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COULOMB EXCITATION AND ANGULAR CORRELATION

R. K. Osborn
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PHYSICS DIVISION

COULOMB EXCITATION AND ANGULAR CORRELATION

R K Osborn
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Date Issued FEB 23 1954

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The direction of emission of the γ-rays emitted by nuclei which have been excited by Coulomb interaction with charged particles is correlated with the direction of incidence of these particles and the angle through which they are scattered In practice it is convenient to carry out the envisaged experiment so that the scattered particle is not observed This is assumed to be the case in the following The results of a calculation for the correlation between the incident beam direction and a subsequent single γ-ray for the case of quadrupole excitation have been published by Alder and Winther[1] It is the purpose of this report to present the details of this calculation, and to generalize the results to cover the case of the triple correlation between the incident beam direction and two subsequent γ-rays, either or both of which may be mixed

The basic assumption upon which the work presented herein rests is that the collision may be treated by the classical impact parameter method A careful study of the significance of this approximation has been made by Huby and Newns[2], particularly as to its effect upon total excitation

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1 K Alder and A Winther, Phys Rev , 91, 1578 (1953), referred to hereafter as AW

2 R Huby and H C Newns, Proc Phys Soc of London, 64A, 619 (1951)

cross sections We merely point out here the obvious implication that the kinetic energy of the system before collision be sufficiently below the Coulomb barrier to ensure that no nuclear reactions take place via compound nucleus formation

We consider the sequence of events whereby a nucleus of charge Z_2e in the angular momentum state $(J_o\ m_o)$ is excited to the angular momentum state $(J_1\ m_1)$ by Coulomb interaction with a particle of charge Z_1e whose direction of incidence is specified by the unit vector, $\hat{k}_1$, the nucleus then undergoing transitions $(J_1m_1) \longrightarrow (J_2m_2) \longrightarrow (J_3m_3)$ by emission of γ-rays in the directions $\hat{k}_2$ and $\hat{k}_3$, respectively The correlation function for these events may be expressed as

$$W(\hat{k}_1, \hat{k}_2, \hat{k}_3) = \sum_{m_1m_2m_1'm_2'} E_{m_1m_2'}(\hat{k}_3)\, S_{m_2m_2',m_1m_1'}(\hat{k}_2)\, F_{m_1'm_1}(\hat{k}_1) \qquad (1)$$

where

$$E_{m_2m_2'}(\hat{k}_3) = \sum_{m_3} (J_3m_3 \mid H(k_3) \mid J_2m_2)^* (J_3m_3 \mid H(k_3) \mid J_2m_2') \qquad (1a)$$

$$S_{m_2m_2',m_1m_1'}(\hat{k}_2) = (J_2m_2 \mid H(k_2) \mid J_1m_1)^* (J_2m_2' \mid H(k_2) \mid J_1m_1') \qquad (1b)$$

$$F_{m_1'm_1}(\hat{k}_1) = \sum_{m_o} (j_1m_1 \mid H(k_1) \mid J_om_o)^* (J_1m_1' \mid H(k_1) \mid J_om_o) \qquad (1c)$$

The density matrix E and the coupling coefficients S need not be discussed here[3] They may be expressed in the form

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3 See L C Biedenharn and M E Rose, Rev Mod Phys 25, 729 (1953), referred to hereafter as BR

$$E_{m_2 m_2'}(\hat{k}_3) = \sum_{L_3 L_3' \nu_3} F_{\nu_3}(L_3 L_3' J_3 J_2)(J_3 \| L_3 \| J_2)^* (J_3 \| L_3' \| J_2)$$

$$(-1)^{L_3+L_3'-J_2+m_2'} C^{J_2 J_2 \nu_3}_{m_2,-m_2'} D^{\nu_3}_{m_2-m_2',0}(\hat{k}_3), \tag{2}$$

$$S_{m_2 m_2', m_1 m_1'}(\hat{k}_2) = \sum_{L_2 L_2' \nu_2} \sqrt{(2L_2+1)(2L_2'+1)}\ (-1)^{m_1-m_2+1} C^{L_2 L_2' \nu_2}_{1,-1}$$

$$C^{J_1 L_2 J_2}_{m_1, m_2-m_1} C^{J_1 L_2' J_2}_{m_1', m_2'-m_1'} C^{L_2 L_2' \nu_2}_{m_1-m_2, m_2'-m_1'} D^{\nu_2}_{m_2'-m_1'+m_1-m_2,0}(\hat{k}_2)$$

