[redacted]

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES

3 4456 0349702 6

CENTRAL RESEARCH LIBRARY
DOCUMENT COLLECTION

AEC RESEARCH AND DEVELOPMENT REPORT

ORNL 1780
Reactors–Research and Power

4A

[redacted] CONFIDENTIAL, UNDOCUMENTED

[redacted]

PHYSICS OF THE HOMOGENEOUS REACTOR TEST STATICS

M. C. Edlund
P. M. Wood

CLASSIFICATION CHANGED TO: DECLASSIFIED
AUTHORITY OF: E. J. Murphy 10-22-59
E. Lackenmeister 19-60

CENTRAL RESEARCH LIBRARY
DOCUMENT COLLECTION

LIBRARY LOAN COPY

DO NOT TRANSFER TO ANOTHER PERSON

If you wish someone else to see this document, send in name with document and the library will arrange a loan.

OAK RIDGE NATIONAL LABORATORY
OPERATED BY
CARBIDE AND CARBON CHEMICALS COMPANY
A DIVISION OF UNION CARBIDE AND CARBON CORPORATION

UCC

POST OFFICE BOX P
OAK RIDGE, TENNESSEE

RESTRICTED DATA
This document contains Restricted Data as defined in the Atomic Energy Act of 1946. Its transmittal or the disclosure of its contents in any manner to an unauthorized person is prohibited.

[redacted]

[redacted]

ORNL- 1780

[redacted]sists of 71 pages.
[redacted], Series A.

4A

Contract No. W-7405-Eng-6

PHYSICS OF THE HOMOGENEOUS REACTOR TEST

- STATICS -

August 27, 1954

M. C. Edlund

and

P. M. Wood

Date Issued: SEP 17 1954

OAK RIDGE NATIONAL LABORATORY
Operated by
CARBIDE AND CARBON CHEMICALS COMPANY
A Division of Union Carbide and Carbon Corporation
Post Office Box P
Oak Ridge, Tennessee

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES

3 4456 0349702 6

[redacted]

INTERNAL DISTRIBUTION

1. C. E. Cent[redacted]
2. Biology Li[redacted]ary
3. Health Phy[redacted]cs Library
4-5. Central Re[redacted]arch Library
6. Reactor Ex[redacted]rimental Engineerin[redacted] Library
7-11. Laboratory [redacted]ecords Department
12. Laboratory [redacted]ecords, ORNL R.C.
13. C. E. Lars[redacted]
14. L. B. Emle[redacted] (K-25)
15. J. P. Murr[redacted] (Y-12)
16. A. M. Wein[redacted]rg
17. E. H. Tay[redacted]r
18. E. D. Shi[redacted]ey
19. J. A. Swa[redacted]out
20. C. P. Kei[redacted]
21. J. H. Fry[redacted] Jr.
22. R. S. Liv[redacted]gston
23. F. C. Von[redacted]rLage
24. W. H. Jor[redacted]n
25. S. C. Lin[redacted]
26. F. L. Cul[redacted]r
27. A. H. Sne[redacted]
28. A. Hollae[redacted]er
29. M. T. Kel[redacted]y
30. G. H. Cle[redacted]tt
31. T. A. Lin[redacted]ln
32. A. S. Hou[redacted]holder
33. C. S. Har[redacted]ll
34. D. W. Car[redacted]vell
35. K. Z. Mor[redacted]n
36. E. M. Kir[redacted]
37. C. E. Wi[redacted]ers
38. J. A. [redacted]ne
39. D. D. [redacted]owen
40. R. A. [redacted]harpie
41. G. E. [redacted] Boyd
42. S. J. [redacted] Cromer
43. M. [redacted]. Skinner
44. R. [redacted]. Aven
45. J. [redacted]N. Baird
46. S. [redacted]E. Beall
47. J. [redacted]el Bengston
48. [redacted] G. Bohlmann
49. [redacted] B. Briggs
50. [redacted] H. Chapman
51. [redacted]. C. Claiborne
52. [redacted]. Coveyou
53. [redacted]. C. Edlund
54. [redacted]. K. Ergen
55. [redacted] R. Gall
56. [redacted] B. Graham
57. [redacted] J. Hairston
58. [redacted] N. Haubenreich
59. [redacted] W. Hill
60. [redacted] R. Kasten
61. [redacted] R. McWherter
62. [redacted] B. Mills
63. [redacted] C. Noderer
64. [redacted] G. Prohammer
65. [redacted] H. Secoy
66. [redacted] H. Thomas
67. [redacted] Tobias
68. [redacted]. Visner
69. [redacted]. A. Welton
70. [redacted]. M. Wood

EXTERNAL DISTRIBUTION

71. AF Plan[redacted] Representative, Burbank
72. AF Plan[redacted] Representative, Seattle
73. AF Plan[redacted] Representative, Wood-Ridge
74. America[redacted]Machine and Foundry Company
75. ANP Pro[redacted]t Office, Fort Worth
76-86. Argonne [redacted]tional Laboratory
87. Armed F[redacted]es Special Weapons Project (Sandia)
88. Armed F[redacted]es Special Weapons Project, Washington
89-93. Atomic E[redacted]rgy Commission, Washington
94. Babcock [redacted]d Wilcox Company
95. Battelle [redacted]emorial Institute
96. Bendix A[redacted]ation Corporation
97-99. Brookhav[redacted] National Laboratory
100. Bureau of Ships

[redacted]

101. Chicago Patent Group
102. Chief of Naval Research
103. Commonwealth Edison Company
104. Department of the Navy - Op-362
105. Detroit Edison Company
106-110. duPont Company, Augusta
111. duPont Company, Wilmington
112. Duquesne Light Company
113. Foster Wheeler Corporation
114-116. General Electric Company (ANPD)
117. General Electric Company (APS)
118-125. General Electric Company, Richland
126. Hanford Operations Office
127. Iowa State College
128-131. Knolls Atomic Power Laboratory
132-133. Los Alamos Scientific Laboratory
134. Nuclear Metals, Inc.
135. Monsanto Chemical Company
136. Mound Laboratory
137. National Advisory Committee for Aeronautics, Cleveland
138. National Advisory Committee for Aeronautics, Washington
139-140. Naval Research Laboratory
141. Newport News Shipbuilding and Dry Dock Company
142. New York Operations Office
143-144. North American Aviation, Inc.
145. Nuclear Development Associates, Inc.
146. Patent Branch, Washington
147-153. Phillips Petroleum Company (NRTS)
154. Powerplant Laboratory (WADC)
155. Pratt & Whitney Aircraft Division (Fox Project)
156. Rand Corporation
157. San Francisco Field Office
158. Sylvania Electric Products, Inc.
159. Tennessee Valley Authority (Dean)
160. USAF Headquarters
161. U. S. Naval Radiological Defense Laboratory
162-163. University of California Radiation Laboratory, Berkeley
164-165. University of California Radiation Laboratory, Livermore
166. Walter Kidde Nuclear Laboratories, Inc.
167-172. Westinghouse Electric Corporation
173-187. Technical Information Service, Oak Ridge
188. Division of Research and Medicine, AEC, ORO
189. Pacific Northwest Power Group
190. Materials Laboratory (WADC)

FOREWORD

This is a report on the statics calculations which have been made for the Homogeneous Reactor Test by members of the Reactor Analysis Section of REED. In particular, acknowledgement is given to L. C. Noderer for his contribution to the development of the harmonics method for estimating critical masses of small D_2O moderated reactors and to T. B. Fowler, J. DiLorenzo and A. Forbes (ANP Division) for computational assistance.

TABLE OF CONTENTS

		Page
1.	Introduction	1
2.	Method for Criticality Calculations	4
3.	Data	14
4.	Critical Concentration	18
5.	Critical Concentration with Ordinary Water as Moderator	28
6.	Neutron Flux Distribution and Adjoint Functions	29
7.	Prompt Neutron Mean Lifetimes	33
8.	Temperature Coefficients of Reactivity	34
9.	Buildup of U-233 in Thorium in Blanket	39
10.	Heat Generation in Blanket Region	41
11.	Breeding of the HRT	44

APPENDIXES

I	- HRT Design Data	52
II	- Nuclear Data	54
III	- Prompt Neutron Mean Lifetime	62

BIBLIOGRAPHY 63

[redacted]

PHYSICS OF THE HOMOGENEOUS REACTOR TEST

- STATICS -

1. Introduction

With the successful conclusion of the Homogeneous Reactor Experiment, a second reactor, the Homogeneous Test Reactor (HRT), is being designed to test various engineering concepts and reactor components which are related to the problem of power production in large aqueous homogeneous reactors. The primary consideration in all aspects of the HRT design is reliable operation and ease of maintenance.

The HRT will be a 5-Mw, two-region reactor operated at 280°C and 2000 psia. A solution of uranyl sulfate and D_2O containing about 10 g of highly enriched uranium per kg of D_2O will be circulated through a 32-inch-diameter core tank, which is centered in a 60-inch-I.D. pressure vessel (Fig. 1), which also serves as the reflector container.

The core and reflector systems will be similar and interconnected through the pressurizers to prevent large pressure differences between the core and reflector. Details of the design and the flowsheet are given by R. B. Briggs et al;[1] a summary of design data is given in Appendix I.

Due to the inherent stability of homogeneous reactors as demonstrated by the Homogeneous Reactor Experiment,[2] the HRT is designed with no external reactivity control other than the concentration of fuel in the core region. The change in the concentration required for criticality of fuel in the core region is therefore the fundamental measure of reactivity involved in a change of reactor conditions such as temperature and pressure.

[redacted]

[redacted]
ORNL-LR-Dwg 2666

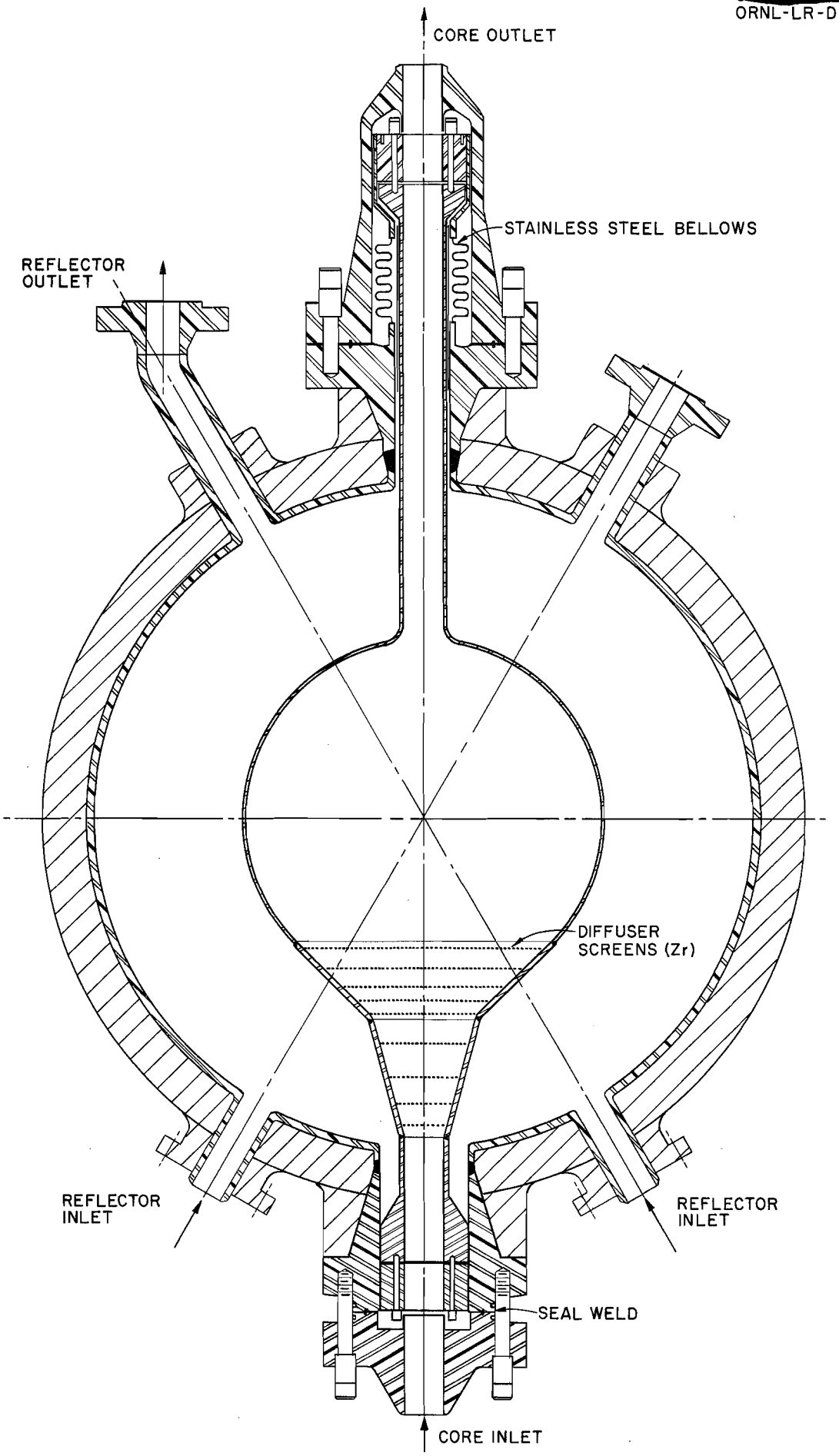

Fig. 1. HRT Core and Pressure Vessel

The initial objective of the HRT is to test a core system which will incorporate several features based on the present concepts of a two-region power breeder.[3]

These features are:

1) A "straight-through" flow pattern for the core. Experiments have shown it to be better than the "rotational" flow used in the HRE for larger reactor cores.
2) $CuSO_4$ catalyst in the core solution to recombine D_2 and O_2.
3) The critical concentration of fuel to be 10 g/kg D_2O or less, which is the range of interest for larger reactors from the viewpoint of chemistry and corrosion. This requirement fixes the smallest core diameter at about 32 inches with a D_2O reflector (Fig. 3).
4) Zircaloy core tank.

During the initial period of testing the operational reliability of the core system, the reflector will contain D_2O in order to bring the critical concentration of fuel below 10 g/kg D_2O. After this period the D_2O may be replaced with blankets containing either thorium or low-enrichment uranium for breeding and conversion experiments.

The statics calculations for the HRT were done for the usual quantities, such as critical masses, total heat sources, neutron generation times, temperature and density coefficients of reactivity, statistical weights and breeding and conversion ratios for D_2O, ThO_2-D_2O slurry and D_2O-UO_2SO_4

reflectors or blankets. The only novel problem encountered in the physics was the development of an adequate method to calculate criticality in small D_2O moderated reactors.

2. Method for Criticality Calculations

Estimates of critical masses of small heavy water moderated reactors depend strongly on the calculated fast neutron leakages which are quite sensitive to the slowing down model used. Since the slowing down in D_2O is best represented by the convolution of an age and a Yukawa kernel,[4] which cannot be approximated well by two or three groups in the usual multigroup technique, a solution of the problem for two-region reactors with spherical symmetry was written as an expansion in the eigenfunctions of the Helmholtz equation which vanish on the outer boundary of the reactor. This development is an extension[5] of an harmonics method discussed by Schweinler[6] and Goertzel and Garabedian.[7]

A comparison of the critical concentration of U-235 predicted by the use of different slowing down kernels shows that the usual two-group method gives results for small D_2O moderated reactors which are substantially lower than those given by use of an age-Yukawa kernel (Fig. 2).

It is natural to attempt a solution of the criticality problem by expansion of the neutron fluxes, slowing down densities, absorptions etc. in the eigenfunctions of the Helmholtz equation which vanish on the outside boundary of the reflector, since the core and reflector have the same moderation properties.

UNCLASSIFIED
ORNL-LR-DWG 303A

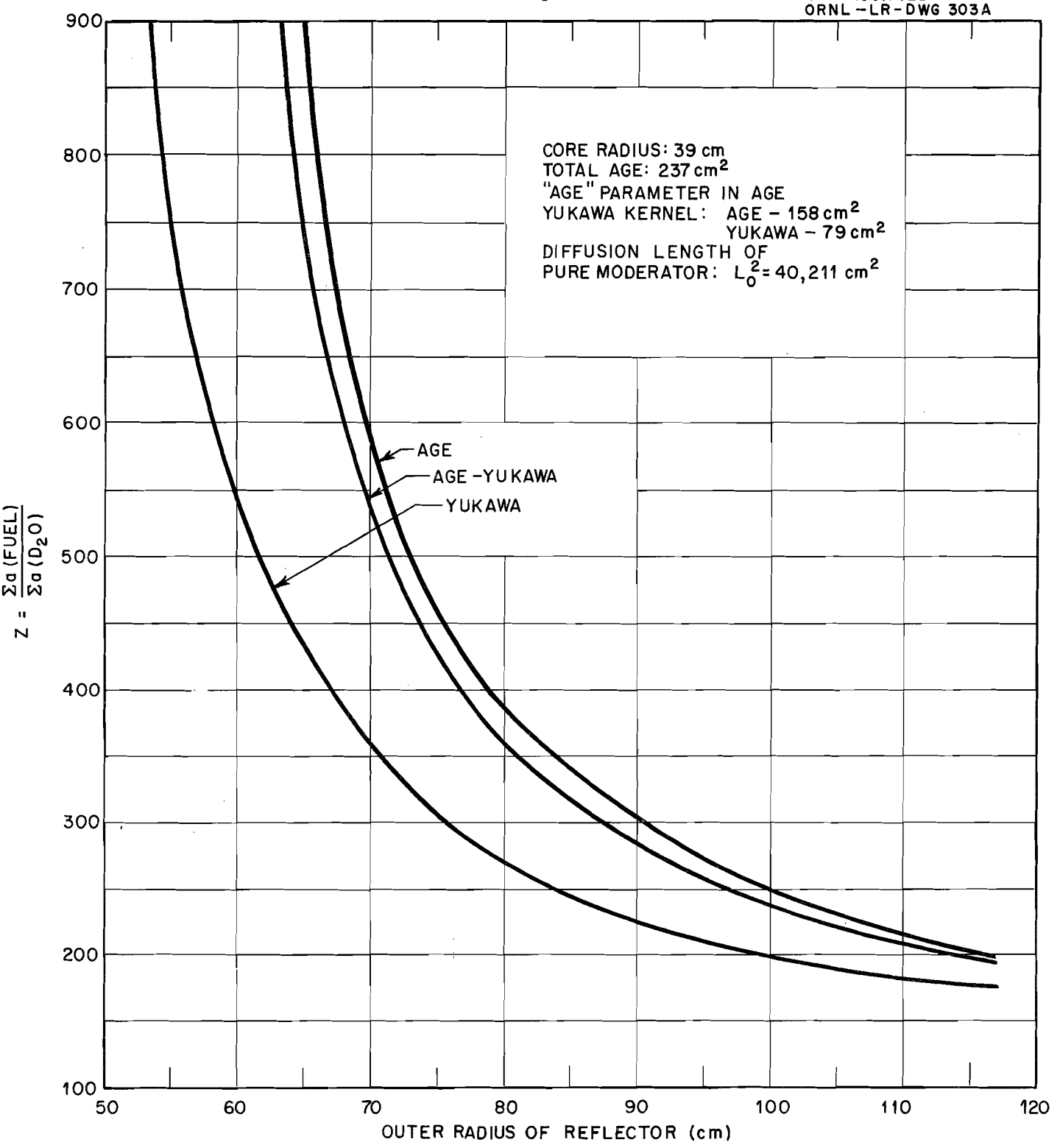

Fig. 2. Comparison of Different Slowing Down Kernels in D_2O.

