is,

$$\sum_{i=1}^{4} [a_i - E(a_i)] = 0 , \qquad \sum_{i=5}^{7} [a_i - E(a_i)] = 0 ,$$

the computed χ^2 will have 2 degrees of freedom.

When the method is applied to the data in Table 2, the estimates are found to be $\hat{N} = 16{,}635$, $\hat{p}_1 = 0.026778$, and $\hat{p}_2 = 0.065138$. The resulting χ^2 is 3.535, which for 2 degrees of freedom is significant at the 17% level. On this evidence, there is no reason to suspect the validity of the hypothesis of independence.

DETERMINATION OF THE MEAN LIFE OF THE BLOOD PLATELET

Origin. J. Furth and T. T. Odell, Biology Division.

Participating Members of Panel. J. Z. Hearon, A. W. Kimball, G. J. Atta.

Background and Status. At time zero, radioactive C^{14}-labeled formate was injected (intraperitoneally) into adult rats. At subsequent times, the number of radioactive counts per platelet, $G(t)$, was determined by direct count on a suitable aliquot.

The equation for $G(t)$ is, in general,

$$(1) \quad G(t) = \int_0^t F(\theta)\, \phi(t - \theta)\, d\theta \quad ,$$

where $F(\theta) = C(\theta)\, R(\theta)/N$, $C(\theta)$ is the number of counts incorporated into a platelet at time θ, $R(\theta)$ is the rate of formation of platelets, and N is the total number of platelets, which is assumed constant. The "mortality function" is

$$\phi(\tau) = \int_\tau^\infty \psi(x)\, dx \quad ,$$

where ψ is the distribution of deaths. It was assumed that

$$F(t) = AC^{-\alpha t} t^2$$

and that

$$\psi(t) = \frac{e^{-t/\beta}}{\alpha!\,\beta^{\alpha+1}} \quad .$$

The constants A, α, and β enter as parameters to be estimated, and the mean life is given by $\bar{t} = \beta(\alpha + 1)$. With these assumptions,

$$(2) \quad G(t) = P_1(t)\, e^{-t/\beta} + P_2(t)\, e^{-\alpha t} \quad ,$$

where P_1 and P_2 are polynomials in t of degree α, whose coefficients are proportional to A and depend upon powers of $1/(\alpha - 1/\beta)$. With the further restriction that $\alpha = 2$, fits of Eq. 2 to the data gave the estimate $\bar{t} = 4.5$ days. Approximate, but suitable, graphical methods for fitting Eq. 2 are being sought. The formulation, experimental, and fitting procedures for the transfer experiment, wherein tagged platelets are transferred into a normal animal, are being worked out. This formulation requires no assumption as to the nature of $F(t)$ and no restriction on α.

EVALUATION OF AN INTERACTION EFFECT IN *TRADESCANTIA* EXPERIMENTS

Origin. D. Schwartz, Biology Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. In x radiation experiments with *tradescantia*, the relationships between dose and certain chromosomal aberrations are well established. The number of isochromatid breaks per cell (y) when plotted against dose (x) yields a straight line through the origin, $y = ax$. On the other hand, the number of exchanges per cell (u) when plotted against dose yields a power curve through the origin, $u = a'x^b$. Attempts have been made in the past to explain these effects on the basis of hit theory. In the present experiments, however, interest was centered on the interaction of environment and time after irradiation as related to the dose-aberration effect.

The data were taken from six experiments in each of which *tradescantia* buds were exposed to varying doses of x radiation. In three of the experiments, exposure took place in air and in the other three, in nitrogen. Each set of three experiments consisted of isochromatid and exchange counts made at 8, 12, and 24 hr after exposure. The appropriate functions were fitted to each set of counts to provide estimates of the parameters and their variances. The estimates are shown

in Table 4. Since the variances were homogeneous, an unweighted analysis of variance was carried out for each set of parameter estimates. These results are shown in Table 5.

The interaction for parameter a of the isochromatid fits is significant at the 5% level and indicates, in this case, a decrease in the difference between the O_2 and N_2 effects as the number of hours after exposure is increased. Neither parameter in the exchange fits manifested an interaction. Most of the main effects were highly significant, but this information was already known.

