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ANALYSIS OF SPHERICAL PRESSURE VESSEL HAVING AN ENERGY SOURCE WITHIN THE WALL

By
R. H. Chapman

October 26, 1954

Oak Ridge National Laboratory
Oak Ridge, Tennessee

Technical Information Service, Oak Ridge, Tennessee

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OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee

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ANALYSIS OF SPHERICAL PRESSURE VESSEL HAVING AN ENERGY SOURCE WITHIN THE WALL

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ABSTRACT

A method is given for determining the optimum thickness of a spherical pressure vessel in which there is an exponential heat source contained within the vessel wall. The method has applications in the design of certain types of nuclear reactors. It is shown how the thermal stress may be estimated for thermal reactors and combined with the pressure stress to obtain the total stress. The solution to a hypothetical design problem is given to illustrate the procedure. Some data are included which should be an aid in the solution to similar problems.

CHAPTER I

GENERAL INTRODUCTION

Frequently in the field of nuclear engineering, the design of pressure vessels requires that serious consideration be given to the thermal stress as well as to the pressure stress. The thermal stress may be relieved by creep and plastic deformation, with no harm being done, provided it is not of a cyclic nature. However, the usual designs are subject to some temperature cycling which could lead to failure of the vessel if the thermal stress is excessive.

It will be shown how the thermal stress may be evaluated and combined with the pressure stress to obtain the total stress. It is known that the thermal stress increases with an increase in vessel thickness while the pressure stress decreases. Obviously their sum will give a minimum value at a particular vessel thickness. The optimum thickness may be of little interest in many cases as a thinner vessel may have a total stress less than the allowable stress. It will be shown how this optimum vessel thickness may be obtained for various design conditions when operating in the steady state region. While the procedure presented is for a spherical vessel, the same general procedure will apply for other shapes.

The thermal stress considered herein is the result of nuclear radiation absorption within the vessel wall. This implies an internal heat source with an exponential distribution. While it is not the purpose of this discussion to present a rigorous theoretical study of nuclear

shielding, a method of determining the heating due to the radiation absorption will be included. It is hoped that one familiar with the fundamentals of nuclear engineering may take this discussion and apply it to the particular problem being investigated.

For simplicity, only pressure stresses and thermal stresses due to radiation absorption are considered in this analysis. No attempt will be made to analyze the localized stresses due to outlets, supports, pressure surges, wind load, weight of fluid, pipeline reactions, etc. These are all dependent on the particular vessel design and are somewhat indeterminate. It should be pointed out that nozzles and connecting pipes may impose high localized stresses if insufficient flexibility is incorporated in the piping design. Some experimental data (1, 2)[1] are available to indicate the magnitude of the stresses around openings in pressure vessels.

Unfortunately much of the work on thermal stresses with an internal heat source is in the classified literature (3). Timoshenko and Goodier (4) give the general equations for thermal stress for any given temperature distribution. They also give the particular solution to the stress equations for a linear temperature distribution. Carter (5) presents some general expressions for several geometries where the heat source is considered constant per unit volume. Luster (6) discusses thermal and pressure stresses in cylinders where the temperature distribution is linear.

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(1) Numbers in parentheses refer to similarly numbered references in bibliography at end of paper.

The effect of thermal stresses is also given along with a method of arriving at the optimum wall thickness for cylinders. Kent (7) considers transient stresses in spheres and cylinders. Durham (8) presents a method of determining heat transfer rates and thermal stresses in several geometries. Criteria for minimizing the thermal stresses based on slab geometry and uniform heating are given along with the corresponding cooling rates.

The discussion presented herein differs from the above cited references in that the thermal stresses are the result of a steady state exponential heat source. The equations of Timoshenko and Goodier are applied in spherical geometry after obtaining an expression for the temperature from the exponential heat source. A simple problem will be worked to illustrate the procedure presented in this discussion. The optimum vessel thickness will be determined for a particular design condition and curves showing the stress and temperature distribution will be given for that thickness.

CHAPTER II

HEAT PRODUCTION

In the classified literature (3) and unclassified literature (8, 9, 10, 11, 12, 13, 14), there are numerous discussions on shielding of reactors. However, very few of these discussions mention the heating effect resulting from the radiation absorption. This is very important in the design of reactors, and, as such, it should be given more prominence in the shielding discussions. The following treatment is not intended as a rigorous theoretical discussion of shielding, but as a practical method of obtaining the heat production within the shield. It should be pointed out that if a more precise method is available, it should be utilized in the determination of the heat production.

The sources of the heat generated within the shield are as follows:

(1) Thermal neutron capture gammas. As thermal neutrons are captured by the materials present within the active region of a reactor, gamma radiation is released. This radiation is often referred to as the primary or pile capture gammas. They are not to be confused with the radioactive decay gammas of the product nuclei. Some of the thermal neutrons which leak out of the active region are captured by the materials present within the shield. Gamma radiation is released in this process also. These gammas are often referred to as the secondary capture gammas.

(2) Fast neutron capture gammas. If fast neutrons are captured by the materials present within the region under consideration, gamma radiation will be released in a similar manner as above.

(3) Elastic scattering of fast neutrons. In an elastic collision, a portion of the neutron's kinetic energy appears as heat.

(4) Inelastic scattering of fast neutrons. In an inelastic collision, a portion of the neutron's kinetic energy appears as gamma radiation. This may occur in the active region as well as in the shield.

(5) Prompt fission gammas. In the fission process, a small portion of the energy released appears immediately as gamma radiation. This radiation is sometimes referred to as the prompt gammas.

(6) Fission product gammas. Some of the fission products resulting from the fission process emit gamma radiation with characteristic half lives after the fissioning takes place. This radiation is sometimes referred to as the delayed gammas.

The pile gammas are attenuated through the active region before they enter the shield which in this case is the reactor vessel. Near the inner boundary of the vessel, the primary radiation may assume a major role in the heat production; whereas, for greater thicknesses the secondary radiation may become the more important of the two. For thermal reactors the effects of the fast neutrons are generally negligible, hence, they will not be included. This will not be true for intermediate or fast reactors where most of the fissions are due to the higher energy neutrons and where most of the neutrons which enter the pressure vessel wall are fast neutrons. The classified literature presents methods of estimating these effects.

In order to obtain the heat production within the vessel wall, it is necessary to know the gamma and neutron currents entering the wall. The total primary gamma current incident on the vessel wall from all

elements of volume is given in the case of a spherical source with self-absorption as

$$I_o = \int_o^R \frac{r\,S(r)}{4\,R^2}\left\{(R+r)F_1\left[\mu(R-r)\right] - (R-r)F_1\left[\mu(R+r)\right] + \frac{1}{\mu}\left[e^{-\mu(R-r)} - e^{-\mu(R+r)}\right]\right\}dr \quad (1)$$

where

I_o = gamma current entering the spherical shield, γ/cm^2 sec.

r = radius at any point within the spherical source, cm.

R = inside radius of the spherical shell, cm.

μ = energy dependent, linear absorption coefficient of the material in the spherical source corrected for multiple scattering, cm^{-1}. See Appendix E.

$S(r)$ = gamma source strength, γ/cm^3sec.

$F_1\left[\mu(R-r)\right]$, $F_1\left[\mu(R+r)\right]$ = exponential integrals defined by $F_1(t) = t\int_t^\infty \frac{e^{-y}}{y^2}\,dy$. See Appendix C.

The derivation of equation (1) is given in Appendix A.

From the information on the size and nature of the reactor, and with an assumed gamma source distribution, equation (1) may be integrated graphically to give the primary gamma current for any particular photon energy. It will be sufficient to assume that the capture and prompt gammas are distributed in proportion to the thermal neutron flux. Inasmuch as the fission product gammas are emitted some time after the fission, the source distribution will depend upon the type of reactor. In a stationary fuel reactor the source distribution will be the same as the thermal neutron flux distribution. The fission product gammas will be emitted uniformly throughout the reactor in most fluid fuel reactors.

Once the gamma currents entering the vessel wall are known, the heat production may be estimated for each of the various photon energies. In order to simplify the problem, slab geometry will be assumed for the purpose of calculating the heat production. This will be sufficient for the calculations provided that the radius is large compared to the thickness. A conservative assumption is made by treating the current, as given by equation (1), as a collimated source as it enters the wall.

The intensity of a collimated beam of photons which has passed through a thin absorber is given as

$$I = I_o e^{-\mu_t x} \tag{2}$$

where

I = intensity of gammas at x, γ/cm^2 sec.

I_o = incident gamma current, γ/cm^2 sec.

μ_t = energy dependent total absorption coefficient, cm^{-1}.

x = thickness of absorber, cm.

For large thicknesses of absorber (2 or 3 attenuation lengths), equation (2) should be modified to include the effect of multiple scattering or buildup as it is commonly known. Several methods of correcting for build-up are available to the designer. The one chosen for this discussion, because of its mathematical simplicity, is to simply divide the total absorption coefficient by 1.2. This reduced value gives a good correlation between the values reported in Appendix E for thin absorbers, where the scattered radiation is negligible, and for thick absorbers where the scattered radiation becomes important.

The energy released at any point is given by the product of the

radiation intensity at the point, the energy released per collision and the energy absorption coefficient. A buildup factor should be considered in evaluating the energy released. With the buildup factor discussed herein, the heat energy at any point is given as

$$H(x) = E_\gamma \, (\mu_t - \sigma_s) \, I_o \, e^{-\mu x} \tag{3}$$

where

$H(x)$ = heat produced per unit volume at any point, Mev/cm^3 sec.

E_γ = energy released per absorption, Mev/γ.

μ = energy dependent absorption coefficient corrected for buildup, cm^{-1}.

$(\mu_t - \sigma_s)$ = energy dependent, energy absorption coefficient, cm^{-1}.

x = distance from face of shield, cm.

I_o = incident gamma current, γ/cm^2 sec.

Equation (3) may be used to estimate the heat production within the vessel wall as a function of the thickness for each of the various photon energies.

In order to obtain the heat production from the neutron reactions, it is necessary to have the neutron currents entering the shield. One method often used is to obtain the thermal and fast neutron flux equations and calculate the net leakage from them. Rather than go through the involved calculations of obtaining the exact flux equations, a simpler method will be discussed here. If one considers the reactor as a bare thermal reactor, the leakages may be obtained in terms of nuclear constants for the particular reactor.

The criticality equation for a bare homogeneous thermal reactor may be written as

$$k_{eff} = \frac{k_{\infty}\, e^{-B^2\tau}}{1 + L^2 B^2} \tag{4}$$

where

k_{eff} = effective multiplication factor for a finite reactor.

k_{∞} = multiplication factor for an infinite reactor.

B^2 = effective geometrical buckling of the reactor, cm^{-2}.

(Note: B^2 may differ for fast and thermal neutrons).

τ = Fermi age or the square of the slowing down length for fast neutrons, cm^2.

L^2 = diffusion length for thermal neutrons, cm^2.

For a finite reactor to remain just critical, k_{eff} must, of necessity, be unity. The value of k_{∞} is fixed for any given reactor by the geometrical and nuclear properties. The factor $e^{-B^2\tau}$ is the probability that a fast neutron does not leak out of a finite reactor while slowing down to thermal energy. For these calculations the exponential may be approximated by $(1 + B^2\tau)^{-1}$. The factor $(1 + L^2 B^2)^{-1}$ is the fraction of the thermalized neutrons which are absorbed as thermal neutrons or, stated differently, the non-leakage probability of the thermal neutrons.

Let ν be the number of fast neutrons produced per fission and p be the probability that a fast neutron will become thermalized before capture. This allows one to calculate the number of thermal and fast neutrons which leak out per fission. Since 1 watt = $3.1 \times 10^{10}\ \frac{\text{fissions}}{\text{sec}}$, the total fissions may be calculated from the given power level, P, in watts.

The current may be estimated by dividing the number of neutrons leaking out by the surface area of the reactor of radius R cm. Defining the fast neutron current entering the shell as $I_{o,f}$ and the thermal neutron current as $I_{o,t}$, one may write the equations for these as

$$I_{o,f} = \left[\frac{3.1 \times 10^{10}\ P\nu}{4\pi R^2}\right]\left[\frac{B^2\tau}{1 + B^2\tau}\right] \tag{5}$$

and

$$I_{o,t} = \left[\frac{3.1 \times 10^{10}\ P\nu_p}{4\pi R^2}\right]\left[\frac{L^2B^2}{(1 + L^2B^2)(1 + B^2\tau)}\right] \tag{6}$$

As stated earlier the heat produced in thermal reactor pressure vessels due to the fast neutron reactions is usually negligible and will be omitted from this discussion. The heat production due to the thermal neutrons is important and must be considered. Enlund (13) has derived an expression for the heat production in a slab due to the capture gammas emitted upon a thermal neutron absorption. His analysis considers the effect of the gamma traveling some distance after the neutron collision before being absorbed. He gives the energy absorption as

$$H(x) = N\ \frac{I_{o,t}}{2}\ (\mu_t - \sigma_s)\ E_\gamma\ F(\beta,\mu_t t) \tag{7}$$

where

N = number of photons of energy E_γ produced per neutron capture.

$F(\beta,\mu_t t)$ = absorption function. See Appendix D.

and other symbols have been previously defined. The above equation does not consider gamma ray buildup. In order to take into account the buildup, this writer chooses to replace the total absorption coefficient by the

reduced absorption coefficient as discussed previously.* Making this change, equation (7) becomes

$$H(x) = N E_\gamma \frac{I_o t}{2} (\mu t - \sigma_s) F(\beta, \mu t) \tag{8}$$

The function $F(\beta,\mu t)$ is defined for three cases: $\beta > 1$; $\beta = 1$; and $\beta < 1$. The derivation of equation (7) and the definition of the functions are given in Appendix D. Curves of the function are presented in the appendix with β as a parameter. Using equation (8) with the appropriate absorption function, the thermal neutron heating effect may be estimated.

The various heating effects may be plotted on semi-log paper and added to give the total heat production as a function of the thickness. The total heating curve may then be approximated by a single exponential; or more exponentials if greater accuracy is desired. The slope and intercept of the exponentials are obtainable from the plot on semi-log paper. For the time being let it be assumed that the total heating is given by

$$H(x) = H_o e^{-mx} \tag{9}$$

where

$H(x)$ = heat production at any point, Mev/cm^3 sec.

H_o = heat production at face of slab, Mev/cm^3 sec.

m = slope of the assumed exponential, cm^{-1}.

x = distance from face of the slab, cm.

It will be shown later how corrections may be made if more than one exponential is needed to approximate the total heat production curve.

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* Since this memorandum was issued, Alexander (17) has issued a memorandum on the application of the NDA Build-up Factors. An equation for the heating is given in terms of build-up factors.

CHAPTER III

TEMPERATURE DISTRIBUTION

A method for estimating the heat production per unit volume within the vessel wall due to nuclear radiation absorption was given in the previous section. Slab geometry was employed in arriving at the expression for the heat production, but, for the remainder of this discussion, spherical geometry will be employed as the basic geometrical consideration. Define the radius at any point within the vessel wall by

$$r \equiv R + x \tag{10}$$

then equation (9) may be written as

$$H(r) = H_R \, e^{-m(r - R)} \tag{11}$$

The differential equation for steady state heat conduction with an internal heat source is given (15) in spherical coordinates as

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = \frac{-H(r)}{k} \tag{12}$$

where

T = temperature at any point, degrees F.

r = radius at any point defined by equation (10), cm.

$H(r)$ = heat production per unit volume at any point defined by equation (11), Mev/cm^3 sec.

k = thermal conductivity of the vessel material assumed to be constant and expressed in consistent units, Mev/cm sec F.

The temperature distribution within the vessel wall is obtained by substituting equation (11) into equation (12) and integrating. The resulting expression is

$$T(r) = -\frac{H_R}{m^3 k}\left(\frac{mr + 2}{r}\right)e^{-m(r - R)} - \frac{C_1}{r} + C_2 \qquad (13)$$

where C_1 and C_2 are constants of integration to be evaluated from the boundary conditions. The integration of equation (12) and the evaluation of the constants are given in Appendix B.

Three sets of boundary conditions have been used to evaluate the constants:

Case I. The outside surface of the vessel is considered as a heat barrier and the inside surface is at some equilibrium temperature.

Case II. The inside surface of the vessel is considered as a heat barrier and the outside surface is at some equilibrium temperature.

Case III. The inside surface and the outside surface are at some equilibrium temperatures which may or may not be the same.

Some comments on the boundary conditions may be of value in determining which case applies to the problem being investigated. Case I considers that all the heat generated within the vessel wall is conducted to the inside surface where it is dissipated by some convenient heat transfer mechanism to the adjoining medium. This condition would be employed where heat losses are important and need to be kept at a minimum. Case II considers the reverse of Case I, i.e., all the heat generated is conducted to the outside surface and dissipated by some convenient

mechanism of heat transfer. This condition would be used for an uninsulated vessel with a large amount of cooling on the outside, or for a vessel insulated on the inside. Case III considers part of the heat generated within the vessel wall as being conducted to both surfaces where it is dissipated by some convenient heat transfer mechanism. A special case of interest is when both surfaces are at the same temperature. However, the practical value of this special case may be limited due to the problem of keeping both surfaces at the same temperature. It should be pointed out that this special case describes the temperature distribution in a thermal shield which may be placed within the vessel in such a manner as to have the same medium cool both sides of the shield.

The various temperature distributions are:

Case I.

$$T(r) = \frac{H_R}{m^3 k}\left\{\frac{A_1 - B_1}{a} + \frac{1}{r}\left[B_1 - (mr + 2)e^{-m(r - R)}\right]\right\} \tag{14}$$

where

$$A_1 = (ma + 2)e^{-m(a - R)}$$

$$B_1 = (m^2b^2 + 2mb + 2)e^{-m(b - R)}$$

$$T(a) = 0 \text{ as a datum}$$

$$\left.\frac{dT}{dr}\right|_{r=b} = 0$$

Case II.

$$T(r) = \frac{H_R}{m^3 k}\left\{\frac{A_2 - B_2}{b} + \frac{1}{r}\left[B_2 - (mr + 2)e^{-m(r - R)}\right]\right\} \tag{15}$$

where

$$A_2 = (mb + 2)e^{-m(b - R)}$$

$$B_2 = (m^2a^2 + 2ma + 2)e^{-m(a - R)}$$

$$T(b) = 0 \text{ as a datum}$$

$$\left.\frac{dT}{dr}\right|_{r=a} = 0$$

Case III.

$$T(r) = \frac{1}{r}\left\{a\, T_a\left(\frac{r-b}{a-b}\right) + b\, T_b\left(\frac{a-r}{a-b}\right) + \frac{H_R}{m^3k}\left[A_3\left(\frac{r-b}{a-b}\right) + B_3\left(\frac{a-r}{a-b}\right) - (mr + 2)e^{-m(r-R)}\right]\right\} \quad (16)$$

where

$$A_3 = (ma + 2)e^{-m(a - R)}$$

$$B_3 = (mb + 2)e^{-m(b - R)}$$

$$T(a) = T_a$$

$$T(b) = T_b$$

Case III. (Special Case)

$$T(r) = \frac{H_R}{m^3k}\left\{\left(\frac{A_3}{a-b}\right)\left(\frac{r-b}{r}\right) - \left(\frac{B_3}{a-b}\right)\left(\frac{r-a}{r}\right) - \left(\frac{mr + 2}{r}\right) e^{-m(r - R)}\right\} \quad (17)$$

where

$$T(a) = T(b) = 0 \text{ as a datum}$$

The heat removal per square ft of surface area may be estimated by evaluating the temperature gradient at the appropriate surface and multiplying by the thermal conductivity.

CHAPTER IV

STRESSES

It was pointed out in an earlier chapter that only pressure stresses and thermal stresses due to radiation absorption would be considered. Only the maximum tangential stresses are necessary in determining the optimum vessel thickness for any given design, and these stresses will occur at either or both of the boundaries. This assumes that the three-dimensional stress system may be treated as a two-dimensional or plane stress problem. Once the optimum thickness has been determined, the three-dimensional stress distribution may be calculated from the equations presented. The stress distribution is mostly of academic interest as the maximum stress may be used as a conservative design criteria in pressure vessels. The value of the maximum allowable stress is given in the various codes under whose jurisdiction the vessel design may fall.

Timoshenko and Goodier (4) give the equations for stresses in thick spheres due to internal pressure as

$$\sigma_{t,P_i} = \frac{P_i \; a^3}{b^3 - a^3} \left[\frac{2r^3 + b^3}{2r^3} \right] \tag{18}$$

and

$$\sigma_{r,P_i} = \frac{P_i \; a^3}{b^3 - a^3} \left[\frac{r^3 - b^3}{r^3} \right] \tag{19}$$

where

σ_{t,P_i} = tangential stress due to internal pressure, psi.

σ_{r,P_i} = radial stress due to internal pressure, psi.

P_i = internal pressure, psi.

a = inside radius of sphere, cm.

b = outside radius of sphere, cm.

r = radius of any point in vessel wall, cm.

Obviously the pressure stresses are principal stresses by virtue of symmetry. It is easily seen that as the vessel thickness increases, the tangential stresses decrease. When the thickness of the vessel is small compared to the radius (ratio of 1 to 10), the vessel may be treated by the membrane theory or the so-called thin sphere equation. This assumes that the vessel acts as a membrane; therefore, the tangential stress is constant across the vessel wall and is given as

$$\sigma_{t,P_i} = \frac{P_i\, a}{2(b-a)} \tag{20}$$

Timoshenko and Goodier (4) give the equations for thermal stresses in hollow spheres for any temperature distribution as

$$\sigma_{t,th} = \frac{2\alpha E}{1-\nu}\left\{\frac{2r^3 + a^3}{2(b^3-a^3)r^3}\int_a^b T(r)r^2dr + \frac{1}{2r^3}\int_a^r T(r)r^2\,dr + \frac{1}{2}\,T(r)\right\} \tag{21}$$

and

$$\sigma_{r,th} = \frac{2\alpha E}{1-\nu}\left\{\frac{r^3 - a^3}{(b^3-a^3)r^3}\int_a^b T(r)r^2\,dr - \frac{1}{r^3}\int_a^r T(r)r^2\,dr\right\} \tag{22}$$

where

$\sigma_{t,th}$ = tangential thermal stress, psi.

$\sigma_{r,th}$ = radial thermal stress, psi.

α = linear coefficient for thermal expansion, in./in. F.

E = modulus of elasticity, psi.

ν = Poisson's ratio.

T(r) = temperature distribution as a function of r, degrees F.

a,b,r = radii as defined in equations (18) and (19).

These stresses are also principal stresses by virtue of symmetry. This fact allows them to be added algebraically to the pressure stresses without using combined stress theory. As the vessel thickness increases, the thermal stresses increase due to the greater temperature gradient required to conduct the heat through the vessel.