$$(J_2 \| L_2 \| J_1)^* (J_2 \| L_2' \| J_1), \tag{3}$$

where the

$$F_{\nu_3}(L_3 L_3' J_3 J_2) = (-1)^{J_3-J_2+1} \sqrt{(2J_2+1)(2L_3+1)(2L_3'+1)}\ C^{L_3 L_3' \nu_3}_{1,-1}$$

$$W(J_2 J_2 L_3 L_3', \nu_3 J_3) \tag{4}$$

have been extensively tabulated[2] The C's and W's are Clebsch-Gordan and Racah coefficients respectively and the D's are the irreducible representations of the rotation group The multipolarities of the first and second γ-rays in the cascade are specified by (L_2, L_2') and (L_3, L_3') respectively The numbers ν_2 and ν_3 must be even integers

The form of the operator $H(k_1)$ appearing in the density matrix F may readily be deduced In view of the above assumptions, we may regard

the nuclear transition made by the target nucleus as a consequence of the Coulomb interaction with the projectile as being induced by a time-dependent potential of the form

$$V(t) = \sum_{i=1}^{A} \frac{Z_1 e\, e_i}{|r_i - r(t)|}, \tag{5}$$

where $r(t)$ specifies the position of the projectile relative to the center of mass of the target at time, t, and the r_i are the position vectors of the charges e_i in the target nucleus measured relative to the nuclear center of mass Then, according to time-dependent perturbation theory, the pertinent operator for determining the nuclear transition rate should simply be

$$H(k_1) = \int_{-\infty}^{\infty} e^{i \frac{\Delta E}{\hbar} t} V(t)\, dt \tag{6}$$

where $\Delta E = E_f - E_i$ is the excitation energy of the target nucleus $H(k_1)$ so defined, of course depends not only upon the incident beam direction but upon the final direction of a particular orbit as well

The potential $V(t)$ is a function of both the parameter of the orbit and the internal coordinates of the target nucleus To obtain a useful separation of this dependence, we expand

$$V(t) = Z_1 e \sum_{i=1}^{A} \frac{e_i}{|r(t) - r_i|}$$

$$= \sum_{\lambda\mu} g_\lambda^\mu(t) B_\lambda^\mu, \tag{7}$$

where

$$g_\lambda^\mu(t) = \frac{4\pi Z_1 e}{2\lambda + 1} \frac{1}{r(t)^{\lambda+1}} Y_\lambda^{\mu *}(\hat{r}(t))$$

$$B_\lambda^\mu = \sum_{i=1}^{A} e_i r_i^\lambda Y_\lambda^\mu(\hat{r}_i)$$

Now the quantities, g_λ^μ, are to be evaluated in a special coordinate system, hence, although the B_λ^μ are explicit functions of the nuclear coordinates only, they still depend implicitly upon the choice of coordinate system in which the orbit is analyzed For purposes of this analysis, we choose our coordinate system as shown in Fig 1

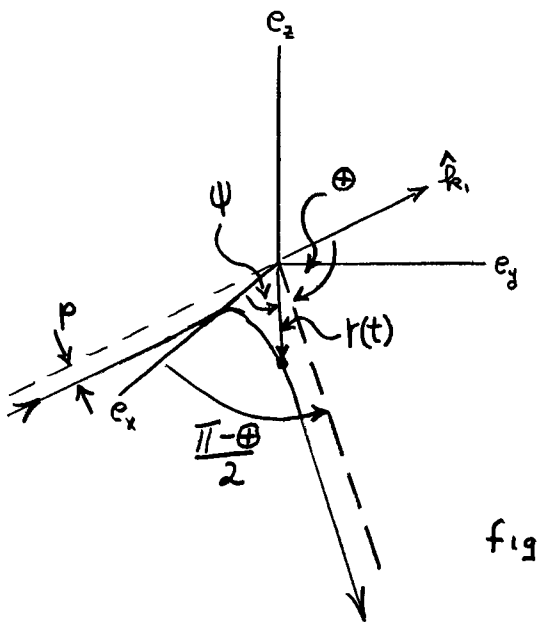

fig 1

The orbit lies in the x-y plane of a coordinate system specified by the unit vectors (e_x, e_y, e_z) such that e_x is in the direction of the point of closest approach The incident beam direction is specified by the unit vector, $\hat{k}_1$ The energy equation is