EFFECT OF X RADIATION ON CHICK TISSUES

Origin. J. R. Totter, Biology Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. Wet-weight determinations of adenine triphosphatase (ATPase) were performed on seven tissues from two groups of chicks. One group of five chicks was exposed to 800 r of x radiation, and the tissues were prepared for analysis within 24 hr after irradiation. The other group of four chicks served as controls. The means are shown in Table 6. An over-all test of the difference between the two groups of chicks was desired.

Since the different tissue measurements on any one chick are correlated, the desired test is given by Hotelling's T^2, which is a multivariate generalization of Student's test for one variate. Let x_{ij} and y_{ik} ($i = 1, \ldots, p$; $j = 1, \ldots, N$; $k = 1, \ldots, M$) be the control and irradiated samples, respectively, where p is the number of tissues, N is the number of chicks in

TABLE 4. PARMAETER ESTIMATES

TIME AFTER EXPOSURE (hr)	ISOCHROMATID BREAKS, a		EXCHANGES, a'		EXCHANGES, b	
	In O_2	In N_2	In O_2	In N_2	In O_2	In N_2
8	0.455	0.160	0.0148	0.00059	1.90	2.13
12	0.451	0.174	0.0111	0.00306	1.73	1.70
24	0.317	0.150	0.0430	0.00258	1.34	1.72

TABLE 5. ANALYSIS OF VARIANCE

SOURCE OF VARIATION	DEGREES OF FREEDOM	MEAN SQUARES		
		a	a'	b
O_2 vs. N_2	1	0.090897	1.6822*	0.05570
Among hours	2	0.003902	0.3041	0.11952
Interaction	2	0.002381	0.1936	0.02101
Pooled error variances		0.0005585	0.1052	0.02670
Degrees of freedom		24	33	33

*Analysis performed on log a'.

TABLE 6. ATPase IN CHICK TISSUE

TISSUE	AVERAGE ATPase (μg)	
	Controls	Irradiated
Brain	27.8	47.7
Breast muscle	21.8	24.8
Heart	5.9	8.9
Kidney	6.0	7.8
Gizzard	2.2	3.8
Leg muscle	9.8	14.0
Liver	4.1	5.3

the control group, and M is the number of chicks in the irradiated group. Further, let

$$\bar{x}_{i\cdot} = \frac{1}{N}\sum_j x_{ij} ,$$

$$\bar{y}_{i\cdot} = \frac{1}{M}\sum_k y_{ik} ,$$

$$(N + M - 2)\, s_{ii'} = \sum_j (x_{ij} - \bar{x}_{i\cdot})(x_{i'j} - \bar{x}_{i'\cdot}) + \sum_k (y_{ik} - \bar{y}_{i\cdot})(y_{i'k} - \bar{y}_{i'\cdot}) .$$

Then T^2 is given by

$$\frac{N + M}{NM} T^2 = \sum_{i,i'} s^{ii'}(\bar{x}_{i\cdot} - \bar{x}_{i'\cdot})(\bar{y}_{i\cdot} - \bar{y}_{i'\cdot}) ,$$

where $[s^{ii'}]$ is the inverse of $[s_{ii'}]$. Under the null hypothesis of equality of covariance matrices and equality of means, the statistic,

$$F = \frac{(N + M - p - 1)T^2}{p(N + M - 2)} ,$$

has the variance ratio distribution with $(N + M - 2)$ and $(N + M - p - 1)$ degrees of freedom. In this problem $F = 1.454$, which with 7 and 1 degrees of freedom is not significant at the 20% level. The lack of significance can be attributed in part to the small sample sizes which result in only one degree of freedom for the denominator of the F-ratio. A slight increase in sample size would substantially increase the sensitivity of the test.

DAMAGE TO TISSUE FROM NEUTRON DISTRIBUTIONS WITH MAXIMUM ENERGIES RANGING FROM 1 TO 300 kev

Origin. J. Neufeld and W. S. Snyder, Health Physics Division.

Participating Members of Panel. P. J. Brown, N. M. Dismuke, J. Moshman.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1435.

Status. The problem described in ORNL-1435 has been changed in some respects and modified in others. A detailed description of the problem, which has been coded for the ORACLE, is given in a memorandum to W. S. Snyder and J. Neufeld from Brown, Dismuke, and Moshman, entitled *Neutron Damage to Human Tissue,* April 2, 1953.

One of the codes records information about each collision of each neutron, and a second code tabulates the data. In order to decrease input and output time, the code and the problem may be revised or simplified before the ORACLE computations are made.