The total stress at any point is given as

$$\sigma_t = \sigma_{t,P_i} + \sigma_{t,th} \tag{23}$$

and

$$\sigma_r = \sigma_{r,P_i} + \sigma_{r,th} \tag{24}$$

Equation (23) has a minimum value for some particular thickness which will provide the optimum stress condition for vessels which have thermal as well as pressure stresses. The usual method of obtaining the maximum or minimum of an equation is too complicated to employ where the stress distribution has both exponential and power terms as is being considered herein. Instead, a graphical method will be used to obtain the minimum.

The maximum tangential stress due to internal pressure is at the inside surface while the maximum tangential thermal stress will occur at the surface which has the lowest temperature. These maximum stresses

may be calculated from the equations presented and their sum plotted versus shell thickness. The optimum shell thickness may be read directly from the curve at the particular thickness where the total stress has its minimum value.

Equations (21) and (22) must be integrated for each of the known temperature distributions as given in Chapter III before the thermal stresses may be evaluated. The results of these integrations will be given for each set of boundary conditions:

<u>Case I:</u>

$$T(r) = \frac{H_R}{m^3 k} \left\{ \frac{A_1 - B_1}{a} + \frac{1}{r} \left[B_1 - (mr + 2)\, e^{-m(r-R)} \right] \right\}$$

$$T(a) = 0 \text{ (as a datum)}; \quad \left. \frac{dT}{dr} \right|_{r = b} = 0$$

$$A_1 = (ma + 2)\, e^{-m(a-R)}; \quad B_1 = (m^2 b^2 + 2mb + 2)\, e^{-m(b-R)}$$

and

$$\int T(r)\, r^2\, dr = \frac{H_R}{m^3 k} \left\{ \left(\frac{A_1 - B_1}{a} \right) \frac{r^3}{3} + \frac{B_1 r^2}{2} + \left(\frac{mr + 2}{m} \right)^2 e^{-m(r-R)} \right\}$$

Substituting the above expressions in equations (21) and (22) gives the tangential thermal stress as

$$\sigma_{t,th} = \frac{\alpha E\, H_R}{(1-\nu)m^3 k}\left\{\begin{array}{l} \dfrac{2r^3 + a^3}{(b^3 - a^3)r^3}\left[\left(\dfrac{A_1 - B_1}{a}\right)\left(\dfrac{b^3 - a^3}{3}\right) + B_1\left(\dfrac{b^2 - a^2}{2}\right)\right. \\ \left. + \left(\dfrac{mb + 2}{m}\right)^2 e^{-m(b-R)} - \left(\dfrac{ma + 2}{m}\right)^2 e^{-m(a-R)}\right] \\ + \dfrac{1}{r^3}\left[\left(\dfrac{A_1 - B_1}{a}\right)\left(\dfrac{r^3 - a^3}{3}\right) + B_1\left(\dfrac{r^2 - a^2}{2}\right)\right. \\ \left. + \left(\dfrac{mr + 2}{m}\right)^2 e^{-m(r-R)} - \left(\dfrac{ma + 2}{m}\right)^2 e^{-m(a-R)}\right] \\ - \left[\dfrac{A_1 - B_1}{a} + \dfrac{B_1}{r} - \left(\dfrac{mr + 2}{r}\right) e^{-m(r-R)}\right] \end{array}\right\} \tag{25}$$

and the radial stress as

$$\sigma_{r,th} = \frac{2\alpha E\, H_R}{(1-\nu)m^3 k}\left\{\begin{array}{l} \dfrac{r^3 - a^3}{(b^3 - a^3)r^3}\left[\left(\dfrac{A_1 - B_1}{a}\right)\left(\dfrac{b^3 - a^3}{3}\right) + B_1\left(\dfrac{b^2 - a^2}{2}\right)\right. \\ \left. + \left(\dfrac{mb + 2}{m}\right)^2 e^{-m(b-R)} - \left(\dfrac{ma + 2}{m}\right)^2 e^{-m(a-R)}\right] \\ - \dfrac{1}{r^3}\left[\left(\dfrac{A_1 - B_1}{a}\right)\left(\dfrac{r^3 - a^3}{3}\right) + B_1\left(\dfrac{r^2 - a^2}{2}\right)\right. \\ \left. + \left(\dfrac{mr + 2}{m}\right)^2 e^{-m(r-R)} - \left(\dfrac{ma + 2}{m}\right)^2 e^{-m(a-R)}\right] \end{array}\right\} \tag{26}$$

The maximum tangential thermal stress for this case is at the inside radius and is given as

$$\sigma_{t,th\ max} = \frac{3\alpha E\ H_R}{(1-\nu)m^3k(b^3-a^3)} \left\{\left(\frac{A_1 - B_1}{a}\right)\left(\frac{b^3 - a^3}{3}\right) + B_1\left(\frac{b^2 - a^2}{2}\right)\right.$$

$$\left. + \left(\frac{mb + 2}{m}\right)^2 e^{-m(b-R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a-R)}\right\} \qquad (27)$$

<u>Case II</u>:

$$T(r) = \frac{H_R}{m^3k}\left\{\frac{A_2 - B_2}{b} + \frac{1}{r}\left[B_2 - (mr + 2)\ e^{-m(r-R)}\right]\right\}$$

$$T(b) = 0 \text{ (as a datum)};\ \left.\frac{dT}{dr}\right|_{r=a} = 0$$

$$A_2 = (mb + 2)\ e^{-m(b-R)}$$

$$B_2 = (m^2a^2 + 2ma + 2)\ e^{-m(a-R)}$$

and

$$\int T(r)\ r^2\ dr = \frac{H_R}{m^3k}\left\{\left(\frac{A_2 - B_2}{b}\right)\frac{r^3}{3} + \frac{B_2\ r^2}{2} + \left(\frac{mr + 2}{m}\right)^2 e^{-m(r-R)}\right\}$$

Substituting the above expressions in equations (21) and (22) gives the tangential thermal stress as

$$\sigma_{t,th} = \frac{\alpha E H_R}{(1-\nu)m^3k} \left\{ \begin{aligned} & \frac{2r^3 + a^3}{(b^3 - a^3)r^3} \left[\left(\frac{A_2 - B_2}{b}\right)\left(\frac{b^3 - a^3}{3}\right) + B_2\left(\frac{b^2 - a^2}{2}\right) \right. \\ & \left. + \left(\frac{mb + 2}{m}\right)^2 e^{-m(b-R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a-R)} \right] \\ & + \frac{1}{r^3} \left[\left(\frac{A_2 - B_2}{b}\right)\left(\frac{r^3 - a^3}{3}\right) + B_2\left(\frac{r^2 - a^2}{2}\right) \right. \\ & \left. + \left(\frac{mr + 2}{m}\right)^2 e^{-m(r-R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a-R)} \right] \\ & - \left[\frac{A_2 - B_2}{b} + \frac{B_2}{r} - \left(\frac{mr + 2}{r}\right) e^{-m(r-R)} \right] \end{aligned} \right\} \quad \ldots\ldots(28)$$

and the radial stress as

$$\sigma_{r,th} = \frac{2\alpha E H_R}{(1-\nu)m^3k} \left\{ \begin{aligned} & \frac{r^3 - a^3}{(b^3 - a^3)r^3} \left[\left(\frac{A_2 - B_2}{b}\right)\left(\frac{b^3 - a^3}{3}\right) + B_2\left(\frac{b^2 - a^2}{2}\right) \right. \\ & \left. + \left(\frac{mb + 2}{m}\right)^2 e^{-m(b-R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a-R)} \right] \\ & - \frac{1}{r^3} \left[\left(\frac{A_2 - B_2}{b}\right)\left(\frac{r^3 - a^3}{3}\right) + B_2 \left(\frac{r^2 - a^2}{2}\right) \right. \\ & \left. + \left(\frac{mr + 2}{m}\right)^2 e^{-m(r-R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a-R)} \right] \end{aligned} \right\} \quad \ldots\ldots(29)$$

The maximum tangential thermal stress for this case is at the outside radius and is given as

$$\sigma_{t,th\ max} = \frac{3\alpha E H_R}{(1-\nu)(b^3-a^3)m^3k} \left\{ \left(\frac{A_2 - B_2}{b}\right)\left(\frac{b^3 - a^3}{3}\right) + B_2\left(\frac{b^2 - a^2}{2}\right) + \left(\frac{mb + 2}{m}\right)^2 e^{-m(b-R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a-R)} \right\} \quad \ldots\ldots (30)$$

<u>Case III</u>:

$$T(r) = \frac{1}{r} \left\{ a\, T_a\left(\frac{r - b}{a - b}\right) + b\, T_b \left(\frac{a - r}{a - b}\right) + \frac{H_R}{m^3k} \left[A_3\left(\frac{r-b}{a-b}\right) + B_3\left(\frac{a-r}{a-b}\right) - (mr + 2)\, e^{-m(r-R)}\right] \right\}$$

$$T(a) = T_a \quad ; \quad T(b) = T_b$$

$$A_3 = (ma + 2)\, e^{-m(a-R)} \quad ; \quad B_3 = (mb + 2)\, e^{-m(b-R)}$$

and

$$\int T(r)r^2dr = \left\{ \left(\frac{aT_a - bT_b}{a - b}\right)\frac{r^3}{3} - ab\left(\frac{T_a - T_b}{a - b}\right)\frac{r^2}{2} + \frac{H_R}{m^3k}\left[\left(\frac{A_3-B_3}{a-b}\right)\frac{r^3}{3} - \left(\frac{bA_3-aB_3}{a-b}\right)\frac{r^2}{2} + \left(\frac{mr+2}{m}\right)^2 e^{-m(r-R)}\right] \right\}$$

Substituting the above expressions in equations (21) and (22) gives the tangential thermal stress as

$$\sigma_{t,th} = \frac{\alpha E}{1-\nu} \left\{ \begin{array}{l} \dfrac{2r^3+a^3}{(b^3-a^3)r^3} \left[\begin{array}{l} \left(\dfrac{aT_a-bT_b}{a-b}\right)\left(\dfrac{b^3-a^3}{3}\right) - ab\left(\dfrac{T_a-T_b}{a-b}\right)\left(\dfrac{b^2-a^2}{2}\right) \\ + \dfrac{H_R}{m^3k}\left\{\left(\dfrac{A_3-B_3}{a-b}\right)\left(\dfrac{b^3-a^3}{3}\right) - \left(\dfrac{bA_3-aB_3}{a-b}\right)\left(\dfrac{b^2-a^2}{2}\right)\right. \\ \left. + \left(\dfrac{mb+2}{m}\right)^2 e^{-m(b-R)} - \left(\dfrac{ma+2}{m}\right)^2 e^{-m(a-R)}\right\} \end{array} \right] \\ + \dfrac{1}{r^3} \left[\begin{array}{l} \left(\dfrac{aT_a-bT_b}{a-b}\right)\left(\dfrac{r^3-a^3}{3}\right) - ab\left(\dfrac{T_a-T_b}{a-b}\right)\left(\dfrac{r^2-a^2}{2}\right) \\ + \dfrac{H_R}{m^3k}\left\{\left(\dfrac{A_3-B_3}{a-b}\right)\left(\dfrac{r^3-a^3}{3}\right) - \left(\dfrac{bA_3-aB_3}{a-b}\right)\left(\dfrac{r^2-a^2}{2}\right)\right. \\ \left. + \left(\dfrac{mr+2}{m}\right)^2 e^{-m(r-R)} - \left(\dfrac{ma+2}{m}\right)^2 e^{-m(a-R)}\right\} \end{array} \right] \\ - \dfrac{1}{r} \left[\begin{array}{l} aT_a\left(\dfrac{r-b}{a-b}\right) + bT_b\left(\dfrac{a-r}{a-b}\right) \\ + \dfrac{H_R}{m^3k}\left\{A_3\left(\dfrac{r-b}{a-b}\right) + B_3\left(\dfrac{a-r}{a-b}\right) - (mr+2)\, e^{-m(r-R)}\right\} \end{array} \right] \end{array} \right\} \quad \ldots\ldots\ (31)$$

and the radial stress as

$$\sigma_{r,th} = \frac{2\alpha E}{1-\nu}\left\{\begin{array}{l} \dfrac{r^3-a^3}{(b^3-a^3)r^3}\left[\left(\dfrac{aT_a - bT_b}{a-b}\right)\left(\dfrac{b^3-a^3}{3}\right) - ab\left(\dfrac{T_a - T_b}{a-b}\right)\left(\dfrac{b^2-a^2}{2}\right)\right. \\ \left. + \dfrac{H_R}{m^3k}\left\{\left(\dfrac{A_3-B_3}{a-b}\right)\left(\dfrac{b^3-a^3}{3}\right) - \left(\dfrac{bA_3 - aB_3}{a-b}\right)\left(\dfrac{b^2-a^2}{2}\right)\right.\right. \\ \left.\left. + \left(\dfrac{mb+2}{m}\right)^2 e^{-m(b-R)} - \left(\dfrac{ma+2}{m}\right)^2 e^{-m(a-R)}\right\}\right] \\ - \dfrac{1}{r^3}\left[\left(\dfrac{aT_a - bT_b}{a-b}\right)\left(\dfrac{r^3-a^3}{3}\right) - ab\left(\dfrac{T_a - T_b}{a-b}\right)\left(\dfrac{r^2-a^2}{2}\right)\right. \\ \left. + \dfrac{H_R}{m^3k}\left\{\left(\dfrac{A_3-B_3}{a-b}\right)\left(\dfrac{r^3-a^3}{3}\right) - \left(\dfrac{bA_3-aB_3}{a-b}\right)\left(\dfrac{r^2-a^2}{2}\right)\right.\right. \\ \left.\left. + \left(\dfrac{mr+2}{m}\right)^2 e^{-m(r-R)} - \left(\dfrac{ma+2}{m}\right)^2 e^{-m(a-R)}\right\}\right] \end{array}\right\} \quad \text{----- (32)}$$

The maximum tangential stress will be at the surface which has the lowest temperature. If this occurs at the inside surface, the maximum stress is given as

$$\sigma_{t,th\ max} = \frac{3\ \alpha\ E\ H_R}{(1-\nu)\ m^3k\ (b^3-a^3)}\left[\left(\frac{A_3-B_3}{a-b}\right)\left(\frac{b^3-a^3}{3}\right) - \left(\frac{bA_3-aB_3}{a-b}\right)\left(\frac{b^2-a^2}{2}\right) + \left(\frac{mb+2}{m}\right)^2 e^{-m(b-R)} - \left(\frac{ma+2}{m}\right)^2 e^{-m(a-R)}\right]$$

$$+ \frac{\alpha E}{(1-\nu)}\ \frac{b}{(a-b)}\left[1 - \frac{3}{2}\ a\left(\frac{b^2-a^2}{b^3-a^3}\right)\right]\left(T_a - T_b\right) \quad \text{----- (33)}$$

If the lowest temperature is at the outside surface, the maximum stress is given as

$$\sigma_{t,th\ max} = \frac{3\,\alpha\,EH_R}{(1-\nu)m^3k(b^3-a^3)}\left[\left(\frac{A_3-B_3}{a-b}\right)\left(\frac{b^3-a^3}{3}\right) - \left(\frac{bA_3-aB_3}{a-b}\right)\left(\frac{b^2-a^2}{2}\right) + \left(\frac{mb+2}{m}\right)^2 e^{-m(b-R)} - \left(\frac{ma+2}{m}\right)^2 e^{-m(a-R)}\right]$$

$$+ \frac{\alpha E}{(1-\nu)}\ \frac{a}{(a-b)}\left[1 - \frac{3}{2}\ b\left(\frac{b^2-a^2}{b^3-a^3}\right)\right]\left(T_a - T_b\right) \qquad \text{-----(34)}$$

It is seen that equations (33) and (34) contain one term which depends upon heat generation within the vessel wall and one term which depends upon the difference in surface temperatures. The heat generation term is identical in both equations whereas the surface temperature dependent term is not. It should be noted that the latter term appears as a linear function of the surface temperature difference. Equations (33) and (34) are for the general case where it will be necessary to obtain the surface temperatures in terms of heat transferred and film coefficients.

To prove that the maximum tangential thermal stress has a minimum value when both surfaces are at the same temperature, it is necessary to prove that the term containing the surface temperature difference is greater than zero for all other cases. For the case where $T_a < T_b$, equation (33) applies. Since $(a-b) < 0$, $(1-\nu) > 0$, and $(T_a - T_b) < 0$, it is necessary to show that $\frac{2}{3} > a\left(\frac{b^2-a^2}{b^3-a^3}\right)$ in order for the surface temperature term to be greater than zero. This may be shown by inequalities in the following manner:

$$\frac{2}{3} > a\left(\frac{b^2-a^2}{b^3-a^3}\right) = \frac{ab + a^2}{b^2+ab+a^2}$$

Reversing the inequality by inverting both sides and breaking the right side into two fractions, one obtains the inequality

$$\frac{1}{2} < \frac{b^2}{a^2 + ab}$$

Now $b > a$, $b^2 > a^2$, $ab > a^2$, and $2b^2 > (a^2 + ab)$. Dividing this inequality by $2(a^2 + ab)$, one obtains the inequality that $\frac{b^2}{a^2 + ab} > \frac{1}{2}$. Therefore, it is concluded that for the case where $T_a < T_b$, the term containing the surface temperature difference is greater than zero. For the case where $T_b < T_a$, equation (34) applies. Since $(a - b) < 0$, $(1 - \nu) > 0$, and $(T_a - T_b) > 0$, it is necessary to show $\frac{2}{3} < b\left(\frac{b^2 - a^2}{b^3 - a^3}\right)$ in order for the surface temperature term to be greater than zero. This may be shown in the following manner:

$$\frac{2}{3} < b\left(\frac{b^2 - a^2}{b^3 - a^3}\right) = \frac{b^2 + ab}{b^2 + ab + a^2}$$

Reversing the inequality by inverting both sides and breaking the right side into two fractions, one obtains the inequality

$$\frac{1}{2} > \frac{a^2}{b^2 + ab}$$

Now $b > a$, $b^2 > a^2$, $ab > a^2$, and $(b^2 + ab) > 2a^2$. Dividing this inequality by $2(b^2 + ab)$, one obtains the inequality that $\frac{1}{2} > \frac{a^2}{b^2 + ab}$. Therefore, it is concluded that for the case where $T_b < T_a$, the term containing the surface temperature difference is greater than zero. For the case where $T_a = T_b$, both equations (33) and (34) give zero for the term containing the surface temperature difference.

It is obvious from the foregoing that the maximum thermal stress is equal to the thermal stress due to the internal heat generation plus the thermal stress due to the difference in surface temperatures which, in itself, will be equal to or greater than zero. Therefore, it is concluded that the maximum tangential thermal stress will have its minimum value when both surfaces are at the same temperature. Thus, for this case and a given inside diameter, the optimum vessel thickness is found from the following expression which gives the minimum value of the maximum tangential stress as

$$\text{Min. } \sigma_{t,max} = \frac{P_i}{2}\left[\frac{2a^3 + b^3}{b^3 - a^3}\right] + \frac{3\alpha E H_R}{(1-\nu)\, m^3 k\, (b^3-a^3)}\left[\left(\frac{A_3 - B_3}{a - b}\right)\left(\frac{b^3 - a^3}{3}\right) - \left(\frac{bA_3 - aB_3}{a - b}\right)\left(\frac{b^2 - a^2}{2}\right) + \left(\frac{mb+2}{m}\right)^2 e^{-m(b-R)} - \left(\frac{ma+2}{m}\right)^2 e^{-m(a-R)}\right] \quad \text{----- (35)}$$

The usual method of obtaining the minimum value of an expression is too complicated to apply to equation (35). Instead, it is recommended that the expression be plotted as a function of vessel thickness, and the particular value of the thickness which gives a minimum value for the total stress be read directly from the curve.

The thermal stress distribution for the case where both surfaces are at the same temperature is obtained from equations (31) and (32) by letting $T_a = T_b = 0$ as a datum. Making that substitution the equations may be written as

$$\sigma_{t,th} = \frac{\alpha\, E\, H_R}{(1-\nu)m^3 k} \left\{ \begin{aligned} & \frac{2r^3+a^3}{(b^3-a^3)r^3}\left[\left(\frac{A_3-B_3}{a-b}\right)\left(\frac{b^3-a^3}{3}\right) - \left(\frac{bA_3-aB_3}{a-b}\right)\left(\frac{b^2-a^2}{2}\right)\right. \\ & \left. + \left(\frac{mb+2}{m}\right)^2 e^{-m(b-R)} - \left(\frac{ma+2}{m}\right)^2 e^{-m(a-R)}\right] \\ & + \frac{1}{r^3}\left[\left(\frac{A_3-B_3}{a-b}\right)\left(\frac{r^3-a^3}{3}\right) - \left(\frac{bA_3-aB_3}{a-b}\right)\left(\frac{r^2-a^2}{2}\right)\right. \\ & \left. + \left(\frac{mr+2}{m}\right)^2 e^{-m(r-R)} - \left(\frac{ma+2}{m}\right)^2 e^{-m(a-R)}\right] \\ & - \frac{1}{r}\left[A_3\left(\frac{r-b}{a-b}\right) + B_3\left(\frac{a-r}{a-b}\right) - (mr+2)\, e^{-m(r-R)}\right] \end{aligned} \right\} \qquad \ldots\ldots(36)$$

and

$$\sigma_{r,th} = \frac{2\alpha\, E\, H_R}{(1-\nu)m^3 k} \left\{ \begin{aligned} & \frac{r^3-a^3}{(b^3-a^3)r^3}\left[\left(\frac{A_3-B_3}{a-b}\right)\left(\frac{b^3-a^3}{3}\right) - \left(\frac{bA_3-aB_3}{a-b}\right)\left(\frac{b^2-a^2}{2}\right)\right. \\ & \left. + \left(\frac{mb+2}{m}\right)^2 e^{-m(b-R)} - \left(\frac{ma+2}{m}\right)^2 e^{-m(a-R)}\right] \\ & - \frac{1}{r^3}\left[\left(\frac{A_3-B_3}{a-b}\right)\left(\frac{r^3-a^3}{3}\right) - \left(\frac{bA_3-aB_3}{a-b}\right)\left(\frac{r^2-a^2}{2}\right)\right. \\ & \left. + \left(\frac{mr+2}{m}\right)^2 e^{-m(r-R)} - \left(\frac{ma+2}{m}\right)^2 e^{-m(a-R)}\right] \end{aligned} \right\} \qquad \ldots\ldots(37)$$

Throughout the previous derivations of the thermal stress equations, it has been assumed that the total heat production function could be approximated by a single exponential expression. If more than

one exponential is necessary in order to get a closer approximation to the heat production function, the same equations will apply by simply evaluating the thermal stress for each exponential, separately, and adding or subtracting them as the case may be. The only limitation to adding the various solutions is when the surface temperatures are specified as T_a and T_b. The effect of the surface temperatures can only be added for one solution in order to maintain the proper boundary conditions. In this manner the thermal stress may be obtained for any particular internal heat source. Generally, two, and seldom more than three, exponentials are required to give a close approximation to the heat production function.