The derivation of this method closely follows CF-51-5-98,[8] with the exception that resonance escape probabilities are allowed to be different in core and reflector, and resonance capture is assumed to take place just above thermal energies. The reactor equations applicable to the entire reactor can be written in the following form in which Σ_a and k are considered to be functions of position;

$$D\nabla^2\phi(\underline{r}) - \Sigma_a(\underline{r})\,\phi(\underline{r}) + q(\underline{r}) = 0 \tag{1}$$

$$q(\underline{r})/p(\underline{r}) = \int_{\text{Reactor}} k(\underline{r}')\,\Sigma_a(\underline{r}')\,\phi(\underline{r}')\,P(\underline{r},\underline{r}')\,dr' \tag{2}$$

where D = thermal diffusion coefficient.

ϕ = thermal neutron flux.

Σ_a = thermal absorption cross section.

q = thermal slowing down density.

p = resonance escape probability.

k = fast neutrons produced per thermal absorption.

$P(\underline{r},\underline{r}')$ = finite slowing down kernel (normalized to unity).

For a reflected reactor it is assumed that P and D are the same in reflector and core. Orthonormal functions $Z_i(r)$ which vanish at the extrapolated boundary and satisfy the wave equation,

$$\nabla^2 Z_i(\underline{r}) + B_i^2\,Z_i(\underline{r}) = 0 \tag{3}$$

are used to expand the following functions:

$$\phi(\underline{r}) = \sum_i C_i\,Z_i(\underline{r}) \tag{4}$$

$$\Sigma_a(\underline{r})\phi(\underline{r}) = \sum_i G_i \, Z_i(\underline{r}) \tag{5}$$

$$k(\underline{r})\,\Sigma_a(\underline{r})\phi(\underline{r}) = \sum_i E_i \, Z_i(\underline{r}) \tag{6}$$

$$q(\underline{r})/p(\underline{r}) = \sum_i A_i \, Z_i(\underline{r}) \tag{7}$$

$$q(\underline{r}) = \sum_i H_i \, Z_i(\underline{r}) \tag{8}$$

By applying the orthonormality relations, it is possible to express E_i and G_i in terms in the C's as follows:

$$G_i = \sum_j (\Sigma_{ac}\lambda_{ij} + \Sigma_{ab}\nu_{ij}) \, C_j \tag{9}$$

$$E_i = \sum_j (k_c\Sigma_{ac}\lambda_{ij} + k_b\Sigma_{ab}\nu_{ij}) \, C_j \tag{10}$$

where

$$\lambda_{ij} = \int_{\text{Core}} Z_i(\underline{r}) \, Z_j(\underline{r}) \, \underline{dr} \tag{11}$$

$$\nu_{ij} = \int_{\text{Reflector}} Z_i(\underline{r}) \, Z_j(\underline{r}) \, \underline{dr} \tag{12}$$

and subscripts c and b refer to core and reflector (blanket) respectively.

Since equations (6) and (10) define the analytic continuation of $k\Sigma_a\phi$ over all space equation (2) can be rewritten

$$q/p = \sum_i E_i \int_{\text{Reactor}} Z_i(\underline{r}')P(\underline{r},\underline{r}')\, \underline{dr}'$$

$$= \sum_i E_i \int_{\substack{\text{all}\\ \text{space}}} Z_i(\underline{r}')P_{\infty}(/\underline{r} - \underline{r}'/)\, \underline{dr}'$$

$$= \sum_i E_i\, \overline{P_{\infty}}\,(B_i^2)\, Z(\underline{r}) \tag{13}$$

where $P_{\infty}(/\underline{r} - \underline{r}'/)$ is the slowing down kernel evaluated at thermal energy in an infinite medium and $\overline{P_{\infty}}(B_i^2)$ is the Fourier transform of $P_{\infty}(/\underline{r} - \underline{r}'/)$. Hence A_i and H_i are given by

$$A_i = \overline{P}(B_i^2)E_i = \overline{P}(B_i^2) \sum_j (k_c\Sigma_{ac}\lambda_{ij} + k_b\Sigma_{ac}\nu_{ij})\, C_j \tag{14}$$

$$H_i = \sum_n (p_c\lambda_{in} + p_b\nu_{in})\, A_n$$

$$= \sum_n \overline{P}(B_n^2)(p_c\lambda_{in} + p_b\nu_{in}) \sum_j (k_c\Sigma_{ac}\lambda_{nj} + k_b\Sigma_{ab}\nu_{nj})\, C_j \tag{15}$$

Substitution of the expansions (4) through (8) in the reactor equations and the linear independence of the $Z_i(\underline{r})$ requires

$$-D\, B_i^2\, C_i - G_i + H_i = 0 \quad \text{for every } i. \tag{16}$$

In the manipulations given above we have solved for the G's and H's in terms of the C's. Hence, upon substitution of (9) and (14) in (16) we obtain an infinite set of linear homogeneous equations in the C's

which is,

$$D B_i^2 C_i + \sum_j (\Sigma_{ac}\lambda_{ij} + \Sigma_{ab}\nu_{ij}) C_j$$

$$- \sum_j \sum_n \bar{P}(B_n^2)(p_c\lambda_{in} + p_b\nu_{in})(k_c\Sigma_{ac}\lambda_{nj} + k_b\Sigma_{ab}\nu_{nj}) C_j = 0 \qquad (17)$$

The criticality condition is therefore the vanishing of the determinant of the coefficients of the C's.

In applications to be made to HRT, all absorption cross sections are expressed in terms of the pure moderator cross section, Σ_{ao}. Upon dividing each element of the critical determinant by Σ_{ao} one obtains,

$$\left| L_o^2 B_i^2 \delta_{ij} + \left[(1 + w + z)\lambda_{ij} + \alpha\nu_{ij}\right] - \sum_n \bar{P}(B_n^2)(p_c\lambda_{in} + p_b\nu_{in})(\eta_c z\lambda_{nj} + k_b\alpha\nu_{nj}) \right| = 0 \qquad (18)$$

for the critical equation, where L_o^2 is the diffusion length of pure moderator (displacement of moderator is small), and z, w and α are core fuel, core poison and blanket absorption cross sections divided by Σ_{ao}.

For reactors with spherical symmetry

$$Z_i(r) = \sqrt{\frac{2}{b}}\,\frac{1}{r}\,\sin\frac{i\pi r}{b} \qquad i = 1, 2, 3 \ldots \qquad (19)$$

then,

$$\lambda_{ij} = \frac{1}{\pi}\left[\frac{\sin\,(i-j)\,\frac{\pi a}{b}}{i-j} - \frac{\sin\,(i+j)\,\frac{\pi a}{b}}{i+j}\right] \qquad (20)$$

and

$$\nu_{ij} = \delta_{ij} - \lambda_{ij} \qquad (21)$$

If there is no multiplication in the blanket (pure D_2O reflector) and no resonance capture in either the core or blanket, the summation over the index n in equation (18) reduces to a single term and the critical equation becomes,

$$\left| (L_o^2 B_i^2 + 1)\, \delta_{ij} + w \lambda_{ij} + (1 - \eta_c \bar{P}(B_i^2))\, \lambda_{ij}\, z \right| = 0 \qquad (22)$$

In this equation everything can be considered fixed except the concentration of fuel which is proportional to z. Hence, the problem is to solve equation (22) or, more generally, equation (18) for z.

A method of finding approximate solutions is indicated by the fact that the diffusion length in D_2O is very large and thus $L_o^2 B_i^2 >> 1$ for all the D_2O moderated reactors of interest. For example, at 20°C $L_o^2 \simeq 10{,}000\ cm^2$ and $L_o^2 B_i^2 > 1$ for reactor radii less than 314 cm. Hence, if we divide each row by $1 + L_o^2 B_i^2$, the elements of the determinant may be written in the form $\delta_{ij} + M_{ij}$, where M_{ij} falls off at least as fast as $1/i^3 j$. The nth approximation to the root z is obtained by setting all the M_{ij} for $i > n$ and $j > n$ equal to zero. The infinite order determinant is thus in the nth order approximation set equal to the $n \times n$ sub-determinant for which the M_{ij}'s have not been neglected. It should be noted that the fuel concentration is not involved in a transcendental way, but is obtained simply by the solution of an nth degree algebraic equation. This method of approximation has turned out to be very convenient for calculating the HRT, since in this case one needs to consider only $n = 2$.

Tests of the Method:

A series of test calculations were made for a Yukawa kernel, for which exact solutions can be obtained by the usual two-group analytical method. The results of these tests for a Yukawa kernel were then used to indicate the magnitude of the errors in the nth approximation for a more general kernel such as the age-Yukawa.

The first series of calculations were made for two core radii and D_2O reflectors of several sizes. The basic data used are: $\eta = 2.09$, age in D_2O at $300^\circ C$ equals 237 cm^2, $L_o^2 = 40{,}211$, $w = 0$ and $\alpha = 0$. The results in terms of z, the ratio of fuel to pure moderator macroscopic absorption cross sections, are given in Table 1 for the various orders of approximation as defined above, where the subscript refers to the order of approximation. The exact two-group values for z are the z's having no subscript.

These results show the convergence to be very rapid for all the cases where $L_o^2 B_i^2 \gg 1$; i.e., for all the outer radii less than about 600 cm. The only case of slow convergence, as is to be expected, is the one with the large reflector (630 cm radius).

As the reflector thickness is increased, the λ_{ij} tend to zero and the z obtained from any finite approximation to the determinant increase without limit.

The calculations were repeated for the 39 cm core radius using an age-Yukawa kernel with the same second moment as the single Yukawa and are recorded in Table 2. The "age" parameters in the age-Yukawa kernel were divided in the ratio 3:2 respectively.

Table 1

Outer Reflector Radius (cm)	Core Radius = 39 cm 52	65	78	117
z	1424.55	434.32	281.40	175.80
z_1	1347.58	396.48	269.45	195.42
z_2	1369.35	422.96	276.96	175.02
z_3	1398.02	431.14	277.01	179.94
z_4	1416.43		279.88	
z_2/z	.961	.974	.984	.996
z_3/z	.981	.993	.984	1.024
z_4/z	.994		.995	

Outer Reflector Radius (cm)	Core Radius = 79 cm 104	130	630
z	57.79	41.02	24.70
z_1	57.71	42.23	153.17
z_2	57.64	40.96	63.39
z_3	57.76	40.98	41.40
z_2/z	.997	.998	2.566
z_3/z	.999	.999	1.676

[redacted]

Table 2

Age-Yukawa Kernel

Outer Reflector Radius (cm)	Core Radius = 39 cm 65	78	117
z_1	601.51	302.20	200.36
z_2	708.01	370.40	192.25
z_3	753.28	371.77	208.16
z_4		381.03	
z_2/z_1	1.18	1.23	.96
*Yukawa z_2/z_1	1.07	1.03	.90
z_3/z_2	1.06	1.00	1.08
Yukawa z_3/z_2	1.02	1.00	1.03
z_4/z_3		1.02	
Yukawa z_4/z_3		1.01	

* The ratios of successive approximations obtained with a Yukawa kernel, Table 1.

Comparison of the ratios of successive approximations for the two kernels show the rate of convergence to be somewhat faster for the Yukawa kernel for the same reactor dimensions. However, since the Yukawa kernel predicts a much lower critical mass than does the age-Yukawa, comparison of rates of convergence should be made not for the same reactor dimensions but for equal z's. This point was not pursued further, since the results already indicate that the mathematical error in the z_2 approximation is probably less than 5 - 10% for reactors like the HRT.

Although the critical concentration is given to sufficient accuracy for the HRT with only the fundamental and the first harmonic, the details of the neutron distribution in the reflector and the neutron leakage out of the reflector are not represented well enough by two modes to permit accurate estimates of the conversion ratios for the case of a ThO_2-D_2O slurry reflector (or blanket). The first harmonic is negative in the blanket, resulting in an underestimate of the neutron leakage; in some cases the leakage was negative. Calculations of the conversion ratios made using the fundamental plus the first three harmonics (4 x 4 approximation) converged very well; tests showed the 4 x 4 approximation to be within 1% for the conversion ratio.

3. Data

Calculations have been made for core solutions of D_2O and UO_2SO_4 and three reflector systems: (1) D_2O, (2) ThO_2-D_2O slurry and (3) low-enrichment UO_2SO_4 solutions. The fuels considered are 93% enriched U-235 and 100% U-233. A heavy water composition of 99.75% D_2O and 0.25% H_2O was assumed in all cases.

Cross Sections and η:

The values of the kT = 2000 m/sec absorption cross section used, with the exception of σ_a(U-233), are those listed in BNL-170,[9] 170A[10] and 170B.[11]

The absorption cross section of U-233 is still somewhat in doubt. The old tabulated kT = 2000 m/sec value is 564 ± 10 b compared with a value of 650 b obtained at Argonne using the fast chopper. If the old value is corrected for a change in the boron cross section from 720 b to 750 b, one obtains σ_a(U-233) = 588 b. On the other hand, use of the Harwell value of 771 b for boron leads to a value of 597 b for U-233. The latter value seems to be in closer agreement with the work of von Dardel, which gives a value of 764 ± 3 b for the boron cross section.

The kT = 2000 m/sec absorption cross sections used are:

U-233 - 600 b	Th - 7.0 b
U-235 - 680 b	S - 0.49 b
U-238 - 2.77 b	D_2O (99.75% D_2O + 0.25% H_2O) - .00273 b/molecule

The values of the fission neutrons emitted per neutron absorbed in U-235 and U-233 were taken to be 2.09 and 2.32 respectively.

All cross sections were assumed to be 1/v except σ_a(U-235), to which the small non-1/v correction given in BNL-170 was applied. Finally all cross sections used in the calculations were averaged over a Maxwellian distribution.

[redacted]

Constants for D_2O:

The "age" parameters for an age-Yukawa slowing down kernel in D_2O at 20°C and density of 1.103 g/cm^3 were obtained from the measurements of Friedman and Wattenburg,[4] who find an age to indium resonance of 106 cm^2, of which 48 and 52 cm^2 are the "ages" in the Yukawa and age kernels respectively. An additional 14 cm^2, which is an estimate of the age from indium resonance to thermal, was added to the age of the Gaussian part of the kernel, giving a total age from fission to thermal of 120 cm^2.

Since the critical concentration of the HRT is strongly temperature dependent, the effect of the chemical binding on the diffusion coefficient of thermal neutrons was calculated as a function of D_2O temperature.[12] Results for a D_2O density of 1.103 g/cm^3 are shown in Table 3.

Table 3

°C	20	50	100	150	200	250	280	300	320
Diff. Coef. (cm)	0.839	0.852	0.872	0.891	0.910	0.926	0.936	0.942	0.951

The value of the diffusion coefficient at 20°C agrees well with experiments which measured thermal neutron relaxation lengths in D_2O with various additions of B_2O_3. Comparisons are given in Table 4.

Table 4

Diffusion coefficient (20°C)	Observer
0.83 ± .03	A. H. Dexter, B. Hamermesh, E. W. Hones, P. A. Morris and G. R. Ringo--ANL-4746, (1951).
0.84 ± .01	D. C. Woods, S. W. Kash and E. Martin--NAA-SR-230 (1953).
0.839	Calculated by L. C. Noderer[12]

ThO_2-D_2O Slurry Blankets:

The age of the thermal neutrons in ThO_2-D_2O slurries for the concentrations less than 1500 g Th/kg D_2O is not very different from the age in pure D_2O. The inelastic scattering compensates for the reduction of D_2O in the slurry. Calculations by Tobias[13] which take into account inelastic scattering in Th indicate that the age of the 1349 gm Th/kg D_2O slurry is about 2% less than for pure D_2O. In the HRT calculations, the age and slowing down process in the ThO_2-D_2O slurry was taken to be the same as for D_2O.

The resonance capture in the thorium was estimated using the resonance integral measurements of Hughes and Eggler.[14] Their results in the concentration range of interest can be represented by the following formula for the effective resonance integral

$$I_{eff} = 8.33 \left(\frac{\Sigma s}{N_{o2}}\right)^{0.253} \text{ barns}$$

where Σ_s/N_{02} is the total scattering cross section per atom of thorium.

Low Enrichment UO_2SO_2 Blankets:

At low concentrations the slowing down is the same as in pure D_2O.

The effective resonance integral, based on measurements of Mitchell,[15] can be expressed in the range of concentrations of interest by,

$$I_{eff} = 3.05 \left(\frac{\Sigma_s}{N_{28}}\right)^{0.471}$$

where Σ_s/N_{28} is the total scattering cross section per atom of U-238.