DISORDERING OF SOLIDS BY HEAVY CORPUSCULAR RADIATION

Origin. J. Neufeld and W. S. Snyder, Health Physics Division.

Participating Member of Panel. C. L. Perry.

Reference. *On the Disordering of Solids by Heavy Corpuscular Radiation* (to be published as an ORNL report).

Background. Neufeld and Snyder have formulated the equation

$$G(E) = 1 + \int_a^E dx \int_a^x K(x,t)\, G(t - a)\, dt$$

for the number $G(E)$ of atoms of a solid displaced by heavy corpuscular radiation (cf, reference given above). They have also found analytical expressions for the upper and lower bounds of $G(E)$.

Status. The analysis in the report referenced above has been checked. Numerical estimates for $G(E)$ could easily be calculated by using the ORACLE. For small values of E, $G(E)$ can be determined exactly by evaluating a formula involving the function

$$\int_b^x \frac{\ln x}{a + x}\, dx ,$$

which has not been extensively tabulated. This function occurs in other applications. Its tabulation could be considered as a project for the ORACLE.

ESTIMATION OF PERCENTAGE POINTS OF DISTRIBUTION OF ELEMENTS IN VARIOUS HUMAN ORGANS

Origin. I. Tipton, Health Physics Division.

Participating Member of Panel. J. Moshman.

Background. Concentrations of different elements are approximately log-normally distributed in various human organs. It is desired to obtain confidence intervals for the 75th and 90th percentiles of the distributions.

Status. Exact formulas for the variance of quartiles of a normal population were found by Hojo.[2] Approximations for large samples of the variance of quartiles and deciles of a normal population are given by Kendall.[3] These formulas will be employed in constructing approximate confidence intervals. The crudity of the data precludes the use of refined techniques.

CALCULATION OF THE EFFICIENCY OF ABSORPTION FOR A SINGLE CRYSTAL

Origin. P. R. Bell and F. K. McGowan, Physics Division.

Participating Members of Panel. N. M. Dismuke, S. E. Ezzell, H. B. Goertzel.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1151 and -1435.

Status. Computation of the problem on the UNIVAC at New York University has been completed. The results are being checked graphically.

SOLID ANGLE OF A CRYSTAL

Origin. P. R. Bell, Physics Division.

Participating Member of Panel. H. B. Goertzel.

Status. The solid angle of a sphere with its center inside a crystal was computed as a function of the radius of the sphere and the results were given to the originator.

CALCULATION OF SOME SHIELDING INTEGRALS

Origin. E. P. Blizard, Physics Division.

Participating Members of Panel. P. J. Brown, H. B. Goertzel, R. C. Weaver.

Background and Status. The desired results were S, F, and S/F, where

$$S = \int_0^t \int_0^t e^{-x_1/a}\, E_1(|x_1 - x_2|)\, e^{-x_2/b}\, dx_1\, dx_2 ,$$

$$F = \int_0^t e^{-(x/a)-(x/b)}\, dx = \frac{1}{a + b}\left\{1 - e^{-t[(1/a)+(1/b)]}\right\},$$

and the parameters $a \equiv \cos\theta_1$ and $b \equiv \cos\theta_2$ are specified.

At $x_1 = x_2$ a singularity is present in the integrand of the integral which defines S. Hence, this integral was solved by numerically integrating by Simpson's method, except for an interval of width ϵ on either side of each singularity. The integral over the omitted interval was found analytically to be given approximately by

$$2[1 - E_2(\epsilon)]\, \frac{ab}{a + b}\left\{1 - e^{-[(a+b)/ab]\,t}\right\},$$

where

$$E_2(\epsilon) = e^{-\epsilon} - \epsilon\, E_1(\epsilon) .$$

(2) T. Hojo, *Biometrika* **23**, 315 (1931).

(3) M. G. Kendall, *The Advanced Theory of Statistics*, p. 225, Vol. I, Griffin, London, 1945.

This solution was of sufficient accuracy in the desired region. The problem was done with the use of a desk calculator and the results were delivered to the originator.

SCATTERING INTEGRALS

Origin. E. B. Johnson and F. C. Maienschein, Physics Division.

Participating Members of Panel. S. E. Ezzell, R. C. Weaver.