$$\frac{p^2}{2\mu} + \frac{Z_1 Z_2 e^2}{r} = E,$$

or

$$\frac{1}{2}(\mu r^2 + \mu r^2 \dot{\psi}^2) + \frac{Z_1 Z_2 e^2}{r} = E,$$

μ = reduced mass It is convenient to parametrize the orbit in terms of a variable, ω , defined by

$$r = a(1 + \epsilon \cosh \omega) \tag{8}$$

where

$$a = Z_1 Z_2 e^2/\mu\sigma^2 ,$$

and

$$\epsilon = \sqrt{1 + (p/a)^2}$$

The quantity ϵ is the eccentricity of the orbit and p is the impact parameter The impact parameter is related to the angular momentum constant for the system by

$$\mu\sigma p = \ell = \mu r^2 \dot{\psi}$$

The other relevant functions of the orbit may be expressed in terms of the parameters a, ϵ and ω according to

$$t = \frac{a}{v} (\omega + \epsilon \sinh \omega), \tag{9}$$

$$\cos \psi = \frac{\epsilon + \cosh \omega}{1 + \epsilon \cosh \omega}, \tag{10}$$

$$\sin \psi = + \frac{\sqrt{\epsilon^2 - 1} \sinh \omega}{1 + \epsilon \cosh \omega} \tag{11}$$

The scattering angle, Θ, is related to ϵ according to

$$\sin \frac{\Theta}{2} = \frac{1}{\epsilon}$$

We may now express the scattering operator in the form

$$H(k_1) = \sum_{\lambda\mu} B_\lambda^\mu \int_{-\infty}^{\infty} e^{i\xi(\omega + \epsilon \sinh \omega)} g_\lambda^\mu(\omega) \frac{a}{v} (1 + \epsilon \cosh \omega) \, d\omega$$

$$\xi = a \Delta E/\hbar v$$

The nuclear operators, B_λ^μ, are here expressed in the (e) coordinate system They are related to the same set of operators in an arbitrary coordinate system, (q), according to

$$B_\lambda^\mu(e) = \sum_\mu B_\lambda^\mu(q) \, D^\lambda_{\mu\mu} \quad (R_{q \to e})$$

where $R_{q \to e}$ is the rotation required to bring the (q) system into the (e) system Thus

$$H(k_1) = \sum_{\lambda\mu'\mu} B_\lambda^\mu \, D^\lambda_{\mu\mu} \int_{-\infty}^{\infty} e^{i\xi(\omega + \epsilon \sinh\omega)} \, \frac{a}{v} \, g_\lambda^\mu(\omega) \, (1 + \epsilon \cosh\omega) d\omega$$

If now we specialize to the case of pure multipole excitation, then

$$H_\lambda(k_1) = \sum_{\mu\mu} B_\lambda^\mu \, D^\lambda_{\mu\mu} \, S_\lambda^\mu(\xi, \epsilon) \tag{12}$$

where, after dropping common factors,

$$S_\lambda^\mu(\xi, \epsilon)$$

$$= \frac{2(-1)^{\mu+1}}{2^\lambda \lambda'} \sqrt{\frac{(\lambda - |\mu|)'}{(\lambda + |\mu|)'}} \left\{ \left(\frac{d}{d\zeta}\right)^{\lambda+|\mu|} (\zeta^2 - 1)^\lambda \right\}_{\zeta=0} \int_{-\infty}^{\infty} \frac{e^{i\xi(\omega + \epsilon \sinh\omega) - i\mu\psi} d\omega}{(1 + \epsilon \cosh\omega)^\lambda} \tag{13}$$

and $\lambda + \mu$ is even These functions are exactly the same as those given in (AW, 7) for the cases $\lambda = 2$, $\mu = 0$ and ± 2 It is to be noted that they are explicitly real If we define

$$N_\lambda^{|\mu|} = \frac{1}{2^\lambda \lambda'} \sqrt{\frac{(\lambda - |\mu|)'}{(\lambda + |\mu|)'}} \left\{ \left(\frac{d}{d\zeta}\right)^{\lambda+|\mu|} (\zeta^\lambda - 1)^\lambda \right\}_{\zeta=0},$$

$$= \frac{(-1)^{\frac{\lambda - |\mu|}{2}}}{2^\lambda} \sqrt{\frac{(\lambda + \mu)' \, (\lambda - \mu)'}{\left(\frac{\lambda+\mu}{2}\right)' \left(\frac{\lambda-\mu}{2}\right)'}} \tag{14a}$$