A summary of all the data as a function of temperature is given in Appendix II.

4. Critical Concentration

Although the reactor will be operated for most of its lifetime with a D_2O reflector, the reactor may be operated for some time with a thorium oxide slurry blanket. There is also a possibility that the reactor may be operated with a low-enrichment uranyl sulfate blanket to test plutonium production. Calculations were therefore made for all three blanket systems.

Both the core and blanket solutions were assumed to be made up with 99.75% D_2O, 0.25% H_2O heavy water. A uniform temperature distribution is assumed throughout the reactor. The core solution is assumed to be either 93% enriched U-235 or 100% U-233 as uranyl sulfate in D_2O. The results are reported as grams U-235 or U-233 per kilogram of moderator.

Hence, the exact enrichment of the fuel is unimportant as the effect of the small amount of U-238 present may be neglected.

It is difficult to treat the core shell and pressure vessel as regions in the harmonics method calculation. The effect of these regions was determined using the Feynman-Welton two-group approximation.[16,17] The effect of the core shell was then taken into account in the harmonics calculations by adding a uniform poison throughout the core region to give the same effect on the critical concentration as the Feynman-Welton result. The pressure vessel was taken into account similarly by increasing the blanket thickness.

In the reactor with 1349 g Th/kg D_2O blanket operating at 280°C, a significant fraction of the fissions in the core are due to resonance absorption in U-235. To ascertain the effect of this on the critical concentration, a multigroup calculation was made for an equivalent bare sphere. (Bare reactor with a radius such to give the same critical concentration as the reflector reactor.) The calculation showed that 37% of the fissions are in the resonance energy region and the net effect is to increase the reactivity by 5%. This is equivalent to lowering the critical concentration by 15 g U-235/kg D_2O.

The uncertainty of the age of thermalized fission neutrons in D_2O, which is about 7%, leads to the rather large uncertainty of 20% in the estimated critical concentration of the HRT with the 1349 g Th/kg D_2O slurry blanket at 280°C. This uncertainty, however, diminishes with decreasing critical concentration, and is about 9% for the HRT with a

D_2O reflector.

Results of the calculations are presented in the following graphs and tables:

Figure 3: Critical concentration of U-235 in the core of spherical two-region D_2O moderated reactors vs. core diameter.

Figure 4: Critical concentration of U-235 in core of HRT with D_2O reflector.

Figure 5: Critical concentration of U-235 in core of HRT with ThO_2-D_2O slurry blanket.

Figure 6: Critical concentration of U-233 in core of HRT with 1349 g Th/kg D_2O ThO_2-D_2O slurry blanket.

Figure 7: Critical concentration of U-235 in core of HRT with UO_2SO_4-D_2O blanket.

Figure 8: Critical concentration of U-235 in core of unreflected HRE.

Table 5: Critical concentration, conversion ratio and blanket power of HRT with UO_2SO_4-D_2O blanket.

[redacted] ORNL-LR-DWG 2667

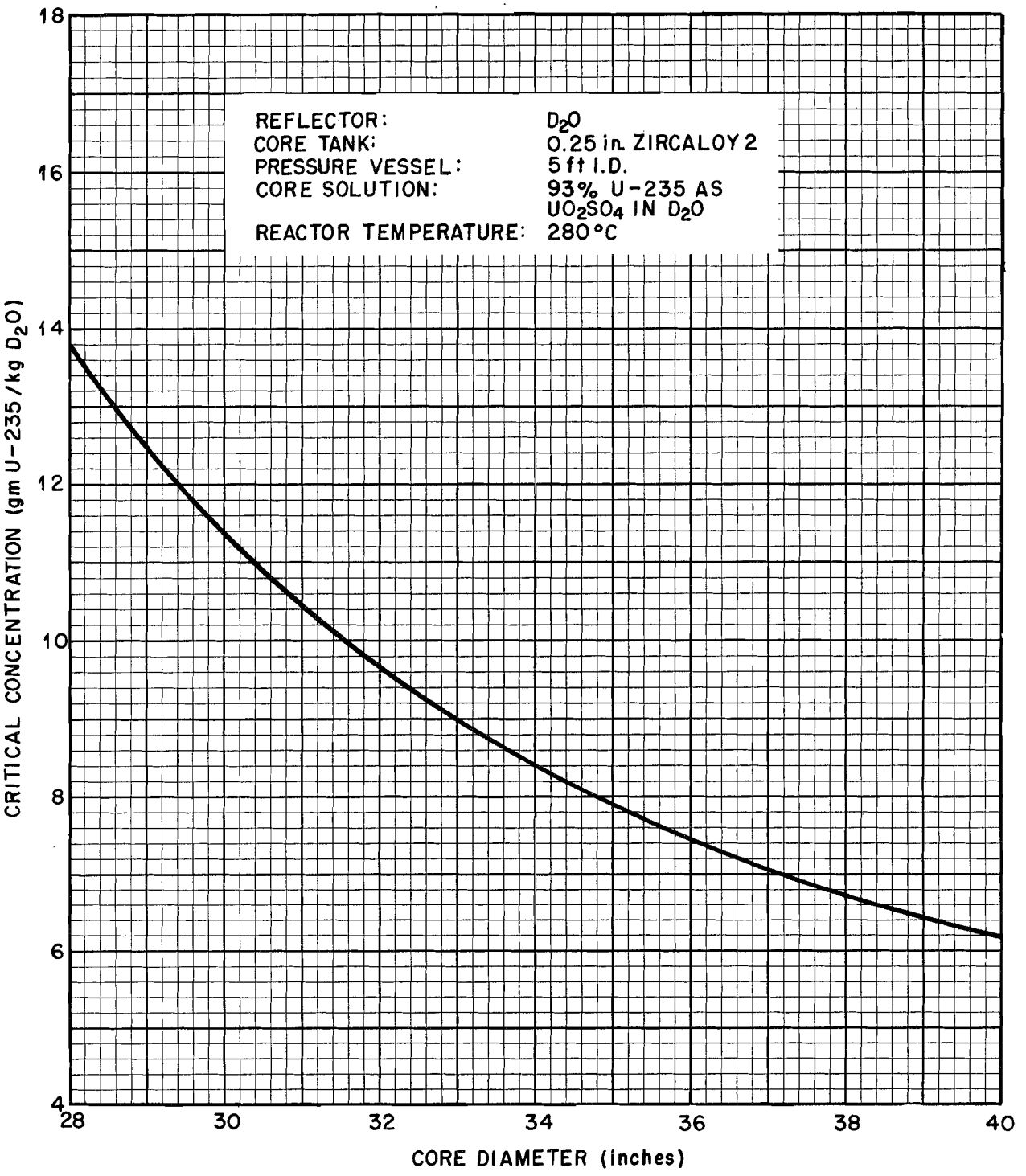

Fig. 3. Critical Concentration of U-235 in Core of Spherical Two Region D_2O Moderated Reactors.

[redacted]

ORNL-LR-DWG 2668

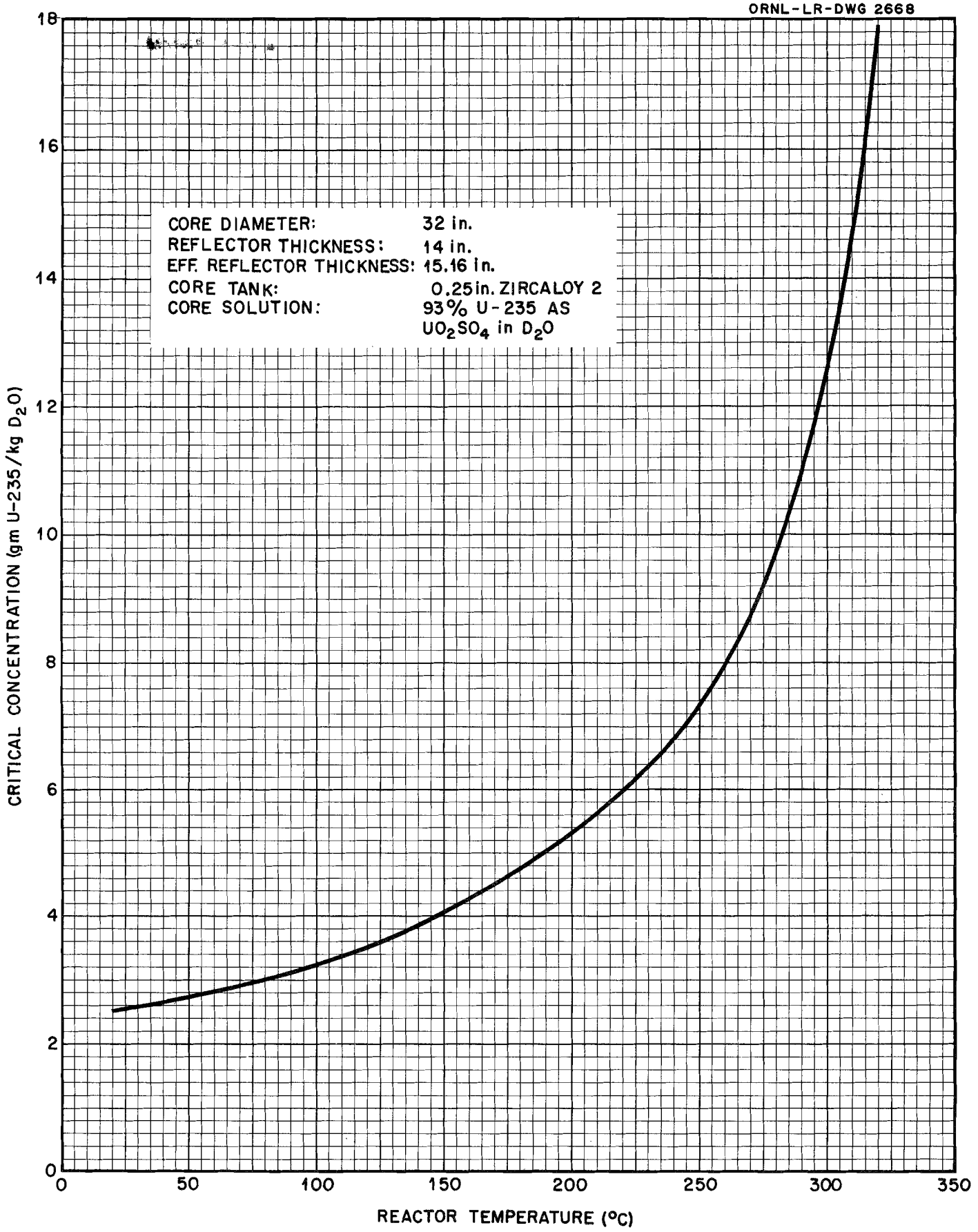

Fig. 4. Critical Concentration of U-235 in Core of HRT with D_2O Reflector.

[redacted]

ORNL-LR-DWG 2669

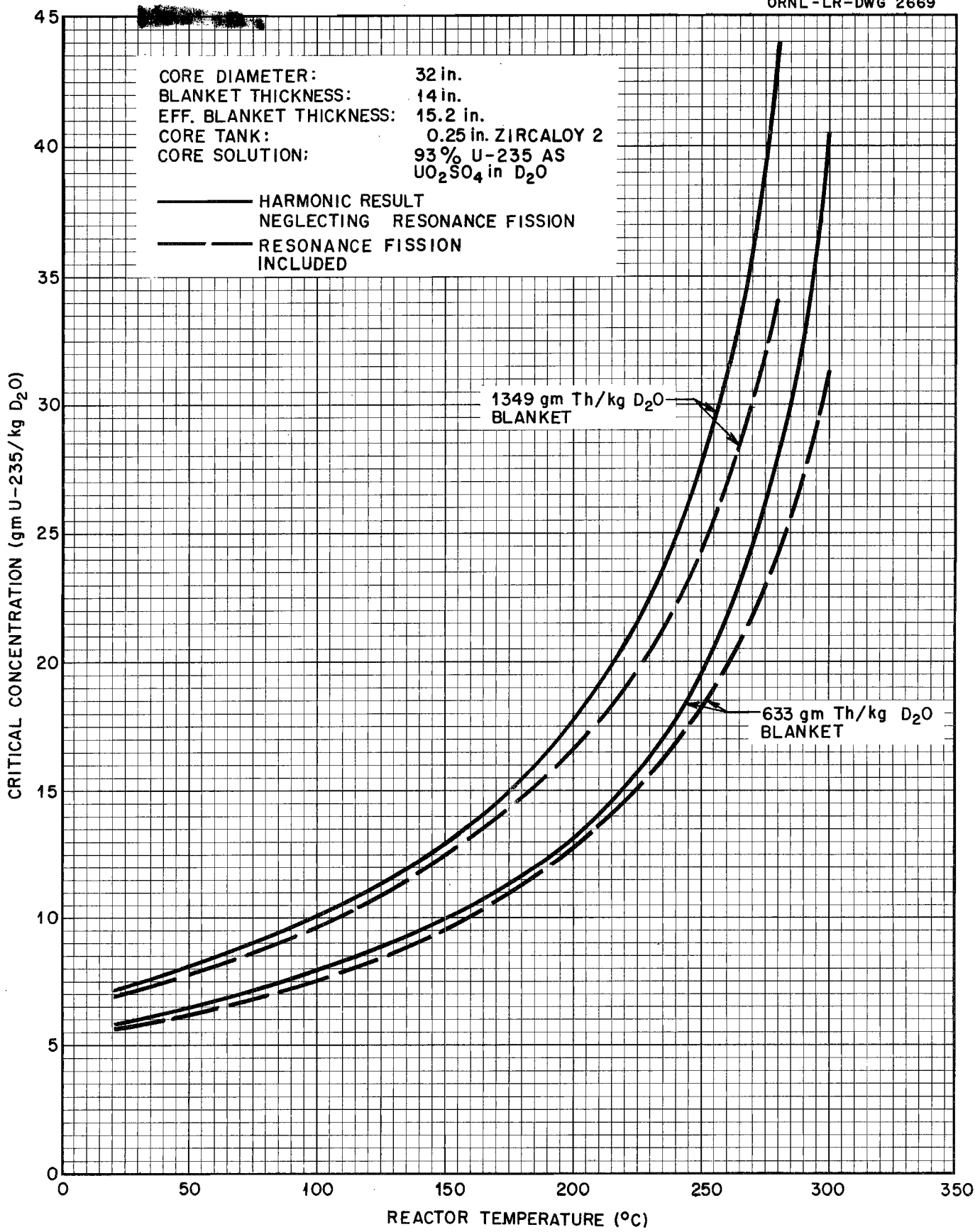

Fig. 5. Critical Concentration of U-235 in Core of HRT with ThO_2 - D_2O Slurry Blanket.

[redacted]

ORNL-LR-DWG 2670

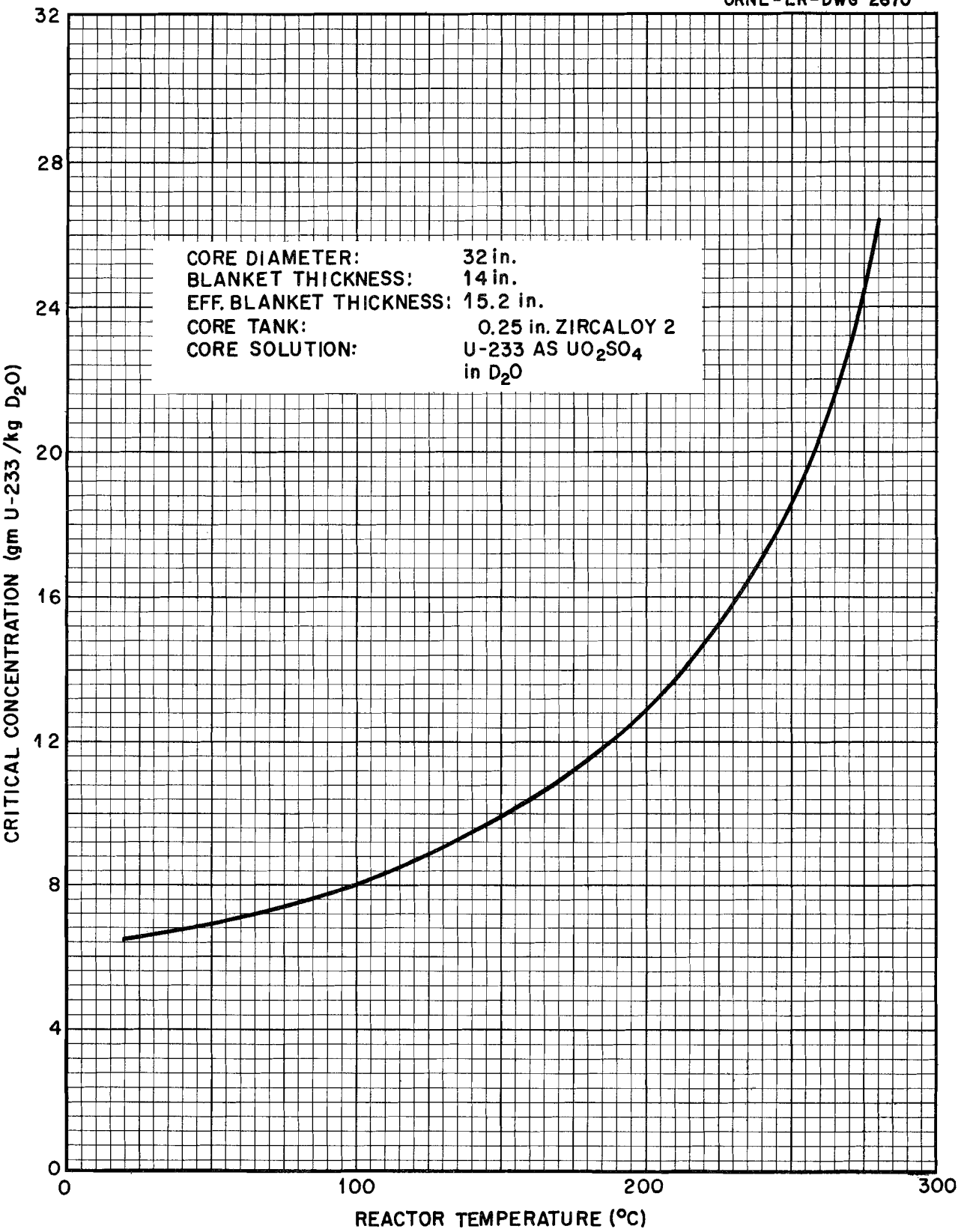

Fig. 6. Critical Concentration of U-233 in Core of HRT with 1349 gm Th/kg D_2O ThO_2 - D_2O Slurry Blanket.