Background and Status. The problem was to calculate the integrals as well as the integrand, at specified values, in the formulas giving the radial-beam scattering and skew-beam scattering of gamma rays. The formulas for the scattering are given, respectively, by

$$\frac{\Phi}{P} = 2\pi N_l \int_{\alpha_a}^{\alpha_b} r^{*2}(\alpha) \sin\alpha \int_{\beta_a}^{\beta_b} R^{-2}(\alpha,\beta)\, F(\alpha,\beta) \int_{E_a}^{E_b} \Gamma^*(\alpha,E)\, k(\alpha,\beta,E)\, dE\, d\beta\, d\alpha$$

and

$$\frac{\Phi}{P} = 2\pi N_l \int_{\alpha_a}^{\alpha_b} r^{*2}(\alpha) \sin\alpha \int_{r_a}^{r_b} R^{-2}(\alpha,r) \int_{E_a}^{E_b} \Gamma^*(\alpha,E)\, k(\alpha,r,E)\, dE\, dr\, d\alpha \ .$$

The functions in the integrand are given either analytically or experimentally. The problem was coded for the UNIVAC. It was run off successfully, and the results were turned over to the originators. A report which gives the details of the problem has been prepared by Maienschein.

CALCULATION OF INTERNAL CONVERSION COEFFICIENTS WITH SCREENING

Origin. M. E. Rose, Physics Division.

Participating Members of Panel. M. R. Arnette, N. D. Given, W. C. Sangren.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1290, -1360, and -1498.

Background and Status. The installation of a UNIVAC and the establishment of an AEC Computing Facility at New York University provide an excellent opportunity for solving long and difficult long-range problems. It was therefore decided that this problem of the calculation of internal conversion coefficients with screening could conveniently be turned over to the Computing Facility. Part I of this problem has been run on the UNIVAC, and some coding has been started for Part II and Part III.

NUMERICAL INTEGRATION OF AN INTEGRAL

Origin. M. E. Rose, Physics Division.

Participating Members of Panel. P. J. Brown, R. C. Weaver.

Background and Status. The problem was to compute

$$N(w',\beta) = \int_{w'}^{w_0} pw\, F(\beta,w)\, dw \ ,$$

where $p = \sqrt{w^2 - 1}$, and

$$F(\beta,w) = \frac{q}{k}\left\{\left[1 + \frac{b}{w} + \frac{a(k^2 - q^2)}{2wq}\right] I_0 + \frac{a}{2wq}(k^2 - q^2)(pI_1 \cos\beta - I_2) + \frac{a}{2wq}(-I_3 - pI_4 \cos\beta + I_5)\right\} .$$

I_0, I_1, I_2, I_3, I_4, and I_5 were functions of the parameters w and β; $q = w_0 - w$; $0 \leq \beta \leq 180°$; w_0 and k were constants; and a and b took on two values each. This was solved with the aid of desk calculators for

$$w' = \frac{1}{4} w_0 \ , \ \frac{1}{2} w_0 \ , \ \frac{3}{4} w_0 \ .$$

EVALUATION OF AN INTEGRAL CONNECTED WITH BETA DECAY

Origin. M. E. Rose, Physics Division.

Participating Member of Panel. H. B. Goertzel.

Status. Attempts at numerical integration indicated the need for more detailed analysis of the problem. It was shown by M. E. Rose that neglecting the finite size of the nucleus, as had been done, led to a nonintegrable singularity.

THREE-REGION THREE-GROUP REACTOR CALCULATION

Origin. M. C. Edlund, Physics Division; L. Cooper, Reactor Experimental Engineering Division.

Participating Members of Panel. S. E. Ezzell, R. C. Weaver.

Background and Status. A three-region three-group spherical reactor calculation has been coded for the UNIVAC. The reactor consists of a core surrounded by a blanket with the blanket, in turn, surrounded by a reflector. The calculation is based upon the work of L. Noderer and A. M. Weinberg. It is expected that this code will be used to calculate many reactors.

TWO INEQUALITIES FOR PERIODIC FUNCTIONS

Origin. W. K. Ergen, Aircraft Nuclear Propulsion Division.

Participating Member of Panel. W. C. Sangren.