$$f(\omega) = \omega + \epsilon \sinh\omega, \tag{14b}$$

and

$$\rho(\omega) = (1 + \epsilon \cosh\omega)^\lambda, \tag{14c}$$

and take account of the fact that ρ is an even function whereas f and ψ are odd functions of ω, we may express

$$S_\lambda^\mu(\xi, \epsilon) = 4\,(-1)^{\mu+1}\, N_\lambda^{|\mu|} \int_0^\infty \frac{\cos\left[\xi f(\omega) - \mu\psi(\omega)\right]}{\rho(\omega)}\, d\omega$$

We also note from this that

$$S_\lambda^{-\mu}(\xi, \epsilon) = S_\lambda^\mu(-\xi, \epsilon)$$

The operator $H(k_1)$ is now in a form such that by application of methods identical to those employed to obtain E or S in the forms quoted herein we find for F

$$F_{m_1'm_1}(\hat{k}_1, \epsilon, \varphi) = \sum_{\nu_1} F_{\nu_1}(\lambda\, J_o\, J_1)\,(-1)^{J_1-m_1+\nu_1}\, C^{J_1 J_1 \nu_1}_{m_1', -m_1}$$

$$\sum_{\mu\mu} (-1)^{\mu+1}\, S_\lambda^\mu\, S_\lambda^\mu\, \frac{C^{\lambda\lambda\nu_1}_{\mu,-\mu}}{C^{\lambda\lambda\nu_1}_{1,-1}}\, D^{\nu_1}_{m_1'-m_1,\mu\,-\mu}\,(R_{q\to e}), \qquad (15)$$

where

$$F_{\nu_1}(\lambda\, J_o\, j_1) = (-1)^{J_o-j_1+1}\sqrt{2j_1+1}\;(2\lambda+1)\, C^{\lambda\lambda\nu_1}_{1,-1}\, W(J_1 J_1 \lambda\lambda, \nu_1 J_o) \qquad (16)$$

We have here indicated explicitly the dependence of $F_{m_1'm_1}$ upon the orientation of the particular orbit in terms of which it was evaluated The angle, φ, is that angle whose variation expresses a rotation of the

orbit about the incident beam direction as an axis If the final direction of motion of the projectile is not observed, then we must average F over all orbits Thus

$$F_{m_1'm_1}(\hat{k}_1) = \int_{\epsilon=1}^{\infty} \int_{\varphi=0}^{2\pi} F_{m_1'm_1}(k_1, \epsilon, \varphi)\, \epsilon\, d\epsilon\, d\varphi$$

$$= \sum_{\nu_1} F_{\nu_1}(\lambda J_0 J_1)\, (-1)^{J_1 - m_1 + \nu_1}\, C^{J_1 J_1 \nu_1}_{m_1', -m_1}$$

$$\sum_{\mu\mu'} (-1)^{\mu+1} \frac{C^{\lambda\lambda\nu_1}_{\mu', -\mu}}{C^{\lambda\lambda\nu_1}_{1,-1}} \int\int S_\lambda^{\mu} S_\lambda^{\mu'} D^{\nu_1}_{m_1'-m_1, \mu-\mu}(R_{q\to e})\, \epsilon\, d\epsilon\, d\varphi \tag{17}$$

since the rotation $R_{q\to e}$ is itself a function of ϵ and φ

We break up the rotation $R_{q\to e}$ into the sum of two rotations $R_{q\to k_1} + R_{k_1\to e}$ where (k_1) represents a fixed coordinate system whose + z-axis is in the direction of the unit vector $\hat{k}_1$ Then

$$D^{\nu_1}_{m_1'-m_1, \mu'-\mu}(R_{q\to e}) = \sum_{\sigma} D^{\nu_1}_{m_1'-m_1, \sigma}(R_{q\to k_1})\, D^{\nu_1}_{\sigma, \mu-\mu}(R_{k_1\to e})$$

The rotation $R_{k_1\to e}$ consists of Euler rotations through φ, $\frac{\pi}{2}$ and $\frac{\Theta - \pi}{2}$ respectively, so that

$$D(R_{k_1\to e}) = D\left(\varphi, \frac{\pi}{2}, \frac{\Theta - \pi}{2}\right)$$