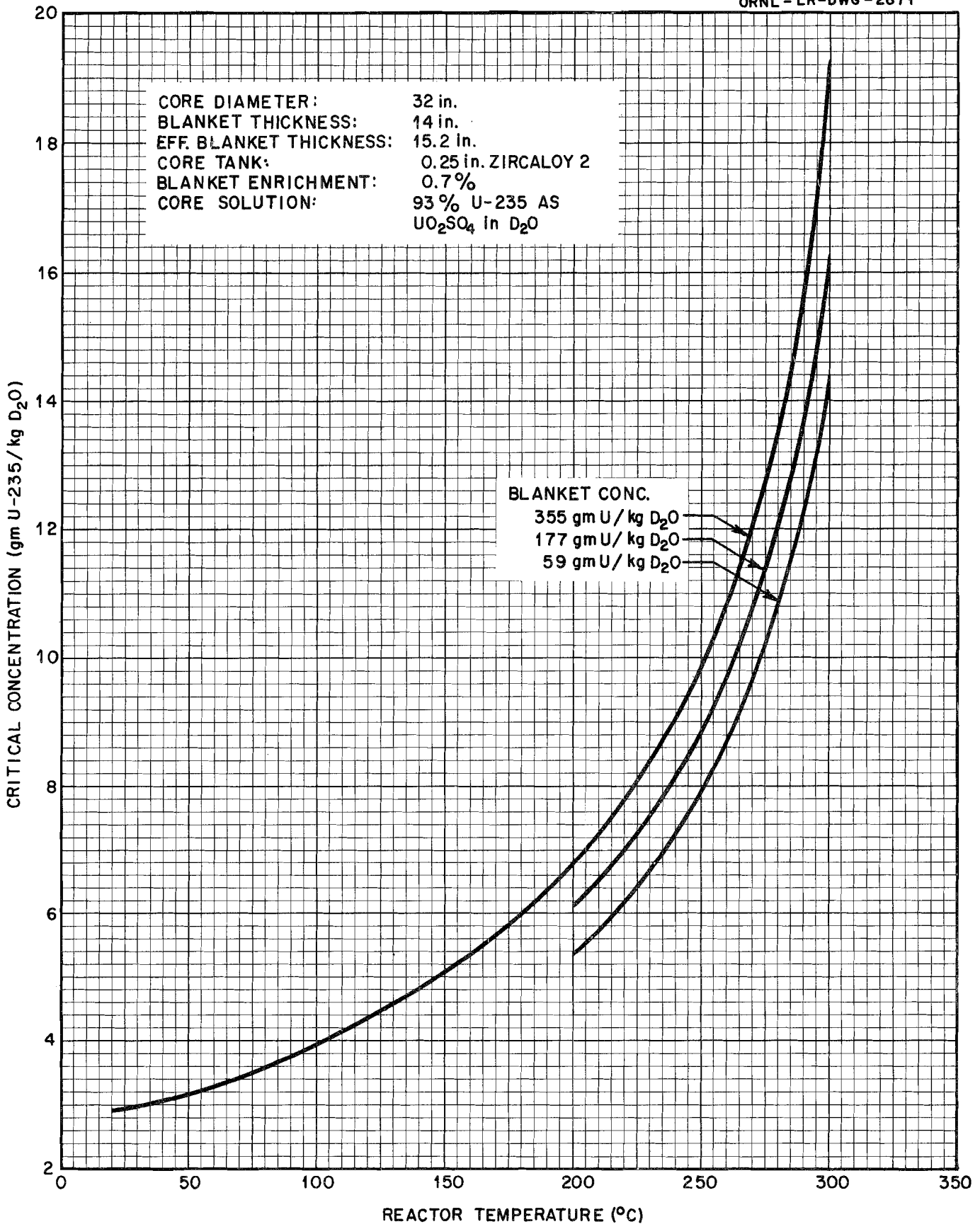

Fig. 7. Critical Concentration of U-235 in Core of HRT with $UO_2SO_4-D_2O$ Blanket.

[redacted]

ORNL-LR-DWG 2672

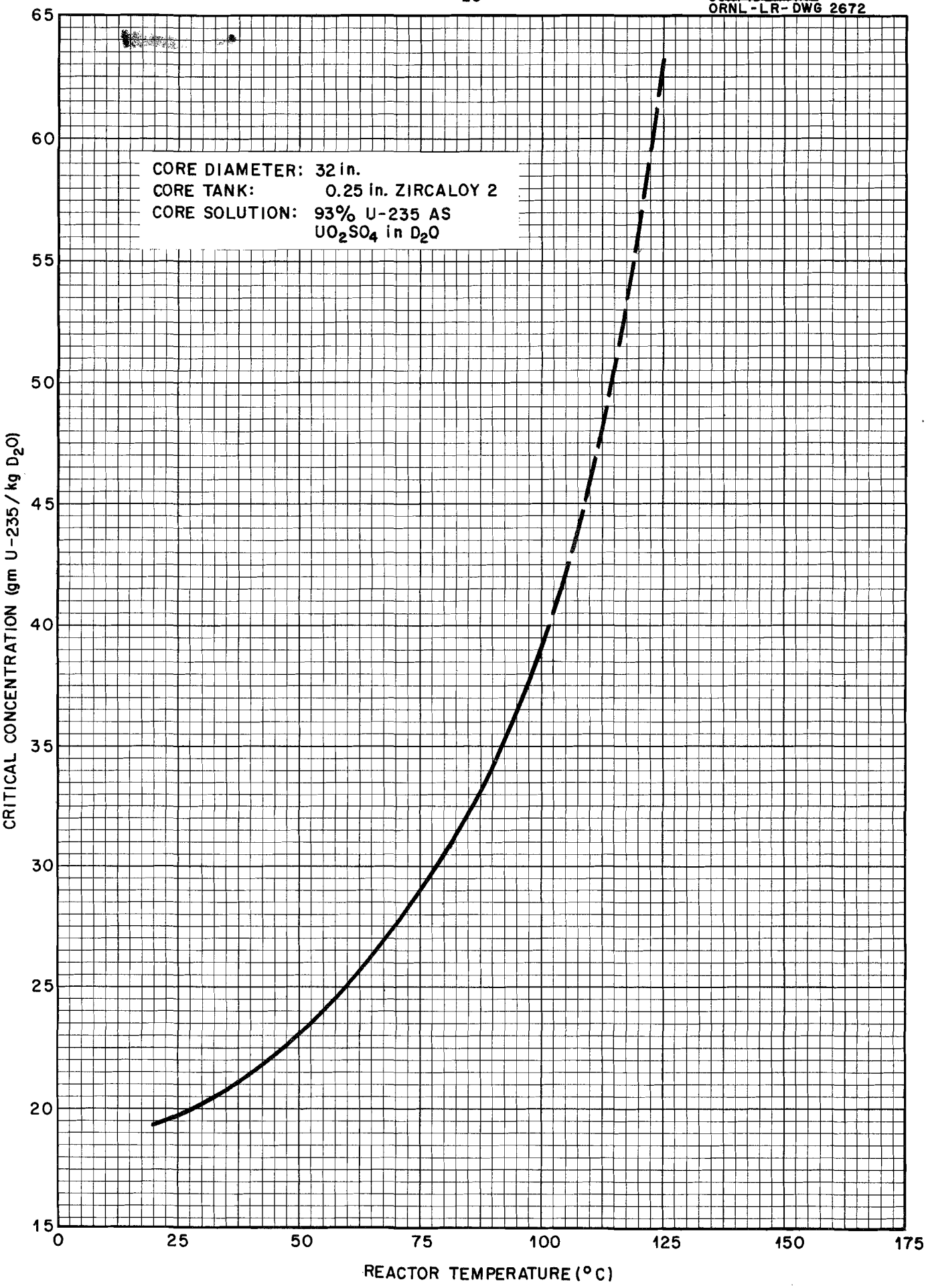

Fig. 8. Critical Concentration of U-235 in Core of Unreflected HRT.

Table 5

Critical Concentration, Conversion Ratio and Blanket Power

of HRT with UO_2SO_4-D_2O Blanket

T °C	Blanket Enrich. Wt % 25	Blanket Conc. g U/kg D_2O	Critical Core Conc. g 25/kg D_2O	Conversion Ratio C_R	Conversion Ratio C_c	$\frac{P_b}{P_c}$	g Pu/day at 5Mw Core Power
200	.7	355.3	6.753	.4976	.7764	.5706	5.2
250	.7	355.3	9.813	.4857	.7021	.4527	4.7
280	.7	355.3	13.900	.4362	.5887	.3544	3.9
300	.7	355.3	19.367	.4173	.5426	.3032	3.6
280	.4	355.3	15.170	.4633	.5556	.2020	3.7
280	.25	355.3	15.836	.4763	.5349	.1249	3.6
200	.7	176.7	6.085	.3638	.4996	.3785	3.35
250	.7	176.7	8.707	.3372	.4355	.2951	2.92
280	.7	176.7	12.084	.3158	.3899	.2369	2.61
300	.7	176.7	16.267	.2978	.3550	.1935	2.38
280	.4	176.7	12.785	.3265	.3687	.1302	2.47
280	.25	176.7	13.834	.3361	.3641	.0838	2.44
200	.7	59.2	5.288	.2008	.2337	.1650	1.56
250	.7	59.2	7.938	.1821	.2038	.1088	1.37
280	.7	59.2	10.853	.1711	.1875	.0961	1.26
300	.7	59.2	14.404	.1667	.1798	.0830	1.20
280	.4	59.2	11.075	.1734	.1826	.0533	1.22
280	.25	59.2	11.199	.1745	.1803	.0331	1.21

$$C_R = \frac{\text{Atoms Pu(239) formed}}{\text{Atom U(235) destroyed in reactor}}$$

$$C_c = \frac{\text{Atoms Pu(239) formed}}{\text{Atom U(235) destroyed in core}}$$

$$\frac{P_b}{P_c} = \frac{\text{Blanket fission power}}{\text{Core power}}$$

5. Critical Concentration With Ordinary Water as Moderator

Because of the high critical concentration in the reactor with the thorium oxide slurry blanket, it might be desirable to operate with ordinary water instead of heavy water to test operational feasibility. Results of the ordinary water reactor at 280°C along with comparable heavy water results are given in the following table.

Table 6

	Critical Core Concentration	
	g 25/kg moderator	g 25/l at 280°
H_2O core - H_2O reflector	20.6	15.50
D_2O core - D_2O reflector	9.69	8.13
H_2O core - 633 g Th/kg H_2O ThO_2-H_2O blanket	21.42	16.11
D_2O core - 633 g Th/kg H_2O ThO_2-D_2O blanket	27.32	22.93

It is evident that the critical concentration for the ordinary water case is insensitive to blanket concentration. The major drawback is that the U-233 production in the thorium oxide blanket is greatly reduced as the conversion ratio is only 0.11 compared to .72 for the D_2O moderated reactor. The ordinary water calculations were made using the Feynman-Welton two-group approximation.[16,17]

6. Neutron Flux Distribution and Adjoint Functions

The flux distributions and adjoint functions were obtained using ordinary two-group diffusion theory.[18] The fast "age" was adjusted in the calculation to give the same critical core concentration as the harmonics method with an age-Yukawa kernel.

The distributions for the reactor operating at a uniform temperature of 280°C were obtained for the case of D_2O reflector, 355 g natural U/kg D_2O UO_2SO_4-D_2O blanket, and 633 g Th/kg D_2O ThO_2, and are plotted in Figures 9, 10, 11 respectively. Values for the thermal flux in the reactor operating at 5 Mw core power are:

Table 7

Thermal Neutron Flux at 5 Mw Core Power, 280°C

Blanket Composition	ϕ_{2c}(max) n cm^{-2} sec^{-1}	$\bar{\phi}_{2c}$ n cm^{-2} sec^{-1}	$\bar{\phi}_{2R}$ n cm^{-2} sec^{-1}	ϕ_2(wall) n cm^{-2} sec^{-1}
D_2O	1.07×10^{14}	8.17×10^{13}	3.82×10^{13}	7.25×10^{13}
ThO_2-D_2O Slurry (633 g Th/kg D_2O)	4.62×10^{13}	2.89×10^{13}	9.05×10^{12}	2.26×10^{13}
UO_2SO_4-D_2O (355 g Nat.U/kg D_2O)	8.75×10^{13}	6.05×10^{13}	1.98×10^{13}	4.80×10^{13}

where: ϕ_2(max) = maximum thermal neutron flux in the core.

$\bar{\phi}_{2c}$ = average thermal neutron flux in the core region.

$\bar{\phi}_{2R}$ = average thermal neutron flux in the blanket region.

ϕ_2(wall) = thermal neutron flux at core tank.

[redacted]

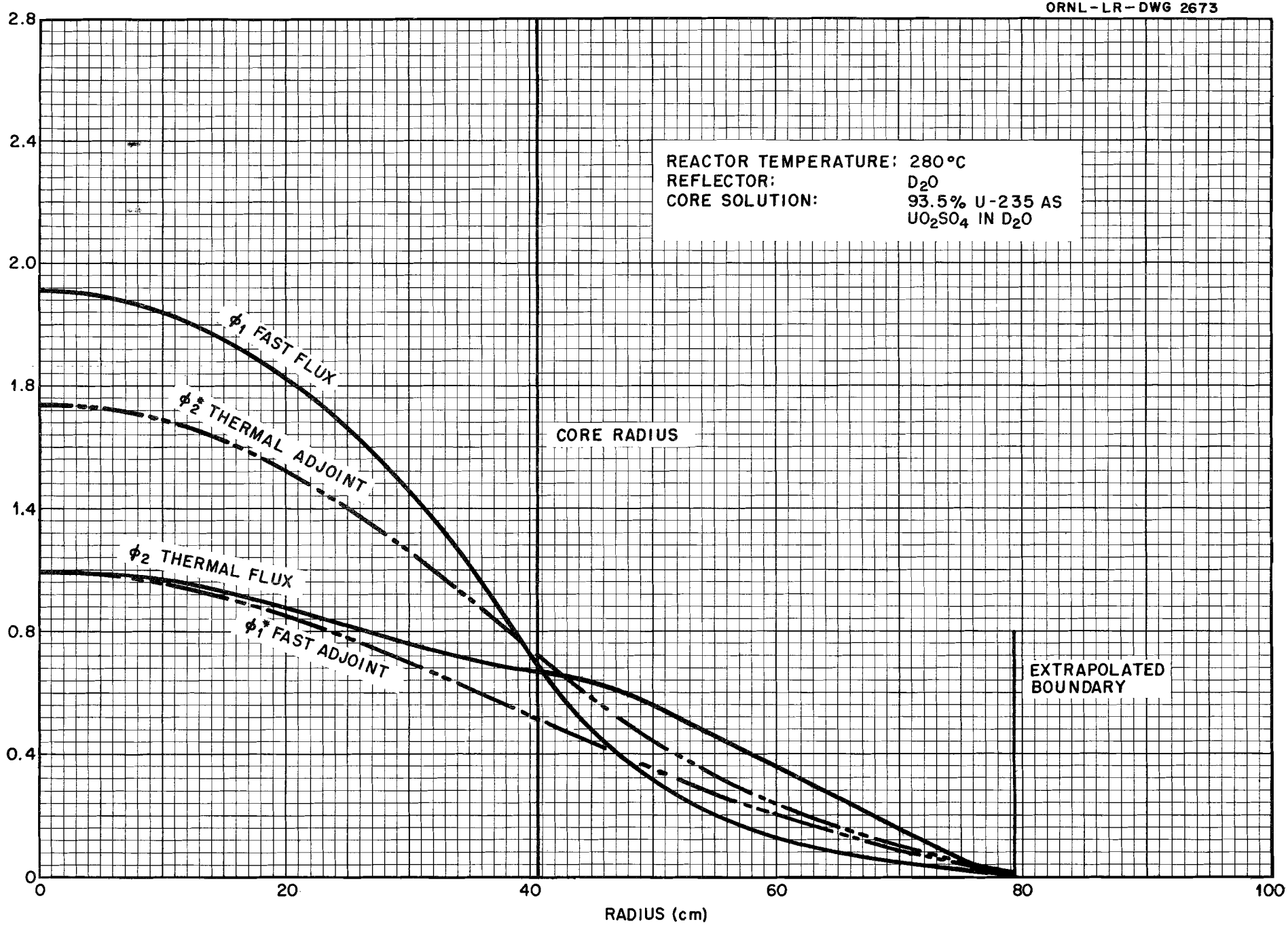

Fig. 9. Flux and Adjoint Functions in HRT with D_2O Reflector.