Background and Status. The two problems submitted by W. K. Ergen have been brought to the attention of a number of mathematicians. Problem 1, below, has, for example, been shown to J. von Neumann and G. Szegö. Since the two inequalities have not yet been proved or disproved, it was decided to submit them to *The American Mathematical Monthly*. It seems likely that they will appear in the "Advanced Problems and Solutions" section of the magazine as a challenge to the readers. The problems were submitted in the following form:

1. Prove or give a counter-example for the inequality

$$\int_0^a ds \int_0^a d\sigma \int_0^p dx \frac{y(x-s)\, y(x-\sigma)}{y(x)} \geqslant a\alpha p Y \quad ,$$

where a and α are positive constants; p is the period of the positive, continuous, and periodic function $y(x)$; and Y is defined by

$$Y = \frac{1}{p} \int_0^p y(x)\, dx \quad .$$

2. Prove or give a counter-example for the inequality

$$\int_0^p z(x) \left\{ \int_0^a f(w) \left[\int_0^w \frac{d}{dx} z(x-s) \frac{d}{dw} f(w-s)\, ds \right] dw \right\} dx > 0 \quad ,$$

where $z(x)$ is continuous and periodic, with period p; and $f(w)$ is positive for $0 < w < a$ and is zero elsewhere.

MINIMUM WEIGHT AND PRESSURE DROP STUDY FOR AN AIR HEAT EXCHANGER

Origin. W. S. Farmer, Reactor Experimental Engineering Division.

Participating Members of Panel. N. D. Given, R. C. Weaver.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1435 and -1498.

Status. The evaluations of the algebraic formulas for pressure changes have been completed. The completed results were given to the originator.

TEMPERATURE DISTRIBUTION IN A CLOSED LOOP

Origin. D. C. Hamilton, Reactor Experimental Engineering Division.

Participating Members of Panel. P. J. Brown, W. C. Sangren.

Background and Status. The differential equation to be solved is

$$\frac{\partial t}{\partial X} = \frac{1}{P}\frac{1}{U_R}\left[\frac{\partial^2 t}{\partial R^2} + \frac{1}{R}\frac{\partial t}{\partial R}\right] ,$$

where $0 \leqslant R \leqslant 1$; $0 \leqslant X \leqslant \ln$; $P = 2, 4, 8$; $U_R = 1$ or $U_R = 6R(1 - R)$. If N_R is the number of radial subdivisions and N_l is the number of axial subdivisions, we can define:

$$F_{y-1} = \left(1 + \frac{1}{2RN_R}\right)\frac{N_R^2}{N_l P U_R} ,$$

$$F_{y+1} = \left(1 - \frac{1}{2RN_R}\right)\frac{N_R^2}{N_l P U_R} ,$$

$$F_y = 1 - (F_{y-1} + F_{y+1}) = 1 - \frac{2N_R^2}{N_l P U_R} ,$$

where $y = 10(1 - R)$.

These are used in solving the difference equation,

$$t_{x+1,y} = F_{y-1}t_{x,y-1} + F_y t_{x,y} + F_{y+1}t_{x,y+1} ,$$

which is associated with the above differential equation. Boundary and initial conditions are given. This problem is being coded for the ORACLE, and answers are anticipated in July.

BOILING REACTOR CRITICALITY CALCULATION

Origin. P. R. Kasten, Reactor Experimental Engineering Division.

Participating Member of Panel. R. C. Weaver.

Status. A modified one-group theory was employed to determine the parameter conditions required for criticality. The parameters involved are moderator-to-fuel ratio, fluid density, fuel enrichment, and reactor buckling. This work was done with the use of a desk calculator for three systems: UO_2SO_4-H_2O, UO_2SO_4-D_2O, and UO_3-H_2O.

LINEAR STABILITY OF BOILING REACTORS

Origin. P. R. Kasten, Reactor Experimental Engineering Division.

Participating Members of Panel. A. S. Householder, W. Givens, V. C. Klema.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1435 and -1498.

Background and Status. The problem has been extended to find all positive roots, if such exist, of

$$f(\nu;\lambda,\mu,\alpha,w^2,\gamma) = 0 ,$$

when f is a polynomial of sixth degree in ν. An upper bound for the roots ν is $2\gamma w^2/\mu$. Replace this upper bound by the next higher power of 2 and write this as $2^{\delta+3}$. Form

$$\nu_0 = 0,\ \nu_1 = 2^\delta,\ \nu_2 = 2^{\delta+1} ,$$

$$\nu_3 = 3 \cdot 2^\delta,\ \ldots,\ \nu_8 = 2^{\delta+3} .$$

The interpolation polynomial is determined by the points $\nu_1, \nu_2, \ldots, \nu_7$ by using Newton's forward-difference polynomial, $f(\nu)$. Form the Sturm sequence $f_0 = f$, $f_1 = f'$, $f_2, \ldots, f_6$, and isolate the roots by successively halving the interval and redetermining the interpolation points together with the polynomial until the desired accuracy is obtained.