If the value of the total stress as calculated by the appropriate equation given herein exceeds the permissible allowable stress for the given vessel material, some means of lowering the thermal stress becomes necessary. This may be achieved by adding a so-called "thermal shield" within the vessel to absorb the radiation before it strikes the pressure vessel. The equations presented will still apply to the problem with some minor assumptions. It is convenient to assume that the inside radius of the thermal shield is the boundary of the reactor described by R throughout this discussion and that the coolant channel between the thermal shield and vessel wall absorbs a negligible amount of the radiation leakage. This means that the pressure stress will go up slightly due to the increased diameter, and the thermal stress will decrease as the actual metal thickness appears as a negative exponential in the thermal stress equations. The thickness of the coolant is neglected in the exponential function. The above procedure will be

demonstrated in a later section.

If it is impractical to maintain both surfaces of the vessel at the same temperature, an optimum vessel thickness may be obtained for the particular boundary conditions imposed upon the vessel. It becomes necessary to apply the appropriate equations as presented herein to arrive at the optimum for the desired conditions in the same manner as outlined for the case where both surfaces are at the same temperature.

CHAPTER V

APPLICATION OF METHOD

The procedure to be followed in the solution to a problem may be outlined as follows:

1. Obtain the radiation leakages incident on the vessel wall by some acceptable manner.
2. Convert the absorbed radiation into heat energy as a function of distance from the face of the shield. Add all the various heat sources and approximate the total heat production by simple exponential functions.
3. Calculate the maximum tangential thermal stress for the particular boundary temperature conditions selected. This should be done for several thickness of practical interest while keeping the inside diameter of the vessel constant.
4. Calculate the maximum tangential stress due to internal pressure for the same thicknesses as the thermal stress was calculated.
5. The above stresses are added and their sum plotted versus shell thickness. The optimum thickness may be read directly from the curve of total stress versus shell thickness.
6. If the total stress for the optimum thickness exceeds the allowable stress, a radiation shield thickness may be employed as a parameter whereby several total stress curves are obtained.

The above outlined procedure will be demonstrated by the solution to a simple problem. The optimum shell thickness will be found for a vessel which has both surfaces at the same temperature. Once the particular thickness is determined, the stress and temperature distribution may be calculated. The values of the various nuclear constants used in the problem are unclassified and may not represent the most accurate data.

The problem solved here is not a design nor is it contemplated as the design for a reactor. For the purpose of illustrating the method, assume a reactor of the following general description. It shall consist of a 6 ft. inside diameter steel sphere operating at 1000 psia and 70 degrees F. The fuel and moderator is a dilute homogeneous, circulating, solution of pure U^{235} and D_2O. The power level of the reactor is taken to be 100,000 kilowatts.

For the purpose of calculating the thermal neutron leakage of this reactor, it may be assumed that the reactor is a so-called bare reactor. The criticality equation may be written as

$$k_{eff} = \frac{\eta \epsilon pf}{(1 + L^2B^2)\,(1 + B^2\tau)} = 1 \qquad (38)$$

where

η = the average number of fast fission neutrons produced per absorption in fuel.

ϵ = the fast neutron fission effect.

f = the thermal utilization = $\dfrac{\Sigma_a(U^{235})}{\Sigma_a(D_2O) + \Sigma_a(U^{235})}$

Σ_a = macroscopic absorption cross section, cm^{-1}.

and the other symbols have been previously defined. It may be noted that equation (38) is the same as equation (4) except that k_∞ has been replaced by $\eta\epsilon pf$ and $e^{-B^2\tau}$ has been approximated by $(1 + B^2\tau)^{-1}$.

For a dilute solution such as specified, it is reasonable to assume ϵ and p to be unity. The Fermi age for the solution may be taken as that for D_2O. However, it is necessary to correct the diffusion length of the solution because of the presence of uranium. The corrected value may be calculated from

$$L^2 = L^2(D_2O) \frac{\Sigma_a(D_2O)}{\Sigma_a(D_2O) + \Sigma_a(U^{235})} \tag{39}$$

Rearranging and substituting, the critical equation becomes

$$\frac{\Sigma_a(D_2O)}{\Sigma_a(U^{235})} = \frac{\eta - (1 + B^2\tau)}{(1 + B^2\tau)\left[1 + B^2L^2(D_2O)\right]} \tag{40}$$

The above equation may be solved for the critical concentration from the values given in Table I as

$$\frac{\Sigma_a(D_2O)}{\Sigma_a(U^{235})} = 0.0653$$

Since the solution is dilute, let it be assumed that the cross section of D_2O is unaffected by the presence of small amounts of uranium. With this

TABLE I

NUCLEAR PROPERTIES[a]

$\tau(D_2O)$	120 cm^2
$L^2(D_2O)$	10,000 cm^2
η	2.1
ν	2.5
$\Sigma_a(D_2O)$	0.0008 cm^{-1}
$\sigma_a(U^{235})$	650 barns
$\sigma_f(U^{235})$	549 barns
$\sigma_c(U^{235})$	101 barns
B^2	0.00118 cm^{-2} [b]
L^2 (solution)	613 cm^2 [c]

[a] Glasstone and Edlund, pp. 83, 127, 183.

[b] B^2 calculated from $\frac{\pi^2}{R^2}$ for a spherical reactor where R includes the extrapolation distance. The extrapolation distance is defined as the distance from the physical boundary of the reactor to the point where the neutron flux would be zero if extrapolated from the boundary with a line whose slope is equal to the slope of the flux at the boundary. The extrapolation distance is usually taken as $0.71\lambda_{tr}$ where λ_{tr} is the transport mean free path for neutrons in the material.

[c] L^2(solution) calculated from equation (39).

assumption $\sum_a(U^{235})$ is calculated to be 0.01225 cm^{-1}. The fission cross section, $\sum_f(U^{235})$, will be $\frac{549}{650} \times \sum_a(U^{235})$ or 0.01035 cm^{-1} and the capture cross section, $\sum_c(U^{235})$, will be $\frac{101}{650} \times \sum_a(U^{235})$ or 0.0019 cm^{-1}.

In order to obtain the gamma source terms for the "pile" gammas, which are proportional to the thermal flux, it will be helpful to obtain the thermal flux expression. For a spherical reactor, this is given as

$$\phi = \frac{A \sin Br}{r} \tag{41}$$

where

ϕ = thermal flux at any radius, neutrons/cm^2sec.

A = constant, neutrons/cm sec.

B = geometrical buckling, cm^{-1}.

r = radius, cm.

It can be shown that the average flux is given by

$$\bar{\phi} = \frac{3A}{\pi R} \tag{42}$$

where R includes the extrapolation distance. The average flux is also given in terms of the power as

$$\bar{\phi} = \frac{3.1 \times 10^{10}\ P}{\sum_f(U^{235})\ V} \tag{43}$$

where P = power level, watts.

V = reactor volume, cm^3.

Since the extrapolation distance for this reactor is small compared to the radius, it is assumed to be negligible. With that assumption

the two expressions for average flux may be combined to give

$$A = \frac{7.75 \times 10^{9} P}{\Sigma_f(U^{235})\ R^2} \tag{44}$$

Upon substituting the values in equation (44), A is found to be 8.96 x 10^{15} neutrons/cm sec. Now equation (41) may be written as

$$\phi = 8.96 \times 10^{15}\ \frac{\sin 0.034r}{r} \tag{45}$$

Equation (1) may be used to calculate the primary gamma currents entering the vessel wall. The source function will be taken proportional to the thermal flux for the capture and prompt fission gammas, and proportional to the average flux for the fission product gammas.

Assuming on the average that one 6 Mev gamma is produced for each non-fission neutron capture in uranium and that the captures in heavy water are negligible, the source function for the capture gammas may be written as

$$S(r) = \Sigma_c(U^{235})\ \phi = 1.71 \times 10^{13}\ \frac{\sin 0.0344r}{r}\ \frac{\gamma}{cm^3 sec} \tag{46}$$

Assuming that five gammas of an average energy of 1 Mev each are emitted as prompt fission gammas, the source function for the prompt fission gammas may be written as

$$S(r) = 5\,\Sigma_f(U^{235})\ \phi = 4.64 \times 10^{14}\ \frac{\sin 0.0344r}{r}\ \frac{\gamma}{cm^3 sec} \tag{47}$$

Assuming that two gammas of an average energy of 2.5 Mev each are emitted as delayed gammas from the fission products, the source function

for the delayed fission gammas may be written as

$$S(r) = 2\Sigma_f(U^{235})\,\bar{\phi} = 1.95 \times 10^{12}\ \frac{\gamma}{cm^3 sec} \tag{48}$$

The gamma absorption coefficients for the solution must be known as a function of photon energy in order to apply equation (3). Some experimental values are given in Appendix E based on thin absorbers and a collimated beam of radiation. For thick absorbers, a buildup factor must be used. The method used in this discussion, as pointed out earlier, is to divide the values as obtained from the curves by 1.2. Since the values for heavy water are not included in the curves of Appendix E, the values for light water may be used and corrected by the ratio of their densities. The corrected values of the coefficients used in performing the calculations in this discussion are given in Table II for the solution and for the vessel.

The values of the exponential integral may be obtained from the curves in Appendix C for use in equation (1).

Table III presents the method chosen to calculate the portion of equation (1) under the integral sign as a function of the radius for the primary capture gammas. The data from the table is given in Figure 1 from which one may graphically integrate to obtain the area underneath the curve. Using the scale factor as given in the figure, the area is evaluated and the 6 Mev capture gamma current is estimated to be 8.95 $\times 10^{11}\ \gamma/cm^2 sec$. In a similar manner the 1 Mev prompt fission gamma current is estimated to be $5.13 \times 10^{12}\ \gamma/cm^2 sec$, and the 2.5 Mev fission product gamma current is estimated to be $9.70 \times 10^{12}\ \gamma/cm^2$ sec. It should

TABLE II

ABSORPTION COEFFICIENTS

Photon Energy	μ(solution)	μ(Fe)	$(\mu_t-\sigma_s)$Fe
1 Mev	0.0705 cm^{-1}	0.388 cm^{-1}	0.213 cm^{-1}
2.5 Mev	0.0438 cm^{-1}	0.244 cm^{-1}	0.163 cm^{-1}
6 Mev	0.0275 cm^{-1}	0.201 cm^{-1}	0.185 cm^{-1}
7 Mev	---	0.197 cm^{-1}	0.189 cm^{-1}

NOMENCLATURE FOR TABLE III

Column	Column Heading	Units
1	r	cm
2	$R + r$	cm
3	$R - r$	cm
4	$\mu(R + r)$	--
5	$\mu(R - r)$	--
6	$F_1\left[\mu(R - r)\right]$	--
7	$F_1\left[\mu(R + r)\right]$	--
8	$(R + r)\ F_1\left[\mu(R - r)\right]$	cm
9	$(R - r)\ F_1\left[\mu(R + r)\right]$	cm
10	$(R + r)\ F_1\left[\mu(R - r)\right] - (R - r)\left[F_1\ \mu(R + r)\right]$	cm
11	$e^{-\mu(R - r)}$	--
12	$e^{-\mu(R + r)}$	--
13	$\frac{1}{\mu}\left[e^{-\mu(R - r)} - e^{-\mu(R + r)}\right]$	cm
14	Br	radians
15	$\sin Br$	--
16	$r\ S(r) \times 10^{-12}$	γ/cm^2 sec
17	$\frac{1}{4R^2}\left\{(R + r)\ F_1\left[\mu(R - r)\right] - (R - r)\ F_1\left[\mu(R + r)\right] + \frac{1}{\mu}\left[e^{-\mu(R - r)} - e^{-\mu(R + r)}\right]\right\} \times 10^4$	cm^{-1}

NOMENCLATURE FOR TABLE III (CONTINUED)

Column	Column Heading	Units
18	$\frac{r\,S(r)}{4R^2}\left\{(R + r)\,F_1\left[\mu(R - r)\right] - (R - r)\,F_1\left[\mu(R + r)\right] + \frac{1}{\mu}\left[e^{-\mu(R - r)} - e^{-\mu(R + r)}\right]\right\} \times 10^{-9}$	γ/cm^3 sec

TABLE III

CALCULATIONS FOR DETERMINING THE 6 MEV GAMMA CURRENT

1	2	3	4	5	6	7	8	9
0.00	91.44	91.44	2.51	2.51	0.019	0.0193	1.737	1.737
10.00	101.44	81.44	2.79	2.24	0.027	0.0136	2.739	1.108
20.00	111.44	71.44	3.06	1.96	0.040	0.0100	4.458	0.714
30.00	121.44	61.44	3.34	1.69	0.056	0.0070	6.801	0.430
40.00	131.44	51.44	3.61	1.41	0.086	0.0051	11.304	0.262
50.00	141.44	41.44	3.89	1.14	0.120	0.0036	16.973	0.149
60.00	151.44	31.44	4.16	0.86	0.183	0.0027	27.714	0.085
70.00	161.44	21.44	4.44	0.59	0.277	0.0019	44.719	0.041
75.00	166.44	16.44	4.58	0.45	0.355	0.0016	59.086	0.026
80.00	171.44	11.44	4.71	0.31	0.460	0.0014	78.862	0.016
85.00	176.44	6.44	4.85	0.18	0.590	0.0012	104.100	0.008
90.00	181.44	1.44	4.99	0.04	0.860	0.0010	156.038	0.001
91.44 = R	182.88	0.00	5.03	0.00	1.000	0.0010	182.880	0.000

TABLE III (CONTINUED)

CALCULATIONS FOR DETERMINING THE 6 MEV GAMMA CURRENT

1	10	11	12	13	14	15	16	17	18
0.00	0.000	0.082085	0.082085	0.000	0.000	0.0000	0.00	0.00	0.00
10.00	1.631	0.106459	0.061421	1.638	0.344	0.3373	5.77	0.98	0.56
20.00	3.744	0.140858	0.046888	3.417	0.688	0.6350	10.86	2.14	2.33
30.00	6.371	0.184520	0.035437	5.421	1.032	0.8583	14.68	3.53	5.18
40.00	11.042	0.244143	0.027052	7.894	1.376	0.9811	16.78	5.66	9.50
50.00	16.824	0.319819	0.020445	10.886	1.720	0.9889	16.91	8.29	14.01
60.00	27.629	0.423162	0.015608	14.820	2.064	0.8810	15.06	12.69	19.11
70.00	44.678	0.554327	0.011796	19.728	2.408	0.6698	11.45	19.26	22.05
75.00	59.060	0.637628	0.010255	22.814	2.580	0.5329	9.11	24.48	22.31
80.00	78.846	0.733447	0.009005	26.343	2.752	0.3802	6.50	31.45	20.45
85.00	104.092	0.835270	0.007828	30.089	2.924	0.2163	3.70	40.12	14.84
90.00	156.037	0.960789	0.006806	34.690	3.096	0.0460	0.79	57.03	4.49
91.44 = R	182.880	1.000000	0.006539	36.126	3.142	0.0000	0.00	65.48	0.00

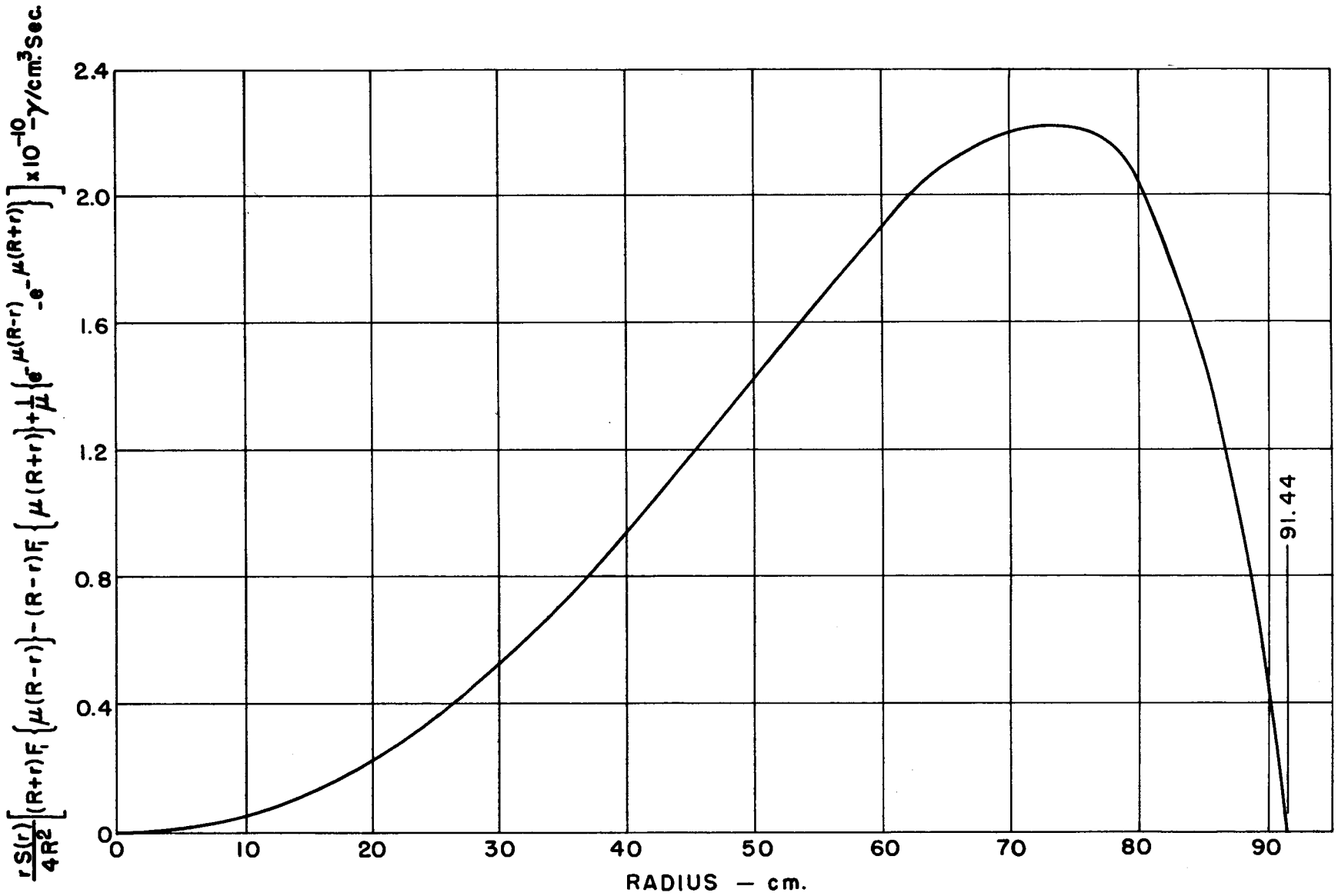

FIGURE I

CURVE FOR DETERMINING 6 MEV GAMMA CURRENT INCIDENT ON VESSEL WALL

be pointed out that the curve in Figure 1 goes to zero at the surface of the active region only because the extrapolation distance was assumed to be negligible in calculating the geometrical buckling. This would not be proper in the solution of a problem of this nature for an actual design.

Equation (6) may be used to estimate the thermal neutron current entering the vessel wall. It should be remembered that the method used herein for calculating the neutron leakage may not be the most accurate one available. However, the method will serve the purpose of obtaining a value of the leakage from which the thermal stresses may be calculated. The diffusion length for thermal neutrons in the solution is calculated to be $613 cm^2$ by use of equation (39) and the values given in Table I. Substituting the proper values in equation (6) gives an estimate of the thermal neutron current entering the vessel as 2.71×10^{13} neutrons/cm^2sec. As pointed out earlier, the effect of fast neutron heating will be neglected in this discussion.

The heating in the vessel wall due to the primary gammas may be calculated as a function of thickness by equation (3) for each photon energy. Using the values as given in Table II for the absorption coefficients and the gamma currents as calculated, the heating due to these gammas is given in Table IV and Figure 2 as a function of shell thickness.

The heating due to the thermal neutrons being absorbed in the shell is calculated by use of equation (8) with the aid of Appendix D. Table IV gives the results of these calculations if the value of β for iron is taken as 3.87. The heating effect is given in Figure 2 as a function of shell thickness.