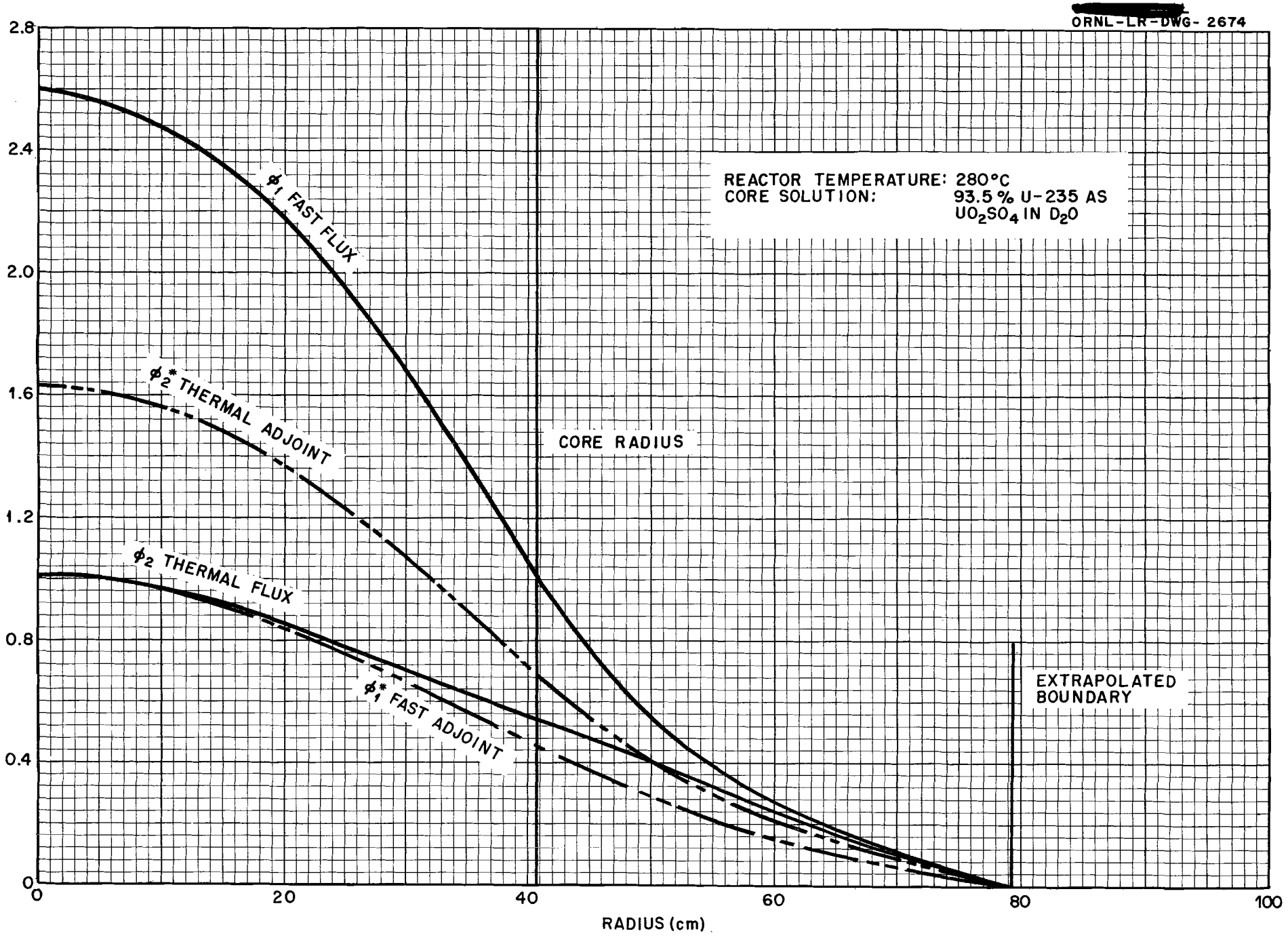

Fig. 10. Flux and Adjoint Functions in HRT with 355 gm U/kg Natural UO_2SO_4-D_2O Blanket.

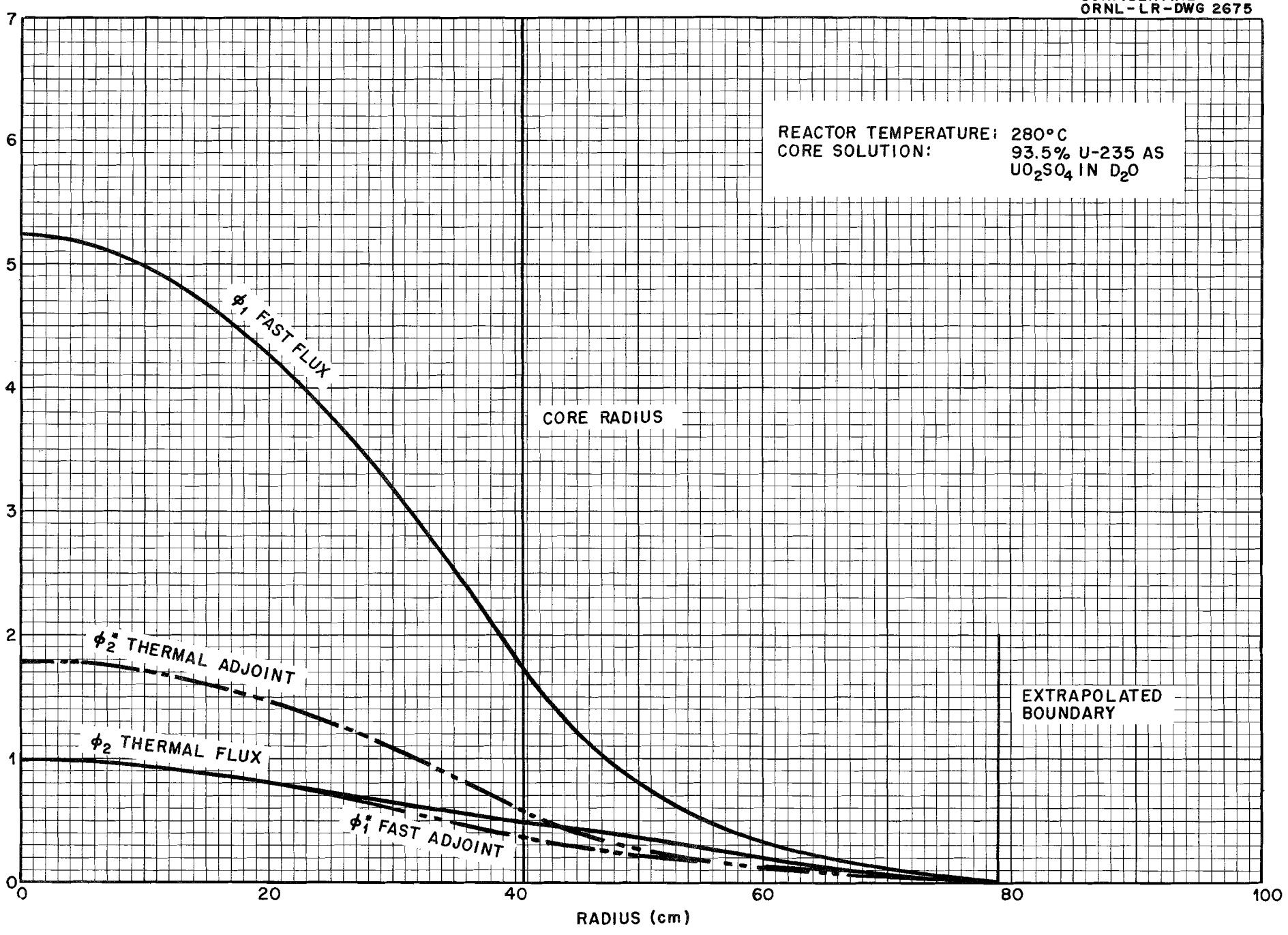

Fig. 11. Flux and Adjoint Functions in HRT with 633 gm Th/kg D_2O ThO_2-D_2O Slurry Blanket.

The neutron leakages from the reactor were obtained from the flux equations. At 5 Mw core power, the neutron leakage evaluated at the inside of the pressure vessel are:

Table 8

Blanket Composition	Fast Leakage n/sec	Thermal Leakage* n/sec
D_2O	5.1×10^{16}	1.6×10^{17}
ThO_2-D_2O Slurry (633 g Th/kg D_2O)	5.8×10^{16}	3.6×10^{16}
UO_2SO_4-D_2O (355 g Nat.U/kg D_2O)	1.1×10^{17}	8.5×10^{16}

* Note: Evaluated at inside of pressure vessel. The thermal leakage is attenuated by a factor of ~ 87 by the pressure vessel.

7. Prompt Neutron Mean Lifetimes

The mean lifetime of prompt neutrons in the reactor was computed using the method of the hypothetical boron addition experiment discussed in Appendix III. The calculation was made for the reactor with a D_2O reflector for a range of temperatures from 20°C to 300°C.

Table 9

Prompt Neutron Mean Lifetimes in HRT

Reactor Temp.	D_2O	BLANKET COMPOSITION Nat. UO_2SO_4-D_2O 355 g U/kg D_2O	ThO_2- D_2O 633 g Th/kg D_2O	ThO_2-D_2O 1349 g Th/kg D_2O
°C	l-sec	l-sec	l-sec	l-sec
20	1.24×10^{-3}	7.5×10^{-4}	4.0×10^{-4}	2.4×10^{-4}
100	1.18×10^{-3}	7.2×10^{-4}	3.8×10^{-4}	2.3×10^{-4}
200	7.8×10^{-4}	4.7×10^{-4}	2.5×10^{-4}	1.5×10^{-4}
280	5.6×10^{-4}	3.4×10^{-4}	1.8×10^{-4}	1.1×10^{-4}
300	4.9×10^{-4}	3.0×10^{-4}	1.5×10^{-4}	0.95×10^{-4}

8. Temperature Coefficient of Reactivity

The magnitude of the negative temperature coefficient of the HRT is important in determining the kinetic stability of the reactor. This quantity may be determined by two methods, i.e. from the slope of the critical concentration vs. temperature curve or from perturbation theory.

In Figure 12 are plotted values for the overall reactor temperature coefficient obtained by taking the slope of the critical concentration vs. temperature curve for the D_2O, 355 g natural U/kg D_2O uranyl sulfate blanket, and the 633 g Th/kg D_2O ThO_2 slurry blanket. The value of the temperature coefficient of core concentration C, $\frac{\partial C}{\partial T}$ multiplied by the core concentration

coefficient of reactivity $\frac{\partial K_{eff}}{\partial C}$ yields the overall temperature coefficient. The core concentration coefficient of reactivity, Figure 13, was obtained by differentiation of the critical equation used in the harmonics method.

Values of the overall temperature coefficients as well as the coefficients for core temperature, blanket temperature, and non-uniform temperature changes were obtained by V. K. Paré[19] using two-group perturbation theory. His values are based on the flux distributions and adjoint functions given in this report.

[redacted]

Table 10

Temperature Coefficient of Reactivity in HRT at 280°C, $^{\circ}C^{-1}$

		BLANKET COMPOSITION	
	D_2O	UO_2SO_4-D_2O 355 g Nat.U/kg D_2O	ThO_2-D_2O 633 g Th/kg D_2O
$\frac{\partial K_{eff}}{\partial T_{uc}}$	-1.81×10^{-3}	-1.71×10^{-3}	-1.91×10^{-3}
$\frac{\partial K_{eff}}{\partial T_{uB}}$	-1.07×10^{-3}	-0.76×10^{-3}	-0.32×10^{-3}
$\frac{\partial K_{eff}}{\partial T_{u}}$	-2.88×10^{-3}	-2.47×10^{-3}	-2.23×10^{-3}
$\frac{\partial K_{eff}}{\partial T_{\phi c}}$	-1.85×10^{-3}	-1.85×10^{-3}	-1.99×10^{-3}

∂T_{uc} = uniform change in core temperature, blanket temperature constant.

∂T_{uB} = uniform change in blanket temperature, core temperature constant.

∂T_{u} = uniform change in both core and blanket temperature.

$\partial T_{\phi c}$ = non-uniform temperature change with shape of heat production. Average core temperature used as measure of change.

[redacted]

ORNL-LR-DWG 2676

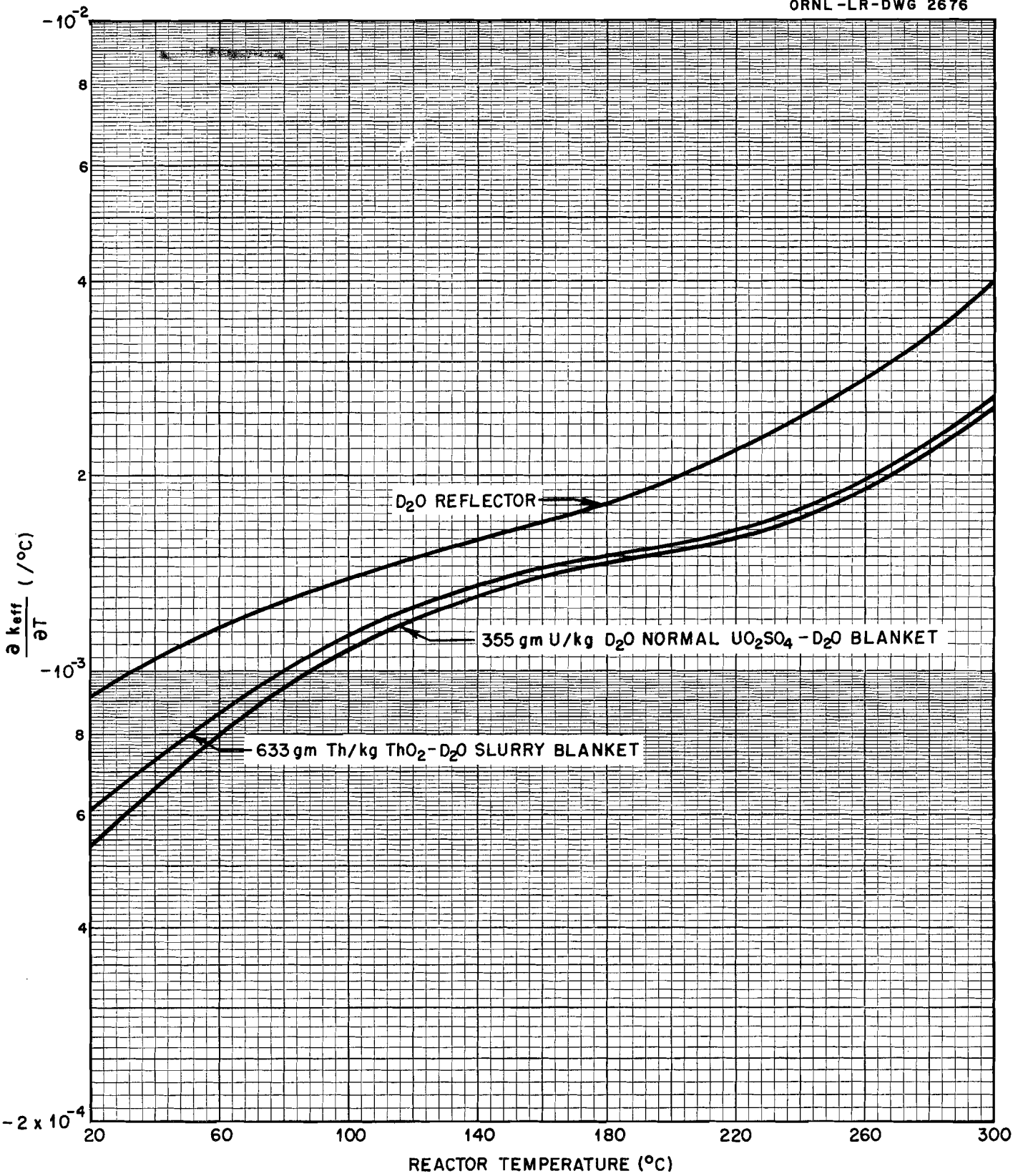

Fig. 12. Temperature Coefficient of Reactivity for HRT.

[redacted]

ORNL-LR-DWG 2677

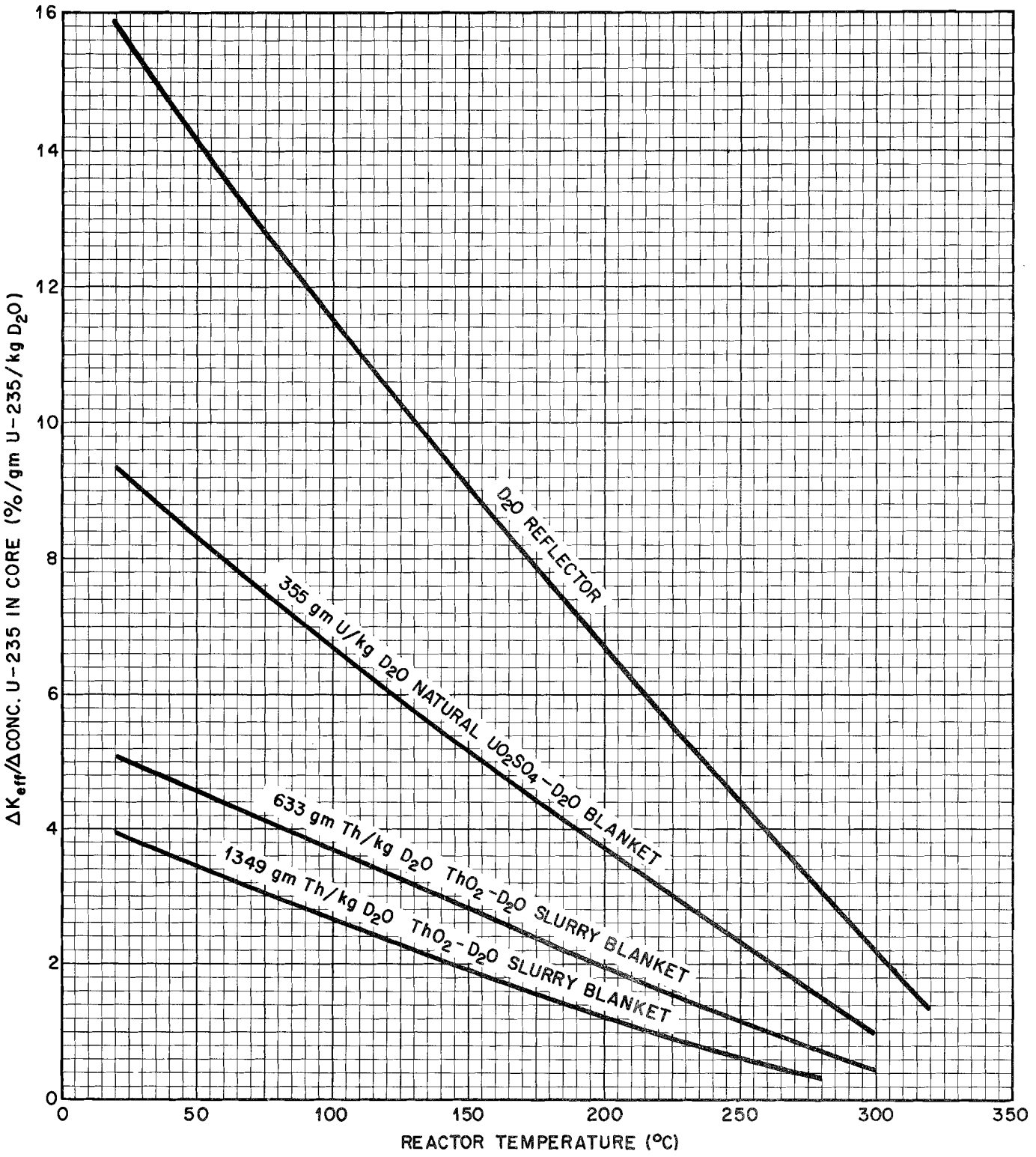

Fig. 13. Core Concentration Coefficient of Reactivity of HRT.