The computations are being programmed for the ORACLE, and it is hoped that the program will be machine-tested in mid-summer.

EIGENVALUE OF A SPHERE WITH A CYLINDRICAL CONTROL ROD

Origin. S. Visner, Reactor Experimental Engineering Division.

Participating Members of Panel. H. B. Goertzel, W. C. Sangren.

Status. The coding necessary to compute the fundamental eigenvalue of the wave equation for a sphere with a black cylindrical control rod through its center has been completed. The problem will be run on the UNIVAC as soon as machine time is available.

HEAT FLOW IN LONG, CIRCULAR PIPES

Origin. H. F. Poppendiek, Reactor Experimental Engineering Division.

Participating Members of Panel. P. J. Brown, N. M. Dismuke, S. E. Ezzell, W. C. Sangren.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1498.

Status. It was desired to solve the differential equations for 16 sets of parameter values. These sets are referred to as Case 1 through Case 16. In the approximation to the differential equations, two lattice spacings were considered. Case 1 was completed for both spacings. The two results were almost identical. Cases 2, 4, and 5 were solved for one spacing. Neither spacing was fine enough for Case 3. UNIVAC time was not available for solving the remaining cases.

Present plans are to recode the problem for the ORACLE and to solve Case 3 by using a finer grid spacing. A few of the remaining cases may be run at that time.

CENTRIFUGE PROBLEM

Origin. K. A. Kraus and J. S. Johnson, Chemistry Division.

Participating Member of Panel. R. C. Weaver.

Background and Status. The log of the ratio of activity coefficients at various radii in the cell needed to be calculated. By using the following equations, this was done with the aid of a desk calculator:

$$\log \frac{y_i + 1}{y_i} = \frac{m\,\omega^2(x_{i+1}^2 - x_i^2)}{2(2.3026)RT} - \log \frac{C_{i+1}}{C_i} ,$$

$$C_{i+1} = \left(\frac{\nabla_x \left[\left(\frac{dn}{dx}\right) - (\rho_{\text{solution}} - \rho_{\text{solvent}})y \right]}{\frac{dm}{dc}} \right) + C_i ,$$

KINETIC EQUATIONS FOR THE HRE

Origin. V. Paré, Reactor Experimental Engineering Division.

Participating Members of Panel. P. J. Brown, E. L. Drabkin, C. P. Hubbard, W. C. Sangren.

Reference. S. Wallach, *Solution of the Pile Kinetic Equations when the Reactivity Is a Linear Function of the Time*, WAPD-13 (Nov. 30, 1950).

Status. The solutions of the reactor kinetic equations when the reactivity is a linear function of time was programmed for the UNIVAC. The coefficients in the equation for the neutron density proved to be such that direct numerical integration of the differential equations does not appear attractive. By using the analytical solutions which appear in WAPD-13 in the form of quadratures, it is, however, possible by numerical integration to obtain the desired results. A few results have been obtained with the use of desk calculators.

where

$$\int_{x_1}^{x_n} C_i x = \int_{x_1}^{x_n} C_0 x$$

and

$$\left(\frac{dn}{dx}\right)_{\text{uncorrected}} = zZ .$$

REFINING THE STRUCTURAL PARAMETERS OF KD_2 AS O_4

Origin. H. Levy, Chemistry Division.

Participating Members of Panel. J. Moshman, R. C. Weaver.

Background. The quantity $F_j (j = 1, 2, \ldots, n)$ is obtained from diffraction measurements. Theoretical considerations state that

$$F_j = \sqrt{A_j^2 + \delta B_j^2} ,$$

where

$$A_j = \sum_{i=1}^{p} x_{5i} \, e^{-a_{1j} x_{5i-1}} \cos 2\pi \left(\sum_{a=2}^{4} a_{aj} x_{5i-a} \right)$$

and

$$B_j = \sum_{i=1}^{p} x_{5i} \, e^{-a_{1j} x_{5i-1}} \sin 2\pi \left(\sum_{a=2}^{4} a_{aj} x_{5i-a} \right) .$$

The value of the quantity δ is either zero or depends upon physical considerations. The variables $x_{5i-a} (i = 1, 2, \ldots, p; a = 2, 3, 4)$ are the coordinates of the deuterium atoms; $x_{5i-1} (i = 1, 2, \ldots, p)$ is a function of temperature; and $x_{5i} (i = 1, 2, \ldots, p)$ is the scattering amplitude of the atoms.