NOMENCLATURE FOR TABLE IV

Column	Column Heading	Units
1	Position	cm
2	μx; Capture Gammas	--
3	$e^{-\mu x}$; Capture Gammas	--
4	$E_\gamma(\mu_t - \sigma_s)I_o\ e^{-\mu x} \times 10^{-11}$; Capture Gammas	Mev/cm^3 sec
5	μx; Prompt Fission Gammas	--
6	$e^{-\mu x}$; Prompt Fission Gammas	--
7	$E_\gamma(\mu_t - \sigma_s)I_o\ e^{-\mu x} \times 10^{-11}$; Prompt Fission Gammas	Mev/cm^3 sec
8	μx; Fission Product Gammas	--
9	$e^{-\mu x}$; Fission Product Gammas	--
10	$E_\gamma(\mu_t - \sigma_s)I_o\ e^{-\mu x} \times 10^{-11}$; Fission Product Gammas	Mev/cm^3 sec
11	μx; Secondary Gammas	--
12	$F(\beta, \mu x)$; Secondary Gammas	--
13	$N\ E_\gamma \frac{I_{o,t}}{2} (\mu_t - \sigma_s)F(\beta, \mu x)10^{-11}$; Secondary Gammas	Mev/cm^3 sec
14	$H(x)$; All Heat Sources	Mev/cm^3 sec

TABLE IV

HEAT PRODUCTION CALCULATIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14
0.0	0.0000	1.0000	9.93	0.000	1.0000	10.93	0.000	1.0000	39.53	0.0000	1.580	283.24	343.63
0.5	0.1005	0.9044	8.98	0.194	0.8237	9.00	0.122	0.8851	34.99	0.0985	1.936	347.06	400.03
1.0	0.2010	0.8179	8.13	0.388	0.6784	7.41	0.244	0.7835	30.97	0.1970	1.790	320.89	367.40
1.5	0.3015	0.7397	7.35	0.582	0.5588	6.11	0.366	0.6935	27.41	0.2955	1.545	276.97	317.84
2.0	0.4020	0.6690	6.65	0.776	0.4602	5.03	0.488	0.6139	24.27	0.3940	1.310	234.84	270.79
3.0	0.6030	0.5472	5.44	1.164	0.3122	3.41	0.732	0.4809	19.01	0.5910	0.913	163.67	191.53
4.0	0.8040	0.4475	4.45	1.552	0.2118	2.31	0.976	0.3768	14.89	0.7880	0.618	110.79	132.44
5.0	1.0050	0.3660	3.64	1.940	0.1437	1.57	1.220	0.2952	11.67	0.9850	0.425	76.19	93.07
6.0	1.2060	0.2994	2.97	2.328	0.0975	1.07	1.464	0.2313	9.14	1.1820	0.302	54.14	67.32
7.0	1.4070	0.2449	2.43	2.716	0.0661	0.72	1.708	0.1812	7.16	1.3790	0.218	39.08	49.39
8.0	1.6080	0.2003	1.99	3.104	0.0449	0.49	1.952	0.1420	5.61	1.5760	0.157	28.14	36.23
10.0	2.0100	0.1340	1.33	3.880	0.0207	0.23	2.440	0.0872	3.45	1.9700	0.082	14.70	19.71
12.0	2.4120	0.0896	0.89	4.656	0.0095	0.10	2.928	0.0535	2.11	2.3640	0.045	8.07	11.17
14.0	2.8140	0.0600	0.60	5.432	0.0044	0.05	3.416	0.0328	1.30	2.7580	0.028	5.02	6.97

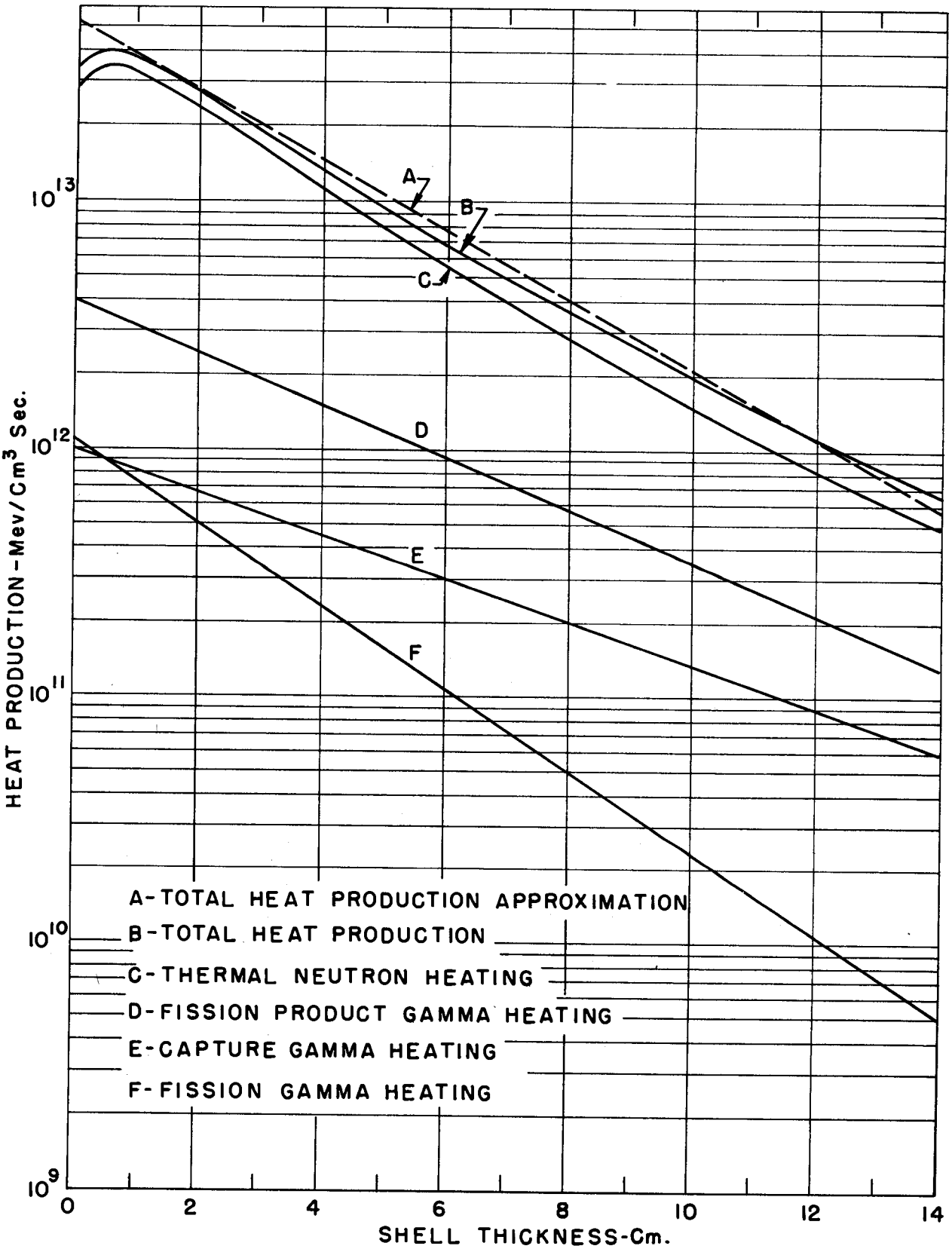

FIGURE 2
HEAT PRODUCTION IN VESSEL WALL

The various heating effects are added and their total is given by curve B in Figure 2. In order to apply the equations for temperature and stress, it is necessary to approximate the total heat production curve by one or more exponential functions. The "hump" in the curve may be obtained by subtracting an exponential if it is thought to be important. For the purpose of this discussion a single exponential as shown by curve A, Figure 2, is used to approximate curve B. The slope of the curve is calculated to be 0.321 cm^{-1}, and the value of the intercept is read from the curve as 5.2×10^{13} Mev/cm^3sec. Thus the total heating at any point is approximated by

$$H(r) = 5.2 \times 10^{13}\, e^{-0.321(r - R)}\ Mev/cm^3sec \qquad (49)$$

The maximum tangential thermal stress and the tangential pressure stress determine the optimum vessel thickness as pointed out previously. It was also shown that the maximum tangential thermal stress has a minimum value when both surfaces are at the same temperature. Therefore, for this condition, equation (35) is used to evaluate the total stress. The pressure stress should be evaluated at the inside radius. These equations are based on R = 36in. = 91.44cm and the value of a and b varied. This enables one to plot the maximum stresses as a function of shell thickness while the diameter of the active region remains constant. The properties of the vessel material used in these calculations are given in Table V.

The maximum stress calculations performed for this discussion are given in Table VI where the thermal shield serves as a parameter. The results are also given by Figure 3. It is seen from curve B of Figure 3

TABLE V

PHYSICAL PROPERTIES OF VESSEL MATERIAL

E - Modulus of Elasticity	30×10^{6} psi
α - Coefficient of expansion	6.30×10^{-6} in/in F
ν - Poisson's ratio	0.3
k - Thermal conductivity	25 Btu/hr ft F $= 1.5 \times 10^{12}$ Mev/cm sec F
σ_a - Allowable stress	13,500 psi

NOMENCLATURE FOR TABLE VI

Column	Column Heading	Units
1	Thermal Shield Thickness	in.
2	R	cm
3	a	in.
4	a	cm
5	b	in.
6	b	cm
7	a^2	cm^2
8	b^2	cm^2
9	a^3	cm^3
10	b^3	cm^3
11	$b^2 - a^2$	cm^2
12	$\frac{b^2 - a^2}{2}$	cm^2
13	$b^3 - a^3$	cm^3
14	$\frac{b^3 - a^3}{3}$	cm^3
15	a - R; See footnote to table.	cm
16	b - R; See footnote to table.	cm
17	a - b	in.
18	a - b	cm
19	m	cm^{-1}
20	ma	--

NOMENCLATURE FOR TABLE VI (CONTINUED)

Column	Column Heading	Units
21	mb	--
22	ma + 2	--
23	mb + 2	--
24	$\frac{ma + 2}{m}$	cm
25	$\frac{mb + 2}{m}$	cm
26	$\left(\frac{ma + a}{m}\right)^2$	cm^2
27	$\left(\frac{mb + 2}{m}\right)^2$	cm^2
28	m(a - R)	--
29	m(b - R)	--
30	$e^{-m(a - R)}$	--
31	$e^{-m(b - R)}$	--
32	$A_3 = (ma + 2)e^{-m(a - R)}$	--
33	$B_3 = (mb + 2)e^{-m(b - R)}$	--
34	bA_3	cm
35	aB_3	cm
36	$A_3 - B_3$	--

NOMENCLATURE FOR TABLE VI (CONTINUED)

Column	Column Heading	Units
37	$\frac{A_3 - B_3}{a - b}$	cm^{-1}
38	$bA_3 - aB_3$	cm
39	$\frac{bA_3 - aB_3}{a - b}$	--
40	$\left(\frac{A_3 - B_3}{a - b}\right)\left(\frac{b^3 - a^3}{3}\right)$	cm^2
41	$\left(\frac{bA_3 - aB_3}{a - b}\right)\left(\frac{b^2 - a^2}{2}\right)$	cm^2
42	$\left(\frac{mb + 2}{m}\right)^2 e^{-m(b - R)}$	cm^2
43	$\left(\frac{ma + 2}{m}\right)^2 e^{-m(a - R)}$	cm^2
44	$\left(\frac{A_3 - B_3}{a - b}\right)\left(\frac{b^3 - a^3}{3}\right) - \left(\frac{bA_3 - aB_3}{a - b}\right)\left(\frac{b^2 - a^2}{2}\right)$	
	$+ \left(\frac{mb + 2}{m}\right)^2 e^{-m(b - R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a - R)}$	cm^2
45	$-\frac{\alpha E H_R}{(1 - \nu)m^3 k}$	psi·cm

NOMENCLATURE FOR TABLE VI (CONTINUED)

Column	Column Heading	Units
46	$\frac{3\alpha E H_R}{(b^3 - a^3)(1 - \nu)m^3 k}$	psi/cm^2
47	$\sigma_{t,th\ max}$	psi
48	$2a^3$	cm^3
49	$2a^3 + b^3$	cm^3
50	$\frac{2a^3 + b^3}{b^3 - a^3}$	--
51	P_i	psi
52	$\sigma_{t,P_i\ max}$	psi
53	$\sigma_{t\ max}$	psi

TABLE VI

CALCULATIONS TO DETERMINE THE MAXIMUM STRESSES IN VESSEL WALL WITH THERMAL SHIELD AS A PARAMETER AND BOTH SURFACES AT THE SAME TEMPERATURE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0.0	91.44	36	91.44	37	93.98	8361	8832	764555	830054	471	235.5	65499	21833.00	0
0.0	91.44	36	91.44	38	96.52	8361	9316	764555	899191	955	477.5	134636	44878.67	0
0.0	91.44	36	91.44	39	99.06	8361	9813	764555	972064	1452	726.0	207509	69169.67	0
0.0	91.44	36	91.44	40	101.60	8361	10323	764555	1048772	1962	981.0	284217	94739.00	0
0.0	91.44	36	91.44	41	104.14	8361	10845	764555	1229413	2484	1242.0	364858	121619.33	0
0.0	91.44	36	91.44	42	106.68	8361	11381	764555	1214085	3020	1510.0	449530	149843.33	0
1.0	91.44	38	96.52	39	99.06	9316	9813	899191	972064	497	248.5	72873	24291.00	2.54
1.0	91.44	38	96.52	40	101.60	9316	10323	899191	1048772	1007	503.5	149581	49860.33	2.54
1.0	91.44	38	96.52	41	104.14	9316	10845	899191	1129413	1529	764.5	230222	76740.67	2.54
1.0	91.44	38	96.52	42	106.68	9316	11381	899191	1214085	2065	1032.5	314894	104964.67	2.54
1.0	91.44	38	96.52	43	109.22	9316	11929	899191	1302886	2613	1306.5	403695	134565.00	2.54
1.0	91.44	38	96.52	44	111.76	9316	12490	899191	1395916	3174	1587.0	496725	165575.00	2.54
2.0	91.44	40	101.60	41	104.14	10323	10845	1048772	1129413	522	261.0	80641	26880.33	5.08
2.0	91.44	40	101.60	42	106.68	10323	11381	1048772	1214085	1058	529.0	165313	55104.33	5.08
2.0	91.44	40	101.60	43	109.22	10323	11929	1048772	1302886	1606	803.0	254114	84704.67	5.08
2.0	91.44	40	101.60	44	111.76	10323	12490	1048772	1395916	2167	1083.5	347144	115714.67	5.08
2.0	91.44	40	101.60	45	114.30	10323	13064	1048772	1493271	2741	1370.5	444499	148166.33	5.08
2.0	91.44	40	101.60	46	116.84	10323	13652	1048772	1595051	3329	1664.5	546279	182093.00	5.08

TABLE VI (CONTINUED)

CALCULATIONS TO DETERMINE THE MAXIMUM STRESSES IN VESSEL WALL WITH THERMAL SHIELD AS A PARAMETER AND BOTH SURFACES AT THE SAME TEMPERATURE

(1)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
0.0	2.54	-1	-2.54	0.321	29.3522	30.1676	31.3522	32.1676	97.6704	100.2106	9540	10042
0.0	5.08	-2	-5.08	0.321	29.3522	30.9829	31.3522	32.9829	97.6704	102.7505	9540	10558
0.0	7.62	-3	-7.62	0.321	29.3522	31.7983	31.3522	33.7983	97.6704	105.2907	9540	11086
0.0	10.16	-4	-10.16	0.321	29.3522	32.6136	31.3522	34.6136	97.6704	107.8305	9540	11627
0.0	12.70	-5	-12.70	0.321	29.3522	33.4289	31.3522	35.4289	97.6704	110.3704	9540	12182
0.0	15.24	-6	-15.24	0.321	29.3522	34.2443	31.3522	36.2443	97.6704	112.9106	9540	12749
1.0	5.08	-1	-2.54	0.321	30.9829	31.7983	32.9829	33.7983	102.7505	105.2907	10558	11086
1.0	7.62	-2	-5.08	0.321	30.9829	32.6136	32.9829	34.6136	102.7505	107.8305	10558	11627
1.0	10.16	-3	-7.62	0.321	30.9829	33.4289	32.9829	35.4289	102.7505	110.3704	10558	12182
1.0	12.70	-4	-10.16	0.321	30.9829	34.2443	32.9829	36.2443	102.7505	112.9106	10558	12749
1.0	15.24	-5	-12.70	0.321	30.9829	35.0596	32.9829	37.0596	102.7505	115.4505	10558	13329
1.0	17.78	-6	-15.24	0.321	30.9829	35.8750	32.9829	37.8750	102.7505	117.9906	10558	13922
2.0	7.62	-1	-2.54	0.321	32.6136	33.4289	34.6136	35.4289	107.8305	110.3704	11627	12182
2.0	10.16	-2	-5.08	0.321	32.6136	34.2443	34.6136	36.2443	107.8305	112.9106	11627	12749
2.0	12.70	-3	-7.62	0.321	32.6136	35.0596	34.6136	37.0596	107.8305	115.4505	11627	13329
2.0	15.24	-4	-10.16	0.321	32.6136	35.8750	34.6136	38.8750	107.8305	117.9906	11627	13922
2.0	17.78	-5	-12.70	0.321	32.6136	36.6903	34.6136	38.6903	107.8305	120.5305	11627	14528
2.0	20.32	-6	-15.24	0.321	32.6136	37.5056	34.6136	39.5056	107.8305	123.0704	11627	15146

TABLE VI (CONTINUED)

CALCULATIONS TO DETERMINE THE MAXIMUM STRESSES IN VESSEL WALL WITH THERMAL SHIELD AS A PARAMETER AND BOTH SURFACES AT THE SAME TEMPERATURE

(1)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)
0.0	0.0000	0.8153	1.000000	0.442507	31.3522	14.2344	2946.4798	1301.5935	17.1178	-6.7393	1644.8863
0.0	0.0000	1.6307	1.000000	0.195792	31.3522	6.4578	3026.1143	590.5012	24.8944	-4.9005	2435.6131
0.0	0.0000	2.4460	1.000000	0.086639	31.3522	2.9283	3105.7489	267.7638	38.4239	-3.7302	2837.9851
0.0	0.0000	3.2614	1.000000	0.038338	31.3522	1.3270	3185.3835	121.3409	30.0252	-2.9552	3064.0426
0.0	0.0000	4.0767	1.000000	0.016963	31.3522	0.6010	3265.0181	54.9554	30.7512	-2.4214	3210.0627
0.0	0.0000	4.8920	1.000000	0.007506	31.3522	0.2720	3344.6527	24.8717	31.0802	-2.0394	3319.7810
1.0	0.8153	1.6307	0.442507	0.195792	14.5929	6.6174	1445.8005	638.7114	7.9778	-3.1409	807.0891
1.0	0.8153	2.4460	0.442507	0.086639	14.5929	2.9989	1482.8723	289.4538	11.5963	-2.2827	1193.4185
1.0	0.8153	3.2614	0.442507	0.038338	14.5952	1.3583	1519.9442	131.1031	13.2369	-1.7371	1388.8411
1.0	0.8153	4.0767	0.442507	0.016963	14.5952	0.6148	1557.0159	59.3212	13.9804	-1.3760	1497.6947
1.0	0.8153	4.8920	0.442507	0.007506	14.5952	0.2782	1594.0877	26.8519	14.3170	-1.1273	1567.2358
1.0	0.8153	5.7074	0.442507	0.003321	14.5952	0.1258	1631.1596	12.1422	14.4694	-0.9494	1619.0174
2.0	1.6307	2.4460	0.195792	0.086639	6.7771	3.0695	705.7672	311.8612	3.7076	-1.4597	393.9060
2.0	1.6307	3.2614	0.195792	0.038338	6.7771	1.3895	722.9810	141.1732	5.3876	-1.0606	581.8078
2.0	1.6307	4.0767	0.195792	0.016963	6.7771	0.6286	740.1949	63.8658	6.1485	-0.8069	676.3291
2.0	1.6307	4.8920	0.195792	0.007506	6.7771	0.2843	757.4087	28.8849	6.4928	-0.6391	728.5238
2.0	1.6307	5.7074	0.195792	0.003321	6.7771	0.1285	774.6225	13.0556	6.6486	-0.5235	761.5669
2.0	1.6307	6.5227	0.195792	0.001470	6.7771	0.0581	791.8364	5.9030	6.7190	-0.4409	785.9334

TABLE VI (CONTINUED)

CALCULATIONS TO DETERMINE THE MAXIMUM STRESSES IN VESSEL WALL WITH THERMAL SHIELD AS A PARAMETER AND BOTH SURFACES AT THE SAME TEMPERATURE

1	39	40	41	42	43	44	45	46
0.0	-647.5930	-147139	-152508	4444	9540	273	282985	12.9613
0.0	-479.4514	-219928	-228938	2067	9540	1537	282985	6.3056
0.0	-372.4390	-258017	-270391	960	9540	3794	282985	4.0912
0.0	-301.5790	-279973	-295849	446	9540	6782	282985	2.9870
0.0	-252.7608	-294489	-313929	207	9540	10107	282985	2.3268
0.0	-217.8334	-305590	-328928	96	9540	13894	282985	1.8885
1.0	-317.7516	-76296	-78961	2171	4672	164	282985	11.6498
1.0	-234.9249	-113816	-118285	1007	4672	804	282985	5.6756
1.0	-182.2626	-133306	-139340	467	4672	1829	282985	3.6875
1.0	-147.4109	-144431	-152202	216	4672	3315	282985	2.6960
1.0	-123.4044	-151695	-161228	100	4672	4961	282985	2.1030
1.0	-106.2347	-157197	-168594	46	4672	6771	282985	1.7091
2.0	-155.0811	-39237	-40476	1055	2276	18	282985	10.5276
2.0	-114.5291	-58444	-60586	489	2276	355	282985	5.1354
2.0	-88.7571	-68348	-71272	226	2276	874	282985	3.3408
2.0	-71.7051	-73953	-77692	104	2276	1567	282985	2.4455
2.0	-59.9659	-77565	-82183	48	2276	2390	282985	1.9099
2.0	-51.5704	-80285	-85839	22	2276	3300	282985	1.5541

TABLE VI (CONTINUED)

CALCULATIONS TO DETERMINE THE MAXIMUM STRESSES IN VESSEL WALL WITH THERMAL SHIELD AS A PARAMETER AND BOTH SURFACES AT THE SAME TEMPERATURE

(1)	(47)	(48)	(49)	(50)	(51)	(52)	(53)
0.0	3538	1529110	2359165	36.018	1000	18039	21577
0.0	9692	1529110	2428301	18.036	1000	9018	18710
0.0	15522	1529110	2501174	12.053	1000	6026	21548
0.0	20258	1529110	2577882	9.070	1000	4535	24793
0.0	23517	1529110	2658523	7.286	1000	3643	27160
0.0	26239	1529110	2743195	6.102	1000	3051	29290
1.0	1911	1798382	2770446	38.017	1000	19085	20996
1.0	4563	1798382	2847154	19.034	1000	9517	14080
1.0	6744	1798382	2927795	12.717	1000	6358	13102
1.0	8937	1798382	3012467	9.566	1000	4783	13720
1.0	10433	1798382	3101268	7.682	1000	3841	14274
1.0	11572	1798382	3194298	6.430	1000	3215	14787
2.0	189	2097544	3226957	40.016	1000	20008	20197
2.0	1823	2097544	3311629	20.032	1000	10016	11839
2.0	2920	2097544	3400430	13.382	1000	6691	9611
2.0	3832	2097544	3493460	10.064	1000	5032	8864
2.0	4565	2097544	3590815	8.082	1000	4041	8606
2.0	5129	2097544	3692595	6.760	1000	3380	8509

* Assuming that the coolant absorbs no radiation, the actual thickness of metal is used for the attenuation of the radiation. However, the larger radius appears in factors involving a and b.