9. Buildup of U-233 in Thorium Blanket

The isotope U-233 is produced by the following decay chain resulting from the absorption of a neutron in thorium.

$$_{90}Th^{232}(n,\gamma) \quad _{90}Th^{233} \xrightarrow[\tau_{1/2} = 23 \text{ min}]{\beta^-} {}_{91}Pa^{233} \xrightarrow[\tau_{1/2} = 27.4 \text{ d}]{\beta^-} {}_{92}U^{233}$$

The half-life of Th-233 is so short that it may be ignored. As the Pa-233 has a significant half-life and also a neutron cross-section of 150 b at kT = 2200 m/sec, it must be considered in the buildup equation.

The equations for buildup of Pa-233 and U-233 in the blanket for the condition that N(02) is constant and for initial values of N(13) and N(23) equal to zero are:

$$N(13) = \left[\frac{\Phi\, \sigma_{eff}(02)\, N(02)}{\lambda_{13} + \sigma_a(13)\Phi}\right]\left[1 - e^{-\left[\lambda_{13} + \sigma_a(13)\,\Phi\right]t}\right]$$

$$N(23) = \left[\frac{\lambda_{13}\,\Phi\,\sigma_{eff}(02)\,N(02)}{(\lambda_{13} + \sigma_a(13)\,\Phi)(\sigma_a(23)\Phi)}\right] \cdot \left\{\frac{\sigma_a(23)\Phi\left[1 - e^{-(\lambda_{13} + \sigma_a(13)\Phi)\,t}\right] - \left[\lambda_{13} + \sigma_a(13)\Phi\right]\left[1 - e^{-\sigma_a(23)\Phi t}\right]}{\left[\sigma_a(23)\Phi - (\lambda_{13} + \sigma_a(13)\Phi\right]}\right\}$$

where subscript

02 refers to $_{90}Th^{232}$

13 refers to $_{91}Pa^{233}$

23 refers to $_{92}U^{233}$

[redacted]

$N(ij)$ = concentration in atoms/cm^3 of the i-j th isotope

λ = decay constant.

$\bar{\Phi}$ = average flux in blanket region including the external loop.

$\sigma_{eff}(02)$ = effective absorption cross-section to take into account resonance captures.

t = time.

At the operating conditions of 5 Mw core power and 280°C, the values of the parameters used were

$$\lambda_{13} = 2.927 \times 10^{-7} \text{ sec.}$$

$$\sigma_{eff}(02) = 7.51 \text{ b for 633 g Th/kg } D_2O \text{ blanket}$$

$$\sigma_{eff}(02) = 7.09 \text{ b for 1349 g Th/kg } D_2O \text{ blanket}$$

$$\sigma_a(23) = 371 \text{ b}$$

$$\sigma_a(13) = 96.8 \text{ b}$$

$$\bar{\Phi} = 9.21 \times 10^{12} \text{ n/cm}^2\text{/sec for 633 g Th/kg } D_2O \text{ blanket}$$

$$\bar{\Phi} = 6.00 \times 10^{12} \text{ n/cm}^2\text{/sec for 1349 g Th/kg } D_2O \text{ blanket.}$$

The ratios $\frac{N(23)}{N(02)}$ and $\frac{N(13)}{N(02)}$ were obtained from the equation for various operating times and converted to concentration in g/kg Th.

As the flux in the blanket is a function of the U-233 concentration, a calculation of the average blanket flux after one year buildup of U-233 was made. The flux was found to differ from the initial flux by only 5% and was therefore assumed to be constant. The U-233 and Pa-233 concentrations as a function of operating time are given in the following figures.

Figure 14: Buildup of U-233 and Pa-233 in 633 g Th/kg D_2O thorium slurry blanket of HRT with U-235 in core.

Figure 15: Buildup of U-233 and Pa-233 in 1349 g Th/kg D_2O thorium slurry blanket of HRT with U-233 in core.

ORNL-LR-DWG 2678

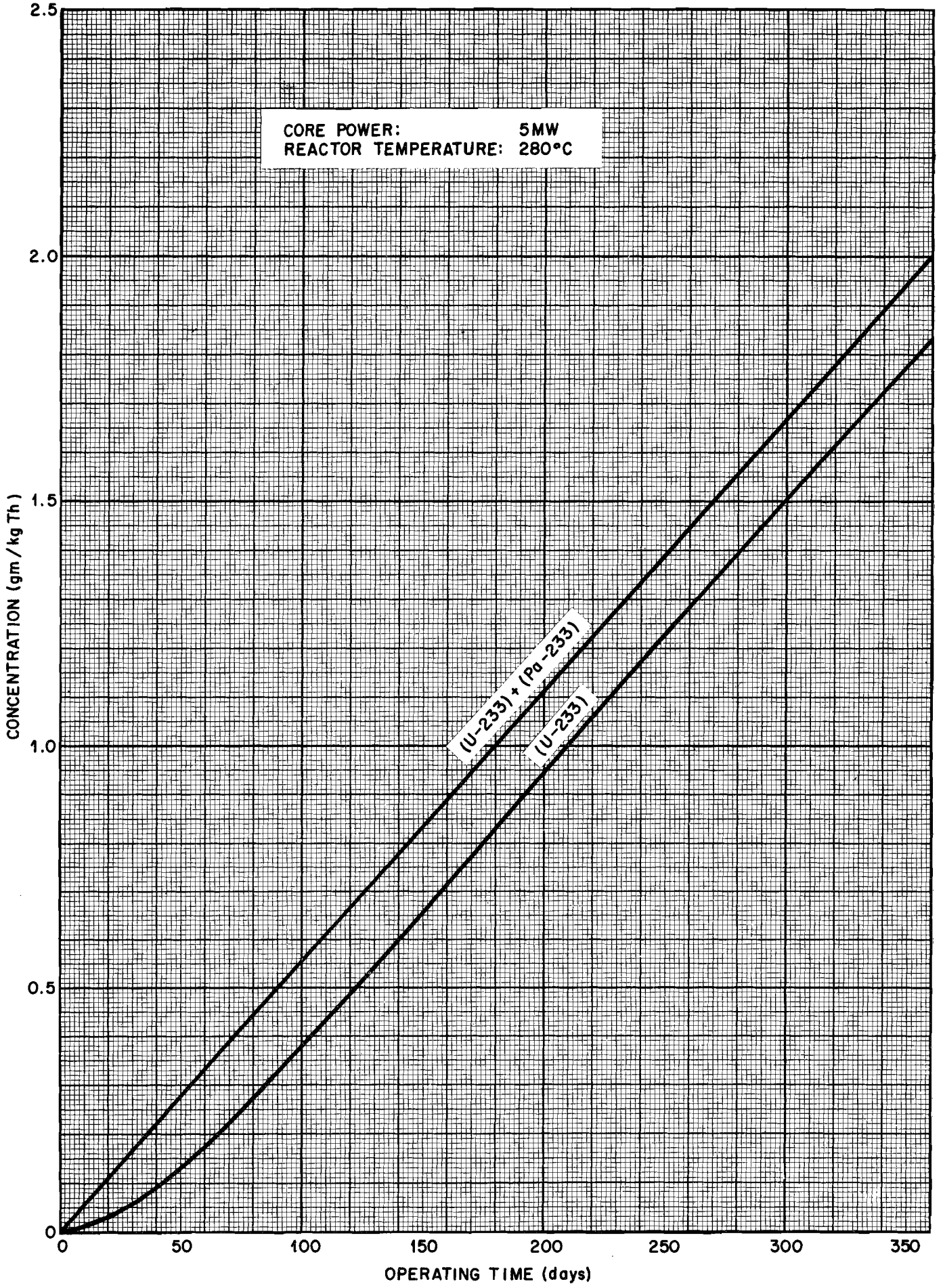

Fig. 14. Buildup of U-233 and Pa-233 in 633 gm Th/kg D_2O ThO_2-D_2O Slurry Blanket of HRT with U-235 in Core.

[redacted]

ORNL-LR-DWG 2679

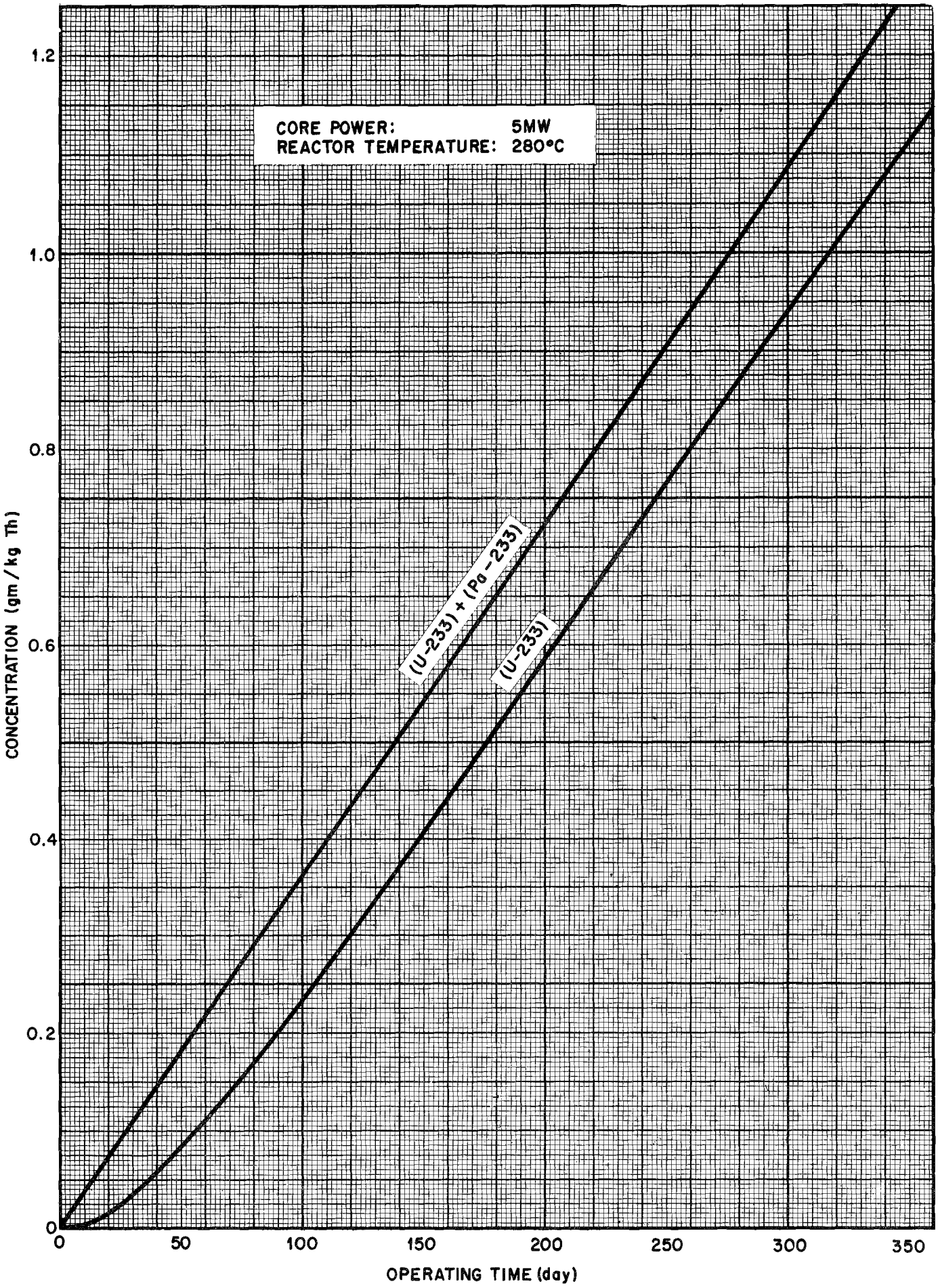

Fig. 15. Buildup of U-233 and Pa-233 in 1349 gm Th/kg D_2O ThO_2-D_2O Slurry Blanket of HRT with U-233 in Core.

10. Heat Generation in the Blanket Region

D_2O BLANKET

The heat generation in the D_2O of the blanket region will be due to the absorption of gamma rays generated in the core region and in the pressure vessel as well as the gamma and beta energy released following neutron absorptions in the D_2O itself. In addition to the energy absorbed in the blanket region, the blanket fluid will be required to carry away the heat generated in the pressure vessel. All calculations are based on the 280°C reactor.

Core Gammas:

In the process of making an estimate of the thermal stresses in the pressure vessel, the following gamma ray currents arising from gamma production in the core were obtained by R. H. Chapman.[20]

Outward current of core gammas at core wall (5Mw core power):

Gamma Energy	Gamma Current
1 mev	8.25×10^{12} photons/cm^2/sec
2.5 mev	5.9×10^{12} " " "
6 mev	7.27×10^{11} " " "

Outward current of core gammas at inside edge of pressure vessel (5 Mw core power):

Gamma Energy	Gamma Current
1 mev	3.55×10^{11} photons/cm^2/sec
2.5 mev	5.27×10^{11} " " "
6 mev	1.01×10^{11} " " "

From these values the result is obtained that the absorption of energy in the blanket region arising from core gammas is 4.01×10^{17} mev/sec.

Neutron Absorptions in Blanket:

The neutron balance obtained in the criticality calculations gives the result that 2.64×10^{15} neutrons/sec are absorbed in the D_2O blanket when the core is operated at 5 Mw. Assuming each absorption gives rise to 6 mev of energy which is absorbed in the D_2O, an energy production rate of 1.58×10^{16} mev/sec is obtained.

Pressure Vessel Gammas:

The neutron leakage from the D_2O blanket into the pressure vessel is 1.58×10^{17} n/sec. Assuming that all of these neutrons are absorbed in the pressure vessel and give rise to 7 mev/absorption of gamma energy, the energy production is 1.1×10^{18} mev/sec. The amount of energy absorbed in the pressure vessel was calculated to be 5500 Btu/hr/ft^2 by R. H. Chapman.[20] This represents 7.91×10^{17} mev/sec for the entire pressure vessel. Assuming that half of the difference between the energy produced and the energy absorbed goes back into the reactor, the D_2O blanket will absorb

$$\frac{1.1 \times 10^{18} - 7.9 \times 10^{17}}{2} \text{ mev/sec} = 1.55 \times 10^{17} \text{ mev/sec}$$

of the pressure vessel gammas.

The total heat that must be carried away by the blanket fluid is then:

Core gammas	4.01×10^{17} mev/sec
Blanket gammas	0.16×10^{17} " "
Pressure vessel absorptions	7.91×10^{17} " "
Pressure vessel gammas absorbed in D_2O	1.55×10^{17} " "
Total:	13.63×10^{17} mev/sec or 218 kw.

THORIUM BLANKET

The gamma heat generation in the 633 g Th/kg D_2O ThO_2 slurry blanket was obtained by the same method as described for the D_2O blanket. The total gamma heat that must be carried away by the blanket fluid is:

Core gammas	5.6×10^{17} mev/sec
Blanket gammas	11.5×10^{17} " "
Pressure vessel absorptions + pressure vessel gammas absorbed in blanket	2.5×10^{17} " "
Total:	19.6×10^{17} mev/sec or 314 kw.

In addition to the heat production from gammas, there will be heat produced by fissions in the U-233 as it builds up in the blanket. The fission heat production was obtained using the U-233 concentrations and fluxes given in section 9.

The fission, gamma, and total heat production are plotted in the following figures.

Figure 16: Blanket power in 633 g Th/kg D_2O ThO_2-D_2O slurry blanket of HRT with U-235 in core.

Figure 17: Blanket power in 1349 g Th/kg D_2O ThO_2- D_2O slurry blanket of HRT with U-233 in core.

URANIUM BLANKET

The heat generation in the blanket region is due largely to fissions occurring in the U-235 present. The ratio of blanket fission power to core power given in Table 5 for various concentrations and enrichments of blanket solution was calculated from the neutron balance obtained in the criticality calculations.

In addition to the fission heat produced in the blanket there will be approximately 300 kw (at 5 Mw core power) of heat produced in the blanket by gamma and beta energy from other sources than fission in the blanket.

11. Breeding in the HRT

Due to its rather small diameter for a D_2O moderated reactor, the HRT operating with U-233 fuel at 280°C and a 1349 g Th/kg D_2O slurry blanket will be a mixed thermal-resonance reactor in which about one-third of the fissions results from neutron absorption in the resonances of U-233. Hence, there exists a considerable uncertainty in the theoretical breeding ratio due to the lack of precise information on the capture to fission ratio, α, for the resonances and the details of the resonance structure.