It is desired to obtain the x_{5i-a} $(i = 1, 2, \ldots, p; a = 0, 1, \ldots, 4)$ which minimizes

$$\sum_{j=1}^{n} w_j \left(F_j - \sqrt{A_j^2 + \delta B_j^2} \right)^2 ,$$

where the w_j are appropriate weights.

The $5p$ variables are not all independent; there exist s linear relations among pairs of variables. The procedure followed linearizes $\sqrt{A_j^2 + \delta B_j^2}$ by a Taylor series about a set of known approximations x^0_{5i-a} $(i = 1, \ldots, p; a = 0, \ldots, 4)$, and by using the s linear relations, there eventually results $5p - s$ linear, algebraic equations, in as many unknowns, whose solutions provide corrections to the original estimates. This procedure may be iterated for further improvement of the estimates.

Status. The series of calculations is being processed by hand in which $p = 10$, $\delta = 0$, $n = 25$, and $a_{4j} = 0$ $(j = 1, 2, \ldots, 25)$. There exist $s = 30$ linear relations among the variables, which leaves ten, final, normal equations to be solved simultaneously. The current calculations are to be used, also, as a guide for coding the procedure for the ORACLE, in complete generality.

DETERMINATION OF FAST NEUTRON FLUX IN THE ORNL GRAPHITE REACTOR

Origin. D. K. Holmes, Solid State Division.

Reference. *Mathematics Panel Quarterly Progress Report,* ORNL-1498.

Participating Members of Panel. S. E. Ezzell, J. Moshman.

Background and Status. The flux was calculated for a region located symmetrically with respect to four fuel rods in the center of the reactor. The flux at two other regions will be evaluated at the UNIVAC installation at New York University.

[redacted]

MULTIGROUP REACTOR CALCULATIONS

Origin. R. A. Charpie, Research Director's Division.

Participating Members of Panel. J. H. Fishel, C. P. Hubbard.

Reference. D. K. Holmes, *The Multigroup Method as Used by the ANP Physics Group*, ANP-58.

Background and Status. In order to speed up the various reactor calculations which are presently being made throughout the Laboratory, it was decided to program the age-diffusion equation in multigroup form for the UNIVAC. The general format of ANP-58 was adopted for this purpose. However, several changes have been made to adapt the IBM thinking of ANP-58 to the high-speed machine.

The computation can be broken up into five sections:

1. computation of the average macroscopic cross sections for each region,
2. computation of constants,
3. calculation of the group fluxes by assuming a fission source which is constant in space,
4. iteration of the source and fluxes until the eigenvalue (multiplication constant) and the fluxes are stationary,
5. edit.

The problem was initially coded for two regions, 30 groups, and 30 space points. It has been run successfully many times. The average number of iterations required for good convergence is five. The running time of an average problem is about 8 minutes.

In order to make the routine more flexible, the number of groups is being increased and four different thermal bases are being added. This will permit the complete calculation, as used by the ANP group, to be done on the UNIVAC. This program is scheduled to be tried at New York University on or about July 15.

A program for computing the time behavior of nonstationary systems has also been coded and run successfully.

CONTROLLED THERMONUCLEAR REACTIONS

Origin. M. Rankin, Electromagnetic Research Division.

Participating Members of Panel. M. R. Arnette, P. J. Brown, N. M. Dismuke, R. C. Weaver.

References: *Electromagnetic Research Division Semiannual Progress Report for Period Ending March 20, 1953*, ORNL-1531, p. 18; letter from J. von Neumann to M. Rankin, April 29, 1953.

Background and Status. The following differential equations have arisen in connection with the theoretical investigation of certain aspects of the feasibility of controlled thermonuclear reactions (cf., ORNL-1531).

Problem 1. The first problem is given by

$$\frac{d^2z}{dy^2} = \frac{1}{k}\left(-1 + \frac{z^3y}{z'}\right) + \frac{z'^2}{z},$$

where $z = 1$ and $dz/dy = 0$ when $y = 0$; the side condition is given by

$$\left.\frac{z^3y}{z'}\right|_{y=y_d} = 3660 ;$$

and k is an arbitrary constant. The desired solution will correspond to a k value which yields a physically reasonable value of y_d.