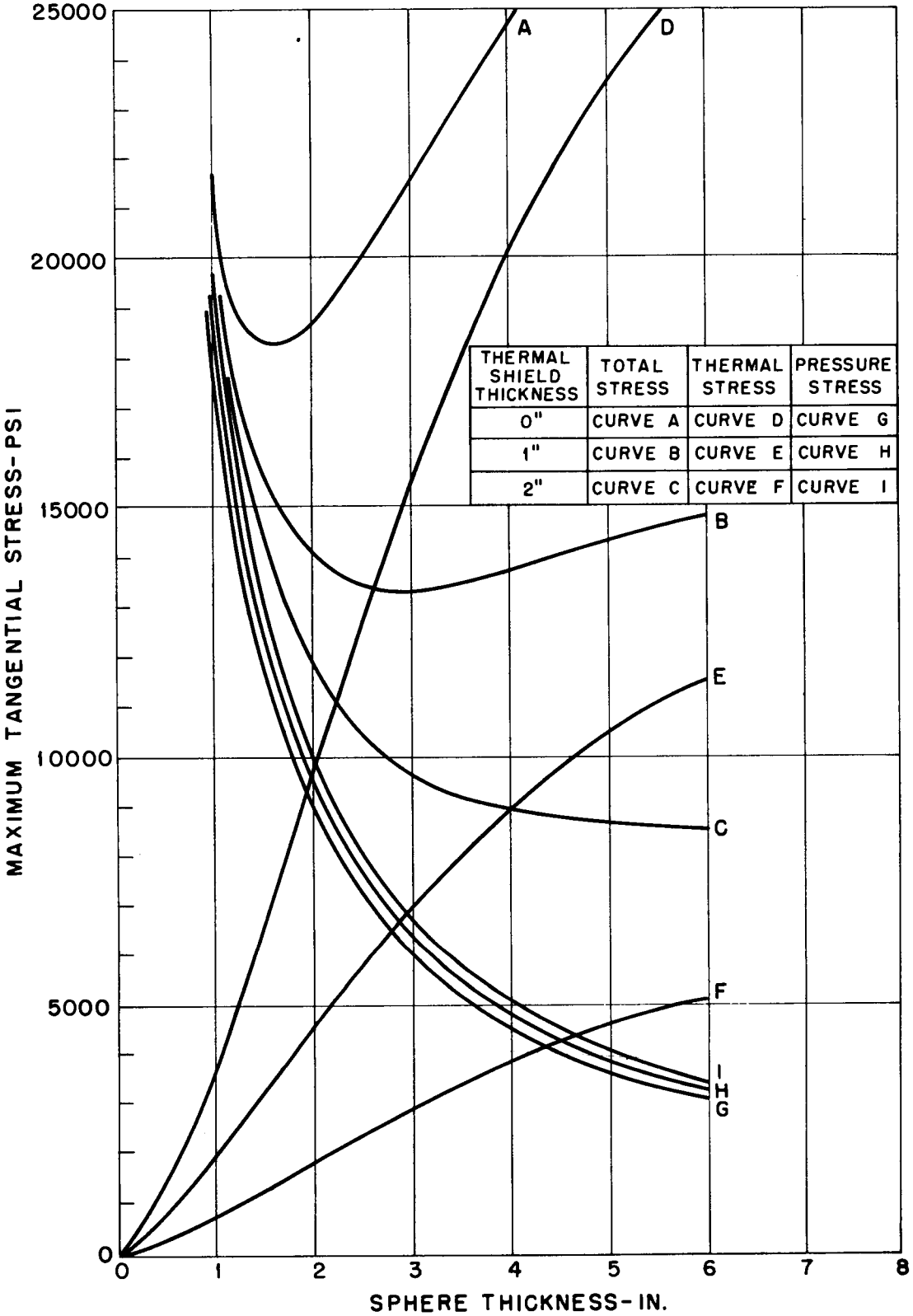

FIGURE 3

MAXIMUM STRESSES IN VESSEL WALL WITH THERMAL SHIELD THICKNESS AS THE PARAMETER AND BOTH SURFACES AT THE SAME TEMPERATURE

that the optimum thickness is about 2.75" for a 1" thermal shield and the total stress is approximately 13,300 psi or slightly less than the allowable stress as given in Table V. It should be noted that the total stress curve for a 2" thermal shield is very flat, and the minimum did not occur within the region of these calculations. It should also be noted that the total stress curve exhibits a sharply defined minimum for the case of zero thermal shield thickness.

In general, the maximum stresses as given by Figure 3 would be all that is necessary in order to insure a safe design. However, for those interested in the stress distribution within the vessel wall, the calculations are presented in Table VII for the case of 1" thermal shield and a 2.75" wall thickness. The tangential stress distribution and the temperature distribution for the vessel wall are given by Figure 4 while the radial stress distribution and temperature distribution are given by Figure 5. One should notice the magnitude of the radial stresses compared to the tangential stresses as the scales for the figures are not the same. The tangential stress and the temperature distribution are given by Figure 6 for the 1" thermal shield.

The maximum temperature in the vessel wall is found to be about 34.8 degrees F above the surface temperature while the maximum temperature in the thermal shield is about 18.9 degrees F above the surface temperature.

Figure 7 shows the variation of the maximum total stress in the tangential direction as a function of the difference in the surface temperatures for the 2.75" wall thickness. It is seen from the figure that the stress rises sharply with a few degrees difference in temperatures. For

NOMENCLATURE FOR TABLE VII

Column	Column Heading	Units
1	r	in.
2	r	cm
3	r^2	cm^2
4	r^3	cm^3
5	$r^2 - a^2$	cm^2
6	$\frac{r^2 - a^2}{2}$	cm^2
7	$r^3 - a^3$	cm^3
8	$\frac{r^3 - a^3}{3}$	cm^3
9	$r - R$; see footnote to table	cm
10	$a - r$	cm
11	$r - b$	cm
12	$\frac{r - b}{a - b}$	--
13	$\frac{a - r}{a - b}$	--
14	$\frac{r^3 - a^3}{b^3 - a^3}$	--
15	$\frac{a^3}{r^3}$	--

NOMENCLATURE FOR TABLE VII (CONTINUED)

Column	Column Heading	Units
16	$\frac{2r^3 + a^3}{r^3}$	--
17	$\frac{a^3}{b^3 - a^3}$	--
18	$\frac{b^3}{r^3}$	--
19	m	cm^{-1}
20	mr	--
21	$mr + 2$	--
22	$\frac{mr + 2}{m}$	cm
23	$\left(\frac{mr + 2}{m}\right)^2$	cm^2
24	$m(r - R)$	--
25	$e^{-m(r - R)}$	--
26	$(mr + 2)e^{-m(r - R)}$	--
27	$\left(\frac{mr + 2}{m}\right)^2 e^{-m(r - R)}$	cm^2
28	$\frac{A_3 - B_3}{a - b}$	cm^{-1}
29	$\frac{bA_3 - aB_3}{a - b}$	--

NOMENCLATURE FOR TABLE VII (CONTINUED)

Column	Column Heading	Units
30	$A_3\left(\frac{r - b}{a - b}\right)$	--
31	$B_3\left(\frac{a - r}{a - b}\right)$	--
32	$\left(\frac{A_3 - B_3}{a - b}\right)\left(\frac{r^3 - a^3}{3}\right)$	cm^2
33	$\left(\frac{bA_3 - aB_3}{a - b}\right)\left(\frac{r^2 - a^2}{2}\right)$	cm^2
34	$\left(\frac{A_3 - B_3}{a - b}\right)\left(\frac{r^3 - a^3}{3}\right) - \left(\frac{bA_3 - aB_3}{a - b}\right)\left(\frac{r^2 - a^2}{2}\right)$ $+ \left(\frac{mr + 2}{m}\right)^2 e^{-m(r - R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a - R)}$	cm^2
35	$A_3\left(\frac{r - b}{a - b}\right) + B_3\left(\frac{a - r}{a - b}\right) - (mr + 2)\, e^{-m(r - R)}$	--
36	$\frac{2r^3 + a^3}{r^3(b^3 - a^3)}\left\{\left(\frac{A_3 - B_3}{a - b}\right)\left(\frac{b^3 - a^3}{3}\right) - \left(\frac{bA_3 - aB_3}{a - b}\right)\left(\frac{b^2 - a^2}{2}\right)\right.$ $\left. + \left(\frac{mb + 2}{m}\right)^2 e^{-m(b - R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a - R)}\right\}$	cm^{-1}

NOMENCLATURE FOR TABLE VII (CONTINUED)

Column	Column Heading	Units
37	$\frac{1}{r^3}\left\{\left(\frac{A_3 - B_3}{a - b}\right)\left(\frac{r^3 - a^3}{3}\right) - \left(\frac{bA_3 - aB_3}{a - b}\right)\left(\frac{r^2 - a^2}{2}\right) + \left(\frac{mr + 2}{m}\right)^2 e^{-m(r - R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a - R)}\right\}$	cm^{-1}
38	$\frac{1}{r}\left\{A_3\left(\frac{r - b}{a - b}\right) + B_3\left(\frac{a - r}{a - b}\right) - (mr + 2)\, e^{-m(r - R)}\right\}$	cm^{-1}
39	$\frac{2r^3 + a^3}{r^3(b^3 - a^3)}\left\{\left(\frac{A_3 - B_3}{a - b}\right)\left(\frac{b^3 - a^3}{3}\right) - \left(\frac{bA_3 - aB_3}{a - b}\right)\left(\frac{b^2 - a^2}{2}\right) + \left(\frac{mb + 2}{m}\right)^2 e^{-m(b - R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a - R)}\right\}$ $+ \frac{1}{r^3}\left\{\left(\frac{A_3 - B_3}{a - b}\right)\left(\frac{r^3 - a^3}{3}\right) - \left(\frac{bA_3 - aB_3}{a - b}\right)\left(\frac{r^2 - a^2}{2}\right) + \left(\frac{mr + 2}{m}\right)^2 e^{-m(r - R)} - \left(\frac{ma + 2}{m}\right)^2 e^{-m(a - R)}\right\}$ $- \frac{1}{r}\left\{A_3\left(\frac{r - b}{a - b}\right) + B_3\left(\frac{a - r}{a - b}\right) - (mr + 2)\, e^{-m(r - R)}\right\}$	cm^{-1}
40	$\frac{\alpha E H_R}{(1 - \nu) m^3 k}$	psi cm

NOMENCLATURE FOR TABLE VII (CONTINUED)

Column	Column Heading	Units
41	$\sigma_{t,th}$	psi
42	$\dfrac{r^3 - a^3}{r^3}$	--
43	$\dfrac{r^3 - a^3}{r^3(b^3 - a^3)}\left\{\left(\dfrac{A_3 - B_3}{a - b}\right)\left(\dfrac{b^3 - a^3}{3}\right) - \left(\dfrac{bA_3 - aB_3}{a - b}\right)\left(\dfrac{b^2 - a^2}{2}\right) + \left(\dfrac{mb + 2}{m}\right)^2 e^{-m(b - R)} - \left(\dfrac{ma + 2}{m}\right)^2 e^{-m(a - R)}\right\}$	cm^{-1}
44	$\dfrac{r^3 - a^3}{r^3(b^3 - a^3)}\left\{\left(\dfrac{A_3 - B_3}{a - b}\right)\left(\dfrac{b^3 - a^3}{3}\right) - \left(\dfrac{bA_3 - aB_3}{a - b}\right)\left(\dfrac{b^2 - a^2}{2}\right) + \left(\dfrac{mb + 2}{m}\right)^2 e^{-m(b - R)} - \left(\dfrac{ma + 2}{m}\right)^2 e^{-m(a - R)}\right\} - \dfrac{1}{r^3}\left\{\left(\dfrac{A_3 - B_3}{a - b}\right)\left(\dfrac{r^3 - a^3}{3}\right) - \left(\dfrac{bA_3 - aB_3}{a - b}\right)\left(\dfrac{r^2 - a^2}{2}\right) + \left(\dfrac{mr + 2}{m}\right)^2 e^{-m(r - R)} - \left(\dfrac{ma + 2}{m}\right)^2 e^{-m(a - R)}\right\}$	cm^{-1}
45	$\dfrac{2\alpha E H_R}{(1 - \nu)m^3 k}$	psi cm
46	$\sigma_{r,th}$	psi

NOMENCLATURE FOR TABLE VII (CONTINUED)

Column	Column Heading	Units
47	$\frac{b^3}{2r^3}$	--
48	$\frac{r^3 - b^3}{r^3}$	--
49	$\frac{2r^3 + b^3}{2r^3}$	--
50	$\left(\frac{a^3}{b^3 - a^3}\right)\left(\frac{r^3 - b^3}{r^3}\right)$	--
51	$\left(\frac{a^3}{b^3 - a^3}\right)\left(\frac{2r^3 + b^3}{2r^3}\right)$	--
52	P_i	psi
53	σ_{t,P_i}	psi
54	σ_{r,P_i}	psi
55	σ_t	psi
56	σ_r	psi
57	$\frac{H_R}{m^3k}$	cm F
58	$T(r)$	^oF

TABLE VII

TEMPERATURE AND STRESS DISTRIBUTION CALCULATIONS FOR OPTIMUM VESSEL THICKNESS

1	2	3	4	5	6	7	8	9	10	11	12	13
38.000 = a	96.5200	9316	899191	0	0.0	0	0.00	2.5400	0.0000	-6.9850	1.0000	0.0000
38.375	97.4725	9501	926075	185	92.5	26884	8961.33	3.4925	-0.9525	-6.0325	0.8636	0.1364
38.750	98.4250	9687	953490	371	185.5	54299	18099.67	4.4450	-1.9050	-5.0800	0.7273	0.2727
39.125	99.3775	9876	981441	560	280.0	82250	27416.67	5.3975	-2.8575	-4.1275	0.5909	0.4091
39.500	100.3300	10066	1009933	750	375.0	110742	36914.00	6.3500	-3.8100	-3.1750	0.4545	0.5455
39.875	101.2825	10258	1038971	942	471.0	139780	46593.33	7.3025	-4.7625	-2.2225	0.3182	0.6818
40.250	102.2350	10452	1068560	1136	568.0	169369	56456.33	8.2550	-5.7150	-1.2700	0.1818	0.8182
40.750 = b	103.5050	10713	1108879	1397	698.5	209688	69896.00	9.5250	-6.9850	0.0000	0.0000	1.0000

TABLE VII (CONTINUED)

TEMPERATURE AND STRESS DISTRIBUTION CALCULATIONS FOR OPTIMUM VESSEL THICKNESS

(1)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
38.000 = a	0.0000	1.0000	3.0000	4.2882	1.2332	0.321	30.9829	32.9829	102.7505	10558	0.8153	0.442507
38.375	0.1282	0.9710	2.9710	4.2882	1.1974	0.321	31.2887	33.2887	103.7031	10754	1.1211	0.325921
38.750	0.2590	0.9431	2.9431	4.2882	1.1630	0.321	31.5944	33.5944	104.6555	10953	1.4268	0.240076
39.125	0.3922	0.9162	2.9162	4.2882	1.1298	0.321	31.9002	33.9002	105.6081	11153	1.7326	0.176824
39.500	0.5281	0.8903	2.8903	4.2882	1.0980	0.321	32.2059	34.2059	106.5604	11355	2.0384	0.130237
39.875	0.6666	0.8655	2.8655	4.2882	1.0673	0.321	32.5117	34.5117	107.5131	11559	2.3441	0.095934
40.250	0.8077	0.8415	2.8415	4.2882	1.0377	0.321	32.8174	34.8174	108.4654	11765	2.6499	0.070658
40.750 = b	1.0000	0.8109	2.8109	4.2882	1.0000	0.321	33.2251	35.2251	109.7355	12042	3.0575	0.047005

TABLE VII (CONTINUED)

TEMPERATURE AND STRESS DISTRIBUTION CALCULATIONS FOR OPTIMUM VESSEL THICKNESS

(1)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)
38.000 = a	14.5952	4672	-1.8525	-193.3942	14.5952	0.0000	0	0	0	0.0000	0.021432	0.000000
38.375	10.8495	3505	-1.8525	-193.3942	12.6044	0.2259	-16601	-17889	121	1.9808	0.021225	0.000131
38.750	8.0652	2630	-1.8525	-193.3942	10.6151	0.4515	-33530	-35875	303	3.0014	0.021025	0.000318
39.125	5.9944	1972	-1.8525	-193.3942	8.6243	0.6774	-50789	-54150	661	3.3073	0.020833	0.000673
39.500	4.4549	1479	-1.8525	-193.3942	6.6335	0.9032	-68383	-72523	947	3.0818	0.020648	0.000938
39.875	3.3108	1109	-1.8525	-193.3942	4.6442	1.1289	-86314	-91089	1212	2.4623	0.020471	0.001167
40.250	2.4601	831	-1.8525	-193.3942	2.6534	1.3548	-104585	-109848	1422	1.5481	0.020300	0.001331
40.750 = b	1.6558	566	-1.8525	-193.3942	0.0000	1.6558	-129482	-135086	1498	0.0000	0.020081	0.001351

TABLE VII (CONTINUED)

TEMPERATURE AND STRESS DISTRIBUTION CALCULATIONS FOR OPTIMUM VESSEL THICKNESS

(1)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)
38.000 = a	0.000000	0.021432	282985	6065	0.0000	0.000000	0.000000	565970	0	0.6166	-0.2332	1.6166
38.375	0.020321	0.001035	282985	293	0.0290	0.000207	0.000076	565970	43	0.5987	-0.1974	1.5987
38.750	0.030494	-0.009151	292985	-2590	0.0569	0.000406	0.000164	565970	93	0.5815	-0.1630	1.5815
39.125	0.033280	-0.011774	282985	-3332	0.0838	0.000599	-0.000074	565970	-42	0.5649	-0.1298	1.5649
39.500	0.030716	-0.009130	282985	-2584	0.1097	0.000784	-0.000154	565970	-87	0.5490	-0.0980	1.5490
39.875	0.024311	-0.002673	282985	-756	0.1345	0.000961	-0.000206	565970	-117	0.5337	-0.0673	1.5337
40.250	0.015142	0.006489	282985	1836	0.1585	0.001132	-0.000199	565970	-113	0.5189	-0.0377	1.5189
40.750 = b	0.000000	0.021432	282985	6065	0.1891	0.001351	0.000000	565970	0	0.5000	0.0000	1.5000

TABLE VII (CONTINUED)

TEMPERATURE AND STRESS DISTRIBUTION CALCULATIONS FOR OPTIMUM VESSEL THICKNESS

1	50	51	52	53	54	55	56	57	58
38.000 = a	-1.000	6.932	1000	6932	-1000	12997	-1000	1048	0.000
38.375	-0.846	6.855	1000	6855	-846	7148	-803	1048	21.296
38.750	-0.699	6.781	1000	6781	-699	4191	-606	1048	31.958
39.125	-0.557	6.710	1000	6710	-557	3378	-599	1048	34.877
39.500	-0.420	6.642	1000	6642	-420	4058	-507	1048	32.190
39.875	-0.289	6.577	1000	6577	-289	5821	-406	1048	25.478
40.250	-0.162	6.513	1000	6513	-162	8349	-275	1048	15.869
40.750 = b	0.000	6.432	1000	6432	0	12497	0	1048	0.000

* Assuming the coolant absorbs no radiation, the actual metal thickness is used as the attenuation distance.