Estimates of the initial breeding ratio of the HRT at 280°C with U-233 fuel, 1349 g Th/kg D_2O blanket based on several assumed average

[redacted]

ORNL-LR-DWG 2680

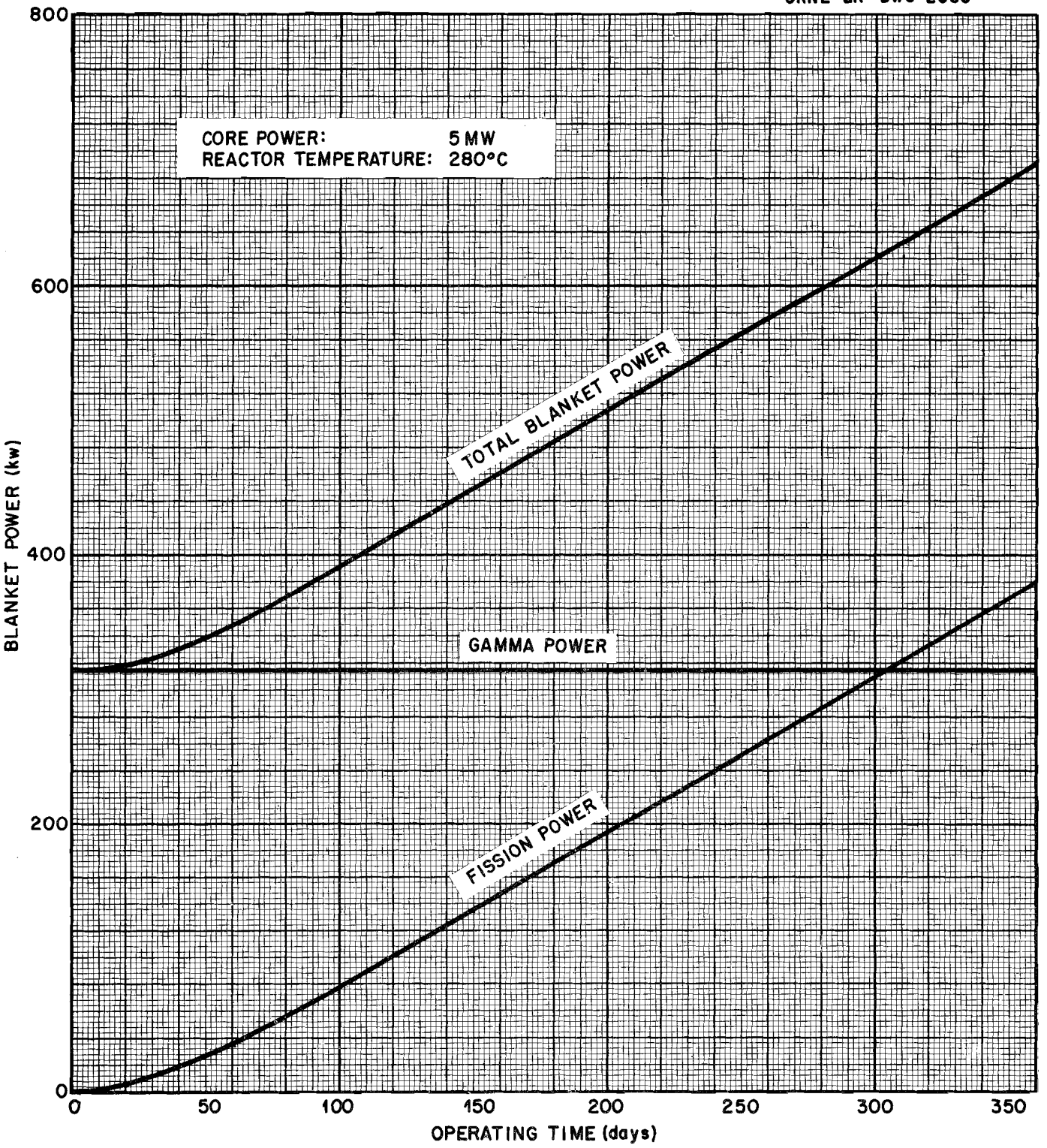

Fig. 16. Blanket Power in 633 gm Th/kg D_2O ThO_2–D_2O Slurry Blanket of HRT with U-235 in Core.

[redacted] ORNL-LR-DWG 2681

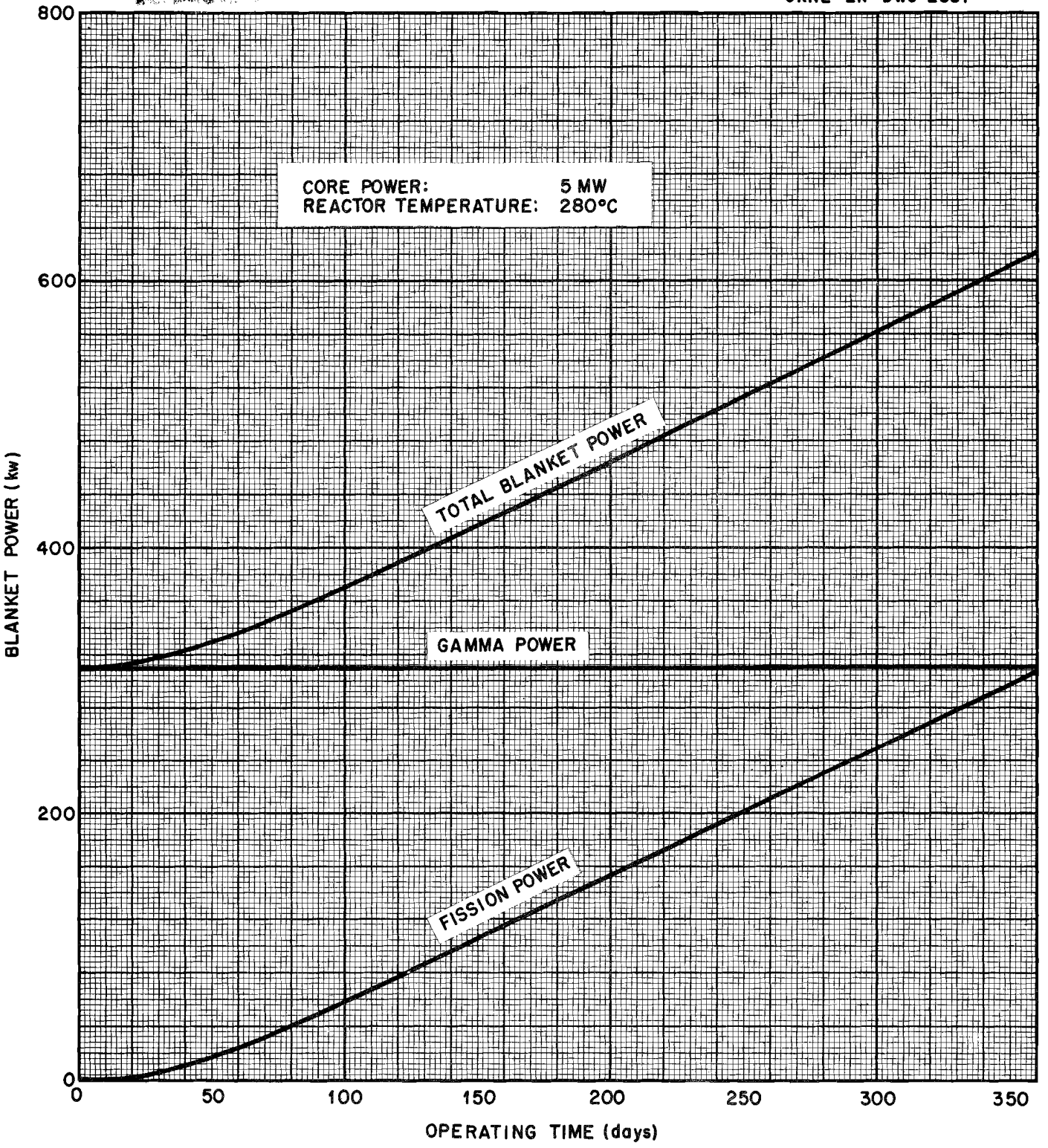

Fig. 17. Blanket Power in 1349 gm Th/kg D_2O $ThO_2 - D_2O$ Slurry Blanket of HRT with U-233 in Core.

values of η for resonance absorption are given in Table 11. A total resonance integral of 400 b and η(thermal) = 2.32 was assumed.

Table 11

η(resonance)	Breeding Ratio
2.32	1.02
2.10	.96
1.90	.92
1.70	.87

The breeding ratio calculated assuming 100% thermal fission is given in Figure 18 as a function of reactor temperature for several blanket concentrations with U-233 in the core.

Although the critical fuel concentration and hence the uncertainty in the breeding ratio can be decreased by lowering the thorium concentration in the blanket, the breeding ratio drops because of increased neutron leakage.

The uncertainty in the breeding ratio can be reduced by lowering the critical concentration. This may be accomplished by:

a) lowering the operating temperature of the reactor to 200°C or less.

b) increasing the core diameter above 32 inches.

The critical concentration of the HRT is markedly dependent on temperature because of its small size and the effect of density on the migration length of neutrons in D_2O. A decrease of the temperature from 280°C to 200°C reduces the critical concentration and hence the uncertainty in the breeding ratio by a factor of $\sim$2. For the HRT at 200°C with U-233

fuel and 1349 g Th/kg D_2O thorium slurry blanket, the conversion ratio is estimated to be 1.12 for η(resonance) = 2.32 and 1.07 for η(resonance) = 1.70.

Breeding at an operating temperature of 280°C probably can only be demonstrated in reactors somewhat larger than the proposed HRT. Calculations show that a reactor with a 40-inch-diameter core and 6-foot pressure vessel has a critical concentration at 280°C of about 10 g U-233/kg D_2O with the 1349 g Th/kg D_2O blanket (Figures 19-20). The breeding ratio is 1.16 assuming η(resonance) = 2.32 and 1.12 for η(resonance) = 1.70. The breeding ratio depends somewhat less sensitively on temperature than in the smaller reactor (Figure 18).

[redacted]

ORNL-LR-DWG-2682

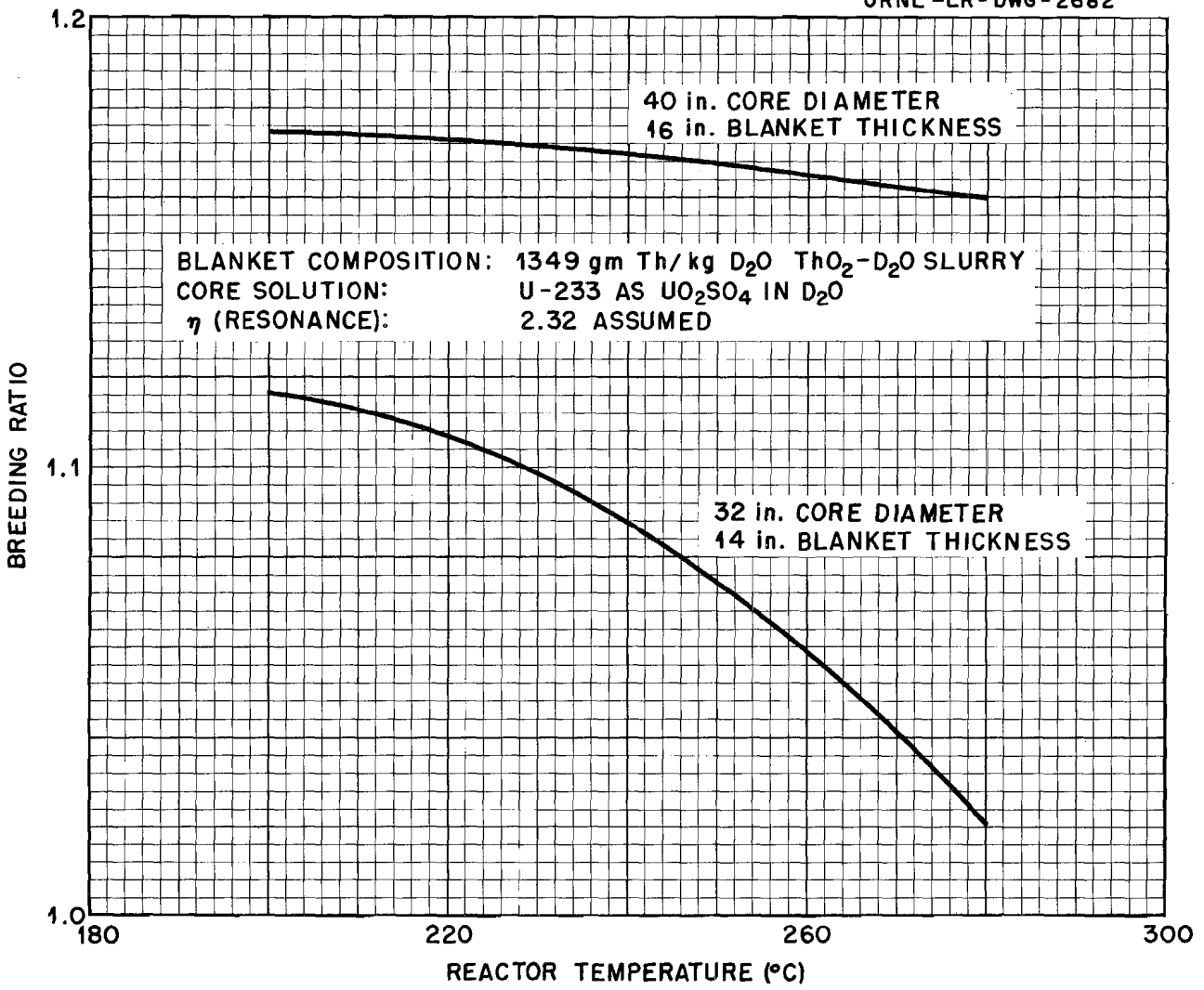

Fig. 18. Breeding Ratio in Spherical Two Region Reactors.

[redacted]

ORNL-LR-DWG 2683

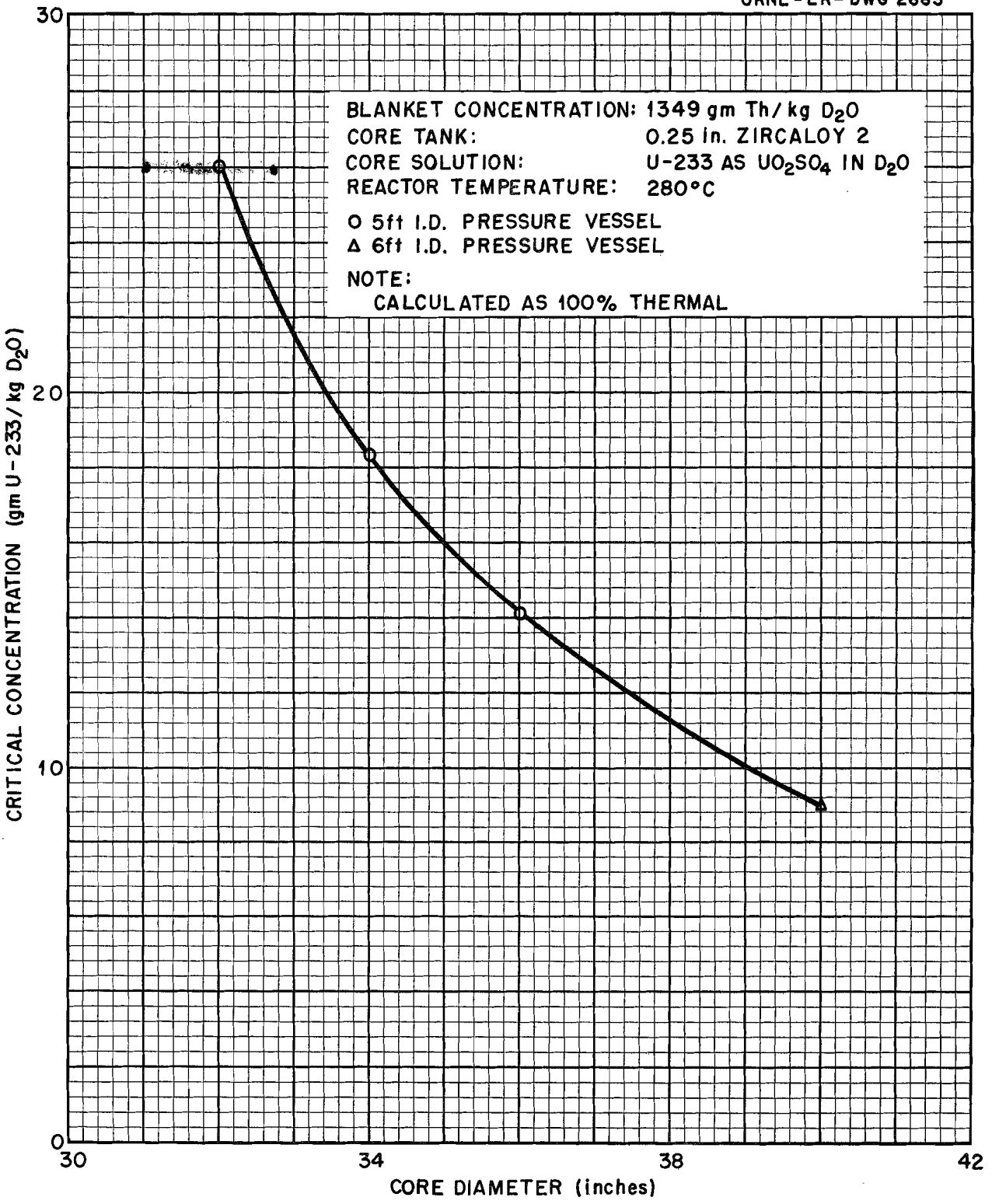

Fig. 19. Critical Concentration of U-233 in Core of Spherical Two Region Reactors with ThO_2-D_2O Slurry Blanket.

