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REVISED Z TABLES OF THE RACAH COEFFICIENTS

L. C. Biedenharn and A. Simon

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REVISED Z TABLES OF THE RACAH COEFFICIENTS

L. C. Biedenharn* and A. Simon

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Previous tables of the Z coefficient[1] did not include the function $Z(l_1\ J_1\ l_2\ J_2,\ 0\ L)$, since this quantity can be expressed in terms of the Clebsch-Gordan coefficient $(l_1\ l_2\ 0\ 0|l_1\ l_2\ L\ 0)$. Experience has shown, however, that these functions are required quite frequently and that the calculation of their numerical values from a table of Clebsch-Gordan coefficients is both cumbersome and inefficient. The present supplement tabulates the function $Z(l_1\ J_1\ l_2\ J_2,\ 0\ L)$ for the same range of arguments as that given previously.[1]

The Z functions appear in expressions for the angular distribution of products of nuclear reactions,[2] and, in several of the formulas, the quantity $Z(l_1\ J_1\ l_2\ J_2,\ 0\ L)$ was replaced by $(l_1\ l_2\ 0\ 0|l_1\ l_2\ L\ 0)$. For convenience, the appropriate substitution required to re-express these equations in terms of the Z coefficient is indicated below.

(1) $Z(l_1\ J_1\ l_2\ J_2,\ 0\ L)$

$$= \delta_{l_1,J_1}\delta_{l_2,J_2}(-i)^{L-l_1+l_2}[(2l_1+1)(2l_2+1)]^{1/2}(l_1\ l_2\ 0\ 0|l_1\ l_2\ L\ 0) \quad .$$

Hence, in Eqs. 6.2, 6.4, 6.6, 7.12, and 7.13 given by Blatt and Biedenharn,[2] the following replacement should be made

$$(2l'+1)(l\ l'\ 0\ 0|l\ l'\ L\ 0)^2 \rightarrow \frac{Z^2(l\ l\ l'\ l',\ 0\ L)}{2l+1} \quad .$$

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[1] L. C. Biedenharn, *Revised Z Tables of the Racah Coefficients*, ORNL-1501 (May 28, 1953)

[2] J. M. Blatt and L. C. Biedenharn, *Revs. Mod. Phys.* **24**, 258 (1952).

TABLE I

$$Z^2\ (l_1\ J_1\ l_2\ J_2,\ 0,\ 0)$$

To find Z take the positive square root of the entry in the table for Z^2 unless this entry is preceded by an asterisk, in which case the negative square root is used

J_2 \ J_1	l_2 \ l_1	0	1	2	3	4
0	0	1				
1	1		*3			
2	2			5		
3	3				*7	
4	4					9

TABLE II

$$Z^2\ (l_1\ J_1\ l_2\ J_2,\ 0,\ 1)$$

To find Z take the positive square root of the entry in the table for Z^2 unless this entry is preceded by an asterisk, in which case the negative square root is used

J_2 \ J_1	l_2 \ l_1	0	1	2	3	4
0	0		3			
1	1	*3		*6		
2	2		6		9	
3	3			*9		*12
4	4				12	

TABLE III

$Z^2\ (l_1\ J_1\ l_2\ J_2,\ 0,\ 2)$

To find Z take the positive square root of the entry in the table for Z^2 unless this entry is preceded by an asterisk, in which case the negative square root is used

J_2 \ J_1	l_2 \ l_1	0	1	2	3	4
0	0			5		
1	1		*6		*9	
2	2	5		50/7		90/7
3	3		*9		*28/3	
4	4			90/7		900/77

TABLE IV

$Z^2\ (l_1\ J_1\ l_2\ J_2,\ 0,\ 3)$

To find Z take the positive square root of the entry in the table for Z^2 unless this entry is preceded by an asterisk, in which case the negative square root is used

J_2 \ J_1	l_2 \ l_1	0	1	2	3	4
0	0				7	
1	1			*9		*12
2	2		9		28/3	
3	3	*7		*28/3		*126/11
4	4		12		126/11	

TABLE V

$$Z^2\ (l_1\ J_1\ l_2\ J_2,\ 0,\ 4)$$

To find Z take the positive square root of the entry in the table for Z^2 unless this entry is preceded by an asterisk, in which case the negative square root is used

J_2 \ J_1	l_2 \ l_1	0	1	2	3	4
0	0					9
1	1				*12	
2	2			90/7		900/77
3	3		*12		*126/11	
4	4	9		900/77		13122/1001

TABLE VI

$$Z^2\ (l_1\ J_1\ l_2\ J_2,\ 0,\ 5)$$

To find Z take the positive square root of the entry in the table for Z^2 unless this entry is preceded by an asterisk, in which case the negative square root is used

J_2 \ J_1	l_2 \ l_1	0	1	2	3	4
0	0					
1	1					*15
2	2				50/3	
3	3			*50/3		*180/13
4	4		15		180/13	

TABLE VII

$Z^2\ (l_1\ J_1\ l_2\ J_2,\ 0,\ 6)$

To find Z take the positive square root of the entry in the table for Z^2 unless this entry is preceded by an asterisk, in which case the negative square root is used

J_2 \ J_1	l_2 \ l_1	0	1	2	3	4
		0	1	2	3	4
0	0					
1	1					
2	2					225/11
3	3				*700/33	
4	4			225/11		180/11

TABLE VIII

$Z^2\ (l_1\ J_1\ l_2\ J_2,\ 0,\ 7)$

To find Z take the positive square root of the entry in the table for Z^2 unless this entry is preceded by an asterisk, in which case the negative square root is used

J_2 \ J_1	l_2 \ l_1	0	1	2	3	4
		0	1	2	3	4
0	0					
1	1					
2	2					
3	3					*3675/143
4	4				3675/143	

TABLE IX

$$Z^2\ (l_1\ J_1\ l_2\ J_2,\ 0,\ 8)$$

To find Z take the positive square root of the entry in the table for Z^2 unless this entry is preceded by an asterisk, in which case the negative square root is used

J_2 \ J_1	l_2 \ l_1	0	1	2	3	4
		0	1	2	3	4
0	0					
1	1					
2	2					
3	3					
4	4					4410/143