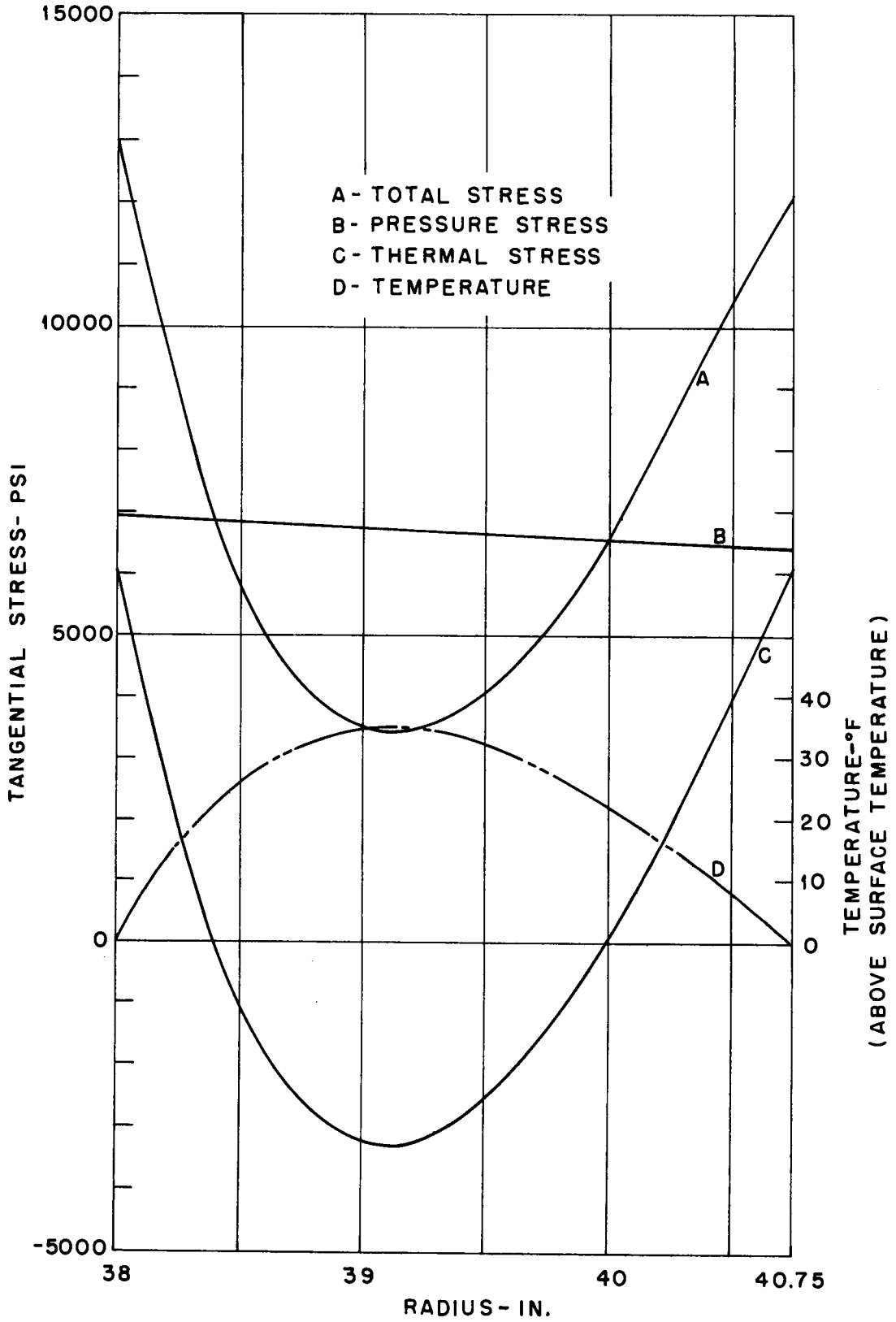

FIGURE 4

TEMPERATURE AND TANGENTIAL STRESS DISTRIBUTION IN OPTIMUM VESSEL THICKNESS

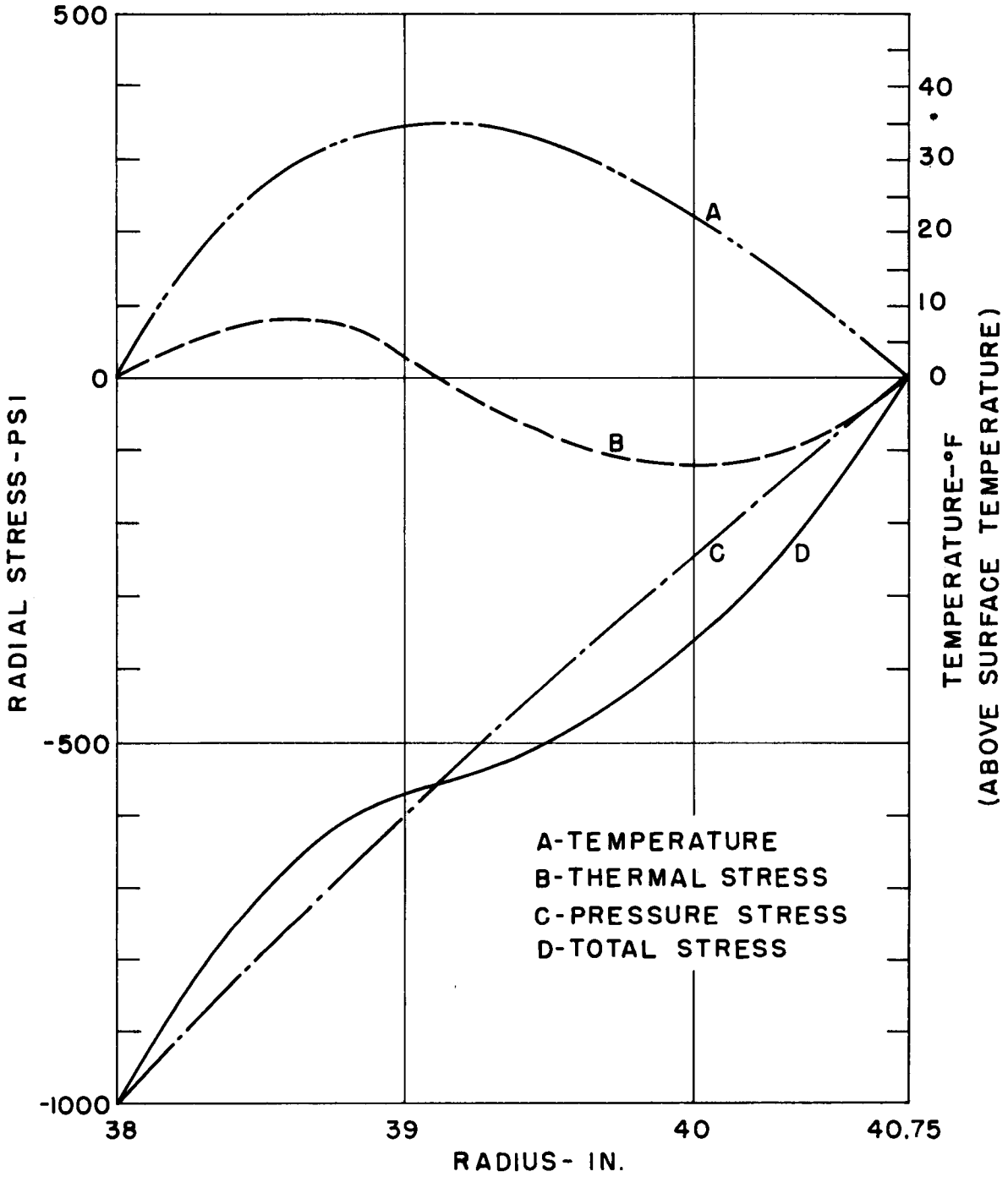

FIGURE 5

TEMPERATURE AND RADIAL STRESS DISTRIBUTION IN OPTIMUM VESSEL THICKNESS

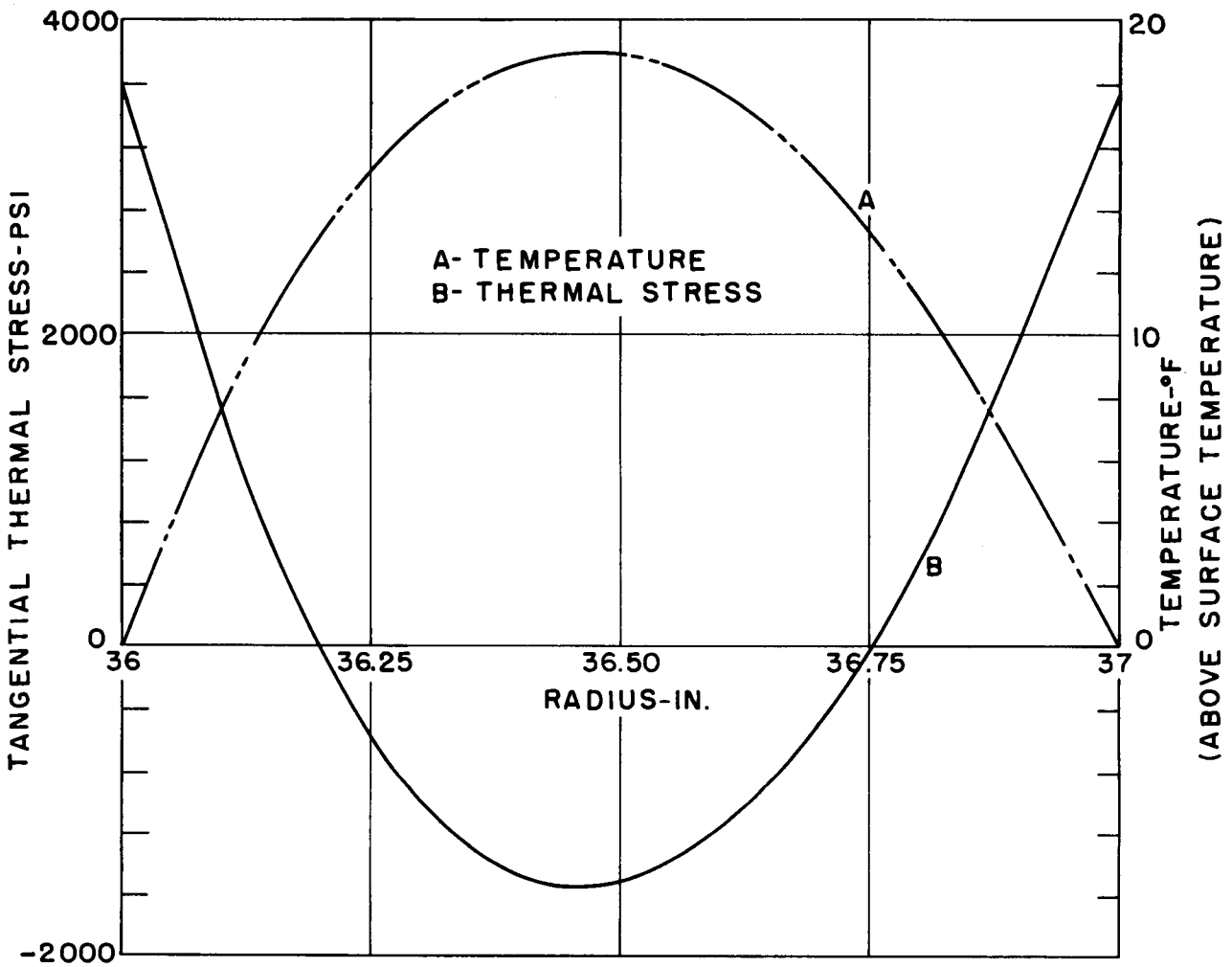

FIGURE 6

THERMAL STRESS AND TEMPERATURE DISTRIBUTION IN THERMAL SHIELD

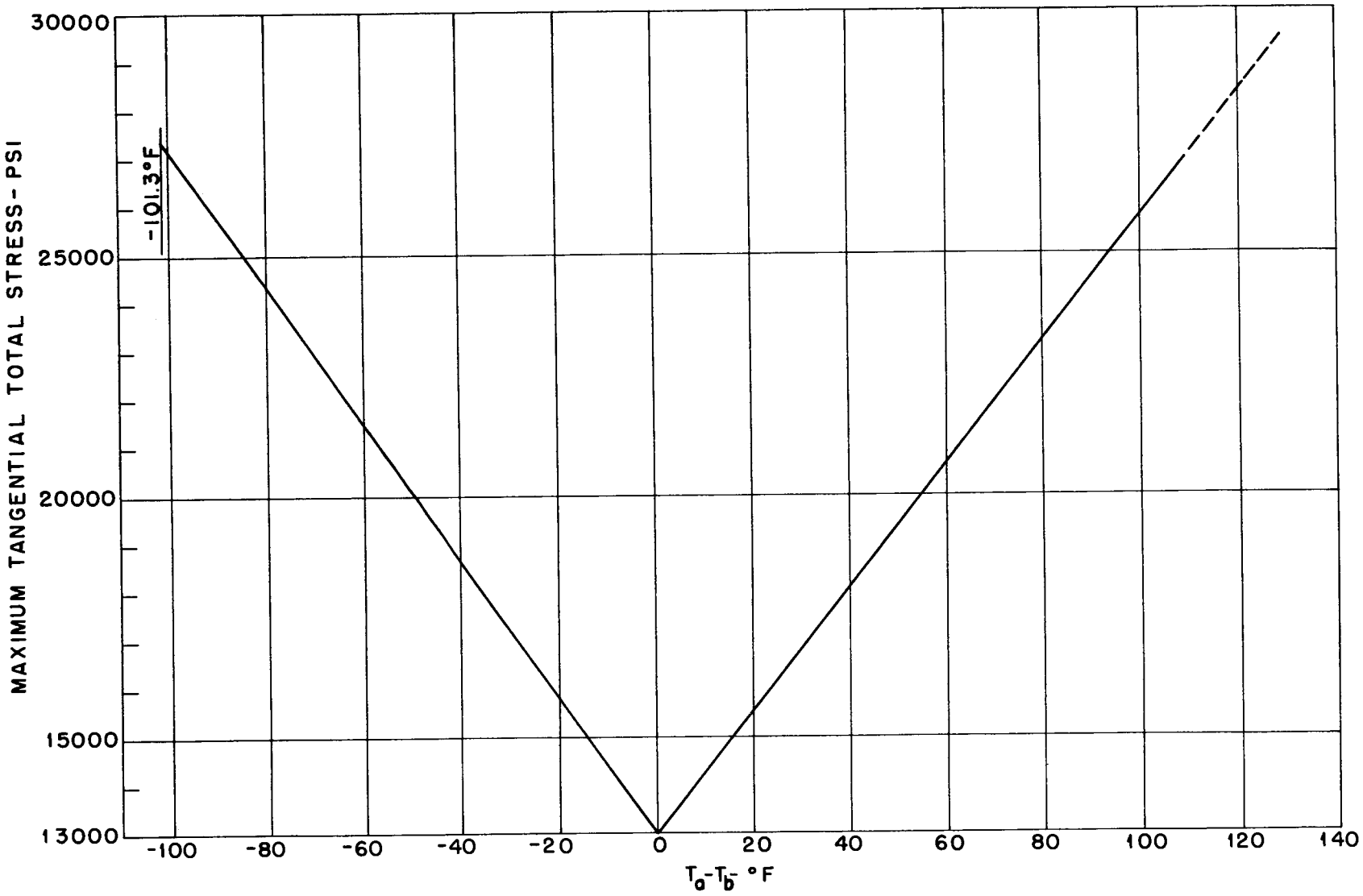

FIGURE 7

MAXIMUM TANGENTIAL STRESS VERSUS SURFACE TEMPERATURE DIFFERENCE FOR 2.75 INCH WALL THICKNESS

instance, if the vessel were perfectly insulated, the difference in surface temperatures would be 101.3 degrees F and the total stress would be 27,380 psi.

CHAPTER VI

DISCUSSION

Obviously there are many ways of arriving at an estimate of the heat production within a body. Many assumptions can be made in order to reduce the amount of calculating necessary to obtain an answer. Several such assumptions have been made in this discussion for the sake of arriving at some numerical values for use in the demonstration of the stress calculations. The seriousness of the problem may shed some light on the amount of simplification which may be tolerated in any given situation.

While the equations for the thermal stresses were derived on the basis of the heat production being described by a single exponential function, this does not limit their usefulness. It may be necessary to approximate the estimated heat production function by addition and/or subtraction of several exponentials in order to "fit" the given curve. If several exponentials are required, the stresses may be calculated for each exponential, as shown, and then added. The one exception would be for the general case where both surface temperatures are determined from the heat transferred to the surrounding mediums. All that would be necessary in this situation (Case III) is to include the effect of the surface temperatures in only one of the exponential functions and the other exponentials would be treated as if both surface temperatures are equal to zero. The terms which contain the surface temperatures are implicit functions of the heat production as shown by equations (33) and (34).

These terms merely serve as a reference for the boundary conditions and their effect cannot be added more than once and still maintain the proper boundary conditions.

The maximum thermal stress (tension) will occur at the points where the temperature is at a minimum, and the minimum thermal stress (compressive) will occur at the points where the temperature is at a maximum. If stress corrosion is a factor in the design, it is possible to impose a compressive thermal stress at the inside surface by controlling the cooling of the vessel. This compressive stress would subtract from the pressure stress to give a net stress which can be made equal to some desired value. However, this would be at the expense of very high tensile stresses at the outside surface.

In performing the stress calculations great care must be exercised in order to obtain consistent results. The usual slide rule accuracy is not sufficient as the differences in very large numbers occur in many places. Quite often these differences are very small compared to the large numbers. It is recommended that mathematical tables and a calculating machine be employed in performing calculations of this type.

The thermal stress in a material is proportional to $\frac{\alpha E}{k}$. Naturally one would like for this number to be small. Often stainless steel is employed for systems operating at high temperatures and containing corrosive mediums because of its strength and corrosion resistant qualities. Unfortunately the factor $\frac{\alpha E}{k}$ for stainless steel is about 2 to 3 times that for carbon steel which means 2 to 3 times the thermal stress. The

use of a thin stainless cladding on carbon steel may be the solution to this problem. However, this suggests another problem in that a thermal stress analysis similar to the one presented here would be desirable for an integrally clad vessel.

Throughout this discussion it has been assumed that the thermal conductivity, the modulus of elasticity, and the coefficient of expansion do not vary with temperature. The stress calculations have been assumed to remain in the elastic region even though they may not. The thermal stress may be relieved by creep. The amount of relief obtained or expected is a matter of conjecture. In the absence of quantitative results it may be desirable to exclude any benefits obtained from the relief of the thermal stresses. It would be helpful to have data to guide one's thinking along these lines.

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APPENDIX

APPENDIX A

RADIATION CURRENT FOR A SPHERICAL SOURCE WITH SELF-ABSORPTION

The gamma current at point Q, Figure 8, is given as I_o γ/cm^2sec. Let S(r) be the source strength per unit volume of material which has an absorption coefficient, μ, dependent upon the gamma energy. One may write the current normal to the surface at Q from an element of volume as

$$dI_o = S(r) \; dV \; \frac{e^{-\mu\rho}}{4\pi\rho^2} \; \sin \psi \tag{50}$$

where

I_o = gamma current at Q, γ/cm^2 sec.

$S(r)$ = source strength, γ/cm^3 sec.

dV = element of volume, cm^3.

$e^{-\mu\rho}$ = linear attenuation factor for the material.

$\frac{1}{4\pi\rho^2}$ = spherical correction factor.

$\sin \psi$ = normal component of the radiation at Q.

For spherical coordinates, the element of volume is given as

$$dV = r^2 \sin \theta \, dr \, d\theta \; d\omega \tag{51}$$

Substituting and integrating with respect to ω , the expression for the total current is obtained as

$$I_o = \int_{r=0}^{r=R} \int_{\theta=0}^{\theta=\pi} \frac{S(r)r^2}{2\rho^2} \; e^{-\mu\rho} \; \sin \theta \sin \psi \, d\theta \, dr \tag{52}$$

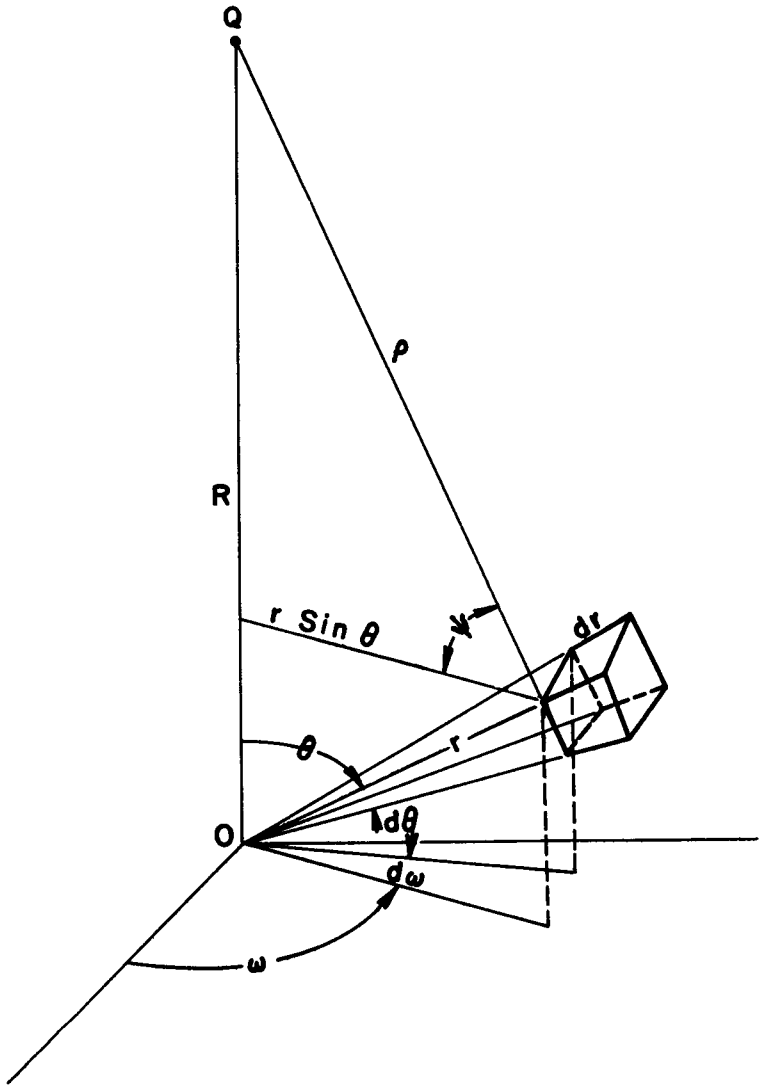

FIGURE 8

DIAGRAM FOR PRIMARY GAMMA CURRENT FOR A SPHERICAL SHELL WITH SELF-ABSORPTION

By applying the law of cosines, it is seen from the figure that

$$\rho^2 = R^2 + r^2 - 2Rr \cos \theta \tag{53}$$

which may be differentiated to give

$$\frac{\rho d\rho}{R} = r \sin \theta \, d\theta \tag{54}$$

From the figure it is also seen that

$$\sin \psi = \frac{R - r \cos\theta}{\rho} \tag{55}$$

Combining equations (53) and (55) gives

$$\sin \psi = \frac{\rho^2 + R^2 - r^2}{2 R\rho} \tag{56}$$

Combining equations (52), (54), and (56), and changing the limits, gives

$$I_o = \int_{r=0}^{r=R} \frac{rS(r)}{4R^2} \int_{\rho=R-r}^{\rho=R+r} \left[\frac{R^2 - r^2}{\rho^2} e^{-\mu\rho} + e^{-\mu\rho} \right] d\rho dr \tag{57}$$

Changing the variable ρ to $\mu\rho$ and rewriting, equation (57) becomes

$$I_o = \int_{r=0}^{r=R} \frac{rS(r)}{4R^2} \left[\int_{\mu\rho=\mu(R-r)}^{\mu\rho=\mu(R+r)} \left(\mu(R^2 - r^2) \frac{e^{-\mu\rho}}{(\mu\rho)^2} \right) d(\mu\rho) \right.$$

$$\left. + \frac{1}{\mu} \int_{\mu\rho=\mu(R-r)}^{\mu\rho=\mu(R+r)} \left(e^{-\mu\rho} \right) d(\mu\rho) \right] dr \tag{58}$$

The integrals within the brackets may be changed to the form

$$F_1(t) \equiv t \int_t^{\infty} \frac{e^{-y}}{y^2}\, dy \tag{59}$$

by writing equation (58) in the following manner

$$I_o = \int_{r=0}^{r=R} \frac{rS(r)}{4R^2} \left[\mu(R^2 - r^2) \int_{\mu\rho=\mu(R-r)}^{\infty} \left(\frac{e^{-\mu\rho}}{(\mu\rho)^2} \right) d(\mu\rho) \right.$$

$$\left. -\mu(R^2 - r^2) \int_{\mu\rho=\mu(R+r)}^{\infty} \left(\frac{e^{-\mu\rho}}{(\mu\rho)^2} \right) d(\mu\rho) + \frac{1}{\mu} \int_{\mu\rho=\mu(R-r)}^{\mu\rho=\mu(R+r)} \left(e^{-\mu\rho} \right) d(\mu\rho) \right] dr \tag{60}$$

Two of the integrals in equation (60) are of the same form as equation (59) and may be evaluated from the information given in Appendix C. Carrying out the integration and writing equation (60) in terms of the F_1 functions, the current at Q is

$$I_o = \int_{r=0}^{r=R} \frac{rS(r)}{4R^2} \left\{ (R+r)\, F_1\left[\mu(R-r)\right] - (R-r)\, F_1\left[\mu(R+r)\right] \right.$$

$$\left. + \frac{1}{\mu}\left[e^{-\mu(R-r)} - e^{-\mu(R+r)} \right] \right\} dr \tag{61}$$

Equation (61) may be graphically integrated for any given source distribution, S(r).

APPENDIX B

DERIVATION OF TEMPERATURE DISTRIBUTION

The differential equation for steady state heat conduction with an internal heat source may be written in spherical coordinates as

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = \frac{-H(r)}{k} \tag{62}$$

where

T = temperature, degree F.

r = radius, cm.

$H(r)$ = heat source, Mev/cm^{3}sec.

k = thermal conductivity of the material assumed independent of temperature, Mev/cm sec F.

Letting $Y = \frac{dT}{dr}$, $\frac{dy}{dr} = \frac{d^2T}{dr^2}$ and substituting

$H_R e^{-m(r-R)}$ for $H(r)$, the above equation becomes

$$\frac{dy}{dr} + \frac{2}{r}y = -\frac{H_R e^{-m(r-R)}}{k} \tag{63}$$

which has as a solution

$$y = e^{-\int\frac{2}{r}dr}\left[\int e^{\int\frac{2}{r}dr}\left(-\frac{H_R e^{-(r-R)}}{k}\right)dr + C_1\right] \tag{64}$$

Carrying out the indicated operations

$$y = \frac{dT}{dr} = \frac{1}{r^2}\left[-\frac{H_R}{k}\int r^2 \; e^{-m(r-R)} \; dr + C_1\right] \tag{65}$$

Further simplication gives an expression for the temperature gradient as

$$\frac{dT}{dr} = \frac{H_R}{mk} e^{-m(r-R)} + \frac{C_1}{r^2} + \frac{2H_R}{m^3k}\left(\frac{e^{-m(r-R)}}{r^2}\right) + \frac{2H_R}{m^2k}\left(\frac{e^{-m(r-R)}}{r}\right) \tag{66}$$

The temperature as a function of the radius is obtained by integrating equation (66) term by term. In performing the integration of equation (66), the exponential integrals of the third and fourth terms cancel when the third term is integrated by parts. After some manipulation, the expression for the temperature is given as

$$T(r) = -\frac{H_R}{m^3k}\left(\frac{mr+2}{r}\right)e^{-m(r-R)} - \frac{C_1}{r} + C_2 \tag{67}$$

where the constants of integration must be evaluated from the appropriate boundary conditions.