The integral in (17) now assumes the form

$$\sum_{\sigma} D^{\nu_1}_{m_1'-m_1,\sigma} \int\int D^{\nu_1}_{\sigma,\mu-\mu}(\varphi, \tfrac{\pi}{2}, \tfrac{\Theta-\pi}{2})\, S^{\mu}_{\lambda}\, S^{\mu}_{\lambda}\, \epsilon\, d\epsilon\, d\varphi$$

$$= D^{\nu_1}_{m_1'-m_1,0}(\hat{k}_1) \int S^{\mu}_{\lambda}\, S^{\mu}_{\lambda}\, D^{\nu_1}_{0,\mu'-\mu}(0, \tfrac{\pi}{2}, \tfrac{\Theta-\pi}{2})\, \epsilon\, d\epsilon$$

We define the particle parameters

$$K_{\lambda\nu_1}(\xi) = \sum_{\mu\mu} (-1)^{\mu+1} \frac{C^{\lambda\lambda\nu_1}_{\mu',-\mu}}{C^{\lambda\lambda\nu_1}_{1,-1}} \int S^{\mu}_{\lambda}\, S^{\mu'}_{\lambda}\, D^{\nu_1}_{0,\mu-\mu}(0,\tfrac{\pi}{2},\tfrac{\Theta-\pi}{2})\, \epsilon\, d\epsilon$$

$$= \sum_{\mu\mu'} (-1)^{\mu+1} \frac{C^{\lambda\lambda\nu_1}_{\mu',-\mu}}{C^{\lambda\lambda\nu_1}_{1,-1}}\, N^{|\mu-\mu|}_{\nu_1} \int S^{\mu}_{\lambda}\, S^{\mu}_{\lambda}\, e^{i(\mu-\mu)\frac{\pi-\Theta}{2}}\, \epsilon\, d\epsilon$$

$$= \sum_{\mu\mu} (-1)^{\mu+1} \frac{C^{\lambda\lambda\nu_1}_{\mu,-\mu}}{C^{\lambda\lambda\nu_1}_{1,-1}}\, N^{|\mu'-\mu|}_{\nu_1}\, I^{\mu\mu}_{\lambda}(\xi) \tag{18}$$

where

$$I^{\mu\mu}_{\lambda}(\xi) = \int S^{\mu}_{\lambda}\, S^{\mu}_{\lambda}\, \cos\left[(\mu-\mu)\frac{\pi-\Theta}{2}\right] \epsilon\, d\epsilon \tag{19}$$

Because of the coefficients $N^{|\mu-\mu|}_{\nu_1}$, previously defined in (14a), we see that (18) vanishes unless ν_1 is an even integer We may express the

density matrix F as

$$F_{m_1'm_1}(\hat{k}_1) = \sum_{\nu_1} F_{\nu_1}(\lambda\, J_o\, j_1)\,(-1)^{J_1-m_1}\, C^{J_1 J_1 \nu_1}_{m_1',-m_1}$$

$$D^{\nu_1}_{m_1'-m_1,0}(\hat{k}_1)\; a_{\lambda\nu_1}(\xi) \tag{20}$$

where we have introduced the normalized particle parameter,

$$a_{\lambda\nu_1}(\xi) = K_{\lambda\nu_1}(\xi)/K_{\lambda 0}(\xi) \tag{21}$$

These have been evaluated by (AW) for the special cases $\lambda = 2$, $\nu = 2$ and 4

We may now put E, S and F together to obtain the correlation function for the case in which both γ-ray transitions involve the mixtures $(L_2, L_2 + 1)$ and $(L_3, L_3 + 1)$

$$W(\hat{k}_1\hat{k}_2\hat{k}_3) = \sum_{\nu_1\nu_2\nu_3} F_{\nu_1}(\lambda J_o J_1)\Lambda_{\nu_1\nu_2\nu_3}(\hat{k}_1\hat{k}_2\hat{k}_3)\, a_{\lambda\nu_1}(\xi)$$

$$\Big\{ F_{\nu_3}(L_3 J_3 J_2) + \delta_1^2 F_{\nu_3}(L_3{+}1\; J_3 J_2)$$

$$+ 2\,\delta_1 (-1)^{J_3-j_2+1} \sqrt{(2j_2+1)(2L_3+1)(2L_3+3)}\; G_{\nu_3}(L_3 L_3{+}1\;\; J_3 J_2)\Big\}$$

$$\times \Big\{ (2L_2+1)\,\Gamma(J_1 L_2 J_2, \nu_1\nu_2\nu_3) + \delta_2^2 (2L_2+3)\,\Gamma(J_1 L_2{+}1\; J_2, \nu_1\nu_2\nu_3)$$

$$+ 2\,\delta_2 \sqrt{(2L_2+1)(2L_2+3)}\,\Pi(J_1 L_2 L_2{+}1\; j_2, \nu_1\nu_2\nu_3)\Big\} \tag{22}$$