Problem 2. The second problem is given by

$$\frac{d^2z}{d\eta^2} = -1 + \lambda z\sqrt{\frac{z\eta}{z'}} + \frac{z'^2}{z},$$

where $z = 1$ and $dz/d\eta = 0$ when $\eta = 0$; the side condition is given by

$$\lambda z\left.\sqrt{\frac{z\eta}{z'}}\right]_{\eta=\eta_d} = \sqrt{61} ;$$

λ is an arbitrary constant. The desired solution will correspond to a λ-value which yields a physically reasonable value of η_d.

Solutions of Problem 1 for several values of the parameter k were obtained on the AEC UNIVAC in December 1952. Subtracting-off difficulties were encountered, as expected, with small values of k. This difficulty was expected, because, for $k \ll 1$,

$$\frac{z^3 y}{z'} \approx 1$$

and

$$\frac{1}{k}\left(-1 + \frac{z^3 y}{z'}\right) \approx \frac{z'^2}{z} \quad .$$

An attempt was made to carry the solution beyond the troublesome point by using a Taylor series solution. However, the differential equation remains almost singular far from the origin. For $k \ll 1$, a new approach had to be applied in order to investigate Problem 1 (cf., letter from von Neumann).

Problem 2 has been coded for the ORACLE for a few values of the parameter λ. This code is now being checked.

TRANSLATION FORMULA FOR THE LID TANK FACILITY

Origin. J. D. Flynn, Physics Division.

Participating Member of Panel. J. Moshman.

References. J. D. Flynn and C. L. Storrs, *Neutron Measurements Around the GE-ANP Outlet Air Duct as a Function of Length of Transition Section*, ORNL CF-52-10-10 (Oct. 3, 1952); E. P. Blizard, *Introduction to Shield Design, Part I*, ORNL CF-51-10-70 (Jan. 30, 1952).

Background. Typical lid tank procedure involves the placement of a dry tank containing various shield materials with interspersed air spaces against the source and then measuring the thermal neutron flux at varying distances in the water. It is desired to derive the curve that would represent the flux if the air in the dry tank were replaced with water. A simple translation is not sufficient.

Let

z = distance to a point in the water from the source,
v = total width of air void,
a = radius of source,
λ = relaxation length in water,
s = total width of shield,
$D_v(z)$ = flux at z with air void,
$D(z)$ = flux at z with no air void.

From the reference reports, the approximate translational formula can be derived:

$$D(z - v) \approx D_v(z)\left[1 + \frac{4\lambda v(2z' + v)(z' + \lambda)}{a^2 z'^2 + 4\lambda z'^2(z' + \lambda)}\right] \quad .$$

Status. Experiments have been run with various configurations of shield and air where the flux was measured and then measured again when the air was replaced with water. Preliminary calculations indicate only partial agreement with the equation given above. The discrepancies are being investigated.

NUMERICAL QUADRATURE OF SOLUTION TO PARTIAL DIFFERENTIAL EQUATION

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Background. The integral solution to

$$(1) \quad \frac{\partial^2\psi}{\partial R^2} + \frac{\sin\theta}{R^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\psi}{\partial\theta}\right) = -\xi R \sin\theta \quad ,$$

subject to $\psi = 0$ along $\theta = 0$, $\theta = \pi$, $R = 0$, and $R = 1$, is

$$(2) \quad \psi = \frac{1}{2\pi} \int_0^1 \int_0^\pi \xi R_+ \, (RR_+ \sin\theta \sin\phi)^{1/2} \, [m_1^{3/2} \, C(m_1) - m_2^{3/2} \, C(m_2)] \, d\phi \, dR_+ \, ,$$

where ξ is a function of R_+ and ϕ,

$$C(m) = \frac{2 - m}{m^2} K(m) - \frac{2}{m^2} E(m) \, ,$$

and $K(m)$ and $E(m)$ are complete elliptic integrals of the first and second kinds, respectively.

Status. The function $\psi(R,\theta)$ is to be evaluated for

$$R = 0.1 \ (0.1) \ 0.9$$

and

$$\theta = \frac{\pi}{24} \left(\frac{\pi}{24}\right) \frac{23\pi}{24}$$

on the UNIVAC. The coding of the problem is now in progress. Equation 1 is one part of a system of nonlinear partial differential equations.