Three sets of boundary conditions will be used to evaluate the constants:

Case I:

$$\frac{dT}{dr} = 0 \; ; \quad r = b$$

$$T = 0 \text{ (as a datum)}; \quad r = a \tag{68}$$

The constants are found from equations (66), (67), and (68) as

$$C_1 = -\frac{H_R}{m^3k}(m^2b^2 + 2mb + 2)e^{-m(b-R)}$$

and $$C_2 = \frac{H_R}{am^3k}\left\{(ma + 2)e^{-m(a-R)} - (m^2b^2 + 2mb + 2)e^{-m(b-R)}\right\}$$

Letting $A_1 = (ma + 2)e^{-m(a-R)}$ and $B_1 = (m^2b^2 + 2mb + 2)e^{-m(b-R)}$

then

$$C_1 = -\frac{H_R}{m^3K}B_1 \quad \text{and } C_2 = \frac{H_R}{m^3k}\left(\frac{A_1 - B_1}{a}\right)$$

Substituting these values into equation (67) gives the temperature for a hollow sphere which has an exponential heat source and is perfectly insulated on the outside surface as

$$T(r) = \frac{H_R}{m^3k}\left[\frac{A_1 - B_1}{a} + \frac{B_1}{r} - \left(\frac{mr + 2}{r}\right)e^{-m(r-R)}\right] \tag{69}$$

<u>Case II:</u>

$$\frac{dT}{dr} = 0 \quad ; r = a$$

$$T = 0 \text{ (as a datum)}; \quad r = b \tag{70}$$

The constants for these conditions are found from equation (66), (67), and (70) as

$$C_1 = -\frac{H_R}{m^3k}(m^2a^2 + 2ma + 2)e^{-m(a-R)}$$

and

$$C_2 = \frac{H_R}{bm^3k}\left\{(mb + 2)e^{-m(b-R)} - (m^2a^2 + 2ma + 2)e^{-m(a-R)}\right\}$$

Letting $A_2 = (mb + 2)e^{-m(b-R)}$ and $B_2 = (m^2a^2 + 2ma + 2)e^{-m(a-R)}$

then

$$C_1 = -\frac{H_R}{m^3k}B_2 \quad \text{and} \quad C_2 = \frac{H_R}{m^3k}\left(\frac{A_2 - B_2}{b}\right)$$

Substituting these values into equation (67) gives the temperature for a hollow sphere which has an exponential heat source and is perfectly insulated on the inside surface as

$$T(r) = \frac{H_R}{m^3k}\left[\frac{A_2 - B_2}{b} + \frac{B_2}{r} - \left(\frac{mr + 2}{r}\right) e^{-m(r-R)}\right] \tag{71}$$

<u>Case III:</u>

$$\begin{aligned} T &= T_a \;;\; r = a \\ T &= T_b \;;\; r = b \end{aligned} \tag{72}$$

The constants for these conditions are found from equations (66), (67), and (72) as

$$C_1 = \frac{ab}{a - b}\left\{(T_a - T_b) + \frac{H_R}{m^3k}\left[\left(\frac{ma + 2}{a}\right)e^{-m(a-R)} - \left(\frac{mb + 2}{b}\right)e^{-m(b-R)}\right]\right\}$$

and

$$C_2 = \frac{1}{a - b}\left\{(aT_a - bT_b) + \frac{H_R}{m^3k}\left[(ma + 2)e^{-m(a-R)} - (mb + 2)e^{-m(b-R)}\right]\right\}$$

Letting $A_3 = (ma + 2)e^{-m(a-R)}$ and $B_3 = (mb + 2)\, e^{-m(b-R)}$

then $$C_1 = \frac{ab}{a - b}\left\{(T_a - T_b) + \frac{H_R}{m^3k}\left(\frac{A_3}{a} - \frac{B_3}{b}\right)\right\}$$

and $$C_2 = \frac{1}{a - b}\left\{(aT_a - aT_b) + \frac{H_R}{m^3k}(A_3 - B_3)\right\}$$

Substituting these values into equation (67) gives the temperature for a hollow sphere which has an exponential heat source and the surface temperatures specified as T_a and T_b. The temperature distribution is given as

$$T(r) = \frac{1}{r}\left\{aT_a\left(\frac{r - b}{a - b}\right) + bT_b\left(\frac{a - r}{a - b}\right) + \frac{H_R}{m^3k}\left[A_3\left(\frac{r - b}{a - b}\right) + B_3\left(\frac{a - r}{a - b}\right) - (mr+2)e^{-m(r-R)}\right]\right\} \quad (73)$$

A special case of interest is when both surfaces are at the same temperature. If that is true, then the temperature at any point is given by

$$T(r) = T_a + \frac{H_R}{m^3k}\,\frac{1}{r}\left\{A_3\left(\frac{r - b}{a - b}\right) + B_3\left(\frac{a - r}{a - b}\right) - (mr+2)e^{-m(r-R)}\right\} \quad (74)$$

APPENDIX C

F - FUNCTIONS

Frequently in shielding calculations the so-called F-functions appear. These functions are defined by

$$F_n(t) \equiv t^n \int_t^{\infty} y^{-(n+1)} \; e^{-y} \; dy \qquad (75)$$

For n = 0, the function is the exponential integral which is tabulated in the WPA Math Tables as $-\,Ei(-x)$. For $n > 1$, the following expression, obtained by integrating the above integral by parts, is helpful in evaluating the functions.

$$F_n(t) \equiv \frac{1}{n}\left[e^{-t} - t\,F_{n-1}(t)\right] \qquad (76)$$

A useful property of the functions is $F_n(0) = \frac{1}{n}$ for $n \geq 1$. Figures 9 and 10 give the value of $F_n(t)$ for $n = 1$ and $0 \leq t \leq 6.5$ calculated from the above expression with the aid of mathematical tables.

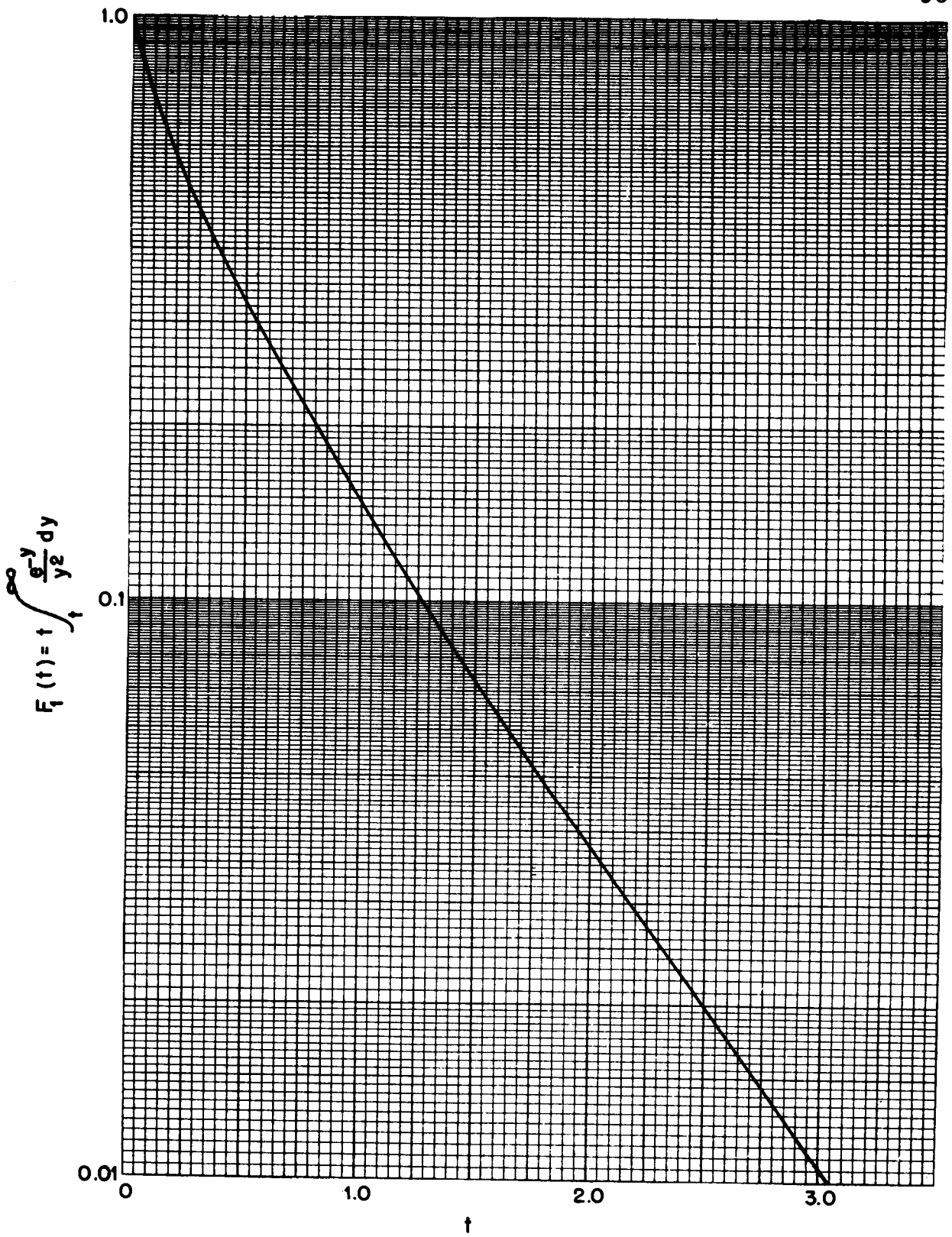

FIGURE 9

$$F_1(t) = t\int_t^\infty \frac{e^{-y}}{y^2}\,dy$$

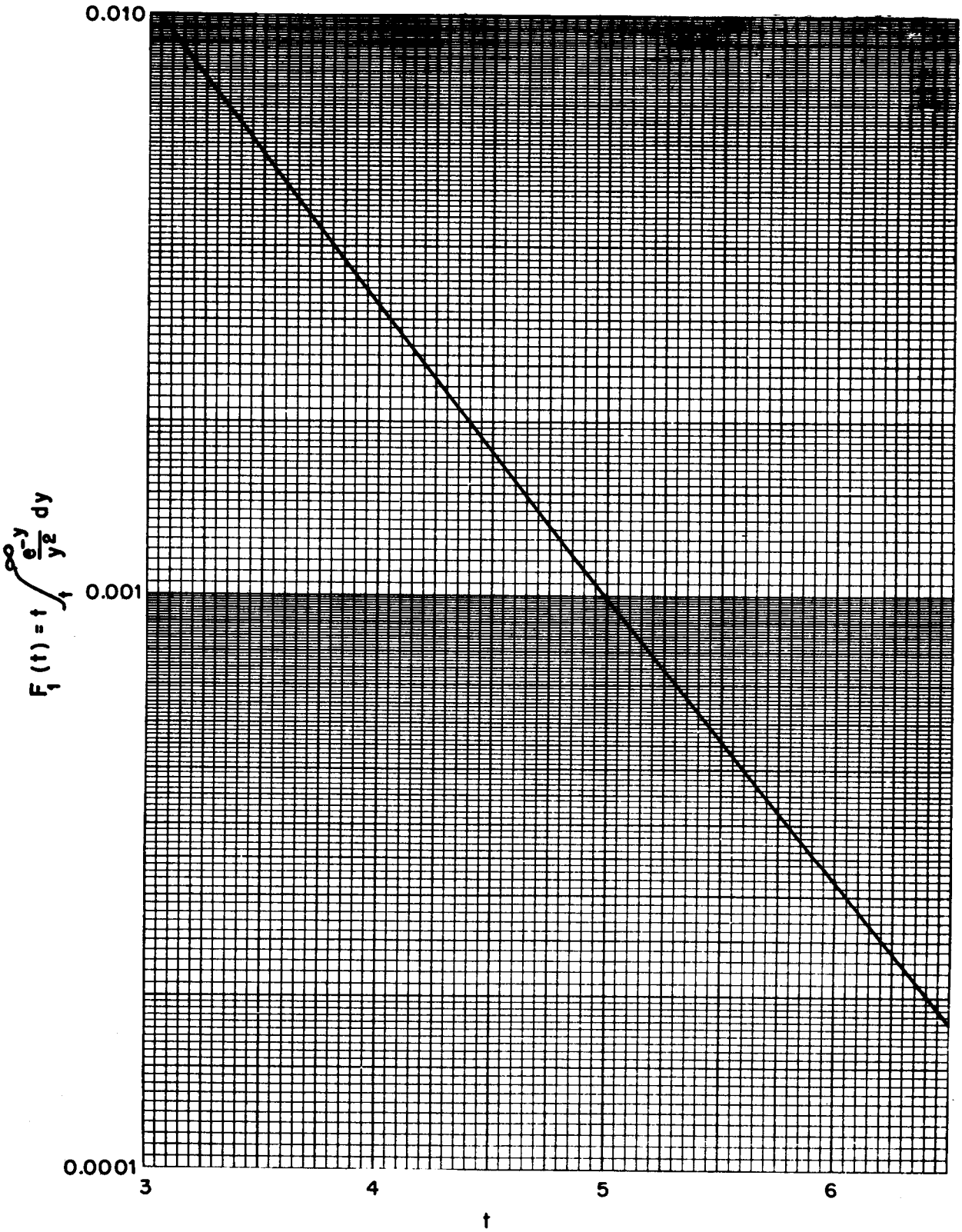

FIGURE 10

$$F_1(t) = t\int_t^\infty \frac{e^{-y}}{y^2}\,dy$$

APPENDIX D

SECONDARY CAPTURE GAMMA HEATING

Enlund (13) presented a solution for the heating in a slab due to thermal neutrons being captured within the slab. Upon the absorption of a thermal neutron, gamma rays of a particular energy spectrum are emitted. The gammas travel some distance before they are absorbed to produce heat at the point of absorption. A brief summary of the derivation is given here.

Consider an infinite plane source of neutrons emitting a neutron current, $I_{o,t}$ into an infinite plane slab of absorber. The neutron flux within the absorber is given as

$$\varphi = \frac{I_{o,t}\, e^{-Kx}}{KD} \tag{77}$$

when

φ = neutron flux, neutrons/cm^2 sec.

$I_{o,t}$ = neutron current incident on the slab, neutrons/cm^{2}sec.

$K = \sqrt{3\Sigma_a \Sigma_{tr}}$, cm^{-1}.

Σ_a = neutron absorption cross section, cm^{-1}.

Σ_{tr} = neutron transport cross section, cm^{-1}.

D = diffusion coefficient, cm.

x = distance from face of slab to point of neutron absorption, cm.

The number of neutrons absorbed at x, Figure 11, is given as

$$\Sigma_a \varphi = \frac{\Sigma_a I_{o,t} e^{-Kx}}{KD} = I_{o,t} K e^{-Kx} \tag{78}$$

Letting N be the number of gammas of energy E_γ produced per neutron absorption, $(\mu_t - \sigma_s)$ be the energy absorption coefficient for energy E_γ, and μ be the linear absorption coefficient for energy E_γ, the heating is given by

$$H(x) = NE_\gamma I_{o,t} K(\mu_t - \sigma_s) \int_{x=0}^{x=\infty} \int_{\rho=0}^{\rho=\infty} \frac{e^{-Kx} \; e^{-\mu h} \; 2\pi\rho d\rho dx}{4\pi h^2} \tag{79}$$

Making the substitution $w = 1/\cos\theta$, and, from the figure,

$$h = \frac{t - x}{\cos\theta}, \quad \rho = (t - x) \tan\theta, \text{ then } \quad \frac{\rho d\rho}{h^2} = \frac{\sin\theta \, d\theta}{\cos\theta} = \frac{dw}{w} .$$

The boundary conditions may be stated as

$$\left\{ \begin{array}{lllll} \theta = 0 \; ; & w = +1 \; ; & \text{and} & x = 0 \\ \theta = \pi/2 \; ; & w = +\infty \; ; & \text{and} & x = t \end{array} \right\} \begin{array}{l} \text{as } \theta \text{ increases from} \\ 0 \text{ to } \pi/2 \end{array}$$

and

$$\left\{ \begin{array}{lllll} \theta = \pi/2 \; ; & w = -\infty \; ; & \text{and} & x = t \\ \theta = \pi \; ; & w = -1 \; ; & \text{and} & x = \infty \end{array} \right\} \begin{array}{l} \text{as } \theta \text{ increases from} \\ \pi/2 \text{ to } \pi \end{array}$$

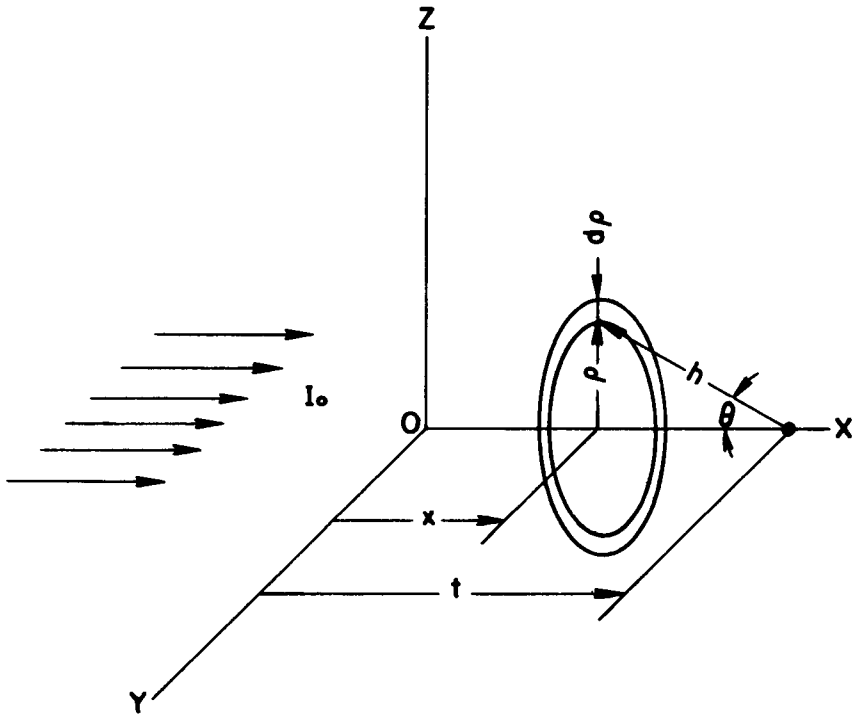

FIGURE 11

DIAGRAM FOR NEUTRON CAPTURE AND SECONDARY GAMMA ABSORPTION FOR AN INFINITE SLAB

The expression for the heating may be written as

$$H(x) = NE_\gamma \frac{I_{o,t}}{2} K(\mu_t - \sigma_s)\left[\int_{x=0}^{x=t}\int_{w=+1}^{w=+\infty}\left(\frac{e^{-Kx}\, e^{-(t-x)w}\, dw\, dx}{w}\right) + \int_{x=t}^{x=\infty}\int_{w=-\infty}^{w=-1}\left(\frac{e^{-Kx}\, e^{-(t-x)w}\, dw\, dx}{w}\right)\right] \tag{80}$$

The integration of (80) may be carried out and the results written as

$$H(x) = NE_\gamma \frac{I_{o,t}}{2} (\mu_t - \sigma_s)\ F(\beta,\mu t)\ \text{Mev/cm}^3\text{sec} \tag{81}$$

where $\beta = K/\mu$. The value of the function $F(\beta,\mu t)$ is defined as follows:

(1) $K > \mu$; $\beta = K/\mu > 1$

$$F(\beta,\mu t) = e^{-\beta\mu t}\left\{e^{\beta\mu t}\left[-\,Ei(-\mu t)\right] + Ei(\mu t\,[\beta - 1]) + \ln\frac{\beta+1}{\beta-1}\right\} \tag{82}$$

Note that the limit of $F(\beta,\mu t) = \ln(\beta + 1)$ as $t \to 0$.

(2) $K = \mu$; $\beta = K/\mu = 1$

$$F(\beta,\mu t) = e^{-\mu t}\left\{e^{\mu t}\left[-\,Ei(-\mu t)\right] + \ln 2\gamma + \ln \mu t\right\} \tag{83}$$

where $\ln 2\gamma = 1.2703$

Note that the limit of $F(\beta,\mu t) = \ln 2$ as $t \to 0$.

(3) $K < \mu$; $\beta = K/\mu < 1$

$$F(\beta,\mu t) = e^{-\beta\mu t} \left\{ e^{\beta\mu t} \left[- Ei(-\mu t) \right] - \left[-Ei(-\mu t [1 - \beta]) \right] + \ln \frac{1 + \beta}{1 - \beta} \right\} \qquad (84)$$

Note that the limit of $F(\beta,\mu t) = \ln (1 + \beta)$ as $t \rightarrow 0$. The Ei functions are the exponential integrals as tabulated in the WPA Mathematical Tables. A family of curves may be plotted for the function $F(\beta,\mu t)$ with β as the parameter and μt as the independent variable to aid in the solution of problems. Such a family of curves appeared in the original article. This writer has extended the range of the original curves to greater values of μt and several additional values of β. The extended curves are given by Figure 12 for use in estimating the heating. It should be noted that as $\beta \rightarrow 0$, $\mu \rightarrow \infty$ and the heating occurs at the point of the neutron absorption. Also as $\beta \rightarrow \infty$, $\mu \rightarrow 0$ and the heating approaches its minimum value.