The functions $F_\nu(LjJ')$, $G_\nu(LL'JJ')$, $\Gamma(JLj',\nu\nu'\nu'')$ and $\Lambda_{\nu\nu\nu'}$ have been defined in (BR 69b), (BR 70d), (BR 138a) and (BR 138b) respectively We have further defined

$$\Pi(J_1L_2L_2'j_2,\nu_1\nu_2\nu_3) = C_{1,-1}^{L_2L_2'\nu_2}(2\nu_3+1)^{-\frac{1}{2}}$$

$$\sum_S (2S+1)\, W(\nu_1\nu_2j_2J_2,\nu_3S)\, W(\nu_2SL_2'J_1,J_2L_2)\, W(J_1J_1SJ_2,\nu_1L_2)$$

$$= (-1)^{J_1+J_2+L_2}\, C_{1,-1}^{L_2L_2'\nu_2}(2\nu_3+1)^{-\frac{1}{2}}\, X(J_1L_2J_2,J_1L_2'j_2,\nu_1\nu_2\nu_3)^{4} \qquad (23)$$

The mixture ratios are

$$\delta_1 = -\frac{(J_3\|L_3+1\|J_2)}{(J_3\|L_3\|J_2)} = \frac{(J_2\|L_3+1\|J_3)}{(J_2\|L_3\|j_3)}$$

and

$$\delta_2 = \frac{(j_2\|L_2+1\|j_1)}{(J_2\|L_2\|J_1)}$$

In the event that the first γ-ray (intermediate radiation) is not observed, we have

$$W(k_1,k_3) = \int W(k_1k_2k_3)\, d\Omega(k_2)$$

$$\sum_{L_3L_3'\nu} (-1)^{L_3+L_3'}\, F_{\nu_3}(L_3L_3'J_3j_2)\, F_{\nu_1}(\lambda J_oJ_1)$$

$$(j_3\|L_3\|J_2)^*\,(j_3\|L_3'\|J_2)\, W(J_1J_1j_2j_2,\nu L_2)\, P_\nu(\hat{k}_1\hat{k}_3)\, a_{\lambda\nu}$$

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4 For a definition of the X coefficient see U Fano, National Bureau of Standards Report No 1214

This expression for the case in which the observed γ-ray is pure is the same as (BR, 141) except for the factors $a_{\lambda\nu}(\xi)$

Finally, if there is no intermediate γ-ray so that $L_2 = 0$ and $J_1 = j_2$, we have

$$W(k_1 k_3) = \omega_I + \delta^2 \omega_{II} + 2\delta \omega_{III},$$

where

$$\omega_I = \sum_\nu A_\nu(\lambda L_3)\, P_\nu(\hat{k}_1 \hat{k}_2)\, a_{\lambda\nu}(\xi)$$

$$\omega_{II} = \sum_\nu A_\nu(\lambda L_3+1)\, P_\nu(k_1 k_3)\, a_{\lambda\nu}(\xi)$$

$$\omega_{III} = (-1)^{j_3-J_2+1} \sqrt{(2J_2+1)(2L_3+1)(2L_3+3)}$$

$$\sum_\nu G_\nu(L_3 L_3+1\ J_3 J_2)\, F_\nu(\lambda J_0 j_2)\, P_\nu(\hat{k}_1 \hat{k}_3)\, a_{\lambda\nu}(\xi),$$

and where

$$\delta = -\frac{(j_3 \| L_3+1 \| j_2)}{(J_3 \| L_3 \| j_2)}$$

for the case in which the γ-transition contains the mixture L_3 and L_3+1 The coefficients A_ν are defined in (BR, 69a) If the γ-transition is pure one simply has

$$W = \sum_\nu A_\nu\, a_{\lambda\nu}(\xi)\, P_\nu(\hat{k}_1 \hat{k}_3)$$

as given by (AW, 8)