[redacted]

ORNL-LR-DWG 2684

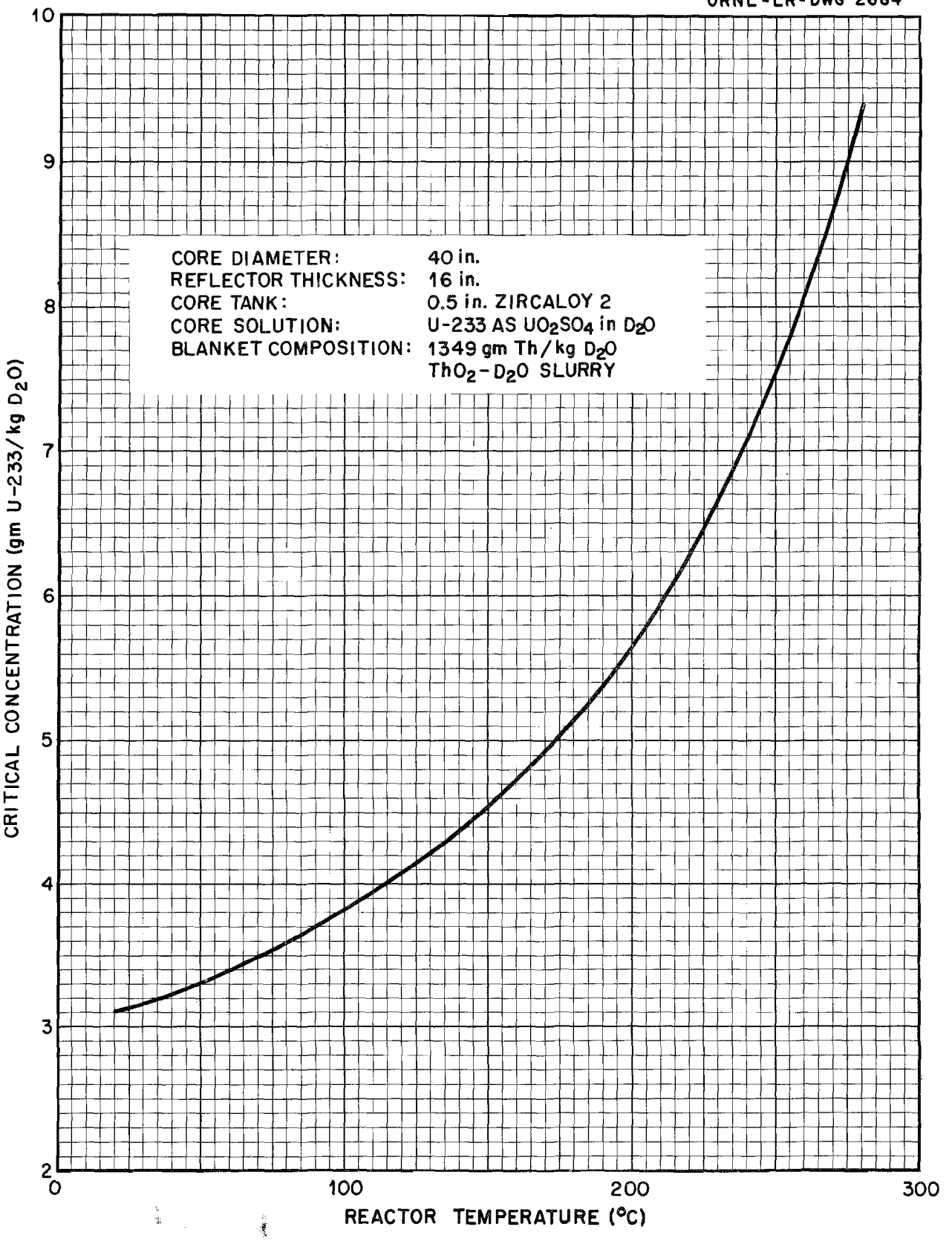

Fig. 20. Critical Concentration of U-233 in Core of A Two Region Breeder.

APPENDIX I - HRT DESIGN DATA

Extracted from HRT Design Data, Issue No. 2, by F. C. Zapp and R. E. Aven.[21]

The following design data are for the Homogeneous Reactor Test (HRT). Since the blanket solution has been changed to D_2O, the present data are for a two-region power producer having an overall gross design power rating of 5220 kw.

General Reactor Data

Gross power level (design)	5,220 kw
Core and blanket pressure	2000 psia
Core solution	UO_2SO_4-D_2O + $CuSO_4$
Core power	5000 kw
Core tank I.D.	32 in.
Core tank thickness	1/4 in.
Core tank material	Zircaloy-2
Core volume	290 liters
Core specific power	17.3 kw/liter
Fuel circulating rate	400 gpm at 256°C
Core inlet temperature	256°C (494°F)
Core outlet temperature	300°C (572°F)
Core average temperature	280°C (536°F)
Blanket solution	D_2O + $CuSO_4$
Blanket power	220 kw
Blanket vessel I.D.	60 in.
Blanket vessel thickness	4.4 in.
Blanket vessel material	Stainless steel clad carbon steel
Blanket volume	1550 liters
Blanket specific power	0.142 kw/liter
Blanket circulating rate	230 gpm at 278°C
Blanket inlet temperature	278°C (533°F)
Blanket outlet temperature	282°C (539°F)
Blanket average temperature	280°C (536°F)

APPENDIX II - NUCLEAR DATA

Summary of data used in the calculations:

Table 12. Microscopic cross sections as a function of reactor temperature.

Table 13. Nuclear constants for the heavy water moderator assumed. The values of D_2O density were obtained from R. E. Aven, ORNL CF-54-1-161.

Table 14. Constants for 632.7 g Th/kg D_2O (500 g Th/liter at 280°C) ThO_2-D_2O blanket.

Table 15. Constants for 1349 g Th/kg D_2O (1000 g Th/liter at 280°C) ThO_2-D_2O blanket.

Table 16. Constants for 355.3 g U/kg D_2O normal uranyl sulfate-D_2O blanket.

Table 17. Resonance escape probability in uranyl sulfate-D_2O solutions.

Table 18. Resonance Escape probability in ThO_2-D_2O slurries.

[redacted]

TABLE 12[1]

Cross Sections

T °C	$\sigma_a(28)$ b	$\sigma_a(25)$ b	$\sigma_a(D_2O)$ b/molecule	$\sigma_a(O_2SO_4)$ b	$\sigma_a(Th)$ b	K[2]
20	2.4557	593.2	.00242	.4344	6.2057	4.773×10^{-2}
50	2.3389	560.1	.002305	.4137	5.9105	4.814×10^{-2}
100	2.1765	516.1	.002145	.3850	5.5001	4.862×10^{-2}
150	2.0438	481.25	.002014	.3615	5.1648	4.889×10^{-2}
200	1.9327	451.9	.001905	.3419	4.8842	4.931×10^{-2}
250	1.8380	427.1	.001812	.3251	4.6449	4.962×10^{-2}
280	1.7875	414.0	.001762	.3162	4.5171	4.977×10^{-2}
300	1.7560	406.0	.001731	.3106	4.4376	4.985×10^{-2}
320	1.7261	398.9	.001701	.3053	4.3621	4.989×10^{-2}

(1) All cross sections have been corrected for a Maxwellian flux distribution.

(2) Used to convert $\frac{\Sigma_a(25)}{\Sigma_a(D_2O)}$ to g 25/kg D_2O.

Note: The number of significant figures listed in this table does not imply that the absolute values are that well known.

[redacted]

[redacted]

TABLE 13

NUCLEAR CONSTANTS FOR 99.75% D_2O - 0.25% H_2O

T	$\rho_a(D_2O)$ at 2000 psia	D_2 at Const. ρ	D_2	Σ_a	L_o^2	τ_1	τ_2
°C	g/cm³	cm	cm	/cm	cm²	cm²	cm²
20	1.1106	0.83880	0.83306	8.078×10^{-5}	$1.031 \times 10^{+4}$	71.02	47.345
50	1.100	0.85200	0.85432	7.620×10^{-5}	$1.121 \times 10^{+4}$	72.39	48.26
100	1.066	0.87190	0.90208	6.873×10^{-5}	$1.313 \times 10^{+4}$	77.07	51.38
150	1.016	0.89104	0.96734	6.150×10^{-5}	$1.573 \times 10^{+4}$	84.86	56.57
200	0.9594	0.90964	1.04579	5.492×10^{-5}	$1.904 \times 10^{+4}$	95.17	63.44
250	0.8935	0.92589	1.14301	4.864×10^{-5}	$2.350 \times 10^{+4}$	109.73	73.15
280	0.8395	0.93646	1.23039	4.445×10^{-5}	$2.768 \times 10^{+4}$	124.29	82.86
300	0.7959	0.94201	1.30544	4.140×10^{-5}	$3.153 \times 10^{+4}$	138.27	92.18
320	0.748	0.95087	1.40215	3.8243×10^{-5}	$3.666 \times 10^{+4}$	156.56	104.37

[redacted]

[redacted]

TABLE 14

CONSTANTS FOR 632.7 g Th/Kg D_2O BLANKET:

Resonance Escape Probability p = .7258

T	g Th/liter	VF (D_2O)	Σ_a(Th)	Σ_a(D_2O)	Σ_a(slurry)
°C		cm^3 D_2O/cm^3 slurry	/cm	/cm	/cm
20	647	.9241	1.0408×10^{-2}	7.465×10^{-5}	1.0483×10^{-2}
50	640	.9197	9.8122×10^{-3}	7.008×10^{-5}	9.8823×10^{-3}
100	625	.9266	8.9160×10^{-3}	6.369×10^{-5}	8.9797×10^{-3}
150	598	.9298	8.0065×10^{-3}	5.718×10^{-5}	8.0637×10^{-3}
200	567	.9335	7.1778×10^{-3}	5.127×10^{-5}	7.2291×10^{-3}
250	530	.9378	6.3863×10^{-3}	4.561×10^{-5}	6.4319×10^{-3}
280	500	.9413	5.8575×10^{-3}	4.184×10^{-5}	5.8993×10^{-3}
300	475	.9442	5.4724×10^{-3}	3.909×10^{-5}	5.5115×10^{-3}
320	448	.9474	5.0725×10^{-3}	3.623×10^{-5}	5.1087×10^{-3}

[redacted]

[redacted]

TABLE 15

CONSTANTS FOR 1349 g Th/Kg D_2O ThO_2 SLURRY BLANKET

Resonance Escape Probability p = .6252

T°C	g Th/liter	VF (D_2O) cm^3 D_2O/cm^3 slurry	$\sum_a$(Th)/cm	$\sum_a$(D_2O)/cm	$\sum_a$(slurry)/cm
20	1274.6	.85033	2.0514×10^{-2}	6.869×10^{-5}	2.0583×10^{-2}
100	1230.9	.85546	1.7559×10^{-2}	5.880×10^{-5}	1.7617×10^{-2}
200	1124	.86801	1.4238×10^{-2}	4.767×10^{-5}	1.4285×10^{-2}
250	1056	.87596	1.2725×10^{-2}	4.261×10^{-5}	1.2767×10^{-2}
280	1000	.88257	1.1715×10^{-2}	3.923×10^{-5}	1.1754×10^{-2}

[redacted]

[redacted]

TABLE 16

CONSTANTS FOR 355.3 g U/Kg D_2O NORMAL (.7115% ENRICHED) URANIUM BLANKET

Resonance Escape Probability = .7496

T°C	g U/liter	g D_2O/cm^3 solution	VF-cm^3 D_2O/cm^3 solution	k_b	$\Sigma_a(25)$	$\Sigma_a(28)$	Σ_a(soln.)
20	362.29	1.0196	.9181	1.2360	3.917×10^{-3}	2.234×10^{-3}	6.624×10^{-3}
50	359.85	1.0128	.9207				
100	352.06	.9908	.9294	1.2266	3.312×10^{-3}	1.924×10^{-3}	5.643×10^{-3}
150	340.55	.9584	.9434				
200	327.55	.9218	.9609	1.2195	2.698×10^{-3}	1.590×10^{-3}	4.624×10^{-3}
250	312.40	.8792	.9840	1.2163	2.432×10^{-3}	1.442×10^{-3}	4.178×10^{-3}
280	300.00	.8443	1.0057	1.2147	2.264×10^{-3}	1.347×10^{-3}	3.895×10^{-3}
300	289.99	.8161	1.0254	1.2138	2.146×10^{-3}	1.279×10^{-3}	3.695×10^{-3}
320	278.98	.7851	1.0497				

Note: $k_b = \frac{K_\infty}{p} = \frac{\eta(25)\Sigma_a(25)}{\Sigma_a(\text{soln.})}$

Σ_a(soln.) = macroscopic absorption cross section of solution.

[redacted]

TABLE 17

RESONANCE ESCAPE PROBABILITY p

IN URANYL SULFATE-D_2O SOLUTION

$$p = \exp\left\{-\frac{N(28)\ I(28)}{\sum_i \xi_i \Sigma_{is}}\right\} ; \qquad I(28) = 3.05\left[\frac{\Sigma_s}{N(28)}\right]^{.471}$$

Constants used at resonance were:

	σ_s(barns)	ξ	$\xi\sigma_s$
U	9	.00838	0.075
O	3.7	.120	0.444
D_2O	9.97	.5024	5.009

$\frac{N(D_2O)}{N(U\text{-}238)}$	g U/kg D_2O	p
20	594.3	.6811
30	396.2	.7363
40	297.2	.7699
50	237.7	.7934
60	198.1	.8112
70	169.8	.8249
100	118.9	.8532
150	79.2	.8800
200	59.4	.8962

[redacted]

TABLE 18

RESONANCE ESCAPE PROBABILITY p

IN ThO_2-D_2O SLURRIES

$$p \approx \exp\left\{-\frac{N(02)/N(D_2O)\ I(02)}{(\xi\ \sigma_s)_{D_2O}}\right\} \quad ; \quad I(02) = 8.33\left\{\frac{\sigma_s(D_2O)}{N(02)/N(D_2O)}\right\}^{0.253}$$

where $\sigma_s(D_2O)$, resonance = 10.5 b

$\xi\ \sigma_s(D_2O)$, resonance = 5.37 b

(02) signifies $_{90}Th^{232}$

G Th/kg D_2O	p
100	.9224
200	.8732
300	.8324
400	.7966
500	.7643
632.7	.7258
1349	.6252

APPENDIX III

PROMPT NEUTRON MEAN LIFETIME

The prompt neutron lifetime can be readily calculated by a simple artifice first proposed by R. P. Feynman. Suppose a reactor with a uniform distribution of 1/v absorber is made critical. Then, if the absorber is removed the power would increase at a rate $\exp(\Sigma_a vt)$, where Σ_a is the cross section of the 1/v absorber. Comparing this rate with $\exp\left(\frac{k_{eff}-1}{\tau} t\right)$, one obtains

$$k_{eff} = 1 + \tau \Sigma_{ao} v_o$$

where $\Sigma_a = \frac{\Sigma_{ao} v_o}{v}$ and τ is the prompt neutron mean lifetime.

Upon differentiation, one obtains

$$\tau = \frac{1}{v_o} \frac{\partial k_{eff}}{\partial \Sigma_{ao}}$$ which can be computed by adding a fictitious amount of 1/v absorber to the reactor to find the effect on critical concentration. The change in critical concentration can be converted to reactivity using the concentration coefficient of reactivity for the reactor, Fig. 13. For a bare reactor, the method reduces to the familiar result,

$$\tau = \frac{1}{v_{th} \Sigma_a (1 + L^2 B^2)} .$$

BIBLIOGRAPHY

1. R. B. Briggs et al, "HRP Quarterly Progress Report for Period Ending January 21, 1954. ORNL-1678, p. 21.

2. S. E. Beall et al, "HRP Quarterly Progress Report for Period Ending October 13, 1953." ORNL-1658, p. 3.

3. R. B. Briggs, "Aqueous Homogeneous Reactors for Producing Central Station Power." ORNL-1642, May 13, 1954.

4. F. L. Friedman and A. Wattenberg, "Age of Fission Neutrons in D_2O." CP-3453, March 6, 1946.

5. M. C. Edlund and L. C. Noderer, "An Harmonics Method Applied to D_2O Moderated Reactors." ORNL-CF-54-3-120, March 18, 1954.

6. H. C. Schweinler, "Variational Method for Calculating Pile Properties." MON-P-152, August 8, 1946.

7. G. Goertzel and H. L. Garabedian, "Method of Solution of the Critical Mass Problem for a Thermal Pile with Slowing Down Properties Independent of Position." ORNL-30, June 1, 1948.

8. A. M. Weinberg and L. C. Noderer, "Theory of Neutron Chain Reactions." ORNL-CF-51-5-98, Volume II, Part I.

9. AEC Neutron Cross-Section Advisory Group, Neutron Cross-Sections, BNL-170, May 15, 1952.

10. AEC Neutron Cross-Section Advisory Group, Supplement 1 to Neutron Cross-Sections. BNL-170A, March 1, 1952.

11. AEC Neutron Cross-Section Advisory Group, Supplement 2 to Neutron Cross-Sections, BNL-170B, April 1, 1953.

12. L. C. Noderer, "Temperature Dependence of the Neutron Diffusion Coefficient in Heavy Water." ORNL-CF-54-4-142, April 20, 1954.

13. M. Tobias, "The Age of Neutrons in a Thorium Oxide Heavy Water Slurry." ORNL-CF-54-8-49.

14. D. J. Hughes and C. Eggler, "Resonance Absorption in Thorium." CP-3093, July 11, 1945.

15. A. C. G. Mitchell et al, "Resonance Absorption of Uranium in Mixtures." CP-1676, March 4, 1944.

16. R. P. Feynman and T. A. Welton, "The Calculation of Critical Masses Including the Effects of the Distribution of Neutron Energies," LA-524, January 21, 1947.

17. M. C. Edlund, "A Two-Group Two-Region Approximation for Blanket Breeder Reactors." ORNL-CF-53-5-119, May 19, 1953.

18. S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory. PP. 238-247, D. Van Nostrand Company.

19. V. K. Pare', "Temperature Coefficients and Maximum Rates of Increase of Reactivity for HRT." ORNL-CF-54-6-200, June 15, 1954.

20. R. H. Chapman and R. D. Cheverton, "Preliminary Design of HRT Reactor Vessel." ORNL-CF-54-8-35, August 9, 1954.

21. F. C. Zapp and R. E. Aven, "HRT Design Data, Issue No. 2." ORNL-CF-54-8-53, August 11, 1954.