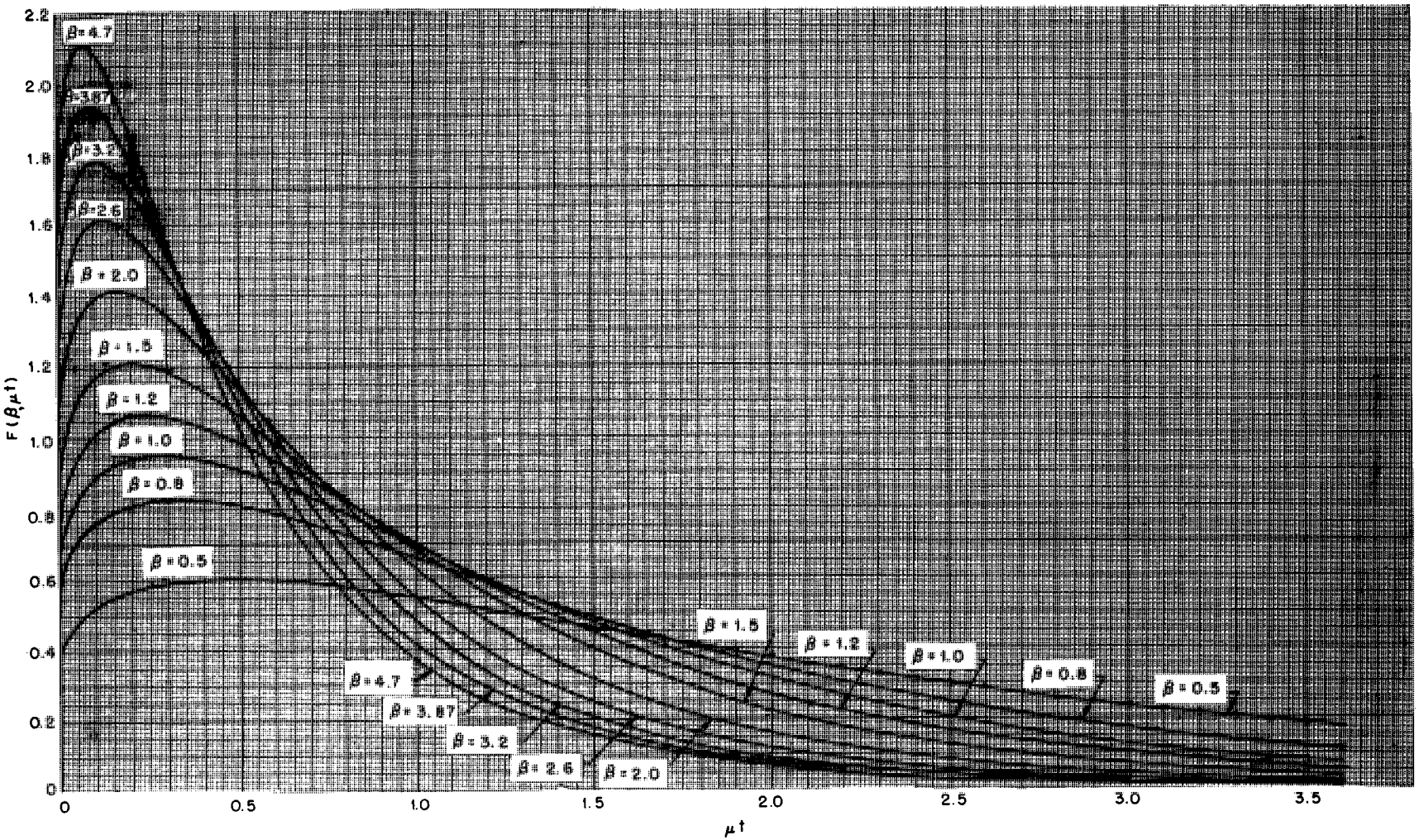

FIGURE 12

FUNCTIONS FOR CALCULATING SECONDARY GAMMA HEATING

APPENDIX E

ABSORPTION COEFFICIENTS

Reference (16) gives a brief discussion of the various absorption coefficients along with values of the coefficients versus energy for several materials. The curves from the reference are reproduced herein for completeness. The values given on the curves are for thin absorbers, hence, the necessity for using a buildup factor is pointed out. Figures 13 and 14 give the linear mass absorption coefficients as a function of photon energy, while Figures 15 and 16 give the energy absorption coefficients.

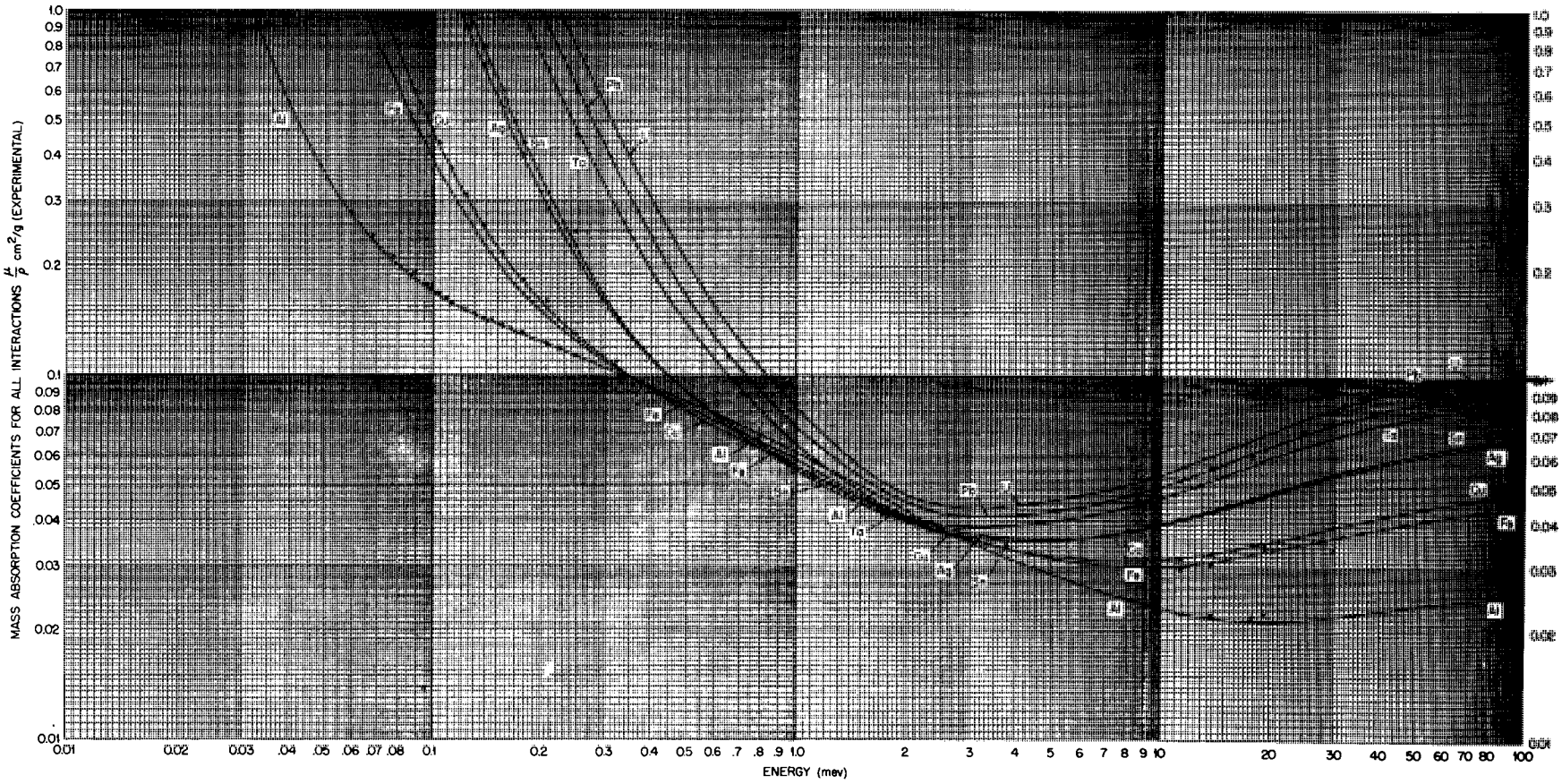

FIGURE 13

MASS ABSORPTION COEFFICIENTS
(MACROSCOPIC TOTAL CROSS SECTION DIVIDED BY DENSITY)

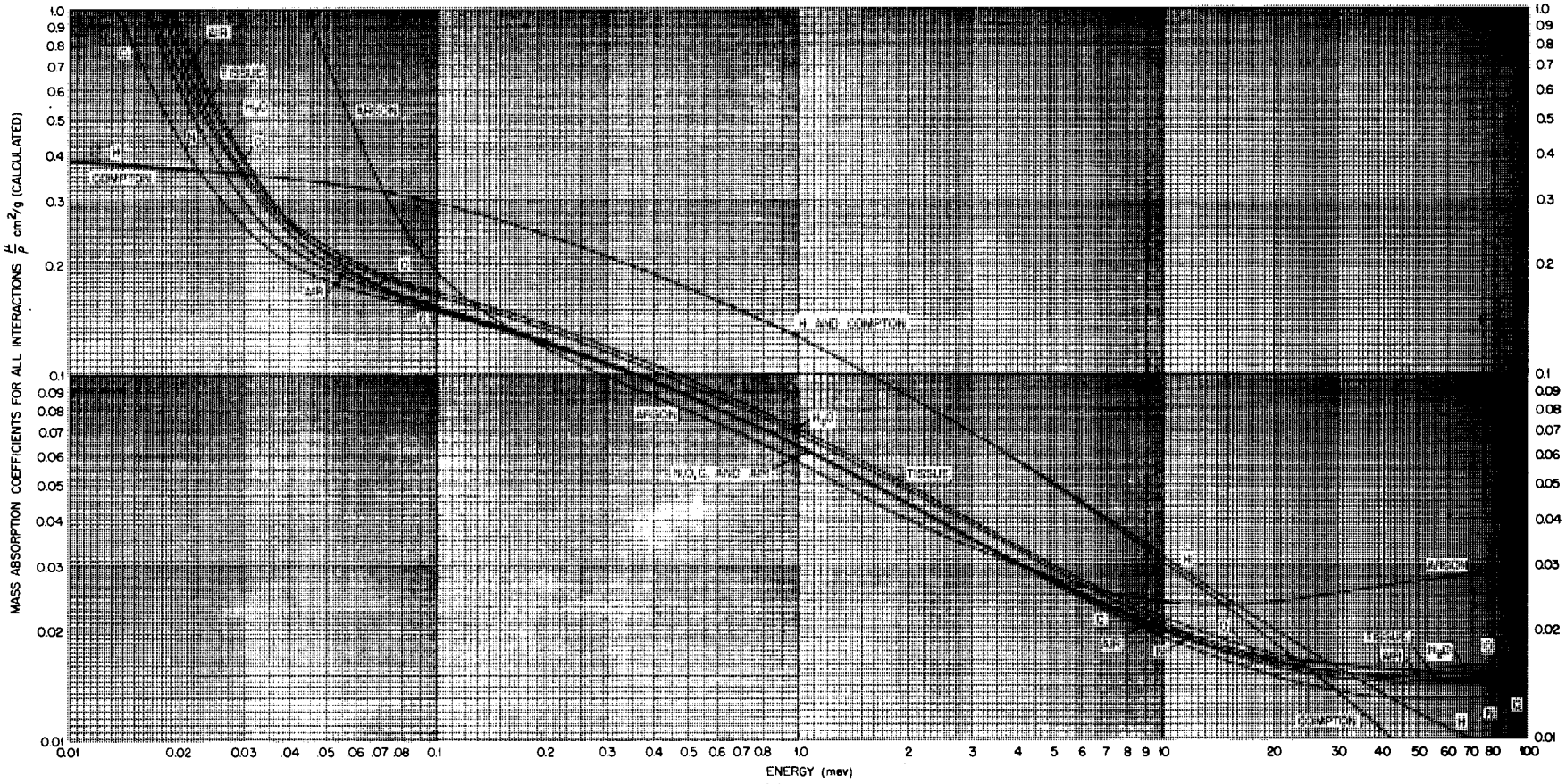

FIGURE 14
MASS ABSORPTION COEFFICIENTS
(MACROSCOPIC TOTAL CROSS SECTION DIVIDED BY DENSITY)

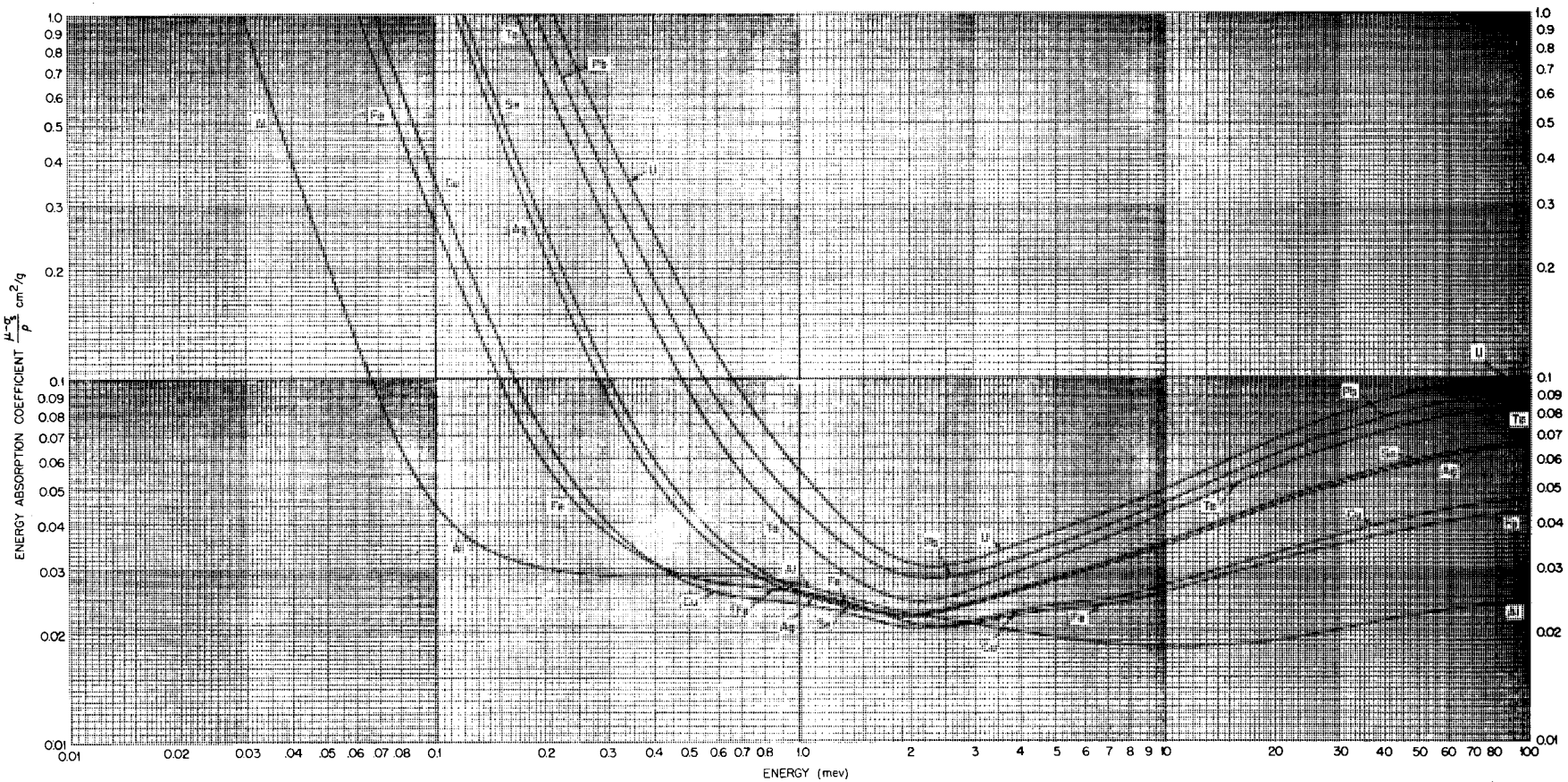

FIGURE 15

ENERGY ABSORPTION COEFFICIENTS
(MACROSCOPIC CROSS SECTION FOR ENERGY ABSORPTION DIVIDED BY DENSITY)

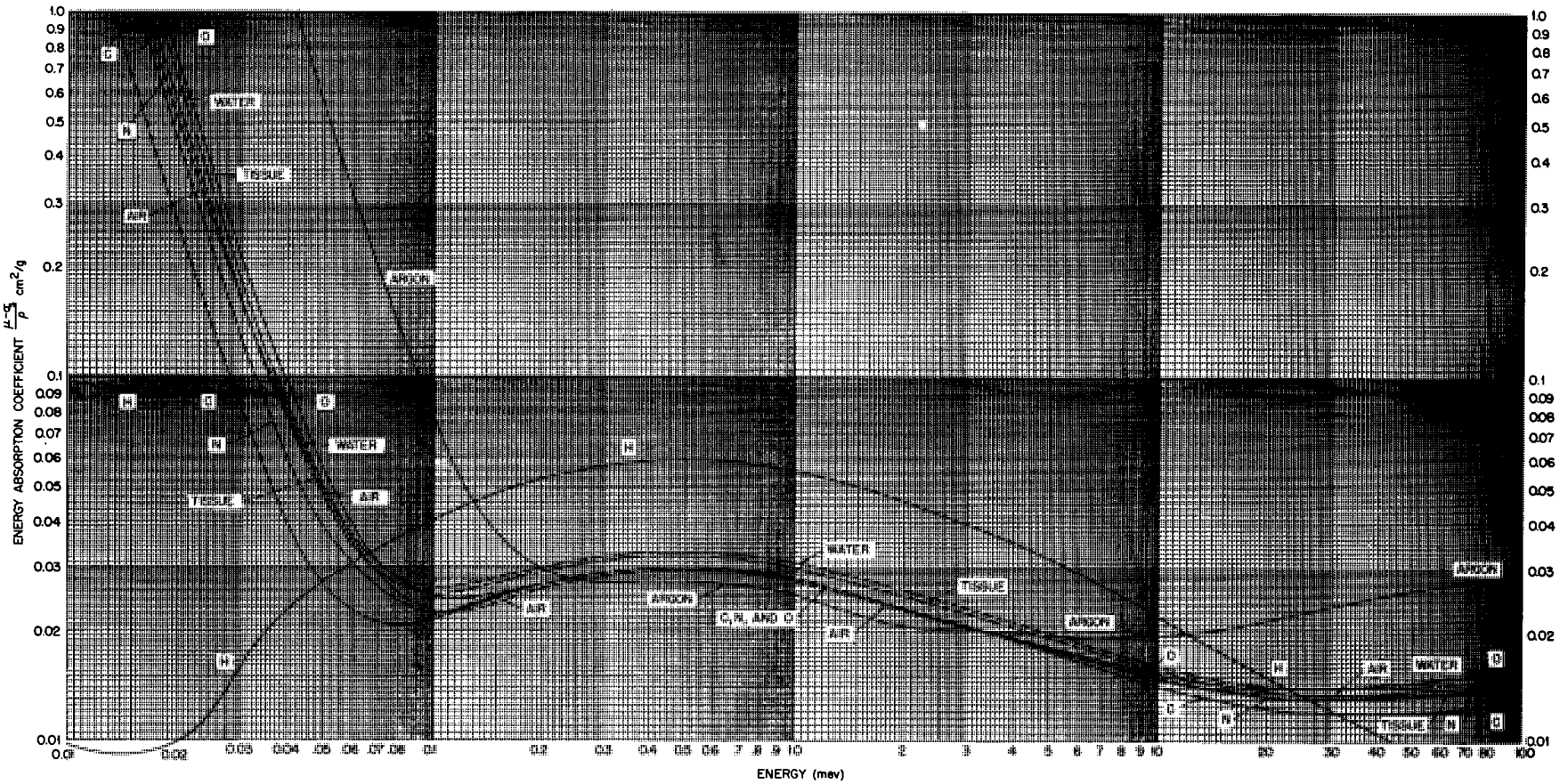

FIGURE 16
ENERGY ABSORPTION COEFFICIENTS
(MACROSCOPIC CROSS SECTION FOR ENERGY ABSORPTION DIVIDED BY DENSITY)

NOMENCLATURE

A	constant, neutrons/cm sec.
A_1	constant = $(ma + 2)e^{-m(a - R)}$
A_2	constant = $(mb + 2)e^{-m(b - R)}$
A_3	A_1
a	inside radius of pressure retaining vessel, cm.
B^2	geometrical buckling of reactor, cm^{-2}.
B_1	constant = $(m^2b^2 + 2mb + 2)e^{-m(b - R)}$
B_2	constant = $(m^2a^2 + 2ma + 2)e^{-m(a - R)}$
B_3	A_2
b	outside radius of pressure retaining vessel, cm.
C_1	constant of integration, degree F-cm.
C_2	constant of integration, degree F.
D	thermal neutron diffusion coefficient, cm.
E	modulus of elasticity of vessel material, psi.
E_γ	gamma energy, Mev.
F, F_n, F_1	prefix designating a function.
f	thermal utilization of the reactor, $\dfrac{\Sigma_a(U^{235})}{\Sigma_a(D_2O)+\Sigma_a(U^{235})}$
$H(x)$	heat production per unit volume at any point in an infinite slab, Mev/cm^3sec.
H_o	heat production per unit volume at face of slab, Mev/cm^3sec.
$H(r)$	heat production per unit volume at any radius within vessel wall, Mev/cm^3sec.
H_R	Heat production per unit volume at inside boundary of reactor, Mev/cm^3sec.

h	distance, cm.
I	radiation intensity at any point, cm^{-2} sec^{-1}.
I_o	incident radiation striking shield, cm^{-2} sec^{-1}.
$I_{o,f}$	fast neutron current entering vessel, neutrons/cm^2sec.
$I_{o,t}$	thermal neutron current entering vessel, neutrons/cm^2 sec.
K	neutron coefficient = $\sqrt{3\ \Sigma_a\ \Sigma_{tr}}$ cm^{-1}.
k	thermal conductivity of vessel material, Mev/cm sec F. (Note: 1 Btu/hr ft F = 6×10^{10} Mev/cm sec F.)
k_{oo}	multiplication factor for an infinite reactor.
k_{eff}	effective multiplication factor for a finite reactor.
L^2	diffusion length of thermal neutrons, cm^2.
$L^2(D_2O)$	diffusion length of thermal neutrons in heavy water, cm^2.
m	slope of heat production function, cm^{-1}.
N	number of gammas produced per neutron absorption.
n	integer.
P	power level of reactor, watts.
P_i	internal pressure, psi.
p	probability.
R	outside radius of active region of reactor, cm. (Note: In the absence of a thermal shield R = a.)
r	radius, cm.
$S(r)$	gamma source strength, γ/cm^3 sec.
T	temperature, degree F.
T_a	temperature of inside surface of pressure vessel, degree F.
T_b	temperature of outside surface of pressure vessel, degree F.
$T(r)$	temperature at any point within vessel wall, degree F.

t	distance, cm.
V	volume of reactor active region, cm^3.
w	$1/\cos\theta$.
x	distance from face of slab, cm.
y	variable.
z	variable.
α	coefficient of thermal expansion of vessel material, in/in F.
β	K/μ
ϵ	fast neutron fission effect.
η	average number of fast fission neutrons produced per absorption in fuel.
θ	angle, radians.
λ_{tr}	neutron transport mean free path, cm.
μ	energy dependent gamma absorption coefficient corrected for buildup, cm^{-1}.
μ_t	energy dependent gamma total absorption coefficient, cm^{-1}.
$(\mu_t-\sigma_s)$	energy dependent gamma energy absorption coefficient, cm^{-1}.
ν	Poisson's ratio.
ρ	distance, cm.
Σ_a	macroscopic neutron absorption cross section, cm^{-1}.
Σ_c	macroscopic neutron capture cross section, cm^{-1}.
Σ_f	macroscopic neutron fission cross section, cm^{-1}.
$\sigma_a(U^{235})$	microscopic neutron absorption cross section, barns.
$\sigma_c(U^{235})$	microscopic neutron capture cross section, barns.
$\sigma_f(U^{235})$	microscopic neutron fission cross section, barns.

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σ_a allowable stress, psi.

σ_r total radial stress at any point, psi.

$\sigma_{r,\,p_i}$ radial pressure stress at any point, psi.

$\sigma_{r,\,th}$ radial thermal stress at any point, psi.

σ_t total tangential stress at any point, psi.

$\sigma_{t,\,p_i}$ tangential pressure stress at any point, psi.

$\sigma_{t,\,th}$ tangential thermal stress at any point, psi.

$\sigma_{t,th\,max}$ maximum tangential thermal stress, psi.

τ Fermi age of fast neutrons, cm^2.

ν number of fast neutrons produced per fission.

φ neutron flux, neutrons/cm^2sec.

$\bar{\varphi}$ average thermal neutron flux, neutrons/cm^2sec.

ψ angle, radians.

ω angle, radians.

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